Fast finite element modelling of bridled leading-edge inflatable kites

Thesis midterm | Patrick Roeleveld

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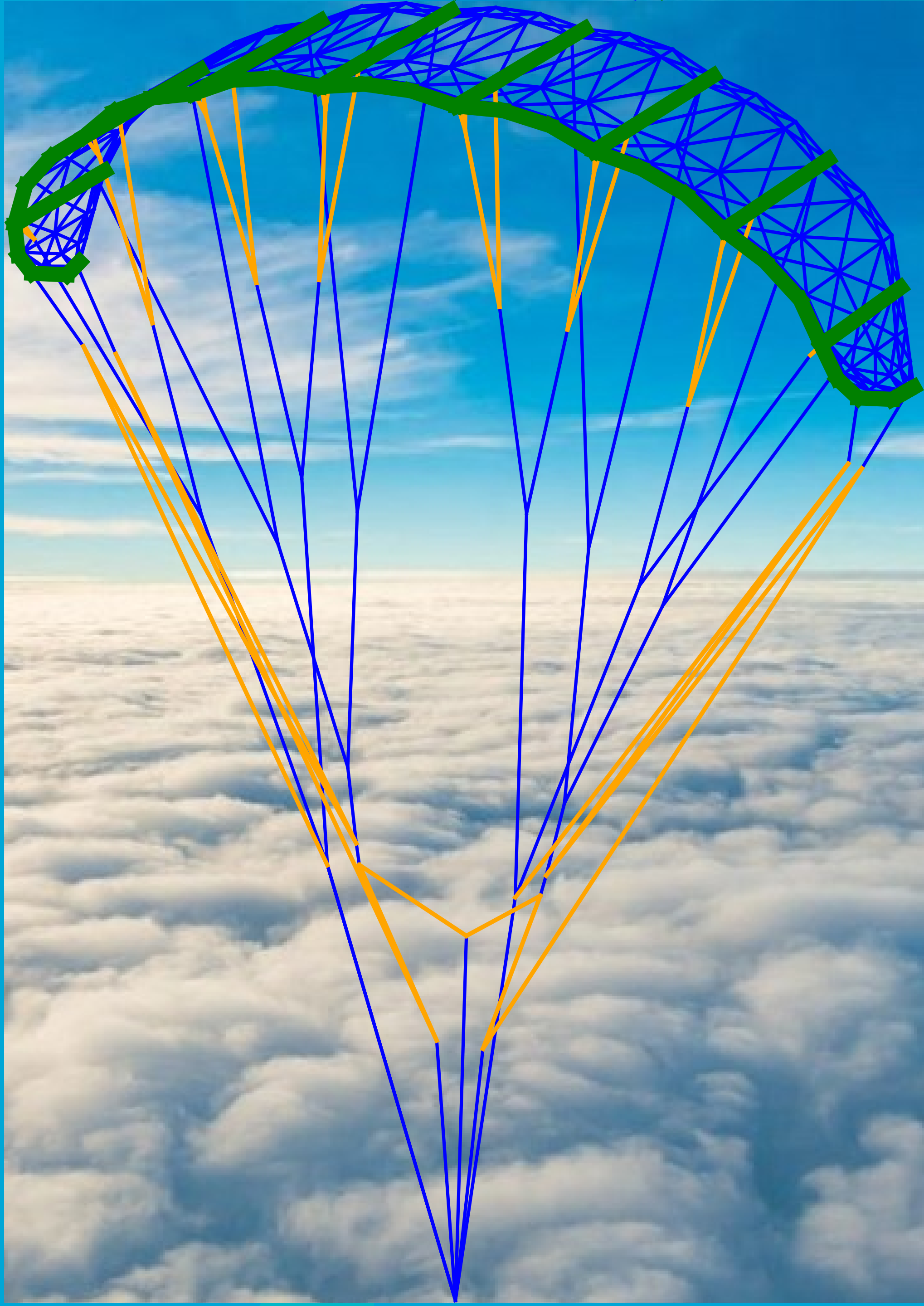

