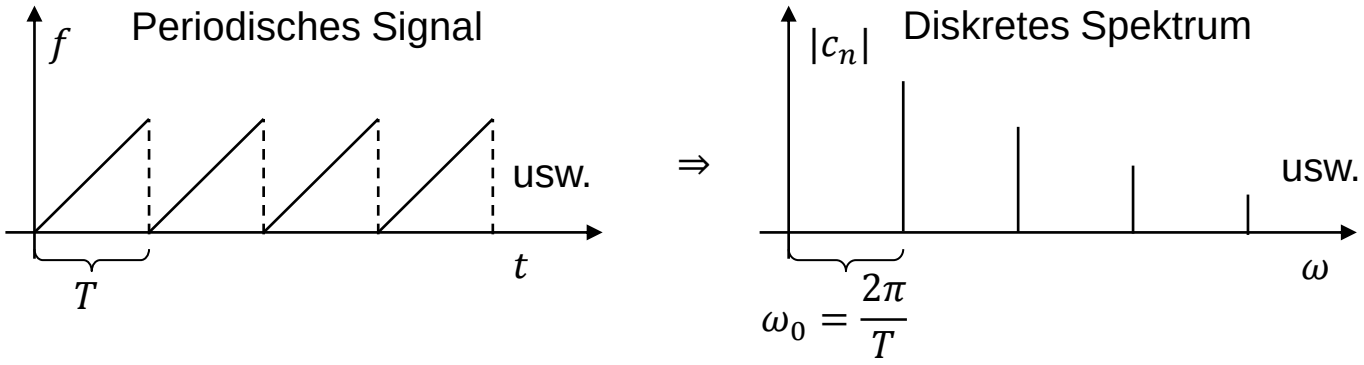
Mathematik 3

Diskrete Fouriertransformation

Wintersemester 2013/14

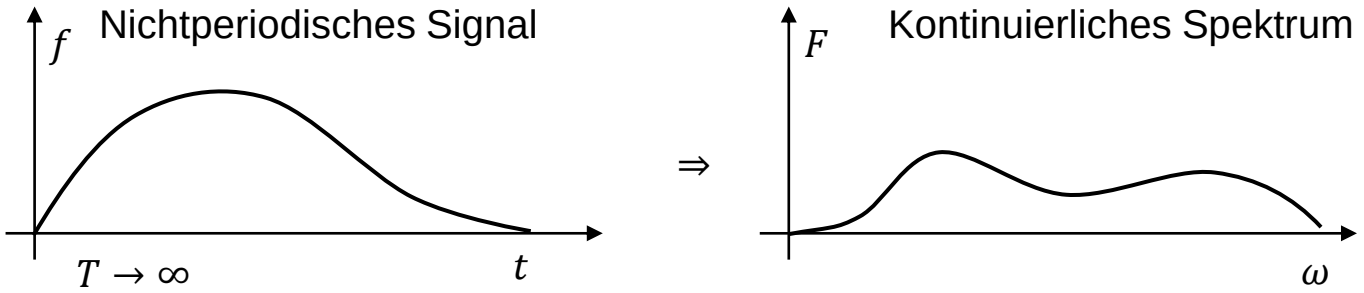
Erinnerung: Fourierreihe und Fouriertransformation

Fourier-Reihe



Fourier-Transformation

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \cdot e^{j\,\omega\,t}\, d\omega \qquad F(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\,\omega\,t}\, dt$$



Diskrete Fouriertransformation

➔ **Diskrete Fouriertransformation und –rücktransformation**

DFT
$$F(n) = \sum_{k=0}^{N-1} f(k) \cdot e^{-j\frac{2\pi}{N}kn}$$

IDFT
$$f(k) = \frac{1}{N} \sum_{n=0}^{N-1} F(n) \cdot e^{j\frac{2\pi}{N}kn}$$

$$\omega_0 = \frac{2\pi}{N \cdot T_A}$$

➔ **Andere Schreibweise**

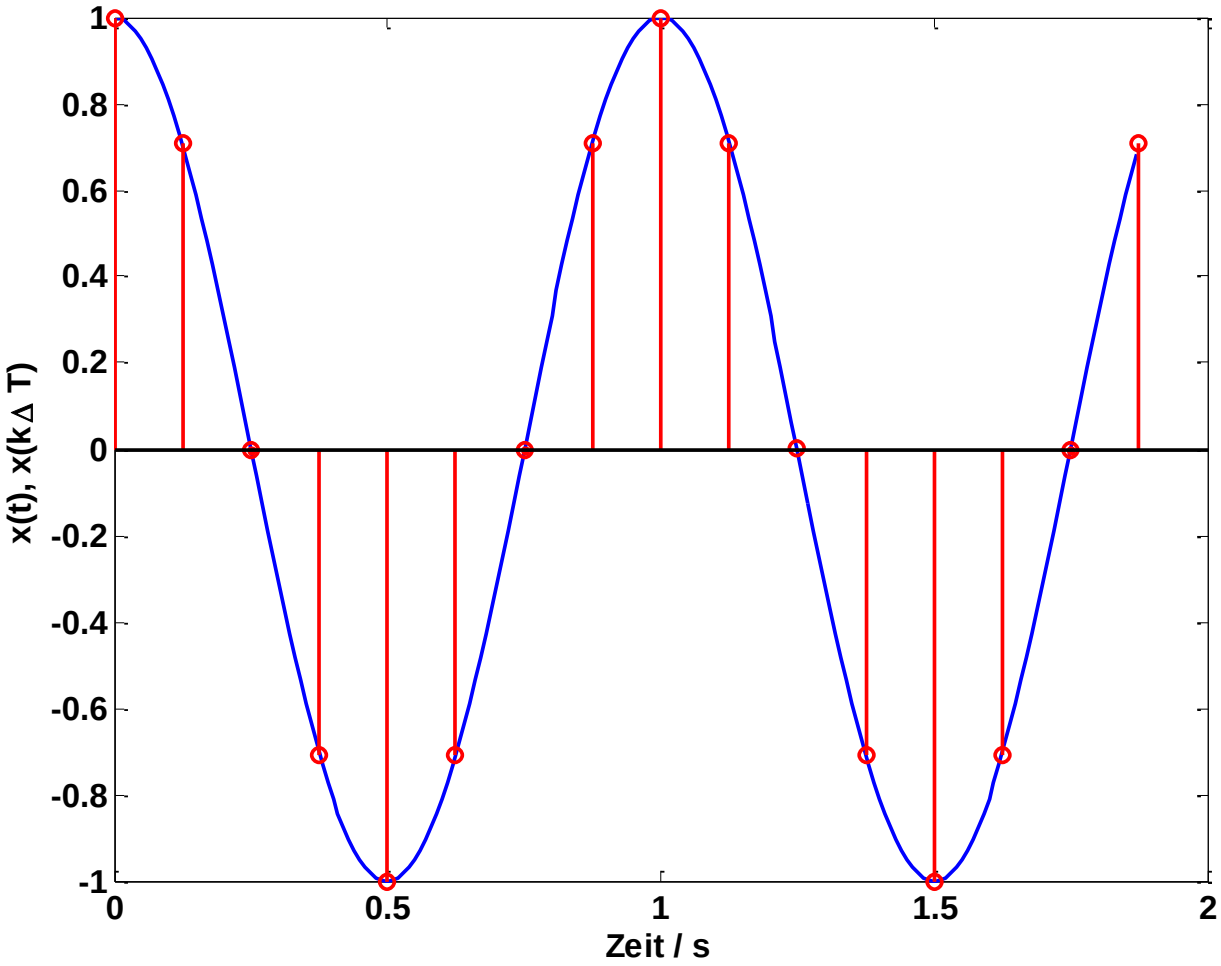
DFT
$$F(n) = \sum_{k=0}^{N-1} f(k) \cdot W_N^{kn}$$

