1) $A = \begin{bmatrix} 1 & 3 & -1 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} = \left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & -4 & 0 \\ 0 & 0 & 0 & 1 & -1 \end{array}\right]$

$$\begin{bmatrix} 1 & 3 & 0 & 0 & -4 \\ 0 & 0 & 1 & 0 & -4 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$x_1 + 3x_2 - 4x_5 = 0$ $\quad x_2, x_5$ free

$x_3 - 4x_5 = 0$

$x_4 - x_5 = 0$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} x_2 + \begin{bmatrix} 4 \\ 0 \\ 4 \\ 1 \\ 1 \end{bmatrix} x_5 \qquad \left(\begin{bmatrix} -3 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 0 \\ 4 \\ 1 \\ 1 \end{bmatrix} \right)$$

b) Dim(Nul(A)) is 2 since there are two vectors

c) You take the reduced-echelon form of A and see which ore are the pivot columns, in this case its 3

d) $\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -4 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -4 & 1 \end{bmatrix}$ Hence Col A = 3

Nul(A) = column of A - col A

$5 - 3$

$= 2$

2) a) for a set of vectors $[v_1, v_2, \ldots, v_n)$ the properties it must have is that it spans V and is linearly independent

B) $\begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 7 \\ -9 \end{bmatrix}$ If you can prove that these set of vectors are independent then they are a basis for $\mathbb{R}^3$, these vectors are linearly independent and they span $\mathbb{R}^3$ so they are a basis

~~C) An invertible matrix cannot be a basis for $\mathbb{R}^n$ because an invertible matrix's vector doesn't equal because Determinant cannot be zero for a invertible matrix~~

C) If we have an invertible matrix then the columns must be independent and any independent vector of n or more vectors will span the space so yes it will form a basis of $\mathbb{R}^n$

3)

$$3x_1 - 7x_2 + 9x_3 - 5x_4 = 9$$
$$3x_1 - 9x_2 + 12x_3 - 9x_4 = 6$$
$$3x_2 - 6x_3 + 6x_4 = 3$$

$$\left[\begin{array}{cccc|c} 3 & -7 & 9 & -5 & 9 \\ 3 & -9 & 12 & -9 & 6 \\ 0 & 3 & 0 & 6 & 3 \end{array}\right] \xrightarrow{R_1/3} \left[\begin{array}{cccc|c} 1 & -7/3 & 9/3 & -5/3 & 9/3 \\ 3 & -9 & 12 & -9 & 6 \\ 0 & 3 & 0 & 6 & 3 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & -7/3 & 9/3 & -5/3 & 9/3 \\ 0 & -2 & 4 & -4 & -2 \\ 0 & 3 & 0 & 6 & 3 \end{array}\right] \xrightarrow{R_2/-2} \left[\begin{array}{cccc|c} 1 & -7/3 & 9/3 & -5/3 & 9/3 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 3 & 0 & 6 & 3 \end{array}\right]$$

[illegible]

$$\left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & 5 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \longrightarrow \left[\begin{array}{cccc|c} 1 & 0 & -2 & 3 & 5 \\ 0 & 1 & -2 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc|c} 1 & 0 & 0 & 3 & 5 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{array}\right]$$

$x_1 = 5 - 3x_4$

$x_2 = 1 - 2x_4$

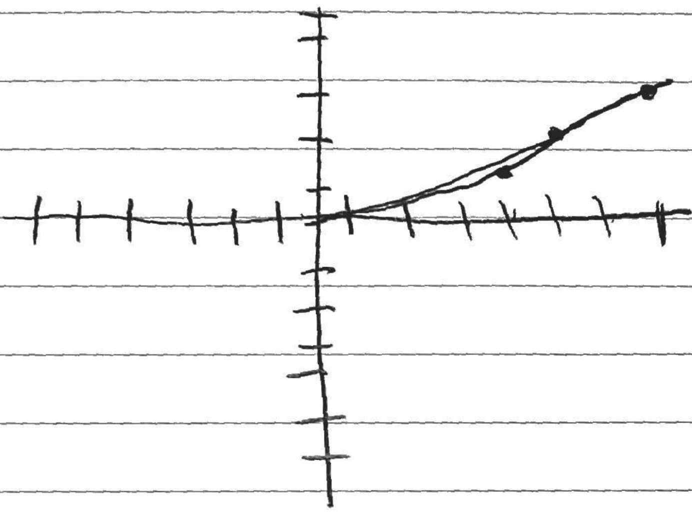
$x_3 = 0$

x_4 – free

$\begin{bmatrix}6\\3\end{bmatrix}$ $\begin{bmatrix}4\\1\end{bmatrix}$ $\begin{bmatrix}5\\2\end{bmatrix}$

$s=3$ $s=1$ $s=2$

4) a)



b) 1) vector 0 must be in S

2) $\bar{s}_1 + \bar{s}_2$ must be in S

3) cS must be in S

c) No it cannot be a subspace since there is no zero vector made possible with the given $\begin{bmatrix}3+s\\s\end{bmatrix}$ vector

5) B) R^2 $\quad B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \end{bmatrix} \right\}$

If $V = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$ what is $[V]_B$

$$\begin{bmatrix} 1 \\ 2 \end{bmatrix} x + \begin{bmatrix} 1 \\ 1 \end{bmatrix} y = \begin{bmatrix} 6 \\ -2 \end{bmatrix}$$

$$X_B = \begin{bmatrix} 3 \\ -1 \end{bmatrix}$$

A) $B = \{1 + x, 3 + x^2, -2x + x^2\}$ $\quad [p(x)]_B = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$

$1 + x = 3$

$3 + x^2 = -1$

$-2x + x^2 = 1$

$p(x) = \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}$

6) $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|ccc} 0 & -1 & 0 & 1 & 0 & 0 \\ -1 & -1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & \cancel{1} & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{array}\right] \xrightarrow{\text{1st column}} X \left[\begin{array}{ccc|ccc} 1 & 1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] \longrightarrow \left[\begin{array}{ccc|ccc} 1 & 0 & -1 & 1 & -1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right] = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$$

B) $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $A = \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix}$

$$Ax = B \Rightarrow A^{-1}B = x \quad \begin{bmatrix} 0 & -1 & 0 \\ -1 & -1 & 1 \\ -1 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$x = \begin{bmatrix} 2 \\ -1 \\ 2 \end{bmatrix}$$

C

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7) $\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 2 \\ 0 & 4 & -3 & 3 \\ 0 & 1 & 0 & 1 \end{bmatrix} \xrightarrow{2nd \cancel{\times} 3rd} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 0 & 5 & 2 \\ 0 & 4 & -3 & 3 \\ 0 & 1 & 0 & 1 \end{bmatrix}$

2nd column gone under

$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & -3 & 3 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 3/4 & 1/4 \end{bmatrix} \xrightarrow{3^{rd} \text{ row}} \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 4 & -3 & 3 \\ 0 & 0 & 5 & 2 \\ 0 & 0 & 0 & -1/20 \end{bmatrix}$

$2 \cdot 4 \cdot 5 \cdot -1/20 = 2$

B) Yes, the Matrix is Invertible