Feb. 2nd, 2026. DM halos #1

Questions:

#1) collisional ⇒ strong gravitational interactions between individual stars are important
collisionless ⇒ individual interactions, even close encounters, are not important

#2) (a) t_{relax} = time required to change a star's velocity by 100% $\approx n_{relax} t_{cross}$
t_{dyn} = crossing time for individual stars $\approx t_{cross}$

where $n_{relax} = \dfrac{N}{8 \ln N}$; $t_{cross} \simeq \dfrac{2\pi R}{v}$

with R = galaxy size & v = star velocity

(b) For a typical galaxy, $N \approx 10^{11}$ & $t_{cross} \sim 100$ Myr

⇒ $t_{relax} \sim 5 \times 10^{10}$ yr ~ 5 million times the age of our Universe!
↳ two-body relaxation cannot be the mechanism through which galaxies reach equilibrium; otherwise they would be far from equilibrium today!

#3) (a) The fundamental assumption of the virial theorem is that the system is in equilibrium, or $\dfrac{dG}{dt} = 0$
where G = virial quantity $= \sum_{i=1}^{N} w_i \vec{x}_i \cdot \vec{v}_i$

(b) $2K + W = 0$
virial theorem

#4) Collisionless Boltzmann eqn:
$$\frac{\partial f(\vec{x}, \vec{v}, t)}{\partial t} + \dot{\vec{x}} \frac{\partial f(\vec{x}, \vec{v}, t)}{\partial \vec{x}} + \dot{\vec{v}} \frac{\partial f(\vec{x}, \vec{v}, t)}{\partial \vec{v}} = 0$$

describes the behavior of the distribn fctn (f) under the influence of grav.

#5) Jeans eqns are the collisionless Boltzmann eqn multiplied by powers of $\vec{x}$ or $\vec{v}$ and integrated over a part of phase space.
They are moment equations of the Boltzmann eqn.

They are useful in astronomy because astro observations are usually performed at fixed $\vec{x}$ / location in the sky. Thus, multiplying the collisionless Boltzmann eqn by $\vec{v}$ and integrating over all components of the velocity connects to observables!

#6) β = orbital anisotropy = difference in the velocity distrib^{n} in different directions.

$= 1 - \frac{\sigma_\theta^2 + \sigma_\phi^2}{2\sigma_r^2}$

$\underline{\beta = 0}$: if the spread of motion in r, θ, ϕ are all the same
$\Rightarrow$ isotropic

$\underline{\beta \to 1}$: if the spread of motion in $r \gg$ in θ & ϕ
$\Rightarrow$ radially biased

$\underline{\beta \to -\infty}$: if all orbits are circular : $\sigma_r = 0$
$\Rightarrow$ tangentially biased

#7) Given a certain mass model and $\beta(r)$ profile, one can calculate the solutions to Jeans equations, which connect back to velocity dispersions (ie, observables).

↳ one can then do forward modeling: given a certain physical model for DM halo structure (eg, NFW profile), what is the expected stellar velocity dispersion? And how does it compare to data?

↳ (inverse modeling: given the data, how can I fit my physical model to it?)