SIGUE, JP-FA4

Github:
https://github.com/PatrickSigue/APM1110/blob/main/FA4/SIGUE%2C-JP-FA4.md

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A geospatial analysis system has four sensors supplying images. The percentage of images supplied by each sensor and the percentage of images relevant to a query are shown in the following table.

```r
images <- data.frame(sensor = 1:4, Supplied = c(0.15, 0.2, 0.25, 0.4), Relevant = c(0.5,
    0.6, 0.8, 0.85))
images
```

```
##   sensor Supplied Relevant
## 1      1     0.15     0.50
## 2      2     0.20     0.60
## 3      3     0.25     0.80
## 4      4     0.40     0.85
```

What is the overall percentage of relevant images?

*R = Relevant, S = Supplied

$$P(R) = P(R|S_1) \cdot P(S_1) + P(R|S_2) \cdot P(S_2) + P(R|S_3) \cdot P(S_3) + P(R|S_4) \cdot P(S_4)$$

```r
prob_rel <- sum(images$Supplied * images$Relevant)

cat("The overall percentage of relevant images is ", prob_rel * 100, "%.", sep = "")
```

```
## The overall percentage of relevant images is 73.5%.
```

A fair coin is tossed twice. Let E_1 be the event that both tosses have the same outcome, that is $E_1 = (HH, TT)$. Let E_2 be the event that the first toss is a head, that is, $E_2 = (HH, HT)$. Let E_3 be the event that the second toss is a head, that it, $E_3 = (TH, HH)$. Show that $E_1, E_2,$ and E_3 are pairwise independent but not mutually independent.

Sample Space

$$S = HH, HT, TH, TT$$

Since the coin is fair, each outcome has an equal probability:

$$P(HH) = P(HT) = P(TH) = P(TT) = \frac{1}{4}$$

Events and Their Probabilities

1. $E_1 = (HH, TT)$
   - $P(E_1) = P(HH) + P(TT) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
2. $E_2 = (HH, HT)$
   - $P(E_2) = P(HH) + P(HT) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$
3. $E_3 = (TH, HH)$
   - $P(E_3) = P(TH) + P(HH) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2}$

Check for Pairwise Independence

Events A and B are independent if:
$$P(A \cap B) = P(A)P(B)$$

1. E_1 and E_2
   $E_1 \cap E_2 = \{HH\}$
   $P(E_1 \cap E_2) = P(HH) = \frac{1}{4}$
   $P(E_1)P(E_2) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
   Since $P(E_1 \cap E_2) = P(E_1)P(E_2)$, E_1 and E_2 are independent.

2. E_1 and E_3
   $E_1 \cap E_3 = \{HH\}$
   $P(E_1 \cap E_3) = P(HH) = \frac{1}{4}$
   $P(E_1)P(E_3) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
   Since $P(E_1 \cap E_3) = P(E_1)P(E_3)$, E_1 and E_3 are independent.

3. E_2 and E_3
   $E_2 \cap E_3 = \{HH\}$
   $P(E_2 \cap E_3) = P(HH) = \frac{1}{4}$
   $P(E_2)P(E_3) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
   Since $P(E_2 \cap E_3) = P(E_2)P(E_3)$, E_2 and E_3 are independent.

Check for Mutual Independence

$E_1, E_2, \ldots, E_k$ are mutually independent if:

$$P(E_{i_1} \cap E_{i_2} \cap \cdots \cap E_{i_k}) = P(E_{i_1})P(E_{i_2}) \cdots P(E_{i_k})$$

From the sets:
$E_1 \cap E_2 \cap E_3 = \{HH\}$
$P(E_1 \cap E_2 \cap E_3) = P(HH) = \frac{1}{4}$

Calculate $P(E_1)P(E_2)P(E_3)$:
$P(E_1)P(E_2)P(E_3) = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$

Since $P(E_1 \cap E_2 \cap E_3) \neq P(E_1)P(E_2)P(E_3)$, the events are not mutually independent.