XCS10011

UNIVERSITY OF DUBLIN
TRINITY COLLEGE

Faculty of Engineering, Mathematics and Science

School of Computer Science & Statistics

JF Integrated Computer Science Programme
JF BA (Mod) Business and Computing
JF BA (Mod) CSLL

Trinity Term 2012

Mathematics

Friday, 4 May 2012 **RDS – MAIN HALL** **14:00 –17:00**

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Instructions to Candidates:
Answer TWO questions from SECTION A and TWO questions from SECTION B; FOUR questions in total to be answered.

All questions carry equal marks.
Each question is scored out of a total of 25 marks

Answer each SECTION in a separate answer book.

Log tables and graph paper are available from the invigilators, if required.

You may not start this examination until you are instructed to do so by the Invigilator.

Materials permitted for this examination:
Non-programmable calculators are permitted for this examination — please indicate the make and model of your calculator on each answer book used.

SECTION A

1. (a) Solve the following systems of linear equations

i)

$$\begin{aligned} 4x + 4y + 1z &= 13 \\ y + 2x - 3z &= 1 \\ 4z + x + y &= 7 \end{aligned}$$

ii)

$$\begin{aligned} x_1 + 2x_2 + 4x_3 &= 3 \\ x_1 + 3x_2 + 7x_3 &= 5 \\ 3x_1 + 4x_2 + 6x_3 &= 5 \end{aligned}$$

[12 marks]

(b) Find the conditions that a_1, a_2, a_3 and a_4 must satisfy for the following system to be consistent:

$$\begin{aligned} 2x + 3y - z + w &= a_1 \\ x + 5y + z - 2w &= a_2 \\ -x + 2y + 2z - 3w &= a_3 \\ 3x + y - 3z + 4w &= a_4 \end{aligned}$$

[8 marks]

(c) Solve the following system of equations using Cramer's Rule

$$\begin{aligned} 2x + 5y &= -1 \\ x + y &= 1 \end{aligned}$$

[5 marks]

[25 marks]

2. (a) i) Calculate the determinant of the matrix, A, given below

$$A = \begin{pmatrix} 3 & -2 & 1 \\ 2 & -1 & 2 \\ 1 & -2 & -4 \end{pmatrix}$$

ii) Hence, or otherwise, calculate the inverse of A.

[12 marks]

(b) Calculate the eigenvalues and associated eigenvectors of the following matrix:

$$\begin{pmatrix} 2 & 2 & 3 \\ 1 & 2 & 1 \\ 2 & -2 & 1 \end{pmatrix}$$

[13 marks]

[25 marks]

3. (a) Calculate the following integrals:

i) $\displaystyle\int \frac{e^{5x}}{(1+e^{5x})^3}\,dx$

ii) $\displaystyle\int x^5 \ln(2x)\,dx$

[6 marks]

(b) Use partial fractions to evaluate

$$\int \frac{3x+10}{x^2+7x+12}\,dx$$

[8 marks]

(c) Using a standard Taylor series, write down the Taylor series about 0 for the function

$$f(x) = \ln(1-x),$$

giving all terms up to and including that for x^4.
Note: The Standard Taylor Series about 0 are given on the following page.

[4 marks]

(d) Using another standard Taylor series, and your answer to part (b), find the Taylor series about 0 for the function

$$g(x) = (1-x)e^x - \ln(1-x),$$

giving all terms up to and including that for x^4.

[4 marks]

(e) Use the corresponding Taylor polynomial of order 4 to estimate the value of $g(0.2)$ to four significant figures. Comment on the expected accurracy of your result.

[3 marks]

[25 marks]

Standard Taylor Series about 0

$$
\begin{aligned}
\sin x &= x - \frac{1}{3!}x^3 + \frac{1}{5!}x^5 - \frac{1}{7!}x^7 + \frac{1}{9!}x^9 + \dots, \quad \text{for } x \in \mathbf{R} \\
\cos x &= 1 - \frac{1}{2!}x^2 + \frac{1}{4!}x^4 - \frac{1}{6!}x^6 + \frac{1}{8!}x^8 - \dots, \quad \text{for } x \in \mathbf{R} \\
e^x &= 1 + x + \frac{1}{2!}x^2 + \frac{1}{3!}x^3 + \frac{1}{4!}x^4 + \dots, \quad \text{for } x \in \mathbf{R} \\
\ln(1+x) &= x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \dots, \quad \text{for } -1 < x < 1 \\
\frac{1}{1-x} &= 1 + x + x^2 + x^3 + x^4 + \dots, \quad \text{for } -1 < x < 1 \\
(1+x)^\alpha &= 1 + \alpha x + \frac{\alpha(\alpha-1)}{2!}x^2 + \frac{\alpha(\alpha-1)(\alpha-2)}{3!}x^3 + \dots,
\end{aligned}
$$

for $-1 < x < 1$ and for any $\alpha \in \mathbf{R}$.

