Patrick Thomasma
10/23/20

Math 341 Midterm

1) $A = \begin{bmatrix} 1 & -1 & 1 \\ 0 & 2 & 1 \end{bmatrix}$ $u = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$ $v = \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}$

a) Au

$$5\begin{bmatrix} 1 \\ 0 \end{bmatrix} + -2\begin{bmatrix} -1 \\ 2 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5(1) + -2(-1) + 3(1) \\ 5(0) + -2(2) + 3(1) \end{bmatrix} = \begin{bmatrix} 10 \\ -1 \end{bmatrix}$$

B) $2u + v$

$$2\begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} = \begin{matrix} 10 \\ -4 \\ 6 \end{matrix} + \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ 6 \end{bmatrix}$$

2) A)

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 5 \\ 0 & -1 & -1 & -3 \\ 0 & 1 & 7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim$$

B) No.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c) No

D) x_1 is leading, x_2 is leading, X_3 is [redacted]

System is inconsistent

E) None

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2) A)

$$\left[\begin{array}{ccc|c} 1 & 3 & 4 & 5 \\ 0 & -1 & -1 & -3 \\ 0 & 0 & 7 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \sim$$

B) No

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c) No

D) x_1 is leading, x_2 is leading, x_3 is ~~leading~~ leading

System is inconsistent

E) None

3) $x + y + z = 30$

$-2x - y - 3z = -55$

$x + 2z = 25$

a) $\begin{bmatrix} 1 & 1 & 1 & 30 \\ -2 & -1 & -3 & -55 \\ 1 & 0 & 2 & 25 \end{bmatrix} \underset{\sim}{R_1 \leftrightarrow R_2} \begin{bmatrix} -2 & -1 & -3 & -55 \\ 1 & 1 & 1 & 30 \\ 1 & 0 & 2 & 25 \end{bmatrix}$

$\underset{\sim}{R_2 + \frac{1}{2}R_1} \begin{bmatrix} -2 & 1 & -3 & -55 \\ 0 & \frac{1}{2} & -\frac{1}{2} & \frac{5}{2} \\ 1 & 0 & 2 & 25 \end{bmatrix} \underset{\sim}{R_3 + \frac{1}{2}R_1} \begin{bmatrix} -2 & -1 & -3 & -55 \\ 0 & \frac{1}{2} & -\frac{1}{2} & \frac{5}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} & -\frac{5}{2} \end{bmatrix}$

$\underset{\sim}{R_3 + R_2} \begin{bmatrix} -2 & -1 & -3 & -55 \\ 0 & \frac{1}{2} & -\frac{1}{2} & \frac{5}{2} \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $2R_2$ $\begin{bmatrix} -2 & -1 & -3 & -55 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\underset{\sim}{R_1 + R_3} \begin{bmatrix} -2 & 0 & 2 & 25 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 1 & 1 & 30 \\ -2 & -1 & -3 & 55 \\ 1 & 0 & 2 & 25 \end{bmatrix}$ $R_2 + 2R_1$ $\begin{bmatrix} 1 & 1 & 1 & 30 \\ 0 & 1 & -1 & 5 \\ 1 & 0 & 2 & 25 \end{bmatrix}$

$\underset{\sim}{R_3 + -1(R_1)} \begin{bmatrix} 1 & 1 & 1 & 30 \\ 0 & 1 & -1 & 5 \\ 0 & -1 & 1 & -5 \end{bmatrix} \underset{\sim}{R_2 + R_3} \begin{bmatrix} 1 & 1 & 1 & 30 \\ 0 & 1 & 1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$\underset{\sim}{R_1 + -1(R_2)} \begin{bmatrix} 1 & 0 & 2 & 25 \\ 0 & 1 & -1 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ ✓

B) There'd be Infinitely Many Solutions since x_1 is leading, x_2 is leading, but x_3 is a free Variable Which implies that there are infinite Solutions

4) $A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}$ $B = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$, $S = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

a) $\begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & 0 \end{bmatrix} \xrightarrow{R_2+R_1} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$ $S = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

$\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 3 [\ ]$

B) When Reducing the matrix in Part (a) there wasn't any free Variables only leading Variables which means that the matrix only has 1 unique Solution $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

5) $$\left[\begin{array}{ccc|c} 20 & 1 & -10 & 0 \\ 0 & -1 & 5 & 0 \\ 0 & 0 & 3 & 0 \end{array}\right] \sim \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Yes the vectors are linearly independent because each column has a leading variable and there isn't any free variables which would make the matrix dependent.

b) ~~Yes since the vectors are linearly independent of eachother you can make them linear combinations of eachother since these vectors would be lying in the same plane of If these vectors are $\mathbb{R}(n)$ and one is a linear combo of another then it's $\mathbb{R}(n-1)$ which still works~~

C

b) No, since the definition of a set of vectors that are linearly dependent means that one of the vectors in the set can be a linear combo of the others, If these vectors were linear combos of the other than the set wouldn't be linearly independent.