IDFT
$$f(k) = \frac{1}{N} \sum_{n=0}^{N-1} F(n) \cdot W_N^{-kn}$$

$$W_N = e^{-j\frac{2\pi}{N}} = \sqrt[N]{1}$$

Cosinus, Transformation von zwei Perioden

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \quad N = 16 \quad T = NT_A = 2 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = \pi$$



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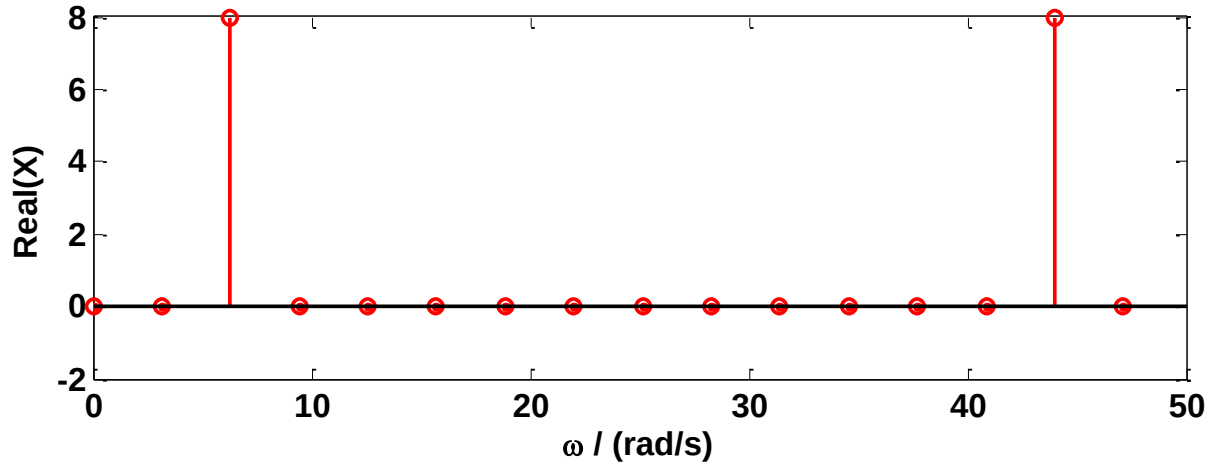
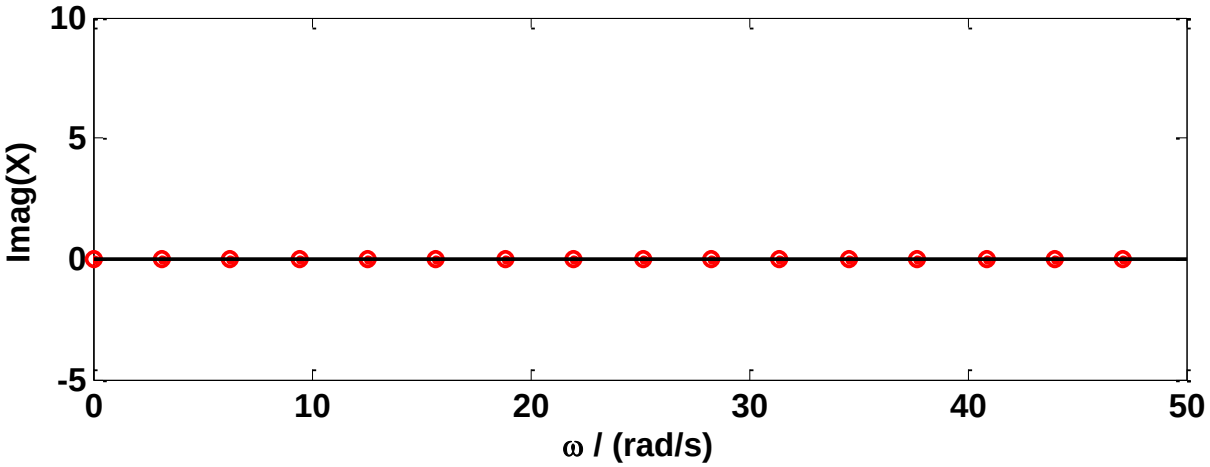
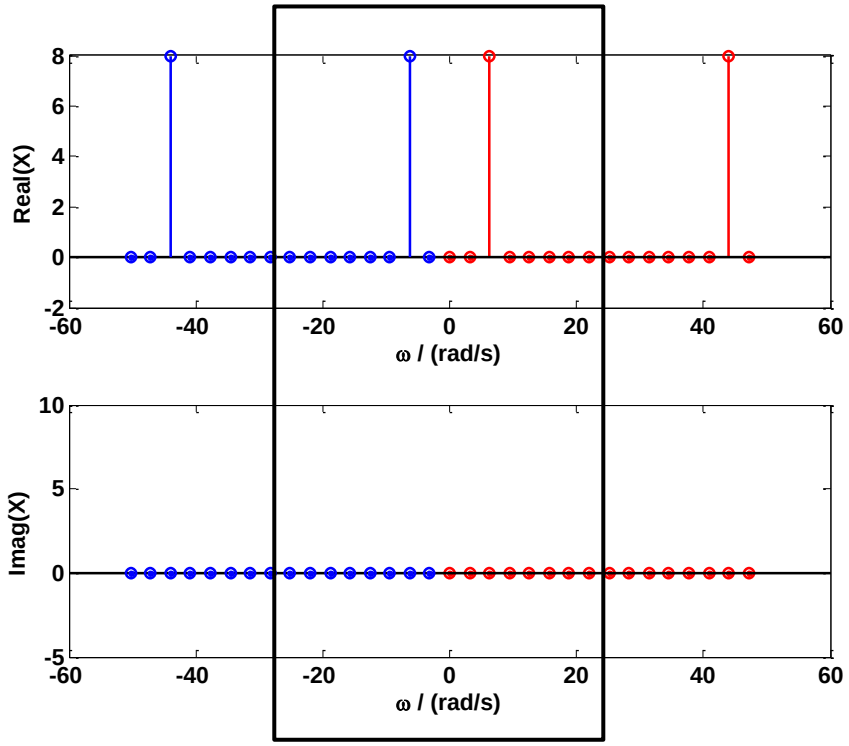