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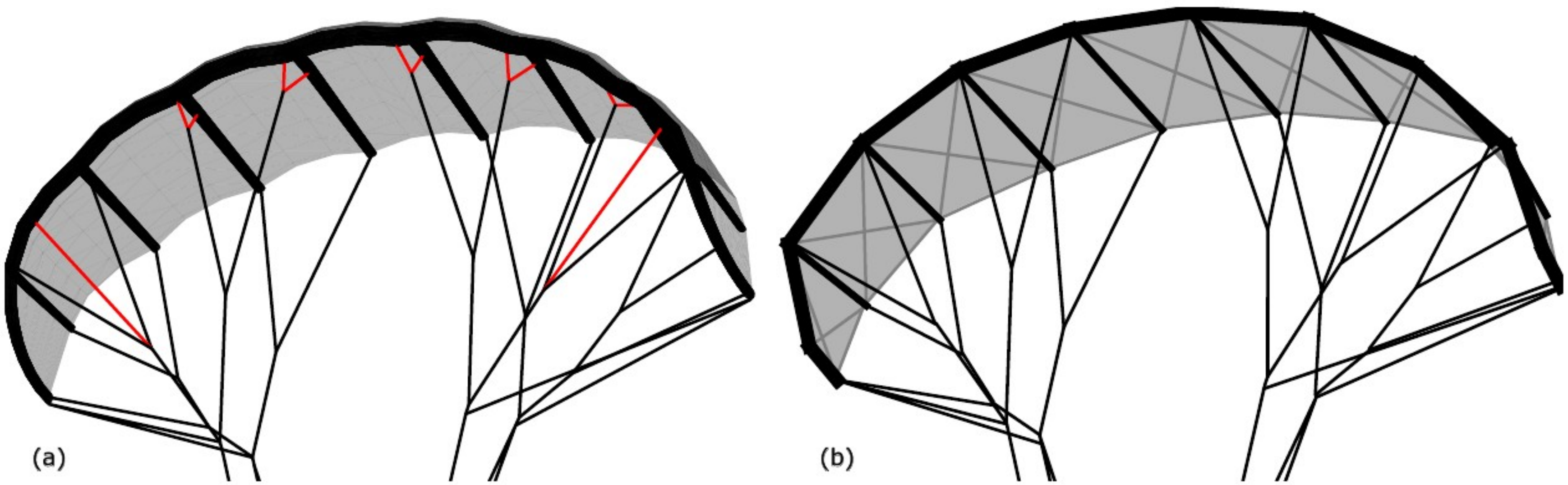
Limitations PSM model

- No bending stiffness in leading edge / struts
- Simplification of bridle line system
- Canopy is flat → unable to model billowing

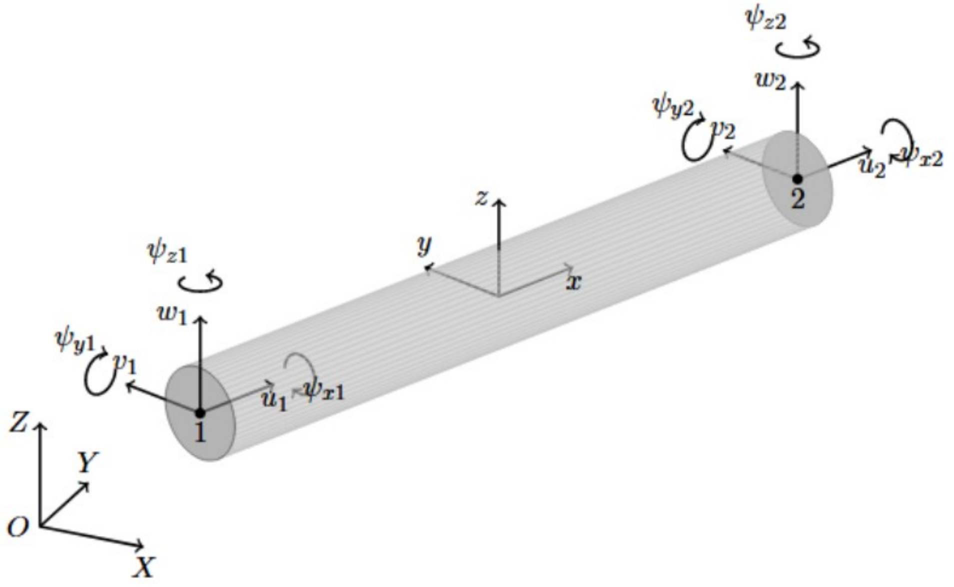


- Modelling a kite is a nonlinear task due to
  - **Geometric nonlinearity** → Shape and orientation of kite and bridle changes
  - **Material nonlinearity** → Canopy material, Inflatable beams
  - **Force nonlinearity** → Aerodynamic forces change due to shape

- Therefore, iteration is required to converge to a solution.
- Each iteration the linear equations are set up and solved

- Model consists of two node elements in 3D space
- 12 Degrees of freedom per element. Stored in displacement vector **u**
- Element stiffness matrix **K**, set up using material properties.
- Element displacements obtained by solving $\mathbf{K}^{\mathsf{e}}\mathbf{u}^{\mathsf{e}} = \mathbf{f}^{\mathsf{e}}$
- Element has a local coordinate frame where the x axis alligns with the element direction.
- Rotation matrix **R** is used to map an element's coordinate system to the global coordinate system. This is updated each iteration (co-rotational method).
- Element's contribution to the global system