SECTION B

4. (a) Let the set operator, $\triangle$, be defined so that

$$X \triangle Y = (X \cup Y) - (X \cap Y)$$

$\overline{X}$ is the complement of X.
Determine by Veitch diagram, or otherwise, whether:

i) $A \triangle B = (A \cup B) \cap (\overline{A} \cup \overline{B})$

ii) $A \cap (B \triangle C) = (A \cap B \cap \overline{C}) \cup (A \cap \overline{B} \cap C)$

iii) $A \cup (B \triangle C) = (A \cup B) \triangle (A \cup C) \triangle A$

[15 marks]

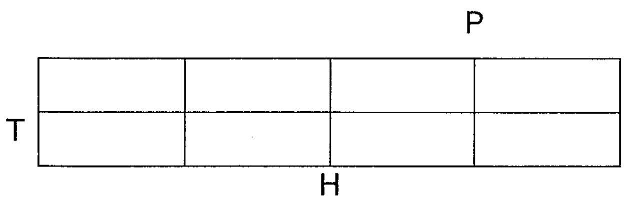
(b) 160 people were surveyed concerning which newpapers they read.

- 60 read the Times, 85 read the Independent , 82 read the Herald
- 20 read the Times and the Herald
- 43 read the Times and Independent
- 25 read the Herald and Independent
- 13 people read all three newspapers.

i) How many people read none of the three newspapers

ii) How many people read just the Times and none of the other two.

The following is a suggested labelling of a Veitch diagram, where:
T: **T**imes P: Inde**p**endent, H: **H**erald.



[10 marks]

[25 marks]

5. (a) Given the truth tables for the boolean operators: $\vee, \wedge, \rightarrow, \equiv$

P	Q	$P \vee Q$	$P \wedge Q$	$P \rightarrow Q$	$P \equiv Q$
F	F	F	F	T	T
F	T	T	F	T	F
T	F	T	F	F	F
T	T	T	T	T	T

determine by Truth Table or otherwise whether the following are Tautologies:

i) $\neg(\neg P \vee \neg Q) \vee \neg(\neg P \vee Q) \equiv P$

ii) $\neg(P \vee Q) \vee \neg(P \wedge Q) \equiv \neg P \vee \neg Q$

iii) $Q \rightarrow P \equiv R \vee \neg R \equiv P \vee Q \equiv P$

iv) $(P \wedge Q) \vee (\neg P \wedge R) \equiv (P \rightarrow Q) \equiv (\neg P \rightarrow R)$

[12 marks]

(b) Consider the abbreviations:

MA: Bill Misses his Appointment
GH: Bill Goes Home.
FD: Bill Feels Downcast.
GJ: Bill Gets the Job.

translate into English the following propositional calculus sentences:

i) $MA \wedge FD \rightarrow \neg GH$

ii) $\neg GJ \rightarrow FD \wedge \neg GH$

iii) $MA \rightarrow GH \rightarrow GJ$

[6 marks]

(c) Let

$P1 : MA \wedge FD \rightarrow \neg GH$
$P2 : \neg GJ \rightarrow FD \wedge \neg GH$

determine by the method of Truth Trees whether the following argument is valid:

$$P1, P2 \vdash Q$$

where $Q : MA \rightarrow GH \rightarrow GJ$
(Note: Truth Tree Rules of Inference are given on the following page)

[7 marks]
[25 marks]

Truth Tree Rules of Inference

Positive Rules

$A \to B$ $\neg A \quad B$	$A \wedge B$ A B	$A \vee B$ $A \quad B$	$A \equiv B$ $A \quad \neg A$ $B \quad \neg B$

Negative Rules

$\neg(A \to B)$ A $\neg B$	$\neg(A \wedge B)$ $\neg A \quad \neg B$	$\neg(A \vee B)$ $\neg A$ $\neg B$	$\neg(A \equiv B)$ $\neg A \quad A$ $B \quad \neg B$

Double Negation

$$\neg\neg A$$
$$A$$

6. (a) Using the techniques of Equational Logic,
Show $\vdash \neg(p \wedge q) \equiv \neg p \vee \neg q$
by completing the following proof:
Proof:

$\neg p \vee \neg q$

$= \{\text{From } \vdash \neg x \vee y \equiv x \vee y \equiv y\}$

$p \vee \neg q \equiv \neg q$

$\ldots$

$\neg(p \equiv q \equiv p \vee q)$

$= \{\text{Golden Rule: } x \wedge y \equiv x \equiv y \equiv x \vee y\}$

$\neg(p \wedge q)$

end Proof. [5 marks]

(b) Let $x = 539, y = 1331, z = 1859$; show that

i) $x \uparrow (y \downarrow z) = (x \uparrow y) \downarrow (x \uparrow z)$
where $a \uparrow b = \max(a, b)$ and $a \downarrow b = \min(a, b)$.

ii) $x \bigtriangleup (y \bigtriangledown z) = (x \bigtriangleup y) \bigtriangledown (x \bigtriangleup y)$
where $a \bigtriangleup b = \text{lcm}(a, b)$ and $a \bigtriangledown b = \gcd(a, b)$
$\gcd(a, b)$ is the greatest common divisor of a and b
$\text{lcm}(a, b)$ is the least common multiple of a and b. [10 marks]

(c) i) Express the fraction, $\dfrac{3159}{3328}$ in its lowest form, i.e. express $\dfrac{3159}{3328}$ as a fraction $\dfrac{a}{b}$ where a and b have no common divisor (except 1).

[4 marks]

ii) Find integers x and y such that

$$3159 \times x - 3328 \times y = \gcd(3328, 3159)$$

where $\gcd(a, b)$ is the greatest common divisor of a and b.

[6 marks]

[25 marks]