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MA-INF 4201 - Artificial Life
Exercise Sheet 5

Assignment 31

Let P_s be the population at start, and P_e be the end population. From the lecture we derive the exponential growth after one year as: $P_e = P_s\left(1 + \frac{1.7}{100}\right)$. After n years we have a population of: $P_e = P_s\left(1 + \frac{1.7}{100}\right)^n$. Since we seek $P_e = 2P_s$, we obtain

$$\begin{aligned}
2P_s &= P_s\left(1 + \frac{1.7}{100}\right)^n && \Leftrightarrow \\
2 &= \left(1 + \frac{1.7}{100}\right)^n && \Leftrightarrow \\
\log(2) &= \log\left(1 + \frac{1.7}{100}\right)n && \Leftrightarrow \\
n &= \frac{\log(2)}{\log\left(1 + \frac{1.7}{100}\right)} && \Rightarrow \\
n &\approx 41.12(a)
\end{aligned}$$

■

Assignment 34

Referring to our last exercise sheet, we constructed an L-system for a spiral basically by adding one unit length to a segment compromising a quarter turn of the spiral. A golden spiral is a spiral where in each quarter turn the golden ratio is added, in order for the spiral to unwind. In other words: Inscribing each quarter turn segment into a bounding square, the area ratio of squares of consecutive quarter turns is the golden ratio. ■

Assignment 35

Suppose the limit exists and call it a: $\lim_{n\to\infty} \frac{F_{n+1}}{F_n} = a$. Then:

$$\begin{aligned}
a = \lim_{n\to\infty} \frac{F_{n+1}}{F_n} = \lim_{n\to\infty} \frac{F_n + F_{n-1}}{F_n} = 1 + \lim_{n\to\infty} \frac{F_{n-1}}{F_n} = 1 + \frac{1}{a} \\
\Rightarrow 0 = a^2 - a - 1
\end{aligned}$$

Using the midnight equation we obtain the solutions: $\frac{1\pm\sqrt{5}}{2}$.

The program implemented in Python is found in the file `fib.py` and takes the number of iterations as parameter `-n`. ■

Assignment 36

The function is periodic, with the phase of the signal generated for the two starting conditions getting in sync in the long term. For $a = 3.75$ we observe the periodicity, but the phases don't get synced (at least for the first 20,000 iterations). A program implementing the function in Python is found in the file `if.py` and takes the number of iterations as parameter `-n`, as well as the paramter a as `-a`. ■