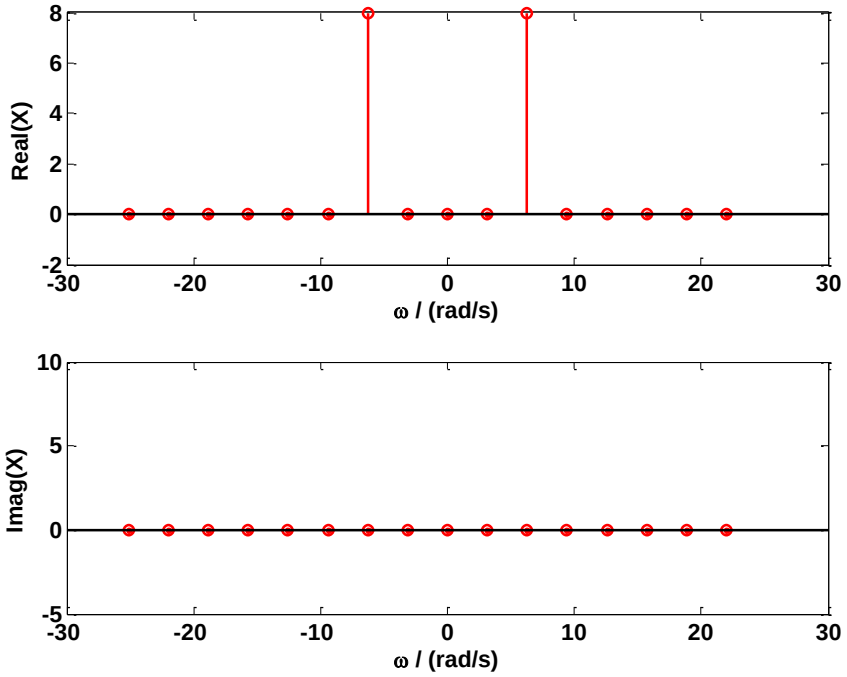


Cosinus, Transformation von zwei Perioden

Periodische Fortsetzung des Spektrums

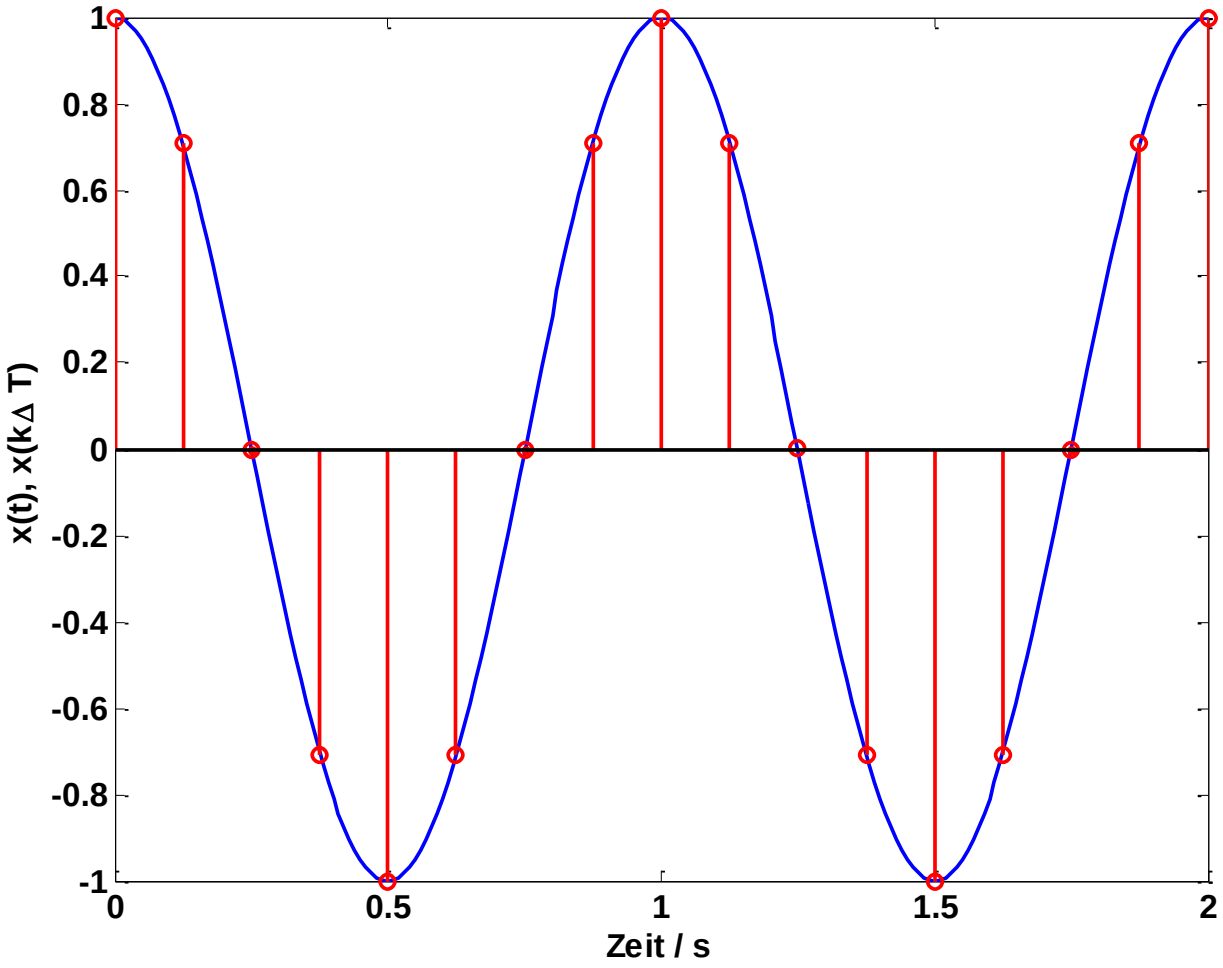


Verschieben der Frequenzen



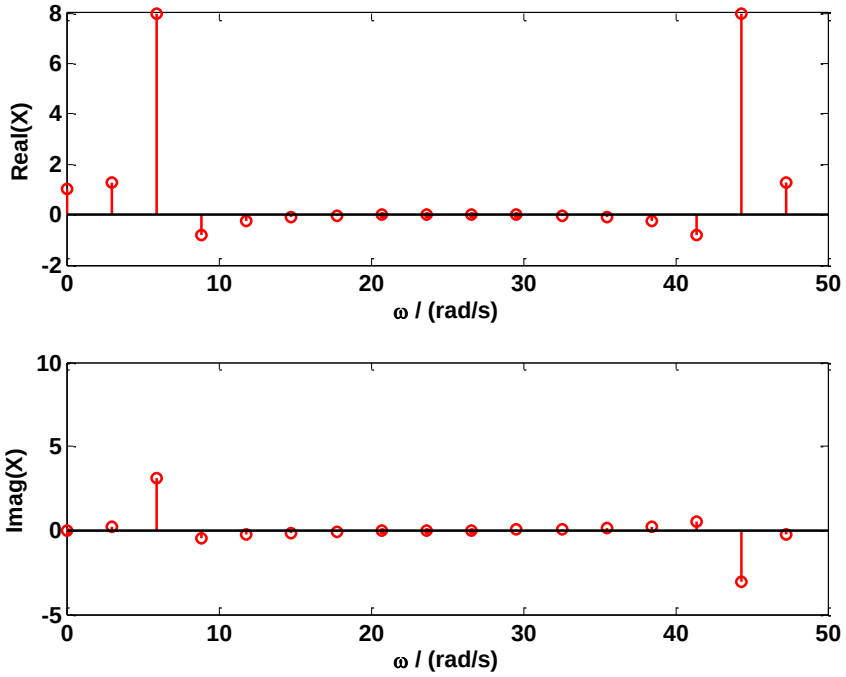
