Math 341 Homework 3 — Due 10/20/2020 before the start of class

Fall 2020, Dr. Honigs

Instructions: Write up all answers neatly and clearly labeled. Turn your homework in on canvas by uploading it as a **single** .pdf file.
Unless stated otherwise, show all your work and explain your reasoning. You are encouraged to work in groups, but your final written solutions must be in your own words.
This homework will be graded out of 50 points.

There are a lot of true/false and justify-type problems on this one! My recommendation for solving these problems is to review the definitions of the concepts involved and test lots of examples with different qualities. For instance, what happens if the matrix you're looking at has more rows than columns? Or more columns than rows?

1. (20 points) Consider the following matrix:

$$A = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 1 & -1 \\ 1 & 3 & 1 \end{bmatrix}$$

(a) (6 points) Describe the set of all solutions to the homogeneous equation $A\mathbf{x} = \mathbf{0}$.

(b) (6 points) Describe the set of all solutions to $A\mathbf{x} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

(c) (4 points) In class we talked about the fact that the set of solutions to any nonhomogeneous matrix equation can be written as the set of solutions to the associated homogeneous matrix equation plus any one solution to the nonhomogeneous matrix. Use the following information

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} \text{ and } \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \text{ are solutions to } A\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$$

and your answer to part (a) to write two different descriptions of all possible solutions to the matrix equation $A\mathbf{x} = \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}$.

(d) (4 points) Now let's talk about why your two descriptions in part (c) describe the same set of vectors. First, why is the vector

$$\begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

included in both descriptions?

Why is the vector

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

included in both descriptions?

2. (15 points) Decide if each of the following statements is always true, sometimes true, or never true. Justify your answers. If the answer is "sometimes", find an example where the statement is true and an example where it's false.

   (a) (5 points) If a matrix A contains a row of 0's, then the vectors given by its columns are linearly independent.

   (b) (5 points) Suppose we're given a matrix A. If the augmented matrix $[A \mid \mathbf{0}]$ has a row of all 0's then the linear system corresponding to $A\mathbf{x} = \mathbf{0}$ has a free variable.

   (c) (5 points) Suppose we're given a matrix A. If A has a row of all 0's then the linear system corresponding to $A\mathbf{x} = \mathbf{0}$ has a free variable.

3. (15 points) Decide if each of the following statements is always true, sometimes true, or never true. Justify your answers. If the answer is "sometimes", find an example where the statement is true and an example where it's false.

   (a) (5 points) If the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly dependent, then the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is linearly dependent.

   (b) (5 points) If the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is linearly independent, then the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is linearly independent.

   (c) (5 points) If the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}, \mathbf{v_4}\}$ is linearly independent, then the set $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_4}\}$ is linearly independent.