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Math Homework # 6

1) Vector 0 is in S
2) $\bar{x} + \bar{y}$ in S
3) $c\bar{x}$ is in S

1) $\begin{bmatrix} a \\ B \\ -a \\ c \end{bmatrix}$

i) $\theta = a = b = c = 0$, ✓

ii) $\alpha = (a_1, b_1, -a_1, c_1)$, $\beta = (a_2, b_2, -a_2, c_2)$

$\alpha + \beta = (a_1 + a_2, b_1 + b_2, -(a_1 + a_2), c_1 + c_2)$
$= (A, B, -A, C)$ ✓

$A = a_1 + a_2$
$B = b_1 + b_2$
$C = c_1 + c_2$

iii) $\lambda x = (A, B, -A, C)$ ✓ $A = \lambda a_1$, $B = \lambda b_1$, $C = \lambda c$

2) 1) Since T is a linear transformation, $T0_v = 0_w$
Therefore, $0_v \in \ker(T)$

2) Let $v, w \in \ker(T)$ Show that $v + w \in \ker(T)$

$T(v+w) = T(v) + T(w) = 0_w + 0_w = 0_w$
T is linear $v, w \in \ker(T)$

Therefore $v + w \in \ker(T)$

3) Let $v \in \ker(T)$, $\lambda \in \mathbb{R}$ Show that $\lambda v \in \ker(T)$

$T(\lambda v) = \lambda T(v) = \lambda 0_w = 0_w$
T linear $v \in \ker(T)$

Therefore $\lambda v \in \ker(T)$

3) $M_{2\times2}$, vectors 2×2 w/ Real entries

a) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ ✓

B) i) $\begin{bmatrix} a & 0 \\ 0 & B \end{bmatrix}$ $\alpha = \begin{pmatrix} a_1, 0 \\ 0, B \end{pmatrix}$ $\beta = \begin{pmatrix} a_2, 0 \\ 0, B_2 \end{pmatrix}$

Yes

ii) $\alpha + \beta = (a_1 + a_2, 0+0, 0+0, B+B_2)$ ✓ $A = a_1 + a_2$, $B = b_1 + B_2$

$\begin{pmatrix} A, 0 \\ 0, B \end{pmatrix}$

iii) $\lambda x = \begin{pmatrix} A, 0 \\ 0, B \end{pmatrix}$ ✓ $A = \lambda a$, $B = \lambda b$, $0 = \lambda 0$

c) $M^{-1}{}_{2\times2} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ a) $a=b=c=d=0$ X

Invertible Matrix has no zero vector

~~$\alpha = \begin{pmatrix} a, B \\ c, d \end{pmatrix}$ $B = \begin{pmatrix} a_2, b_2 \\ c_2, d_2 \end{pmatrix}$ for subspace~~

~~$\alpha + \beta = (a_1 + a_2, b_1 + b_2, c_1 + c_2, d_1 + d_2)$~~ ~~$A = a_1 + a_2$~~

~~$= \begin{pmatrix} A, B \\ C, D \end{pmatrix}$~~ ~~$B = b_1 + b_2$~~ ~~$C = c_1 + c_2$~~ ~~$D = d_1 + d_2$~~

~~$\lambda x = \begin{pmatrix} A, B \\ C, D \end{pmatrix}$~~ ~~$A = \lambda a$, $B = \lambda b$, $C = \lambda c$, $D = \lambda d$~~

4) Vector space = P_3

$T: P_3 \to P_3$ be function

$T(P(x)) \to p'(x)$

Let

$P(x)$ & $q(x) \in P_3$

$P(x) = a_1 x^3 + a_2 x^2 + a_3 x + a_4$

$q(x) = b_1 x^3 + b_2 x^2 + b_3 x + b_4$

$$T(\alpha p(x) + \beta q(x)) = (\alpha p(x) + \beta q(x))'$$
$$= \alpha p'(x) + \beta q'(x)$$
$$= \alpha T(P(x)) + \beta T(q(x)) \checkmark$$

B) $T(1) = 0 = 0 \cdot 1 + 0 \cdot x + 0 \cdot x^2 + 0 x^3$

$T(x) = 1 = 1 \cdot 1 + 0 \cdot x + 0 \cdot x^2 + 0 \cdot x^3$

$T(x^2) = 2x = 0 \cdot 1 + 2x + 0 \cdot x^2 + 0 \cdot x^3$

$T(x^3) = 3x^2 = 0 \cdot 1 + 0x + 0x^2 + 0 \cdot x^3$

$$T = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \{(0,1,0,0)\ (0,0,2,0)\ (0,0,0,3)\}$$

range of T

C) Kernal of T

$T(P(x)) = 0$

$p'(x) = 0$

$P(x) = a_1 x^3 + a_2 x + a_3 x + a_4$