Cosinus, ein Abtastwert mehr

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \quad N = 17$$

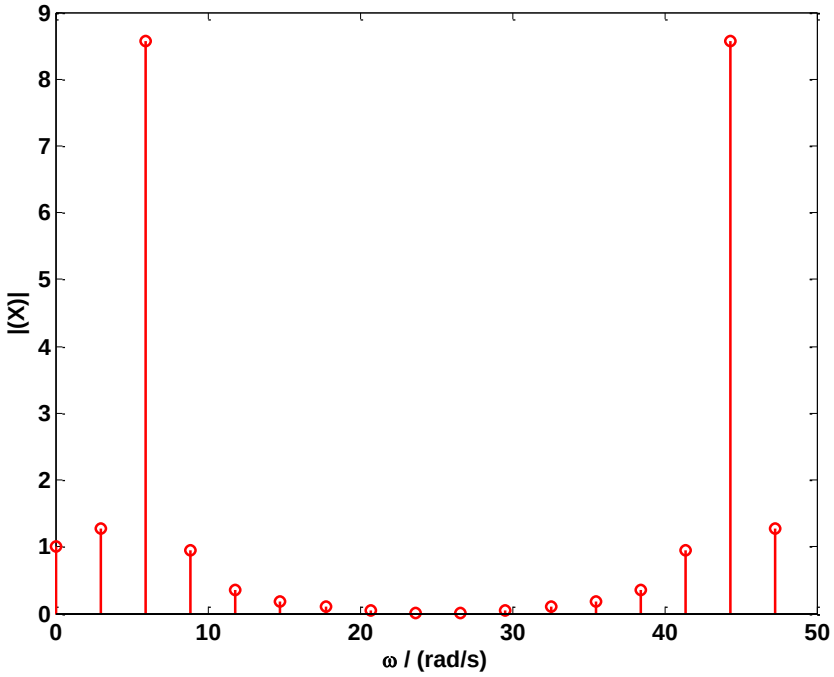


Cosinus, ein Abtastwert mehr

Diskrete Fouriertransformation

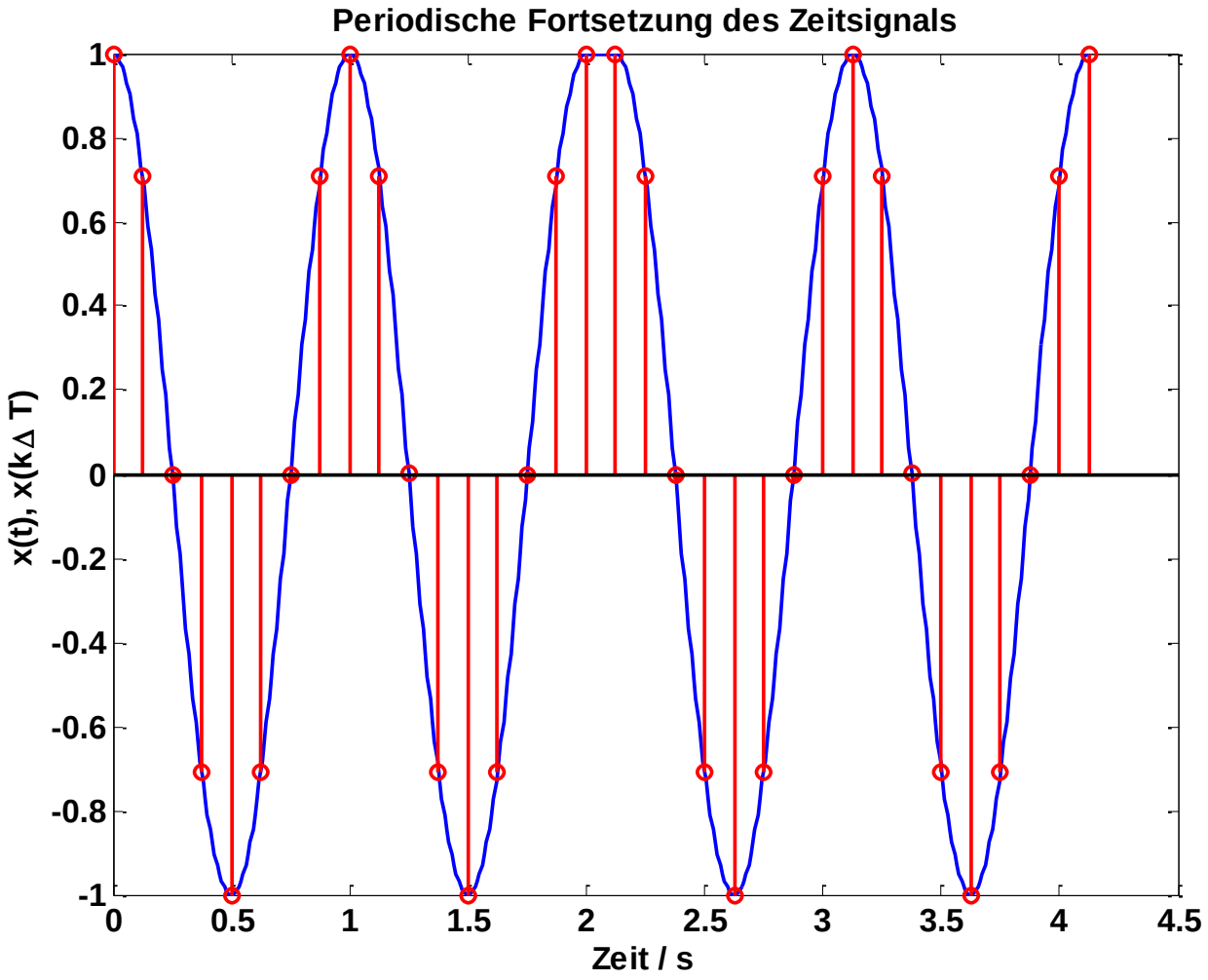


