UNIVERSITY OF DUBLIN

XCS1BA11

TRINITY COLLEGE

Faculty of Engineering, Mathematics & Science

School of Computer Science & Statistics

Junior Freshman Computer Science
Junior Freshman CSLL

Trinity Term 2008

Mathematics (1BA1)

Tuesday, May 20, 2008 **Exam Hall** **14:00 – 17:00**

Hugh Gibbons, Meriel Huggard

Attempt five questions out of six

All questions carry equal marks

Log tables are available from the invigilators, if required.

The HANDBOOK OF MATHEMATICS of Computer Science is available from the invigilators, if required.

Non-programmable calculators are permitted for this examination, please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

1. **Linear Algebra**

Let the matrix

$$M = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 2 & 2 \\ 0 & 2 & 3 \end{bmatrix}$$

(a) Show that the characteristic equation of M is

$$\lambda^3 - 6\lambda^2 + 3\lambda + 10 = 0.$$

(5 marks)

(b) One of the eigenvalues of M is -1, find the other eigenvalues of M.

(4 marks)

(c) Show the matrix M is a solution of the equation:

$$M^3 - 6M^2 + 3M + 10I_3 = 0_3.$$

where I_3 is the 3×3 identity matrix and 0_3 is the 3×3 zero matrix.

Note: Using Horner's method we can write the polynomial

$$M^3 - 6M^2 + 3M + 10I_3$$

as

$$((M - 6I_3)M + 3I_3)M + 10I_3.$$

(6 marks)

(d) Since

$$M^3 - 6M^2 + 3M + 10I_3 = 0_3$$

we get that

$$I_3 = \frac{1}{10}(-M^3 + 6M^2 - 3M)$$

hence

$$M^{-1} = \frac{1}{10}(-M^2 + 6M - 3I_3).$$

Using this result, find M^{-1}, the inverse of M.

(5 marks)

[20 marks]

2. **Integration**

   (a) Calculate the following integrals

   (i) $\displaystyle\int \frac{1}{2\sqrt{\theta}} \sin(\sqrt{\theta})\ d\theta$ (3 marks)

   (ii) $\displaystyle\int \frac{(\log(x+1))^2}{x+1}\ dx$ (3 marks)

   (iii) $\displaystyle\int \frac{2x}{\sqrt{1-x^4}}\ dx$ (3 marks)

   (iv) $\displaystyle\int e^{2x} \sin x\ dx$ (3 marks)

   (b) Use partial fractions to evaluate $\displaystyle\int \frac{x^2}{(x-1)(x+1)^2}\ dx$

   (8 marks)

   [20 marks]

3. **Taylor Series**

   (a) Write down Taylor's theorem. (5 marks)

   (b) Consider the function $f(x) = x^3 - 2x + 7$. Determine, up to and including the x^2 term, the Taylor expansion of $f(x)$ at the point $x_0 = -2$. Give the equation of the tangent and the quadratic curve that approximates $f(x)$ at -2. (5 marks)

   (c) Explain the Newton-Raphson method. (4 marks)

   (d) Find the root of the equation

   $$x^3 - 2x + 7 = 0$$

   correct to two decimal places using the Newton-Raphson method with your first approximation starting from $x_0 = -2$. (6 marks)

   [20 marks]

4. **Textual Substitution and Propositional Logic**

(a) Perform the following textual substitutions. Be careful with parenthesization and remove any unnecessary parentheses.

(i) $x + y \cdot x[x, y := b + 2, x + 2]$

(ii) $(x + x \cdot y + x \cdot y \cdot z)[x, y := y, x][y := 2 \cdot y]$

(4 marks)

(b) Explain what is meant by the terms "satisfiable" and "valid" in relation to a boolean expression P. (4 marks)

(c) Translate the following English sentences into boolean expressions:

(i) If I go to Paris, then I will visit the Eiffel tower and the Louvre Museum.

(ii) I will go to Paris either by boat or by plane.

(6 marks)

(d) Give the truth table of the following expression:

$$p \wedge q \Rightarrow r \equiv p \Rightarrow (q \Rightarrow r)$$

(6 marks)

[20 marks]

5. **Proofs**

Prove the validity of the following theorems. Using the supplied Handbook of Mathematics, justify each step in your proof by referencing the corresponding theorem numbers (following the scheme introduced in class). You may only reference theorems or axioms of a lower number than the theorem to be proved. Do not use truth tables.

(a) (3.19) $p \not\equiv q \equiv r \equiv p \equiv q \not\equiv r$ (5 Marks)

(b) (3.32) $p \vee q \equiv p \vee \neg q \equiv p$ (5 Marks)

(c) (3.52) $(p \equiv q) \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (5 Marks)

(d) (3.82(a)) $(p \Rightarrow q) \wedge (q \Rightarrow r) \Rightarrow (p \Rightarrow r)$ (5 Marks)

[20 marks]

6. **Predicate Logic**

(a) Translate the following predicate logic formulas into English. First, give a literal translation and then give a sentence in English which captures the meaning of the formula.

(i) $(\exists k : \mathbb{R} | : (\forall i : \mathbb{Z} | : f.i = k))$

(ii) $(\forall x : \mathbb{R} | x \neq m : f.x > f.m)$

(iii) $(\forall x : \mathbb{Z} | : (\exists z : \mathbb{R} | : f.x = z))$

(10 Marks)

(b) Define suitable predicates and functions and then formalize the sentences that follow.

(i) Some integer is larger than 23.

(ii) Real number i is the largest real solution of the equation $f.i = i + 1$.

(iii) No integer is larger than all others.

(10 Marks)

[20 marks]