

Reading Questions Week #2

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January 2026

- 1) Why is the quantity U (defined in eq. 3.22) described as a “pseudo-potential”?

It is described as a pseudo-potential because only some of the accelerations experienced by the particle in the rotating frame can be derived from it. This deviation from a true potential relates to the fact that there cannot be exact conservation of energy and momentum in a system where we ignore the contribution of one of the masses on the two others (which is what we do in the restricted three-body problem).

- 2) In Figures 3.7 and 3.9, the contours show constant values of what quantity? How should we interpret them physically? What is special about the points L1 to L5?

The contours show constant values of zero-velocity. They can be interpreted as dividing lines for which orbits within them are stable, and orbits outside of them are unstable. The points L1 to L5 are special in that an orbiting third body of negligible mass located at those points would have no velocity and no acceleration in the rotating frame.

- 3) Section 3.7 is a detailed stability analysis of the Lagrange points. What are the conclusions of this analysis?

The stability analysis starts off by diagonalizing equations related to displacements about the positions of equilibrium points. It is found that the eigenvalues of the associated matrix will tell you whether or not the equilibrium point is stable (if all the eigenvalues are purely imaginary, then the solution is purely oscillatory, and hence stable). Then, it looks into the specific cases of the collinear Lagrangian points (L1, L2 and L3) and the triangular Lagrangian points (L4 and L5). As it turns out, L1, L2 and L3 are always unstable, and L4 and L5 can be stable provided the secondary mass is small enough.

- 4) What are the differences between tadpole and horseshoe orbits? In Fig 3.18, which orbits are tadpoles and which horseshoes?

In tadpole orbits, the particle stays near its equilibrium point (either L4 or L5; this happens when the particle starts off a small distance away from equilibrium). In horseshoe orbits, the particle moves from L4 to L5 (or vice-versa; this happens when the particle starts off a larger distance away from equilibrium).

- 5) What is the Hill sphere?

The Hill sphere is a sphere of radius $\Delta_H = (\mu_2/3)^{1/3}$ surrounding the secondary mass. It corresponds to the region around μ_2 where its own gravity dominates that of μ_1 .