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Problem Set 3

PHYS 512
FALL 2022

$\boxed{1}$

- For our first integrator, each step requires 4 function calls.
- For 200 points, 199 computations are carried out, which brings the total integration at $4 \times 199 = 796$ function calls.
- For our second integrator, each step requires 11 function calls. If we want a similar number of function calls, we have to use $\frac{796}{11} \approx 72$ points for our integration.

Truncation term :

- Denote our full-step y solution by y_1 & our half-step y solution by y_2
- From numerical Recipes p. 911 the improved numerical estimate of the true solution $y(x+2h)$ is given by

$$y(x+2h) = y_2 + \frac{\Delta}{15} + O(h^6)$$

for $\Delta \equiv y_2 - y_1$ $\Rightarrow$ Should be accurate to 5^{th} order

$\boxed{3}$

(a) We have

$$z - z_0 = a\left((x-x_0)^2 + (y-y_0)^2\right)$$

How to choose other parameters to make the problem linear?

$$\Rightarrow z = a\left((x^2 - 2xx_0 + x_0^2 + y^2 - 2yy_0 + y_0^2)\right) + z_0$$

$$= ax^2 - 2ax_0x + ax_0^2 + ay^2 - 2ay_0y + ay_0^2 + z_0$$

$$= a(x^2+y^2) \underbrace{-2ax_0}_{\equiv\alpha} x \underbrace{-2ay_0}_{\equiv\beta} y + \underbrace{ax_0^2 + ay_0^2 + z_0}_{\gamma}$$

$$\boxed{z = a(x^2+y^2) + \alpha x + \beta y + \gamma}$$

parameters: a

$\alpha = -2ax_0$

$\beta = -2ay_0$

$\gamma = ax_0^2 + ay_0^2 + z_0$