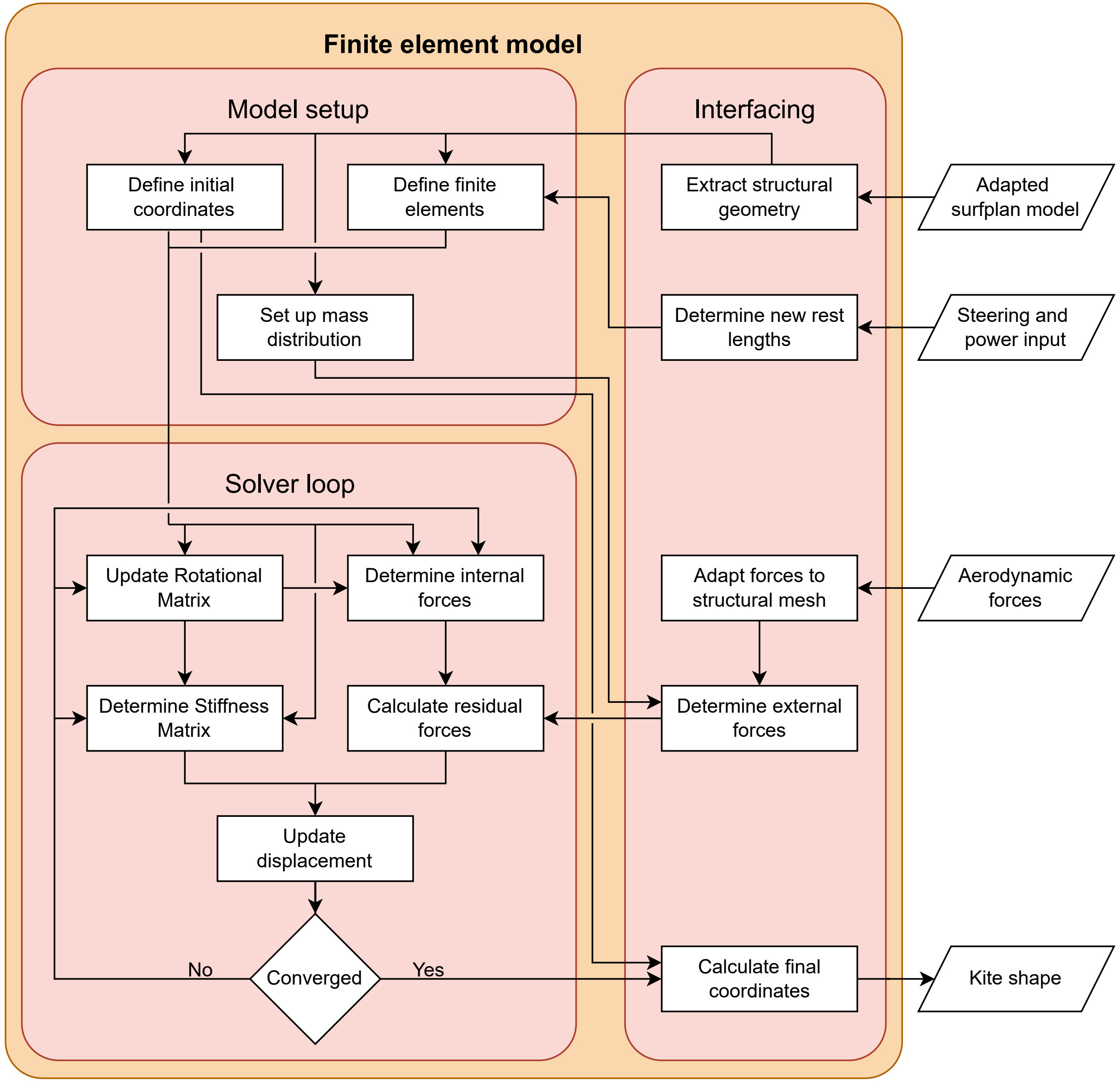
  is then $\mathbf{K}^{\mathsf{g}} = \mathbf{R}^{\mathsf{T}}\mathbf{K}^{\mathsf{e}}\mathbf{R}$



Two node element in 3D space (Bosch 2012)

Nonlinear Finite Element Modelling

- Convergence is reached when internal forces and external forces are balanced.
- Each iteration, the rotational matrix, internal forces and the stiffness matrix are recalculated.
- Geometric non-linearity and material non-linearity covered by this modelling approach
- Force non-linearity resolved by coupling with an aerodynamic solver
- Pyfe3d python library used for setting up finite elements and calculation stiffness matrix