Betragsspektrum



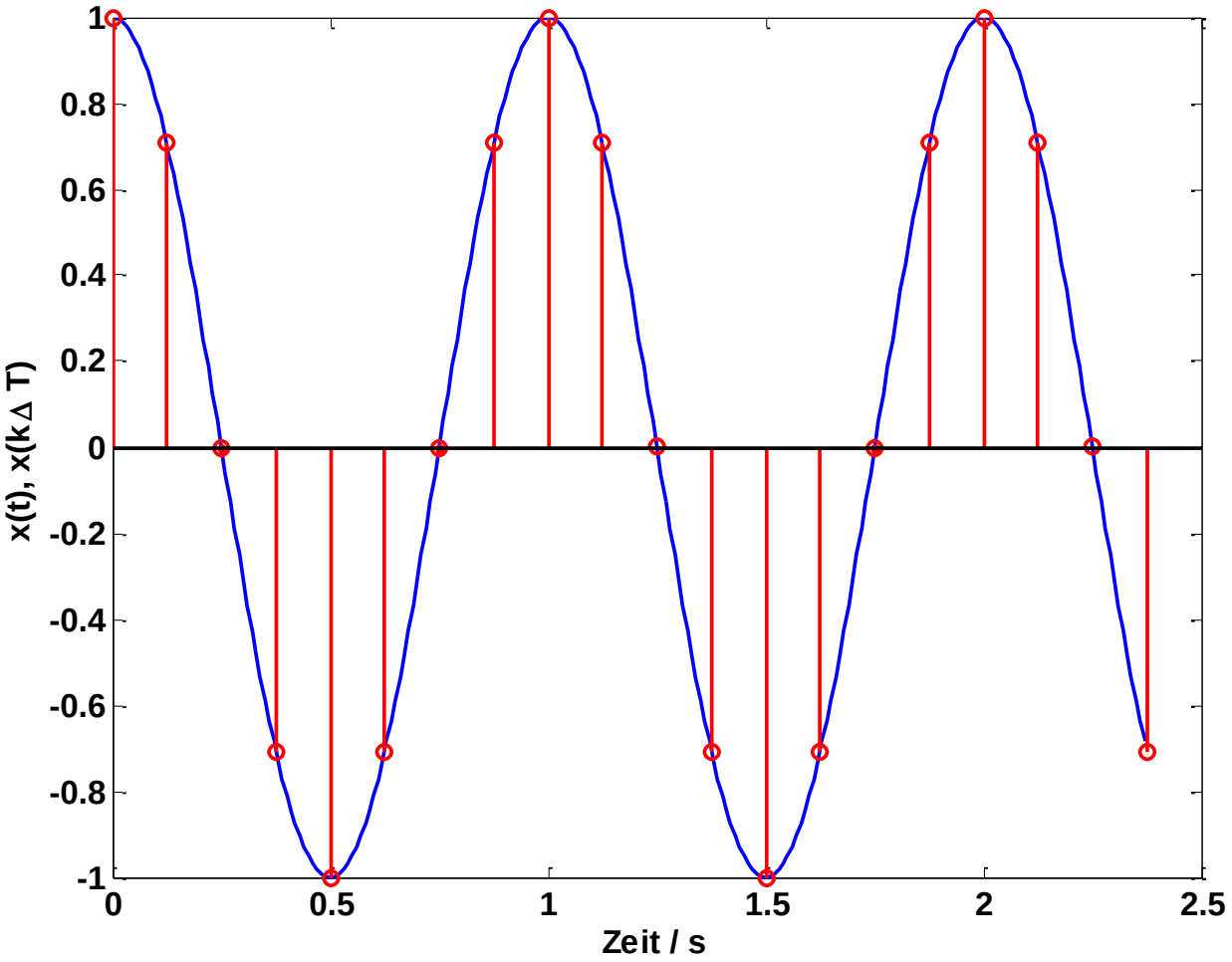
Cosinus, ein Abtastwert mehr

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \qquad N = 17 \;\; T = NT_A = 2{,}125 \Rightarrow \;\; \omega_0 = \frac{2\pi}{T} \approx \frac{16}{17}\pi$$



