UNIVERSITY OF DUBLIN

XCS10011

TRINITY COLLEGE

Faculty of Engineering, Mathematics & Science

School of Computer Science & Statistics

JF BA (Mod) Business and Computing
JF BA (Mod) Computer Science
JF BA (Mod) CSLL

Trinity Term 2010

Mathematics

Thursday, 6 May, 2010 **RDS MAIN HALL** **14.00 – 17.00**

Ms. Meriel Huggard, Dr. Hugh Gibbons

Please answer TWO questions from SECTION A and TWO questions from SECTION B – four questions in total to be answered.

All questions carry equal marks.

Answer each SECTION in a separate answer book.

Log tables and graph paper are available from the invigilators, if required.

The HANDBOOK OF MATHEMATICS of Computer Science is available from the invigilators, if required.

Non-programmable calculators are permitted for this examination – please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

SECTION A

1. (a) Using Veitch or Venn diagrams determine whether:

$$(A \cup B) \cap (A \cup C) \cap (B \cup C) = (A \cap B) \cup (A \cap C) \cup (B \cap C)$$

(6 marks)

(b) In a survey of 200 students taking subjects Computing, Physics and Mathematics, it is found that:

90 take Computing,

60 take Physics,

110 take Mathematics,

20 take Computing and Maths

20 take Computing and Physics and

30 take Mathematics and Physics.

How many students, if any, take all 3 subjects?

(6 marks)

(c) Determine whether the following are Tautologies:

(i) $(P \vee Q) \wedge P \equiv Q$

(ii) $(P \rightarrow Q) \rightarrow R \equiv P \rightarrow (Q \rightarrow R)$

(iii) $(P \vee Q) \rightarrow R \equiv (P \rightarrow R) \wedge (Q \rightarrow R)$

(6 marks)

(d) Determine whether the following argument is valid:

If John is in Dublin then John is in Ireland

therefore

If John is in Dublin then John is in England
or if John is in London then John is in Ireland.

In formalising the argument, let

A : *John is in Dublin*	B : *John is in Ireland*
C : *John is in England*	D : *John is in London*

(7 marks)

[25 marks]

2. (a) Let div and mod be defined as follows:

For $b \neq 0$,

$$a\, div\, b = q \wedge a\, mod\, b = r \quad \equiv \quad a = b * q + r \wedge 0 \leq r < |b|$$

(i) Using these definitions calculate:

(I) $(-33)\, div\, 10$ and $(-33)\, mod\, 10$

(II) $33\, div\, (-10)$ and $33\, mod\, (-10)$

(ii) Determine whether:

$$-(a\, div\, b) = a\, div\, (-b)$$

(13 marks)

(b) (i) Find x, y such that:

$$6751 * x + 3249 * y = gcd(6751, 3249)$$

(ii) Let $a = 34$, $b = 55$ and $c = 89$, show that:

$$lcm(a,\, gcd(b, c)) = gcd(lcm(a, b),\, lcm(a, c))$$

(12 marks)

[25 marks]

3.(a) Find and simplify the unknown precondition, ?, in the following:

(i) $\{?\}y := x - 2\{-1 \leq y \leq 4\}$

(ii) $\{?\}j := j - 1; k := k - 1\{m - 1 \leq k < j < n\}$

(iii) $\{?\}\mathbf{if}\, x < 0\, \mathbf{then}\, y := -x\, \mathbf{else}\, y := x\, \mathbf{fi}\, \{y > 0\}$

(9 marks)

(b) Show that Inv: $x = y + 2 \wedge 0 \leq y \leq N$ is an Invariant of the following loop:

```
{Inv: x = y + 2 ∧ 0 ≤ y ≤ N }
  While  y ≠ N
  do
     x := x + 1;
     y := y + 1
 od
```

(6 marks)

(c) Using the proving techniques of Equational Logic, complete the following proofs:

(i) Show $\vdash \neg(p \wedge q) \equiv \neg p \vee \neg q$

Pf: $\neg p \vee \neg q$

$= \{$prop: $\neg$ $\}$

$(F \equiv p) \vee (F \equiv q)$

$= \{\vee$ distributes over $\equiv\}$

...

(ii) Show $\vdash \neg(p \vee q) \equiv \neg p \wedge \neg q$

Pf: $\neg p \wedge \neg q$

$=$ {By definition of $\wedge$ by Golden Rule }

$\neg p \equiv \neg q \equiv \neg p \vee \neg q$

$=$ {by theorem: $\vdash \neg(p \wedge q) \equiv \neg p \vee \neg q$ }

$\neg p \equiv \neg q \equiv \neg(p \wedge q)$

$=$ {property $\equiv$ }

...

(10 marks)

[25 marks]

SECTION B

4.(a) Explain the meaning of the terms

(i) Consistent Linear System,

(ii) Inconsistent Linear System.

(4 marks)

(b) Solve the following systems of linear equations:

(i)

$$\begin{aligned} -2x_1 + x_2 + 3x_3 &= 0 \\ x_1 + x_2 + 2x_3 &= 11 \\ 2x_1 + 4x_2 - 6x_3 &= 22 \end{aligned}$$

(ii)

$$\begin{aligned} x + y + 3z - w &= 7 \\ x - 2y + z + w &= 2 \\ 3x + 5y + z - 2w &= 1 \end{aligned}$$

(12 marks)

(c) Find the conditions that a_1, a_2, a_3 and a_4 must satisfy for the following system to be consistent:

$$\begin{aligned} 2x + 3y - z + w &= a_1 \\ x + 5y + z - 2w &= a_2 \\ -x + 2y + 2z - 3w &= a_3 \\ 3x + y - 3z + 4w &= a_4 \end{aligned}$$

(9 marks)

[25 marks]

5.(a)(i) Calculate the determinant of the matrix, A, given below.

$$A = \begin{pmatrix} 3 & 4 & 5 \\ 1 & 3 & 2 \\ 1 & 5 & 4 \end{pmatrix}$$

(ii) Hence, or otherwise, calculate the inverse of A

(13 marks)

(b) Calculate the eigenvalues and associated eigenvectors of the following matrix:

$$\begin{pmatrix} 5 & 4 & 2 \\ 4 & 5 & 2 \\ 2 & 2 & 2 \end{pmatrix}$$

(12 marks)

[25 marks]

6.(a) Calculate the following integrals:

(i) $\int e^{5-2x}\, dx$

(ii) $\int \frac{x}{(4x^2+1)^5}\, dx$

(iii) $\int (x+3)e^{2x}\, dx$

(9 marks)

(b) Use partial fractions to evaluate

$$\int \frac{4x+11}{x^2+5x+6}\, dx$$

(6 marks)

(c) Give a brief description of the role of the existential and universal quantifiers in Predicate Logic.

(4 marks)

(d) Symbolise the following statements using appropriate quantifiers and the suggested abbreviations

(i) "No principle is worth the sacrifice of a single human life" (Suggested abbreviations: $W(x) = x$ is worth the sacrifice of a human life)

(ii) "To know me is to love me"(Suggested abbreviations: $K(x) = x$ knows me, $L(x) = x$ loves me)

(iii) "A thing worth having, is a thing worth cheating for" (Suggested abbreviations: $H(x) = x$ is worth having, $C(x) = x$ is worth cheating for)

(6 marks)

[25 marks]