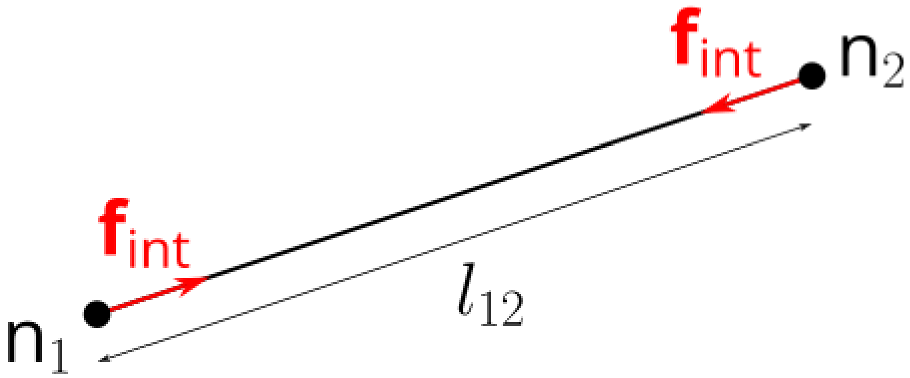


Spring Elements

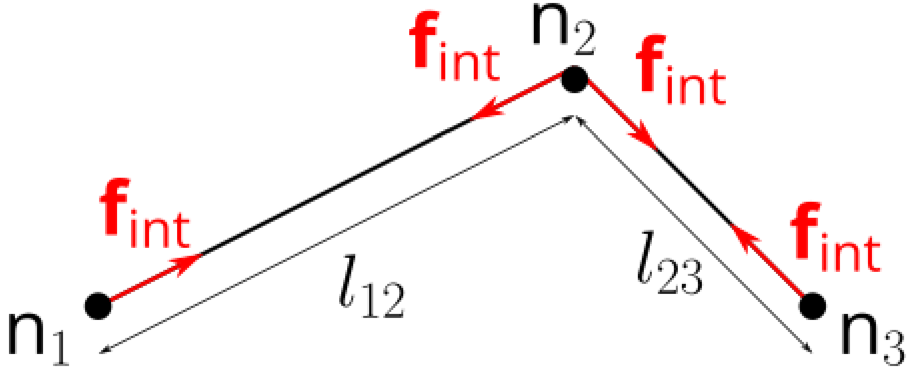
- Spring elements only have stiffness in the element's x axis
- To represent bridle elements, the spring stiffness can be set to 0 when the line is slack

$$k_x = \begin{cases} k_x & \text{if } (l - l_0) \geq 0, \\ 0 & \text{if } (l - l_0) < 0 \end{cases}$$

- Internal force calculation $\mathbf{f}_{\text{int}}^{e,T} = \begin{bmatrix} k_x(l - l_0) & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$
- A pulley can be modelled by combining two spring elements and sharing a single rest length and total length



