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Gabarito $3^{\underline{a}}$ Lista - MAT 137 - Introdução à Álgebra Linear 2016/II

1. $(a) a = -2, b = -9, \quad (b) a = 4/5, b = -2, \quad (c) a = -6, b = 8.$
2. $(a) a = 7, b = -3, c = -2, \quad (b) a = 9, b = -6, c = -12, \quad (c) a = 1, b = 1, c = 6.$
3. $c_1 = 2, c_2 = -1, c_3 = 2.$
4. $c_1 = c_2 = c_3 = 0.$
5. (a) e (b) não são espaços vetorias com estas operações.
6.
7. (a) e (c) são espaços vetoriais, mas (b) e (d) não são espaços vetoriais.
8. $d = 0$ e a, b, c quaisquer números reais.
9.
10. (a) $w = 3u - v$
    (b) Não
    (c) $k = 12$
    (d) $c = 16a + 10b$
11. Somente o ítem (a) é subespaço vetorial.
12. $v_1 = 1u + \frac{11}{3}v + \frac{16}{3}w; \quad v_2 = 3u - \frac{11}{3}v - \frac{10}{3}w; \quad v_3 = 0u + 0v + 0w.$
13. (i) $E = 2A - B + 2C$;
    (ii) Não é possível.
14. $(x, y, z) = x(1, 1, 1) + \frac{-2x+y+z}{2}(0, 1, 1) + \frac{y-z}{2}(0, 1, -1).$
15. $c = \frac{2a}{3} - \frac{4b}{3}.$
16. $k = -8.$
17. $-a + 3b + 5c = 0.$
18.
19. Não.
20. Sim.
21.
22. (a) $\{(2, 1, 0), (0, 0, 1)\}$
    (b) $\{(2, 1, -2)\}$
    (c) $\{(2, 1, -2)\}$

    A resposta não é única.

23. $\{(2,5,0)\}$.

24.

25.

26. (a) Sim;

    (b) Não;

    (c) $\{(1,0,1,0),(0,1,2,0)\}$.

27. $s = \{(x,y,z,t) \in I\!R^4; -17x + 9y + 7z = 0\}$.

28. Não.

29. (a) L.I. se $k \neq 8$ e L.D. se $k = 8$.

    (b) Os vetores são sempre L.D.

30. L.I.

31. (a) L.D.

    (b) L.D.

    (c) L.I.

    (d) L.D.

    (e) L.D.

32.

33.

34.

35.

36.

37.

38.

39. $\lambda = -4$.

40. $B = \left\{ \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \right\}$. O subespaço tem dimensão 4.

41. (a) $dim(W_1) = 3$ e $B = \left\{ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \right\}$

    (b) $dim(W_2) = 1$ e $B = \{(1,1)\}$

    (c) $dim(W_3) = 2$ e $B = \{(1,2,3),(0,0,2)\}$

    (d) $dim(W_4) = 2$ e $B = \{(1,1,1,0),(-1,-1,0,1)\}$.

42. $v_2 = (0,1)$. Basta tomar qualquer vetor que não seja múltiplo de v_1.

43. (a) Não é base.

(b) É base. Podemos escrever $(x,y,z) = y(2,1,-1) + (2y-x)(-1,0,1) + (x-y+z)(0,0,1)$.

(c) É base. Podemos escrever $(x,y,z) = \frac{1}{16}(x+4y-2z)(2,3,-1) + \frac{1}{16}(-3x+4y+6z)(-2,1,1) + \frac{1}{4}(x+2z)(2,0,1)$.

44.

45.

46. $(a)[(6,2)]_B = \begin{bmatrix} 2 \\ 1 \end{bmatrix}, \quad (b)[(6,2)]_B = \begin{bmatrix} -2/3 \\ 10/3 \end{bmatrix}, \quad (c)[(6,2)]_B = \begin{bmatrix} 6 \\ 2 \end{bmatrix}$

47. $(a)[(2,-3,4)]_B = \begin{bmatrix} -2 \\ 1 \\ 4 \end{bmatrix}, \quad (b)[(2,-3,4)]_B = \begin{bmatrix} -3 \\ 11 \\ 6 \end{bmatrix}, \quad (c)[(2,-3,4)]_B = \begin{bmatrix} 0 \\ -1 \\ 1 \end{bmatrix}$

48. $B = \{(1,1,1,0),(1,1,2,1),(2,1,0,3),(0,0,0,1)\}$.

