Fundamentals of Simulation Methods

Exercise 5

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November 19, 2016

5.1. FFT based convolution

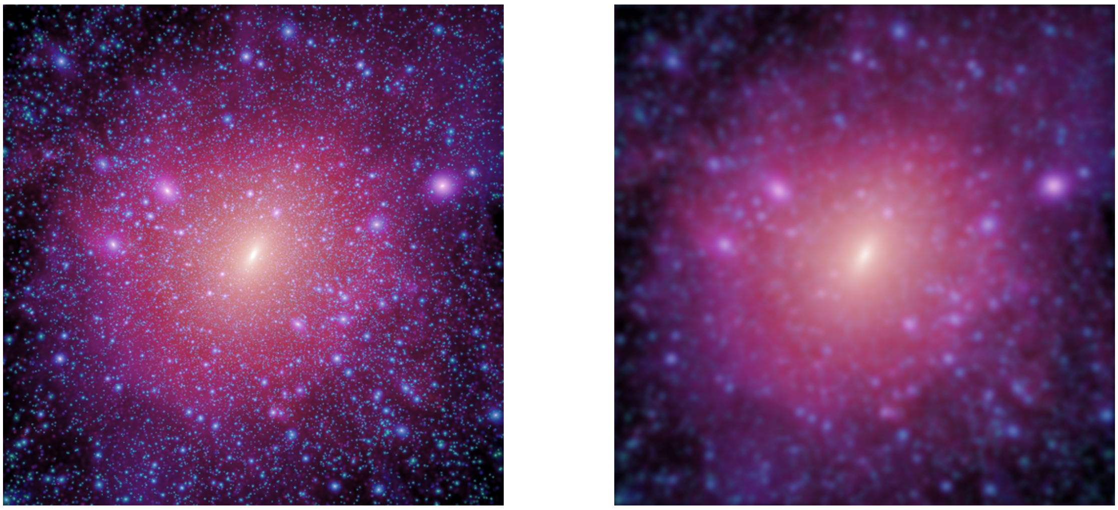
(a) We integrate over the entire plane to obtain the normalisation factor using the coordinate transformation $u = r/h$. There is no dependence on an angle so in polar coordinates the angular integral can be carried out immediately ($\int \mathrm{d}\phi = 2\pi$):

$$\begin{aligned} 1 &= \int\limits_{\mathbb{R}^2} W(|\vec{r}|)\mathrm{d}^2 r \\ &= 2\pi h^2 k \left(\int\limits_0^{1/2} \mathrm{d}u u(1 - 6u^2 + 6u^3) + \int\limits_{1/2}^{1} \mathrm{d}u u 2(1-u)^3 \right) \\ &= 2\pi h^2 k \; 0.0875 \\ \Rightarrow \quad k &= \frac{11.43}{2\pi h^2} \approx 1.819/h^2 \end{aligned}$$

(b) and (c) The sum of all colour channels (red, green, blue) and of all pixels before and after smoothing differ by 241 which corresponds to a relative error of 4 ppm. The integral over the kernel differs from the desired value of 1 by 4×10^{-6} (cf. table 1). This difference could be further reduced by choosing a higher precision for the kernel, for instance long double, and defining the normalisation k with more significant digits (here 14 digits were used). The images before and after smoothing can be found in fig. 1.

before smoothing	62265510.0
after smoothing	62265269.9
kernel integral	0.999996144309

Table 1: Results of checking the consistency of the smoothing procedure.



(a) Original image. (b) Smoothed image.

Figure 1: Comparison of original and smoothed image.