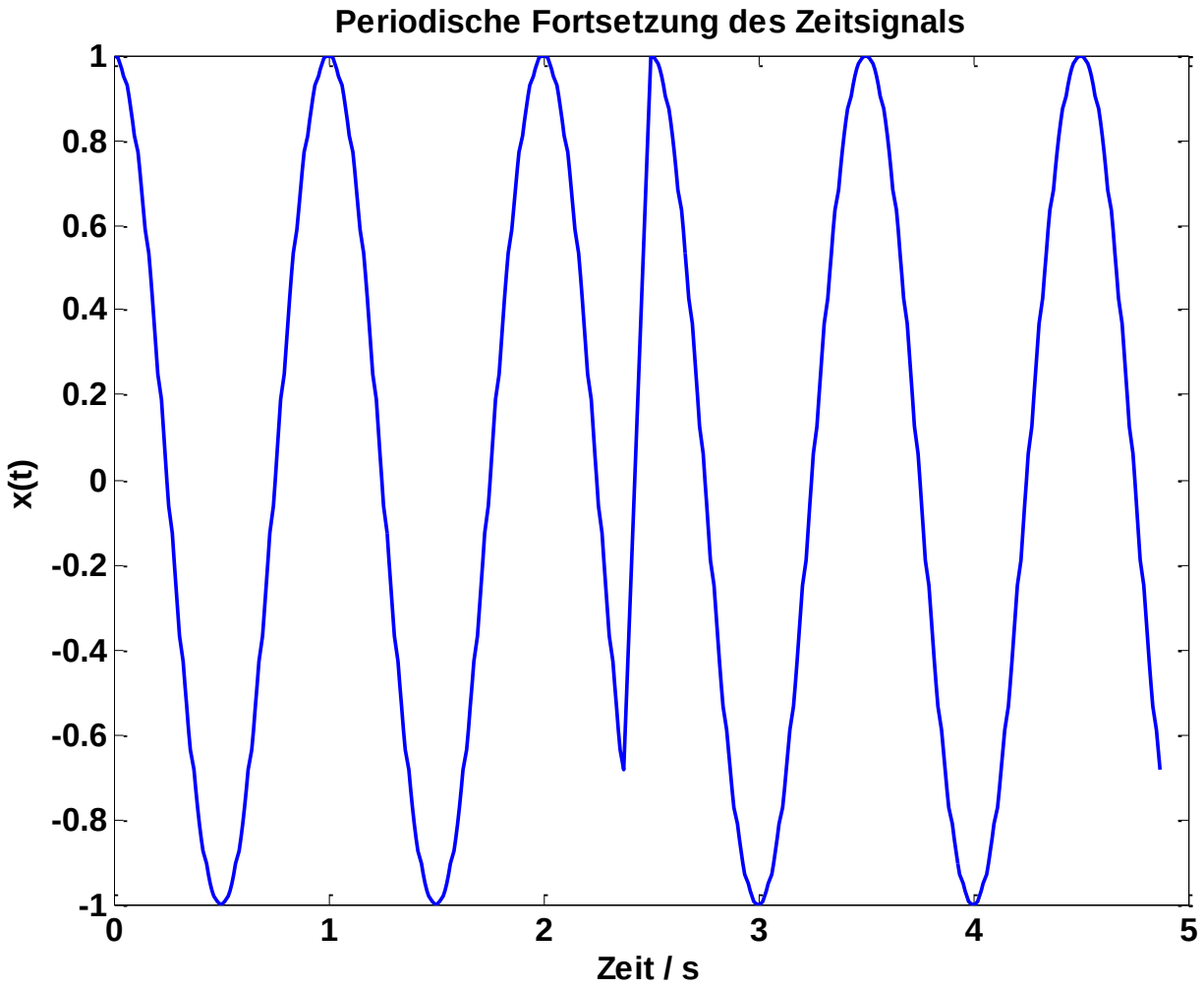
Cosinus, nicht periodisch fortsetzbar

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \qquad N = 20 \quad T = NT_A = 2{,}5 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 0{,}8\pi$$



