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MA-INF 4201 - Artificial Life
Exercise Sheet 5

Assignment 31

Let P_s be the population at start, and P_e be the end population. From the lecture we derive the exponential growth after one year as: $P_e = P_s \left(1 + \frac{1.7}{100}\right)$. After n years we have a population of: $P_e = P_s \left(1 + \frac{1.7}{100}\right)^n$. Since we seek $P_e = 2P_s$, we obtain

$$\begin{aligned} 2P_s &= P_s \left(1 + \frac{1.7}{100}\right)^n && \Leftrightarrow \\ 2 &= \left(1 + \frac{1.7}{100}\right)^n && \Leftrightarrow \\ \log(2) &= \log\left(1 + \frac{1.7}{100}\right) n && \Leftrightarrow \\ n &= \frac{\log(2)}{\log\left(1 + \frac{1.7}{100}\right)} && \Rightarrow \\ n &\approx 41.12(a) \end{aligned}$$

■

Assignment 32

We have

$$\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = 1.681\ldots = \Phi$$

against

$$\lim_{n\to\infty} \frac{\exp(n+1)}{\exp(n)} = e$$

Since $e > \Phi \Rightarrow \exp$ grows faster than Fibonacci sequence. ■

Assignment 33

Shows a stable pattern after 9582 iterations $\Rightarrow$ class II. ■

Assignment 34

Referring to our last exercise sheet, we constructed an L-system for a spiral basically by adding one unit length to a segment compromising a quarter turn of the spiral. A golden spiral is a spiral where in each quarter turn the golden ratio is added, in order for the spiral to unwind. In other words: Inscribing each quarter turn segment into a bounding square, the area ratio of squares of consecutive quarter turns is the golden ratio. ■

Assignment 35

Suppose the limit exists and call it a: $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = a$. Then:

$$a = \lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \lim_{n\to\infty} \frac{F_n + F_{n-1}}{F_n} = 1 + \lim_{n\to\infty} \frac{F_{n-1}}{F_n} = 1 + \frac{1}{a}$$
$$\Rightarrow 0 = a^2 - a - 1$$

Using the midnight equation we obtain the solutions: $\frac{1\pm\sqrt{5}}{2}$.

The program implemented in Python is found in the file `fib.py` and takes the number of iterations as parameter `-n`. ∎

Assignment 36

The function is periodic, with the phase of the signal generated for the two starting conditions getting in sync in the long term. $\Rightarrow$ class II.

For $a = 3.75$ we observe a seamingly periodic behaviour, but the phases don't get synced (at least for the first 20,000 iterations). $\Rightarrow$ class III, deterministic chaos.

A program implementing the function in Python is found in the file `if.py` and takes the number of iterations as parameter `-n`, as well as the paramter a as `-a`. ∎