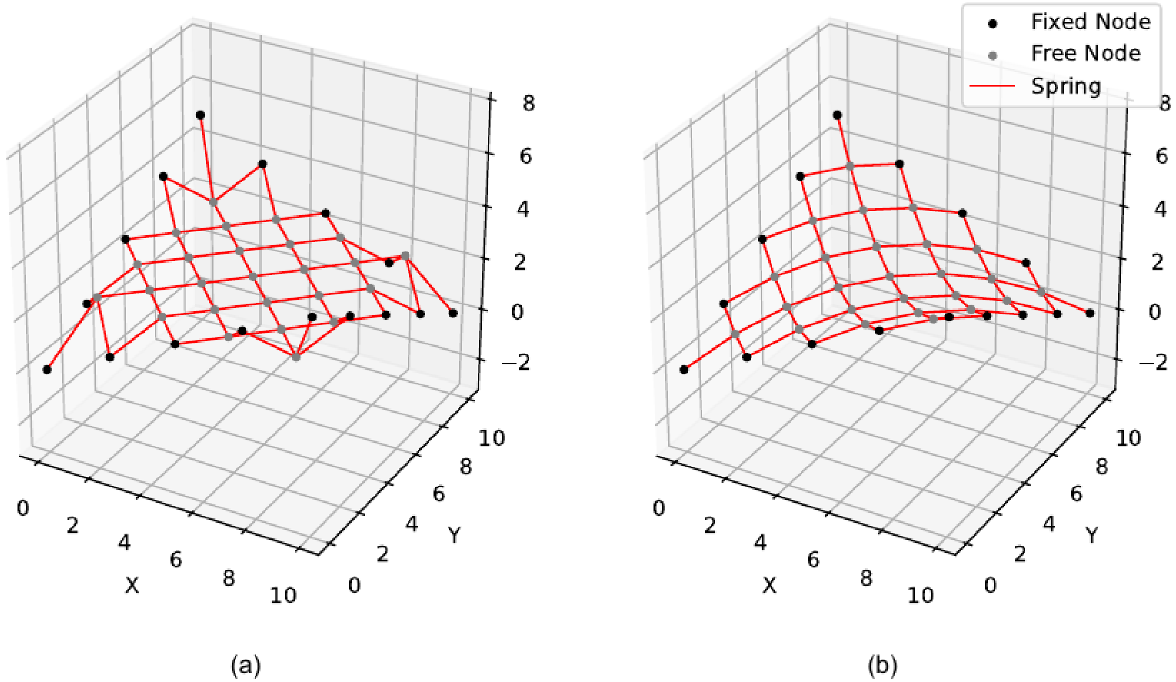
Spring internal force calculation



Pulley internal force calculation

Spring Elements

- Saddle problem for spring verification
- Compared to python PSM solver



Initial shape

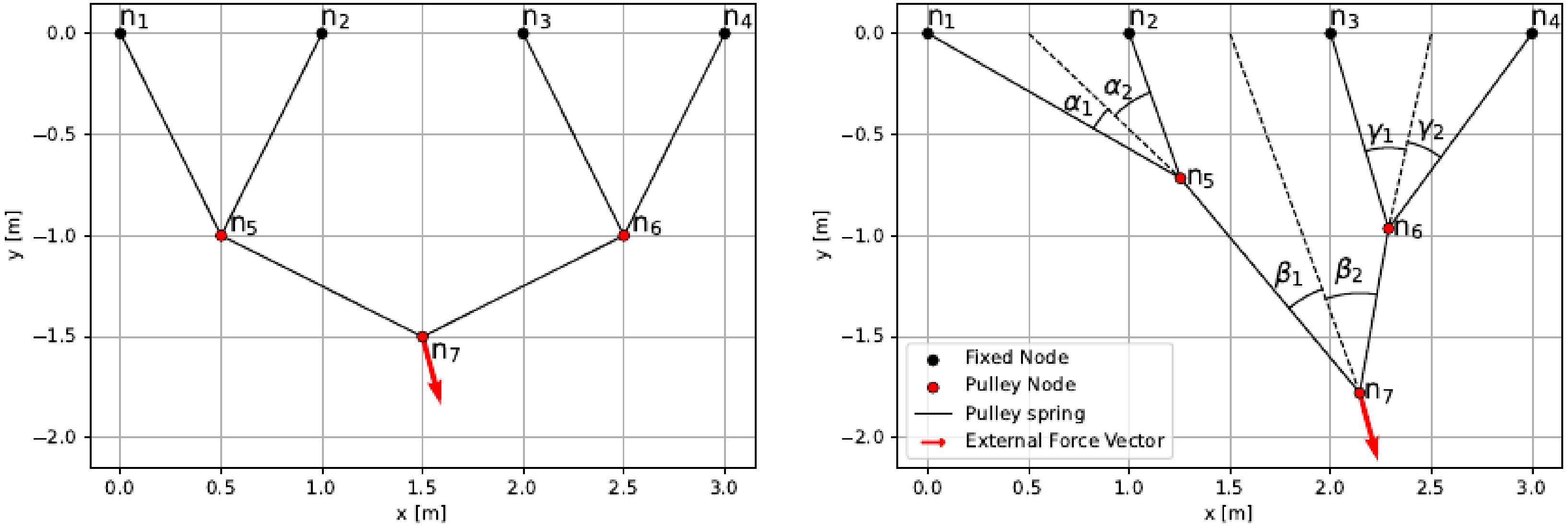
Final shape

Grid size	Number of nodes	Maximum nodal difference [mm]	FEM solver time [s]*	PSM solver time [s]*
3×3	13	0.06	0.03	0.08
5×5	41	0.10	0.11	0.41
7×7	85	0.16	0.27	1.68
9×9	145	0.27	0.71	2.92
11×11	221	0.38	1.61	5.95
13×13	313	0.49	3.09	18.10

*Using a HP Zbook Power G7 mobile workstation

Spring Elements

- Pulley verification by setting up system of pulleys
- For a ideal pulley, angles between the force on the pulley and the corresponding lines are equal



(a) Initial shape

(b) Final shape

α_1 [°]	α_2 [°]	β [°]	β_2 [°]	γ_1 [°]	γ_2 [°]
20.12	20.12	24.54	24.54	26.09	26.09

Beam Elements

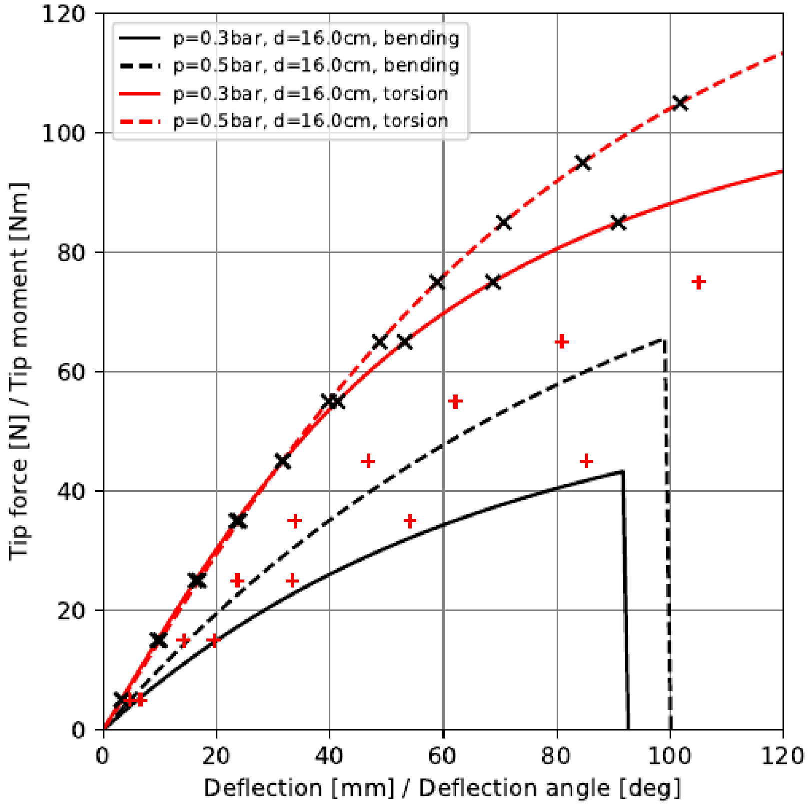
- Timoshenko beam elements used to represent the tubular frame
- Breukels (2011) performed bending and torsional tests on inflatable beams and developed fitted equations to relate pressure, radius and deflection/rotation to tip load/torque

$$P(p,r,\delta) \qquad\qquad T(p,r,\varphi)$$

- Beam properties E and G can then be updated using timoshenko beam theory

$$E(p,r,\delta,\varphi,k) = \frac{P(p,r,\delta)}{3I(\delta - \frac{P(p,r,\delta)}{kAG})}$$

