

Reading Questions Week #3

to: Prof. Andrew Cumming
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1) What do the tidal force vectors look like at the surface of a planet? How does their magnitude depend on position on the surface? Include a labeled sketch in your answer.

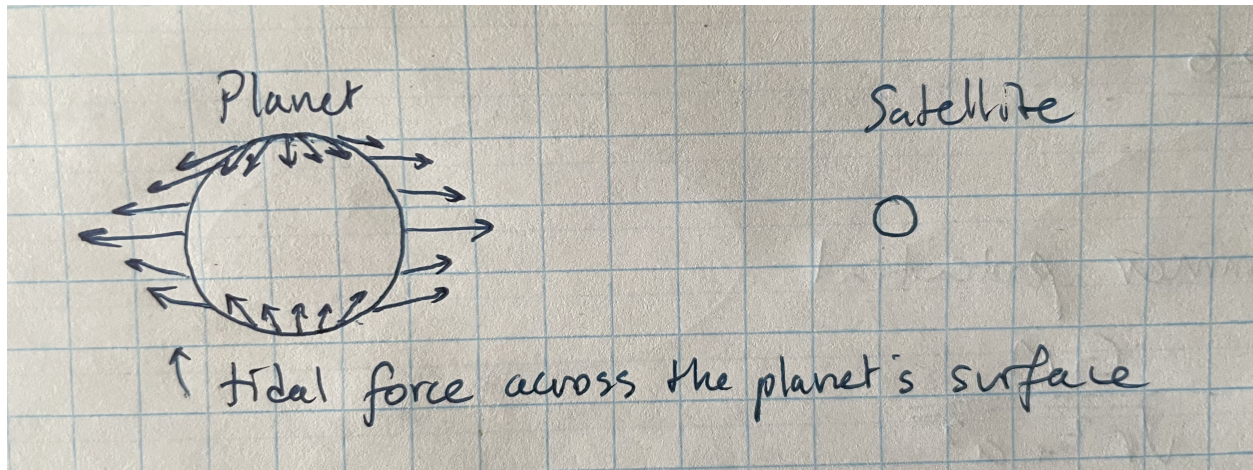


Figure 1: Sketch of a planet, satellite, and the tidal force exerted by the satellite on the surface of the planet. The magnitude of the tidal force is strongest at the closest and farthest points on the surface of the planet with respect to the satellite, and it gradually decreases between those points and the midpoints, such that the tidal forces are minimum at those midpoints (which we could refer to as the points on the planet whose surface is ‘the least normal’ to the distance to the satellite).

2) Identify the three types of tides discussed in the book. What physical motions or effects give rise to each type?

There are fortnightly, semidiurnal, and diurnal tides. They can be mathematically derived from the expansion of the angle between different points on the surface of the planet (P) and the distance between the planet and the satellite (where the satellite is labeled M and has a mean motion n ; cf. $\cos \psi$, Equation 4.15). Fortnightly tides have a frequency $2n$. The semidiurnal tide has a frequency 2Ω , where Ω is the angular speed of the rotating planet. The diurnal tide varies with a frequency Ω . In particular, fortnightly and semidiurnal tides relate to the satellite’s eccentricity.

3) Explain how the composition and internal structure of a planet can be studied using tidal and rotational deformation. What theoretical result introduced in this chapter allows such inferences to be made?

The theoretical result is that both tidal and rotational deformation give rise to a surface that can be modeled with a Legendre polynomial of degree two. While this simplifies things, constraining interior models requires to determine the different moments of inertia (cf. Eq 4.105 with \mathcal{C} and \mathcal{A}). In the case of the Earth, this can be resolved by looking at the effects of the torques by the Sun and the Moon, which cause the Earth’s spin axis to rotate at a rate proportional to $(\mathcal{C} - \mathcal{A})/\mathcal{A}$.

- 4) What is the Roche limit? Explain its physical significance.

The Roche limit defines the distance from which a particle is no longer gravitationally bound to a satellite (in orbit around a planet; *note that I am using the term planet for primary body and satellite for secondary body, but they don't have to be actual planets and satellites, they are just labels for the bigger and smaller bodies*). The different forces that determine the position of the Roche limit are notably relevant to determine the location and formation of planetary rings.

- 5) This text was first published in 1999, after the Pioneer and Voyager missions and during the Galileo mission to Jupiter. Have the “plausible models” for the interiors of the Galilean moons shown in Fig. 4.11 stood the test of time? Briefly justify your answer.

From cross-checking with modern paradigms, here's what I found. *Io*: yes; metal core with magma mantle. *Europa*: roughly yes; metallic core and rock shell, although it is thought there is a subsurface ocean. *Ganymede*: not quite. It is thought that it should have an ice crust, an ocean (missing from the textbook), an ice mantle, a rocky mantle and a metal core. *Callisto*: yes; mostly ice and rock. In summary: the past models mostly missed out on the ocean layers of Jupiter's satellites.

- 6) What does it mean for two planets to be in resonance?

It means that their orbital periods are related by a ratio of small integers. This happens when they exert regular, periodic gravitational influences on each other.

- 7) The Grand Tack hypothesis proposes that Jupiter formed beyond the snow line, migrated inward through interactions with the protoplanetary gas disk, and later reversed its migration outward. Explain how resonant capture played a role in this migration.

Initially, Jupiter starts moving inward as it opens a gas disk in the protoplanetary system. Later, Saturn forms and migrates towards the Sun even faster. As it approaches Jupiter, it gets trapped in a 3:2 mean motion resonance. This occurs because 1) there is resonant coupling that removes angular momentum from Jupiter and 2) Jupiter's and Saturn's orbits are converging. From then on, the two planets move inward in lockstep.

- 8) Why does the Kozai mechanism not change the semi-major axis of the perturbed object?

This is because this mechanism is orbit-averaged, and on average there is no net exchange of energy. Hence, the orbital energy stays constant, and so does the semi-major axis.