PHYS 641 – Problem Set 5

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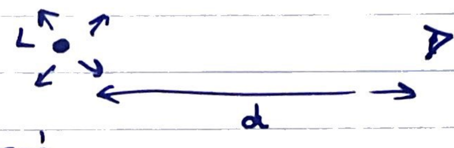
a)

Flux measured some distance d away from a source of luminosity L is:

$$S = \frac{L}{4\pi d^2}$$



$$\Rightarrow S \propto d^{-2} \quad \text{or} \quad d \propto S^{-\frac{1}{2}}$$

On the other hand, the number of sources enclosed in the volume of the sphere of radius d is proportional to the volume:

$$N \propto V$$

which itself is proportional to the radius d:

$$V \propto d^3$$

So

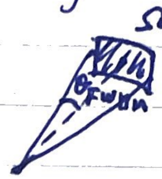
$$N \propto d^3$$

$$\Rightarrow N \propto \left(S^{-\frac{1}{2}}\right)^3 \Rightarrow \boxed{N \propto S^{-\frac{3}{2}}}$$

b)

The confusion limit is at one visible source for every 30 beams

Take the solid angle of the beam Ω



$$\Rightarrow \Omega \propto \pi \left(\frac{\theta_{FWHM}}{2} \right)^2$$

where $\theta_{FWHM} \sim 1.22 \frac{\lambda}{D} = 1.22 \frac{c}{fD}$

We require no more than $\frac{\text{one source}}{\text{30 beams}}$

$$\text{So } N \lesssim \frac{1}{30\Omega}$$

$$\Rightarrow N \lesssim \frac{1}{30 \cdot \left(\pi \left(\frac{\theta_{FWHM}}{2}\right)^2\right)}$$

$$\Rightarrow N \lesssim \frac{1}{30\pi \left(\frac{1.22^2 c^2}{2^2 f^2 D^2}\right)}$$

$$\Rightarrow N \lesssim 0.09 \frac{f^2 D^2}{\pi c^2} \Rightarrow N_{lim} \propto 0.09 \frac{f^2 D^2}{\pi c^2}$$

From a), we know $N \propto S^{-\frac{3}{2}} \Rightarrow S \propto N^{-\frac{2}{3}}$

We take $100 \frac{\text{sources}}{\text{deg}^2}$ above $1\,\text{mJy}$ at $1.4\,\text{GHz}$ $= \frac{100 \text{ sources}}{\left(\frac{\pi}{180}\right)^2 \text{rad}^2} \approx 328\,280 \frac{\text{sources}}{\text{rad}^2}$

$$\text{So } N_{lim} \cdot 328\,280 \frac{\text{sources}}{\text{rad}^2} = 0.09 \frac{f^2 D^2}{\pi c^2}$$

$$\Rightarrow N_{lim} = 2.73 \times 10^{-7} \frac{f^2 D^2}{\pi c^2} \quad \text{So } S_{lim} = \left(2.73 \times 10^{-7} \frac{f^2 D^2}{\pi c^2}\right)^{-\frac{2}{3}}$$

$$\Rightarrow \boxed{S_{lim} \approx 23769 \left(\frac{f^2 D^2}{\pi c^2}\right) \text{ mJy}}$$

Now, take $f = 1.4\,GHz$
$c = 3 \times 10^8 \frac{m}{s}$

VLA_A: $D \sim 30\,km$ so $S_{lim} \sim 7.01 \times 10^{-3}\,mJy$

VLA_D: $D \sim 1\,km$ so $S_{lim} \sim 0.65\,mJy$

GBT: $D \sim 100\,m$ so $S_{lim} \sim 14\,mJy$

FAST: $D \sim 300\,m$ so $S_{lim} \sim 3.3\,mJy$

c)

From the radiometer eqⁿ: $\sigma_c = \frac{SEFD}{\sqrt{\Delta\nu\,\tau}}$

$$\Rightarrow \boxed{\tau = \frac{SEFD^2}{\sigma_c^2 \Delta\nu}}$$

The SEFD is an equivalent of T_{sys} that's scaled by aperture efficiency
Also need to convert $[T_{sys}] = K$ to $[SEFD] = mJy$

$\Rightarrow 2\frac{K}{Jy}$; $\frac{1000\,mJy}{Jy}$; $T_{sys} \sim 25 + 273.15 = 298.15\,K$

So $SEFD \sim \frac{1}{0.7} \cdot 298.15\,K \cdot \frac{1\,Jy}{2K} \cdot \frac{1000\,mJy}{Jy}$

$SEFD \sim 212\,964\,mJy$

$$\Rightarrow \boxed{\tau = \frac{4.5 \times 10^{10}}{\sigma_c^2 \cdot \Delta\nu}}$$

for $\Delta\nu = 500\,MHz$

$\rightarrow \tau \approx \frac{90.7}{\sigma_c^2}$

will be taken as confusion limit

d)

Now take $\Delta\nu = 2\,GHz$

$\Rightarrow \tau \approx \frac{22.7}{\sigma_c^2}$