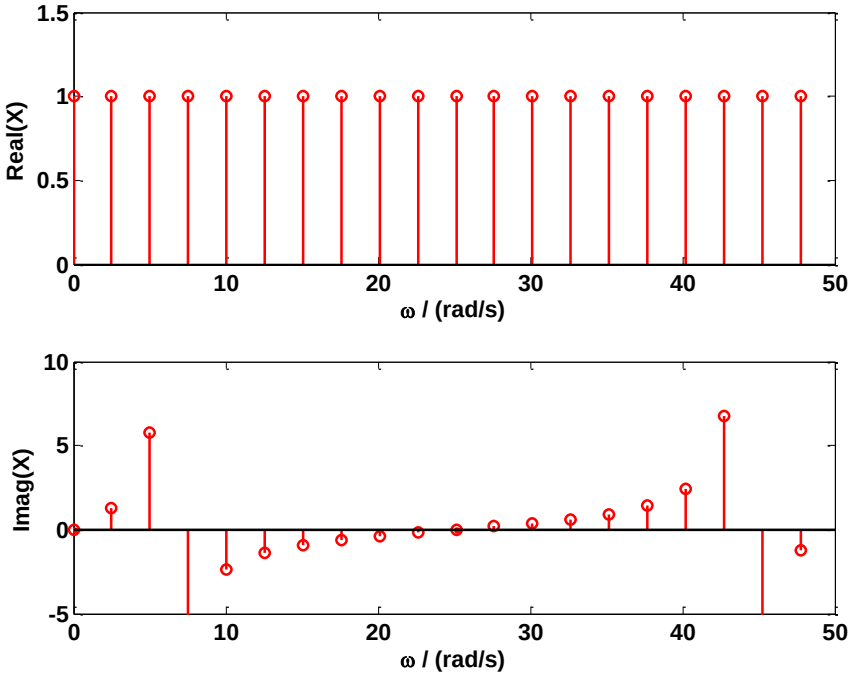
Cosinus, nicht periodisch fortsetzbar

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \qquad N = 20 \quad T = NT_A = 2{,}5 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 0{,}8\pi$$

