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Math 341 HW

$$A = \begin{bmatrix} -2 & 4 & -2 & -4 \\ -2 & -6 & -3 & 1 \\ -3 & 8 & 2 & -3 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$Nul(A) = \{\vec{x} \mid A\vec{x} = \vec{0}\}$
$\{\vec{x} \mid B\vec{x} = \vec{0}\}$

$B\vec{x} = 0$

$$\begin{bmatrix} 1 & 0 & 6 & 5 \\ 0 & 2 & 5 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_3 = t_1$ $\quad t_1, t_2 \in \mathbb{R}$

$x_4 = t_2$

$x_2 = -\frac{5}{2}t_1 - \frac{3}{2}t_2$

$x_1 = -6t_1 - 5t_2$

$Nul(A) = \{(6t_1 - 5t_2, -\frac{5}{2}t_1 - \frac{3}{2}t_2, t_1, t_2) \mid t_1, t_2 \in \mathbb{R}\}$

$Nul(A) = \{t_1(6, -\frac{5}{2}, 1, 0), (-5, -\frac{3}{2}, 0, 1)\}$

Basis $= \{(-6, -\frac{5}{2}, 1, 0), (-5, -\frac{3}{2}, 0, 1)\}$, $Nul(A)$

Basis of $Col(A) = \{(-2, 2, -3), (4, -6, 8)\}$

Math 341 HW

2) a) $T: P^2 \to R^2$

$$p(x) \mapsto \begin{bmatrix} p(1) \\ p(-1) \end{bmatrix}$$

$p(x)$: $p(1) = 0 \wedge p(-1) = 0$

$(x-1)(x+1) = 1x^2 - 1$

b) $R(e^1) = 0x^2 + 0x + 1$

$R(e^2) = 0x^2 + 1x + 0$

$R(e^3) = 1x^2 + 0x + 0$

$T_0 R(e^1) = (1, 1)$

$T_0 R(e^2) = (1, -1)$

$T_0 R(e^3) = (1, 1)$

b)

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \end{bmatrix} \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$a + b + c$

$a - b + c$

$2a + 2c = 0$

$0a \quad 2b + 0c = 0$

$a = -c$

$b = 0$

$ax^2 - a$

$ax^2 + bx + c$

c) $\ker = \{a(x^2 - 1)\}$

$\dim = 1$

3) Let $C = \{1+2x^2,\ 4+x+5x^2,\ x\}$ for $\mathbb{P}_2$ what are C coordinates of $3+2x$

$$3+2x = a(1+2x^2) + B(4+x+5x^2) + C(x)$$

$$\begin{bmatrix} 0 \\ 2 \\ 3 \end{bmatrix} = A\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} + B\begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix} + C\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 \\ 0 & 1 & 1 \\ 2 & 5 & 0 \end{bmatrix}\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 4 & 0 & 3 \\ 0 & 1 & 1 & 2 \\ 2 & 5 & 0 & 0 \end{bmatrix} = \begin{matrix} a=-5 \\ b=2 \\ c=0 \end{matrix} \quad \begin{bmatrix} -5 \\ 2 \\ 0 \end{bmatrix}^{1}_{2x}$$

Coordinates is $-5+2x$

$$3+2x = -5(1+2x^2) + 2(4+x+5x^2) + 0(x)$$

$[-5, 2, 0]$

b) T of $(1,0,0,0) = T(1) = (-1,1)$

T of $(0,1,0,0) = T(x) \quad (1,-1)$

T of $(0,0,1,0) = T(x^2) \quad (1,1)$

T of $(0,0,0,1) = T(x^3) \quad (1,-1)$

$$T \text{ of } \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

c) Rank A + nullity A $= \dim \mathbb{P}^3$, Col(A) dim $= 2$

Nullity $= 4-2$

d) Nul(A) $= 2$

e) $T \circ R \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = 0$

$T \circ R \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} a+b+c+d \\ a-b+c-d \end{pmatrix}$

$a + c = 0 \rightarrow a = -c$

$b + d = 0 \rightarrow b = -d$

$NUL = \{ (a, b, -a, -b) \}$

$= \{ a(1,0,-1,0) + b(0,1,0,-1) \}$

$\Rightarrow \{ (1,0,-1,0), (0,1,0,-1) \}$ basis

dim $= 2$

f) $R(1,0,-1,0) = 1 - x^2$

$R(0,1,0,-1) = x - x^3$

Null $\in (1 - x^2)$

$p(x) = (1-x)(1+x) \Rightarrow p(0) \neq 0$

a) $f_1 = e^{2t}\sin t,\ f_2 = e^{2t}\cos t$

If $W(f_1, f_2) \neq$ then f_1 and f_2 are L.I

Now $W(f_1, f_2) = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} = \begin{vmatrix} e^{2t}\sin t & e^{2t}\cos t \\ e^{2t}\cos t + 2e^{2t}\sin t & -e^{2t}\sin t + 2e^{2t}\cos t \end{vmatrix}$

$= e^{4t}\left(-\sin^2 t + 2\sin t\cos t - \cos^2 t - 2\sin t\cos t\right)$

$= -e^{4t}\left(\sin^2 t + \cos^2 t\right) = -e^{4t} \neq 0 \ \forall\, t \in \mathbb{R}$

$\Rightarrow e^{2t}\sin t$ and $e^{2t}\cos t$ are linearly independent

B) Let $f(t) = c_1 e^{2t} + c_2 e^{2t}\sin t$

$f(0) = 3 \Rightarrow c_1 e^{2t}\cos(0) + c_2 e^{2(0)}\sin(0) = 3$

$\Rightarrow c_1 + 0 + 3 \Rightarrow c_1 = 3$

and, $f\left(\frac{\pi}{2}\right) = 5 \Rightarrow 0 + c_2 e^{\pi} = 5 \Rightarrow c_2 = 5e^{-\pi}$

$f(t) = 3e^{2t}\cos t + 5e^{-\pi}e^{2t}\sin t$

$= 3e^{2t}\cos t + 5e^{2t-\pi}\sin t$

$f''(t) - 4f'(t) + 5f(t) = 0$

$m = \dfrac{4 \pm \sqrt{16 - 4(1)(5)}}{2(1)} = \dfrac{4 \pm \sqrt{16-20}}{2} = \dfrac{4 \pm 2i}{2}$

Sol of (1) is given by $f(t) = e^{2t}\left(c_1\cos t + c_2\sin t\right)$

4) a) $f_1 = e^{2t}\sin t,\ f_2 = e^{2t}\cos t$

If $W(f_1, f_2) \neq$ then f_1 and f_2 are L.I

Now $W(f_1, f_2) = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix} = \begin{vmatrix} e^{2t}\sin t & e^{2t}\cos t \\ e^{2t}\cos t + 2e^{2t}\sin t & -e^{2t}\sin t + 2e^{2t}\cos t \end{vmatrix}$

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Sol of ① is given by $f(t) = e^{2t}(c_1\cos t + c_2\sin t)$