CIS 315, Intermediate Algorithms
Winter 2021

Assignment 1

Due 5 PM Friday January 15, 2021

Problem 1 (5 points): Suppose you are given the adjacency matrix representation M of a directed graph $G = (V, E)$. Note that the size of M is $\Theta(n^2)$. The goal here is to determine if there is a node of G with in-degree $n - 1$ and out-degree 0 (that is, all other nodes point to it and it points to no other node). Give an algorithm to do this which runs in $\Theta(n)$ time (so **not** $\Theta(n^2)$).

Problem 2 (5 points): Suppose you work for a lab which is studying butterflies. It has a sample of n butterflies, $L_1, L_2, \ldots, L_n$. It is believed that all these butterflies belong to two different species. The researchers have made a series of r (note: the value of r can be different from that of n) "determinations" in order to try to find out whether these butterflies indeed belong to two different species. A determination is of the form (i, j), associated with a binary label which indicates either L_i and L_j belong to the same species or they belong to different species. Your job is to give an $O(n + r)$ time algorithm to decide whether all the determinations are "consistent". Being "consistent" means there is no contradiction across all the determinations. For example, "(1,2) same", "(1,3) same", and "(2,3) different" are not consistent.

Problem 3 (5 points): Exercise 22.3-2, from CLRS text.

Problem 4 (5 points): Exercise 22.4-1, from CLRS text.

Problem 5 (5 points): Exercise 3-16, from DPV text.

In addition to the problem statement of 3-16 as in the textbook, we add the following clarification. A set of courses can be taken in one semester if and only if there is no direct and indirect "prerequisite" relationship among any of the courses in this set. "Direct" means a directed edge; "indirect" means a path (following the directions of the edges). Besides, a course can be taken if and only if all of its prerequisite courses were taken in a previous semester.