49. $(a)B = \left\{ \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & -1 \\ -3 & 0 \end{pmatrix} \right\}, \quad (b)B = \left\{ \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}, \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \right\}$.

50. (i) $B = \{(1,-2,5,3),(2,3,1,-4)\}$

(ii) $\{(1,-2,5,3),(2,3,1,-4),(0,0,1,0),(0,0,0,1)\}$

(iii) Nenhum subconjunto da base canônica irá gerar W.

51. $B = \{(-1,3,5,0),(-2,0,7,3)\}$.

52. (a) $B_U = \{(0,1,0),(0,0,1)\}$, $\quad B_V = \{(1,0,0),(0,2,1)\}$, $\quad B_{U\cap V} = \{(0,2,1)\}$.

(b) $B_U = \{(1,-1,4)\}$, $\quad B_V = \{(1,0,0),(1,0,1)\}$, $\quad B_{U\cap V} = \emptyset$.

53. $B_U = \{(1,1,0,-1),(1,2,3,0)\}$, $\quad B_W = \{(1,2,2,-2),(2,3,2,-3)\}$,
$\dim U = \dim W = 2,\ \dim(U\cap W) = 1\ e\ \dim(U+W) = 3$

54. $B_U = \{t^3+t^2, -t^3+t, 1\}$, $\quad B_W = \{t^3+1, t^2+1, t+1\}$,
$B_{U+W} = \{t^3+t^2, t^2+t, t-1, -2\}$, $\quad B_{U\cap W} = \{t^3+t^2+2, -t^3+t\}$.

55. (a) (i) $\alpha = \{\,(1,0,-1),(0,1,-1)\,\}$ e $\dim(U) = 2$.

(ii) $\beta = \{\,(1,0,0),(0,1,0)\,\}$ e $\dim(W) = 2$.

(iii) $\gamma = \{\,(1,0,-1),(0,1,-1),(1,0,0)\,\}$ e $\dim(U+W) = 3$.

(iv) $\delta = \{\,(-1,1,0)\,\}$ e $\dim(U\cap W) = 1$.

(b) (i) $\alpha = \{\,(1,-1,0,0),(0,0,1,1)\,\}$ e $\dim(U) = 2$.

(ii) $\beta = \{\,(1,0,0,0),(0,1,0,0)\,\}$ e $\dim(W) = 2$.

(iii) $\gamma = \{\,(0,0,1,1),(1,0,0,0),(0,1,0,0)\,\}$ e $\dim(U+W) = 3$.

(iv) $\dim(U\cap W) = 1$.

(c) (i) $\alpha = \{\,(0,1,0),(0,0,1)\,\}$ e $\dim(U) = 2$.

(ii) $\beta = \{\,(2,2,0),(1,2,3),(7,12,21)\,\}$ e $\dim(W) = 3$.

(iii) $\gamma = \{\,(1,0,0),(0,1,0),(0,0,1)\,\}$ e $\dim(U+W) = 3$.

(iv) $\delta = \{\,(0,1,0),(0,0,1)\,\}$ e $\dim(U\cap W) = 2$.

(d) (i) $\alpha = \{\,(1,0,0,-1),(0,1,0,1),(0,0,1,-1)\,\}$ e $\dim(U) = 3$.

(ii) $\beta = \{\,(1,0,0,1),(0,1,0,1),(0,0,1,1)\,\}$ e $\dim(W) = 3$.

(iii) $\gamma = \{\,(1,0,0,-1),(0,1,0,1),(0,0,1,-1),(1,0,0,1)\,\}$ e $\dim(U+W) = 4$.

(iv) $\delta = \{\,(1,0,-1,0),(0,1,0,1)\,\}$ e $\dim(U \cap W) = 2$.