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MA-INF 4111 - Intelligent Learning and Analysis Systems: Machine Learning
Exercise Sheet 2

## 3. Properties of the Entropy

(i)

$$\begin{aligned} H(X) = H(p_1 \dots p_n) &= \sum_{i=1}^{n} -p_i \log_2 p_i \\ &= \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i} \\ &= \mathbb{E}\left[\log_2 (X)\right] \leq \log_2 \left(\mathbb{E}\left[X\right]\right) \qquad (1) \\ &= \log_2 \left( \sum_{i=1}^{n} p_i \frac{1}{p_i} \right) \\ &= \log_2 n \end{aligned}$$

We used the well known concavity of the logarithm in applying Jensen's inequality in (1).

(ii)

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