Reading Questions Week #5

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1) Explain dynamical friction in a sentence or two.

Dynamical friction refers to a series of two-body gravitational interactions between a larger mass M with numerous smaller masses m, the result of which is the deceleration of M.

2) Why is the quantity $h(v)$ in equation (19.35) referred to as a "potential"? In going from equation (19.35) to (19.36) an upper limit v_M appears on the integral. Why?

$h(v)$ is a potential because in the same way that the gradient of the gravitational potential Φ is proportional to the gravitational force, the gradient of $h(v)$ is proportional to the frictional force which gives rise to dynamical friction (note equation (19.35) is the expression for the acceleration caused by dynamical friction, which is directly to the force through Newton's second Law). With $h(v)$, the potential is sourced by $-n(v)$, whereas it is with gravity, Φ is sourced by the density $\rho(x)$; so it is a potential in velocity space as opposed to position space for gravity. Also, because it is sourced by a negative density, the field is repulsive. For the upper limit, the reason comes from Newton's shell theorem, whose classical result for gravity (equation (2.22)) shows that only masses contained with r contribute to the gravitational acceleration at r. Equivalently here, only velocities up to v_M contribute to the frictional deceleration of M.

3) Equation (19.42) gives the timescale for dynamical friction to decay the orbit of a massive body of mass M moving in a cluster of total mass M_{tot}. If you wanted to evaluate this timescale, how would you evaluate the factors t_{dyn} and Λ?

We have that $\Lambda = b_{\max} v_{\mathrm{typ}}^2/(G(M+m))$, where $b_{\max}$ is the maximum possible value of the impact parameter and v_{typ} is the typical velocities of objects in the system (depends on the system). To first order, we can approximate $b_{\max}$ as the radius at which the decelerated body is orbiting. We also have that the dynamical time t_{dyn} of a galaxy is the crossing time for individual stars; and the crossing time is $t_{\mathrm{cross}} = 2\pi R/v$, where R is the galaxy size and v is the velocity of the star.

4) What is the main takeaway of the Kirsh et al. paper?

The main takeaway of the paper is that low-mass planets are highly susceptible to inwards migration by the influence of nearby objects called planetesimals, which are about the size of a comet or an asteroid.

5) How would you calculate the location of the inner and outer edges of the feeding zone?

The outer edge of the feeding zone would be about 3.5 Hill radii R_H, where $R_H = a_P(M_P/3M_\odot)^{1/3}$ for a planet of mass M_P at a semi-major axis a_P. For the inner edge, I would have to calculate Tisserand's parameter $C_T = a_P/a + 2\sqrt{a/a_P}\sqrt{1-e^2}\cos i$, where e and i are the planetesimal's eccentricity and inclination, respectively. Then, calculate C_T at the outer edge, and determine the inner radius at which C_T is the same.

6) What is the Hill eccentricity e_H and why is it a useful quantity to look at?

The Hill eccentricity is the orbital eccentricity expressed in units of the Hill factor, it is equal to $e_H = e/\chi$ where $\chi = R_H/a_P$. It is useful because it is a delimiter: for large e_H, planetesimals might miss the planet's gravitation, even when their orbital parameters seemed to otherwise indicate an encounter.

7) Explain the different features that can be seen in the $a - e_H$ lot in Figure 4.

In the top panel, the planet has first had an influence on a population of planetesimals with low eccentricity in its encounter zone. The planet can be located by the clump of points at $a = 25$ AU, which corresponds to nearby particles in horseshoe orbits around it. The angle formed by the V-shaped feature relates to the conservation of C_T for all the particles being scattered to higher eccentricity. As time goes on, the planet moves inwards, and its neighboring particles in horseshoe orbits scatter away. In this process, the planet keeps interacting with more planetesimals at smaller semi-major axes; hence the broadening of the V-shape towards smaller a.