Reading Questions Week #6

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1) (a) Why does Equation 4.2 imply that relativity is not important, and for what three reasons does relativity indeed play a part? (b) Why does capture occur specifically on orbits of high eccentricity? (You may need to do some research into this, but try to think mechanically based off of what else we've covered thus far.)

Relativity seems unimportant in Equation 4.2 because the ratio of the gravitational radius r_g to the gravitational influence radius r_h is very small for most of the stars located in the gravitational influence radius of the SBH. It is misleading because 1) it disregards stars with highly eccentric orbits (whose closest approach is much closer than the semi major axis), which happen to be the ones that mainly feed the SBH, 2) the aspherical distribution of matter in the vicinity of the SBH cause torques which force stars onto those highly eccentric orbits (although there are mechanisms that tend to suppress those torques) and 3) precession of stars due to relativity turn out to be significant enough to be observable (book is from 2013 and mention they should soon be observed; but the GRAVITY collaboration has since released papers showing that indeed, the star S2 near SgrA* is precessing according to GR!)

2) What is the Kerr metric, and what are the constants of motion in Kerr geometry

The Kerr metric describes the geometry of space-time near a rotating and spherically symmetric mass (which is a good model for, e.g., a spinning black hole). Its constants of motion are E_∞, the total energy, L_z, the component of angular momentum along the z-axis and C, the Carter constant (its role is similar to that of the component of the angular momentum parallel to the spin axis of the black hole; as $r \to \infty$, we have $C \to L^2 - L_z^2$, and $C = 0$ is equivalent to restricting motion to the black hole's equatorial plane).

3) What is the difference between the event horizon and the static limit?

The event horizon is the region within which the escape velocity of the black hole becomes larger than the speed of light; the static horizon is the region within which an observer has to move in the positive ϕ direction by the "dragging of inertial frames."

4) Physically, why are there discrepancies in the capture radius and constants of motion between prograde and retrograde orbits for a maximally rotating black hole?

Stability of an orbit of an object in the vicinity of the black hole depends if the object is orbiting in the same direction as the black hole is spinning: if they are rotating in opposite directions, then their rotations are somewhat 'cancelling each other out', and the object is more likely to fall towards the black hole; as such, its ISCO is farther out. In other words, objects in a retrograde orbit have an ISCO radius larger than objects in a prograde orbit. The capture radii for each case are consequently different. Note that if the black hole is not spinning, there would be no difference between the capture radii of prograde and retrograde orbits; this difference is largest when the black hole is maximally spinning.

5) (a) What is capture vs. disruption? What factor(s) determine if a star passing a SMBH is captured or tidally disrupted? (b) Consider Eqns 6.2 and 6.3. Briefly explain why a Sun-like star can only be tidally disrupted by a SMBH with $M_\bullet \leq 10^8 M_\odot$. What if it encounters a more massive SMBH?

(a) Capture is when the black hole swallows the star whole; disruption is when the tidal acceleration due to the black hole near periapsis is greater than the surface gravity of the star, which causes the star to be 'torn apart' by tidal forces exerted by the black hole. The factors that determine whether a star might be captured or disrupted are: the star's mass, the black hole mass, the pericenter and the star's radius. (b) For a SBH with a mass ¡ $10^8 M_\odot$, we have $\eta < 1$ such that its tidal radius is larger than r_p, the pericenter of orbit: the tidal acceleration due to the black hole is larger than the surface gravity of the star, and disruption ensues. If a Sun-like star encounters a more massive SBH, then its gets captured ('swallowed whole').

6) (a) What is the "loss cone"? What happens to stars within it? (b) How does the loss cone relate to the feeding rate of the SMBH?

(a) The loss cone is the region within which either disruption or capture starts for stars of a given type in that region, whichever extends farthest: any star within its loss cone will either experience disruption or capture. (b) Stars within the loss cone are destined to be fed to the black hole within one orbital period, such that the feeding rate of the black hole will be determined by how fast can the loss cone be repopulated with other stars.

7) Why is diffusion in angular momentum more important than diffusion in energy for SMBH feeding?

Stars that diffuse angular momentum or energy will have their orbital radius shrink and contribute to the feeding of the black hole. However, energy-loss mechanisms have not been able to provide efficient loss-cone repopulation to explain the important feeding of SBHs. If we include diffusion by momentum, then all the stars within a sphere of radius r will diffuse into the SBH in one relaxation time at r. While this does not produce a full loss-cone repopulation, it does predict a higher feeding rate than with just energy diffusion alone.