UNIVERSITY OF DUBLIN

XCS1BA11

TRINITY COLLEGE

Faculty of Engineering, Mathematics & Science

School of Computer Science & Statistics

Junior Freshman Computer Science
Junior Freshman CSLL

Trinity Term 2009

Mathematics (1BA1)

Tuesday, 19 May, 2009 **Exam Hall** **14:00 – 17:00**

Meriel Huggard, Hugh Gibbons

Attempt FIVE questions, including at least one question from SECTION B

Answer SECTION A and B in separate answer books

All questions carry equal marks

Log tables are available from the invigilators, if required.

The HANDBOOK OF MATHEMATICS of Computer Science is available from the invigilators, if required.

Non-programmable calculators are permitted for this examination, please indicate the make and model of your calculator on each answer book used.

You may not start this examination until you are instructed to do so by the Invigilator.

SECTION A

1. **Linear Algebra**

(a) Solve the following systems of linear equations:

(i)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= -2 \\ -x_1 + x_2 - 2x_3 &= 3 \\ 2x_1 - x_2 + 3x_3 &= 1 \end{aligned}$$

(ii)

$$\begin{aligned} x_1 - 2x_2 + 3x_3 &= -2 \\ -x_1 + x_2 - 2x_3 &= 3 \\ 2x_1 - x_2 + 3x_3 &= -7 \end{aligned}$$

(12 marks)

(b) Find conditions on a, b and c so that the following linear system is consistent.

$$\begin{aligned} x_1 + x_2 + 2x_3 &= a \\ x_1 + x_3 &= b \\ 2x_1 + x_2 + 3x_3 &= c \end{aligned}$$

(8 marks)

[20 marks]

2. **Determinants, Eigenvalues and Eigenvectors**

(a) Evaluate the determinant of the following matrix:

$$\begin{bmatrix} -2 & 8 & 1 & 4 \\ 3 & 2 & 5 & 1 \\ 1 & 10 & 6 & 5 \\ 4 & -6 & 4 & -3 \end{bmatrix}.$$

(8 marks)

(b) Find the characteristic equation, eigenvalues and eigenvectors associated with the following matrix:

$$\begin{bmatrix} -2 & 2 & 3 \\ -2 & 3 & 2 \\ -4 & 2 & 5 \end{bmatrix}.$$

(12 marks)

[20 marks]

3. **Integration**

(a) Calculate the following integrals

(i) $\displaystyle\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$ (3 marks)

(ii) $\displaystyle\int \frac{\cot x}{\ln(\sin x)}\,dx$ (3 marks)

(iii) $\displaystyle\int_0^1 \frac{x^3}{\sqrt{x^2+1}}\,dx$ (3 marks)

(iv) $\displaystyle\int x\cos(2x+1)\,dx$ (3 marks)

(b) Use partial fractions to evaluate $\displaystyle\int \frac{(x+4)}{x(x^2+3x-10)}\,dx$ (8 marks)

[20 marks]

4. Taylor Series, Newton Raphson and Propositional Logic

(a) Calculate

(i) The third order Taylor polynomial at $x_0 = 1$ for $f(x) = x \ln x$.

(ii) The second order Taylor polynomial at $x_0 = 0$ for $f(x) = \dfrac{x}{e^x - 1}$.

(iii) The fourth order Taylor polynomial at $x_0 = 0$ for $f(x) = x \sin x$.

(9 marks)

(b) Use the Newton-Raphson method to find the positive root of the equation

$$\sin x = x^2$$

correct to three decimal places, using $x_0 = 0.75$ as your first approximation.

(7 marks)

(c) Translate the following English sentences into boolean expressions:

(i) If I am in the Alps then I will either go skiing or snowboarding.

(ii) If I go skiing, then I will need to bring skis and poles.

(4 marks)

[20 marks]

5. **Proofs**

Prove the validity of the following theorems. Using the supplied Handbook of Mathematics, justify each step in your proof by referencing the corresponding theorem numbers (following the scheme introduced in class). You may only reference theorems or axioms of a lower number than the theorem to be proved. Do not use truth tables.

(a) (3.18) $((p \not\equiv q) \equiv r) \equiv (p \not\equiv (q \equiv r))$ (5 Marks)

(b) (3.31) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$ (5 Marks)

(c) (3.43)(a) $p \wedge (p \vee q) \equiv p$ (5 Marks)

(d) (3.62) $p \Rightarrow (q \equiv r) \equiv p \wedge q \equiv p \wedge r$ (5 Marks)

[20 marks]

6. **Predicate Logic**

(a) Translate the following predicate logic formulas into English. First, give a literal translation and then give a sentence in English which captures the meaning of the formula.

(i) $(\exists z : \mathbb{R} | : (\forall i : \mathbb{Z} | : f.j = f(j + i \cdot z)))$

(ii) $(\exists x, y : \mathbb{R} | : f.x < 0 \wedge 0 < f.y \Rightarrow (\exists z : \mathbb{R} | : f.z = 0))$

(iii) $(\forall x : \mathbb{Z} | : (\exists z : \mathbb{Z} | : f.x = z))$

(9 Marks)

(b) Define suitable predicates and functions and then formalize the following quotations:

(i) "No principle is worth the sacrifice of a single life" (Daniel Berrigan)

(ii) "To know me is to love me" (Charlie Brown)

(iii) "A thing worth having is a thing worth cheating for" (W.C. Fields)

(iv) "No one can be happy without causing harm to others" (A. Camus)

(11 Marks)

[20 marks]

SECTION B

7. Let the set operator, $\approx$, be defined as follows:

$$X \approx Y = (X \cap Y) \cup (\overline{X} \cap \overline{Y})$$

where $\overline{X}$ is the complement of X.

(a) Determine, by Venn/Veitch diagram of otherwise, whether:

(i) $A \approx B = (\overline{A} \cup B) \cap (A \cup \overline{B})$

(ii) $(A \cap B) \approx A = (A \cup B) \approx B$

(iii) $A \cup (B \approx C) = (A \cup B) \approx (A \cup C)$

(iv) $A \approx (B \cup C) = (A \approx B) \cup (A \approx C)$

(8 Marks)

(b) Out of 50 TV viewers,

15 watch TG4, 22 watch RTE1, 21 watch RTE2,

6 watch both TG4 and RTE1,

7 watch both TG4 and RTE2,

9 watch both RTE1 and RTE2,

and 3 watch all 3 channels.

(i) How many viewers don't watch any of the channels?

(ii) How many watch just TG4?

(6 Marks)

(c) The subset of $U = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ can be represented as a 'bit-array' where for the subset, B, the representing array, b, of Booleans is such that

$$b(k) \text{ iff } k \in B$$

i.e.

If $k \in B$ then the array item, $b(k)$ is set to True (T) and

if $k \notin B$ the array element $b(k)$ is set to False (F).

e.g. the set $\{0, 1, 3, 9\}$ is represented by the array,

$$[T, T, F, T, F, F, F, F, F, T]$$

Let $A = \{0, 2, 5, 7, 8\}$ and $B = \{1, 2, 3, 6, 8\}$. Determine the bit-array for

(i) $A \cup B$

(ii) $A \cap B$

(iii) $A - B$

(6 Marks)

[20 Marks]

8. (a) Show that $|\mathbb{Z}| = |N|$ (5 Marks)

(b) Show that $|\mathbb{Q}| = |\mathbb{Q}^+|$ (5 Marks)

(c) Show that $|\mathbb{N}| \neq |\mathbb{R}|$ (10 Marks)

[20 marks]