XCS10011

UNIVERSITY OF DUBLIN

TRINITY COLLEGE

Faculty of Engineering, Mathematics and Science

School of Computer Science and Statistics

JF B.A. (Mod.) Computer Science **Trinity Term 2011**
JF B.A. (Mod.) CSLL
JF B.A. (Mod.) Business & Computing

Mathematics

11th May 2011 **Sports Hall** **2pm - 5pm**

Dr. Hugh Gibbons

Instructions to Candidates:

- Attempt **4** questions, **2** from each section
- Please use separate answer books for each section
- All questions are marked out of 25
- Formulae tables and graph paper are available from the invigilators, if required.
- Non-programmable calculators are permitted for this examination.
  Please indicate the make and model of your calculator on each answer book used.
- You may not start this examination until you are instructed to do so by the Invigilator.

Section A

Qs. 1.

a) Let the set operator, $\gg$, be defined so that

$$X \gg Y = \overline{X} \cup Y$$

where $\overline{X}$ is the complement of X.

Determine by Venn or Veitch diagram whether:

i) $A \gg B = \overline{A \cap \overline{B}}$

ii) $A \cap B = \overline{A \gg \overline{B}}$

ii) $A \gg (B \gg C) = (A \cap B) \gg C$

ii) $(A \cup B) \gg C = (A \gg C) \cap (B \gg C)$

[12 marks]

b) 200 tennis fans were asked which of these grand slam tournaments did they attend. The tournaments enquired about were: French Open, Wimbledon and Australian Open.

- 30 people have not been to any of these tournaments.
- 10 people have been to all thee tournaments.
- 25 people have been to Wimbledon and the Australian Open.
- 20 people have been to the French Open and the Australian Open but not to Wimbledon, i.e. they have been exactly to the two tournaments, the French Open and the Australian Open.
- 50 people have been to the Australian Open

- 65 people have been to exactly two of the tournaments, i.e. been to two but not the third one.
- 50 people have been to only to the French Open.
- 130 people have been to the French Open or to the Australian Open.
- 165 people have been to the French Open or to Wimbledon.

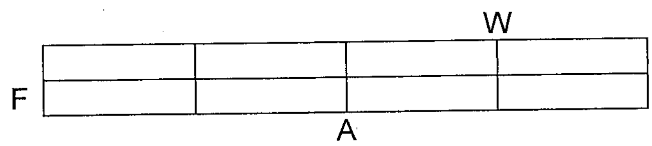
i) How many people have been only to the Australian Open.

ii) How many people have been only to Wimbledon

[13 marks]

The following is a suggested labelling of a Veitch diagram, where

F: **F**rench Open, W: **W**imbledon, A; **A**ustralian Open.



Qs. 2.

a) Express 26741 in terms of powers of primes, i.e. find powers $p1, p2, p3\ldots$ such that

$$26741 = 2^{p1} * 3^{p2} * 5^{p3} * \ldots$$

where $p1 \geq 0, p2 \geq 0, \ldots$

[7 marks]

b) Find x and y such that

11798*x + 3145*y = gcd(11798,3145)

[12 marks]

c) Find n such that $0 \leq n < 17$ and $13^{17} \equiv_{17} n$

i.e. determine $13^{17} \bmod 17$.

[6 marks]

Qs. 3.

a) The conditional operator, →, is defined by a Truth Table as:

P	Q	P→ Q
F	F	T
F	T	T
T	F	F
T	T	T

Determine by Truth Table whether

i) $P \wedge (P \rightarrow Q) = P \wedge Q$

ii) $(P \wedge Q) \rightarrow R = P \rightarrow (Q \rightarrow R)$

iii) $(P \rightarrow Q) \rightarrow R = P \rightarrow (Q \vee R)$

iv) $(P \vee Q) \rightarrow R = (P \rightarrow R) \wedge (Q \rightarrow R)$

[16 Marks]

b) Determine by the method of Truth Trees whether the following argument is valid:

$$A \vee D,\ (\neg B \wedge \neg C) \equiv D,\ B \rightarrow \neg (C \rightarrow A) \ \vdash \ \neg B$$

[9 marks]

Truth Tree Rules of Inference

Positive Rules

A→B	A∧B	A∨B	A≡B
¬A B	A B	A B	A ¬A B ¬B

Negative Rules

¬(A→B)	¬(A∧B)	¬(A∨B)	¬(A≡B)
A ¬B	¬A ¬B	¬A ¬B	¬A A B ¬B

Double Negation

¬¬A
|
A

Section B

Qs. 4.

a) Let a matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 1 & 5 & 7 \end{bmatrix}$$

i) Find the determinant of the matrix, A.

ii) Find the inverse of the matrix, A.

[10 marks]

b) Solve the following system of linear equations:

$$\begin{aligned} x + 2y + 3z &= 3 \\ 2x + 3y + 4z &= 2 \\ x + 5y + 7z &= 1 \end{aligned}$$

[6 marks]

c) Find condtions on a, b and c so that the following linear system is consistent:

$$\begin{aligned} x_1 + x_2 + 2x_3 &= a \\ x_1 + x_3 &= b \\ 2x_1 + x_2 + 3x_3 &= c \end{aligned}$$

In particular, if $a = 1$ and $b = 2$, find the value for c that makes the linear system consistent.

[9 marks]

Qs. 5.

Let the matrix

$$A = \begin{bmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix}$$

a) Find the characteristic equation for the matrix, A.

[4 marks]

b) Find the eigenvalues for the matrix, A.

[6 marks]

c) Find the eigenvectors for the matrix, A.

[9 marks]

d) Let the matrix

$$P = \begin{bmatrix} 1 & \frac{1}{2} & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

Show that

$$P^{-1} * A * P = B$$

where

$$B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$$

[6 marks]

Qs. 6.

i) Calculate the following integrals:

a) $\int \frac{x^2}{\sqrt{x^3+1}}\,dx$

[3 marks]

b) $\int \frac{\cos x}{\sqrt{(1+\sin x)}}\,dx$

[3 marks]

c) $\int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{(1-x)(1+x)}}\,dx$

[4 marks]

d) $\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

[5 marks]

e) $\int x^3 \log x\,dx$

[4 marks]

ii) Using Partial Fractions evaluate: $\int \frac{x+4}{x(x^2+3x-10)}\,dx$

[6 marks]