$$G(p,r,\varphi) = \frac{T(p,r,\varphi)}{2\varphi I}$$

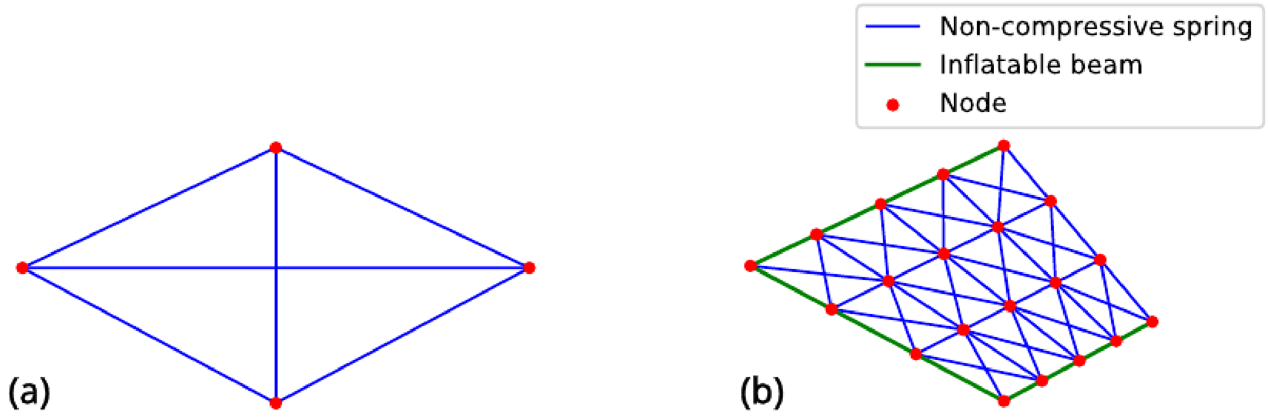


(a)

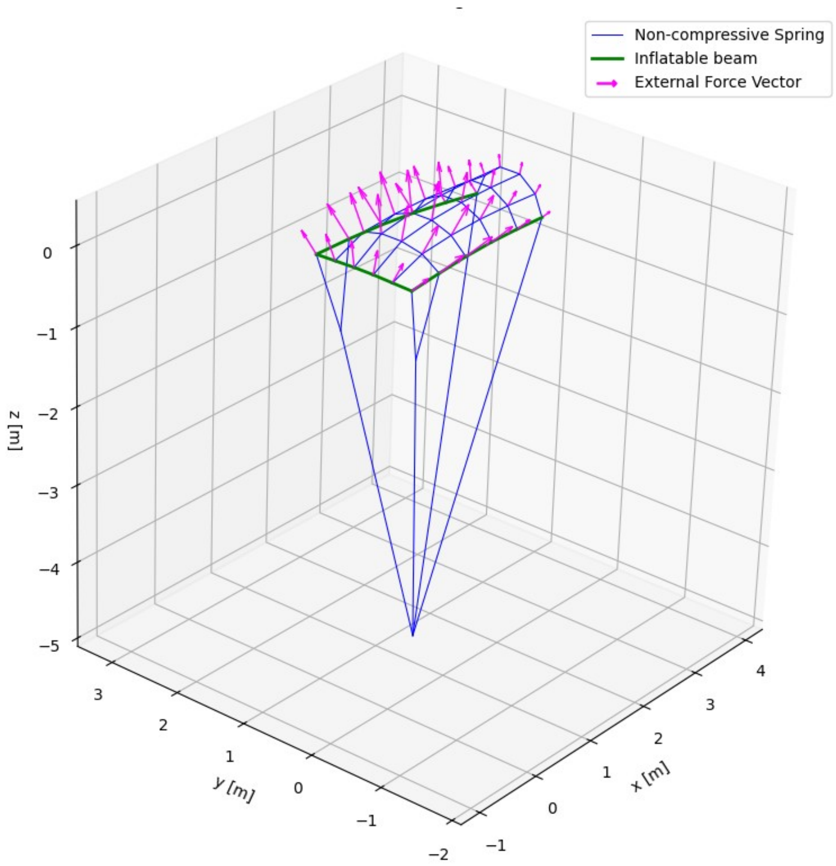
Comparison of a 1 meter beam element with Breukel's equations

Canopy section

- Canopy can be modelled by making quadrilaterals with 6 non-compressive spring connections



