Math 341 Homework 6 Due 11/24/2020 before the start of class
Fall 2020, Dr. Honigs

Instructions: Write up all answers neatly and clearly labeled. Turn your homework in on canvas by uploading it as a **single** .pdf file.
Unless stated otherwise, show all your work and explain your reasoning. You are encouraged to work in groups, but your final written solutions must be in your own words.
This homework will be graded out of 50 points.

1. (10 points) Consider the set W of vectors in $\mathbb{R}^4$ that are of the following form where a, b, c can be any real numbers:

$$\begin{bmatrix} a \\ b \\ -a \\ c \end{bmatrix}$$

   Is W a subspace of $\mathbb{R}^4$? If yes, explain why it satisfies each of the three subspace criteria. If not, give an example showing that it doesn't satisfy at least one of the criteria.

2. (10 points) Show that the kernel of any linear transformation $T : V \to W$ is a subspace of the domain V.

3. (15 points) Consider the vector space $M_{2\times 2}$. Its vectors are 2×2 matrices with real entries. The addition operation is addition of matrices, and scalar multiplication is scalar multiplication of matrices. (Note that multiplying matrices is not part of the structure of this vector space)
   (a) What is the zero vector of this vector space?
   (b) Is the set of all 2×2 diagonal matrices a subspace of $M_{2\times 2}$? Diagonal matrices are matrices that can have any entries on the main diagonal, but all the other entries must be 0.
   (c) Is the set of all 2×2 invertible matrices a subspace of $M_{2\times 2}$?

4. (15 points) Consider the vector space $\mathbb{P}_3$ of all polynomials of degree at most 3 (in just one variable, x, as we've seen in class). Let $T : \mathbb{P}_3 \to \mathbb{P}_3$ be the function where $T(p(x)) = p'(x)$. (That is, take the first derivative).
   (a) Verify that T is a linear transformation.
   (b) What is the range of T? Explain your reasoning. (Hint: This isn't a matrix calculation. You will need to use your knowledge of derivatives.)
   (c) What is the kernel of T? Explain your reasoning.