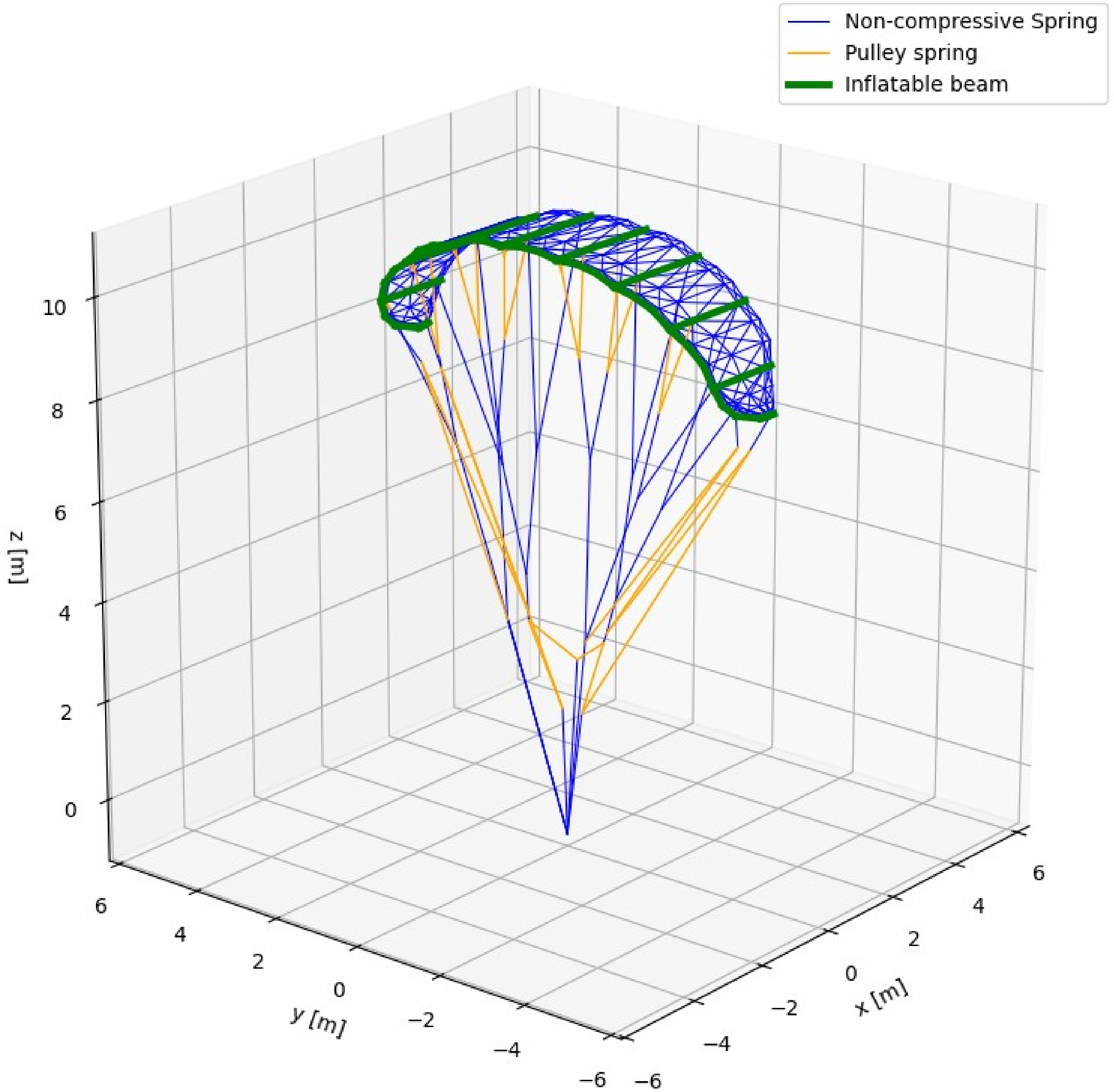
Quadrilateral representaiton of canopy



Example of canopy section with billowing

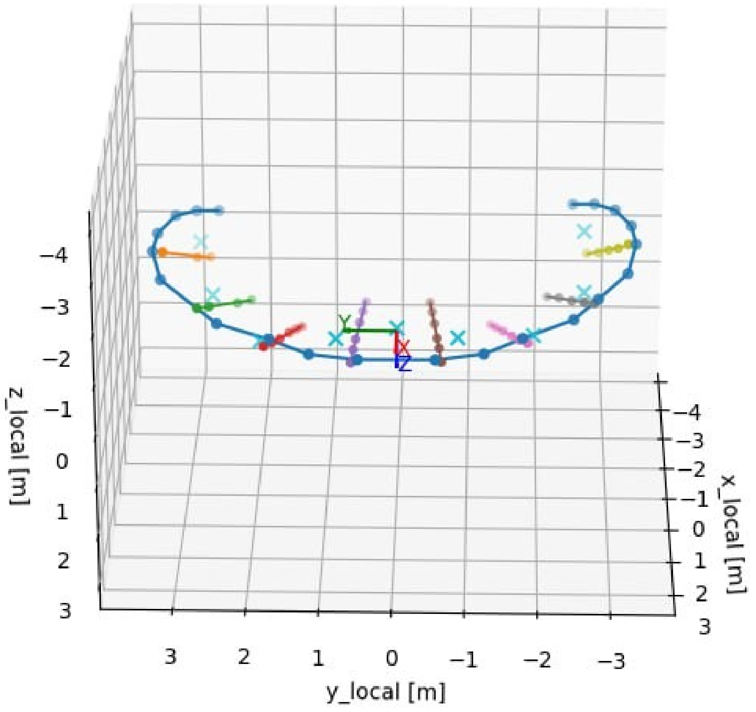
V3 Kite model

- V3 Kite model set up using surfplan → yaml → askite pipeline
- Some manual adaptations on the bridle system required, to get it in line with the recent testing data



Static tests

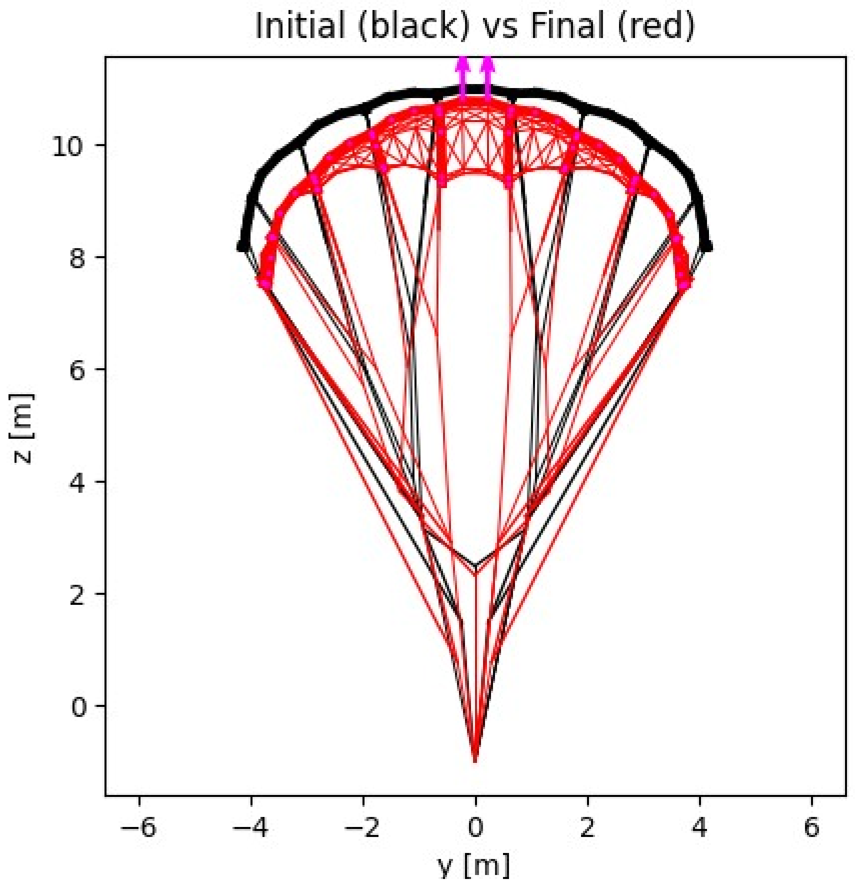
- Static tests performed together with Pim, to get validation data for the model.
- Varied loads, and internal pressure.

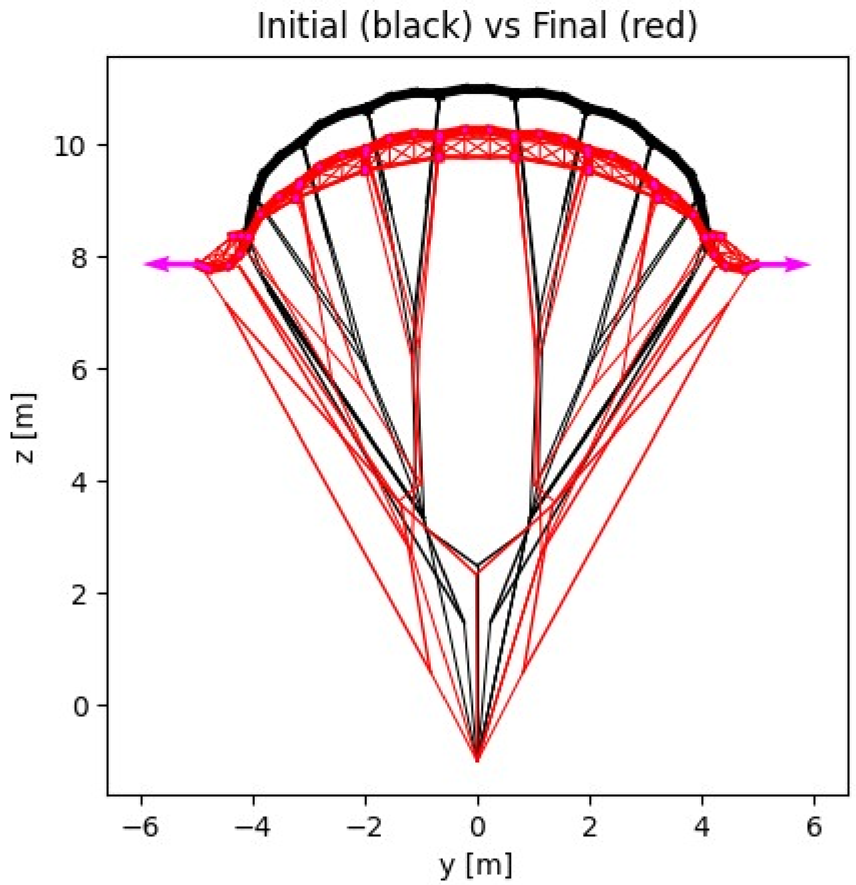


Point load on middle of leading edge

Static tests

- Full comparison still to be done, kite behaviour to input seems visually correct
- Still some convergence issues to iron out





Aerostructural coupling

- Model is to be fitted into existing Aero-structural framework (askite).
- Will be combined with VSM
- FEM framework partly integrated into askite, the simplified kite model can already be resolved with the model in askite