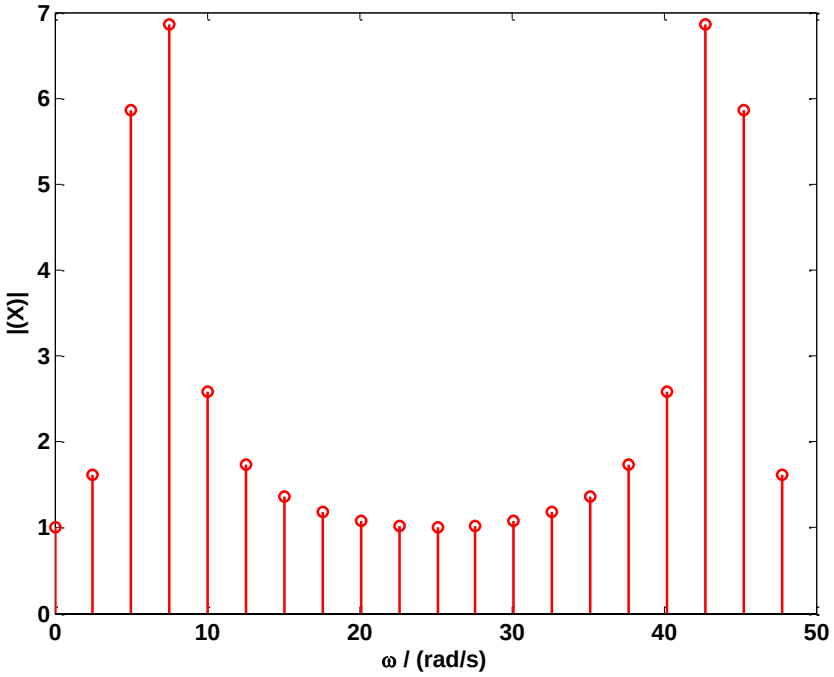


Cosinus, nicht periodisch fortsetzbar

Diskrete Fouriertransformation

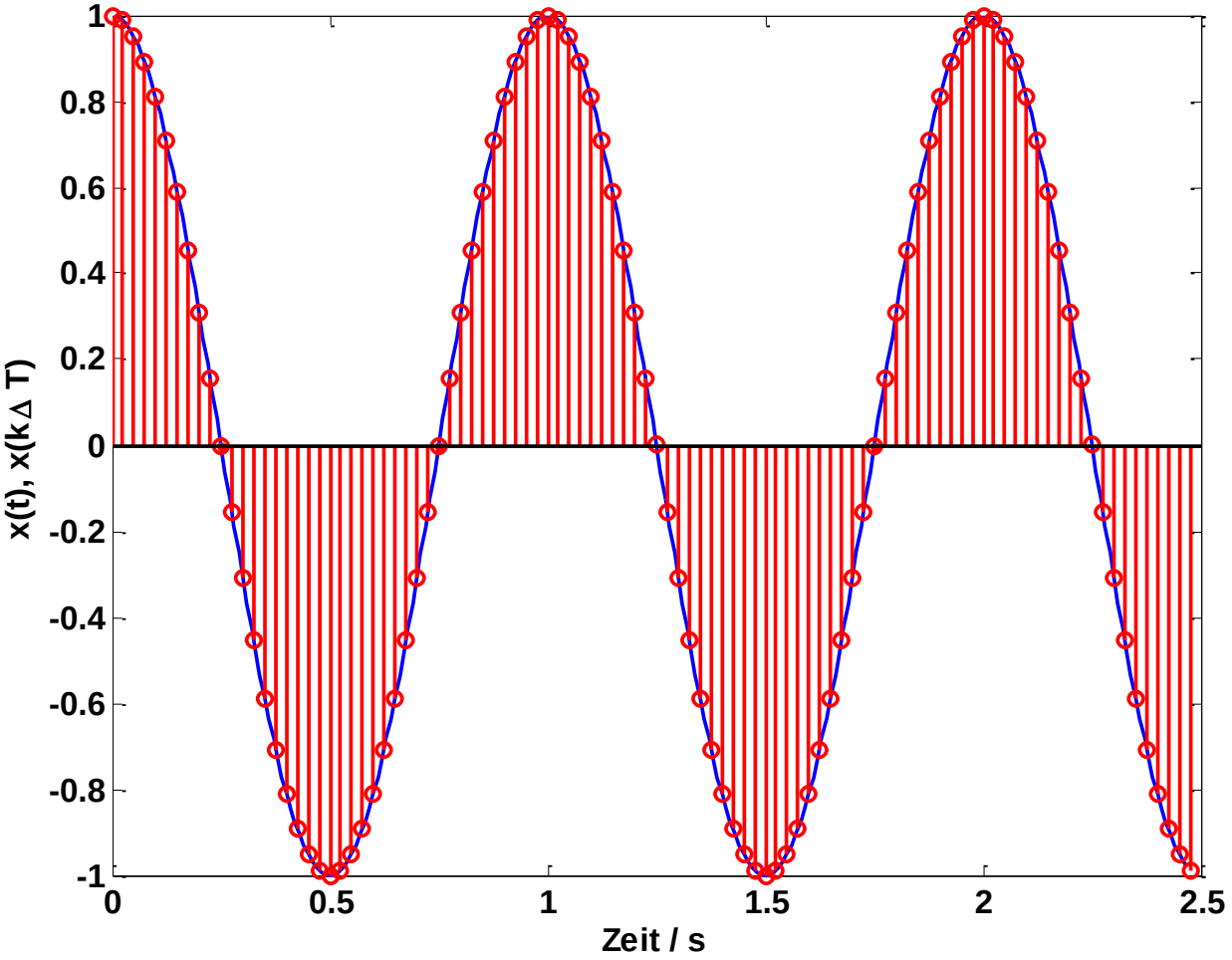


Betragsspektrum



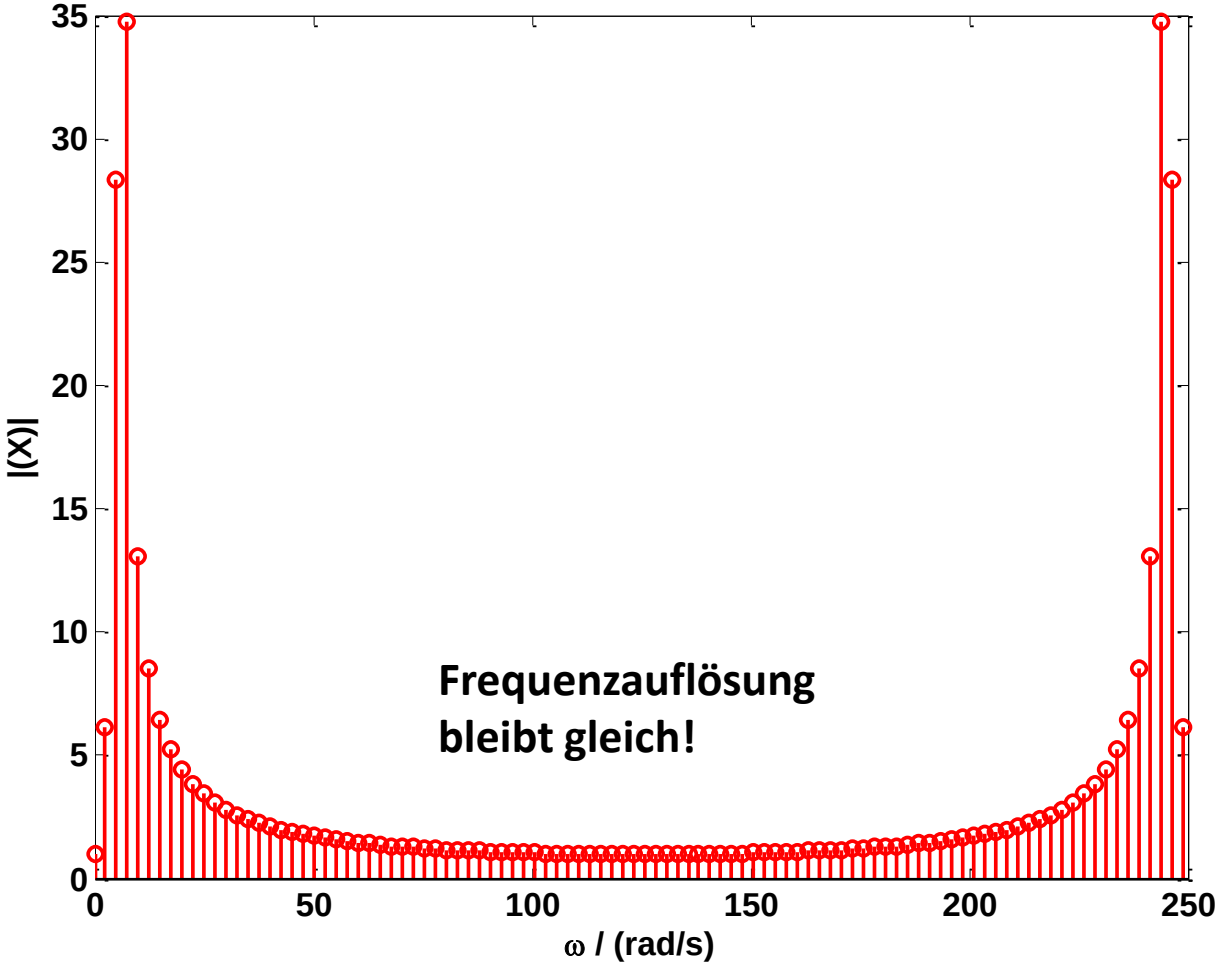
Cosinus, nicht periodisch fortsetzbar

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{40} \quad N = 100 \quad T = NT_A = 2{,}5 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 0{,}8\pi$$



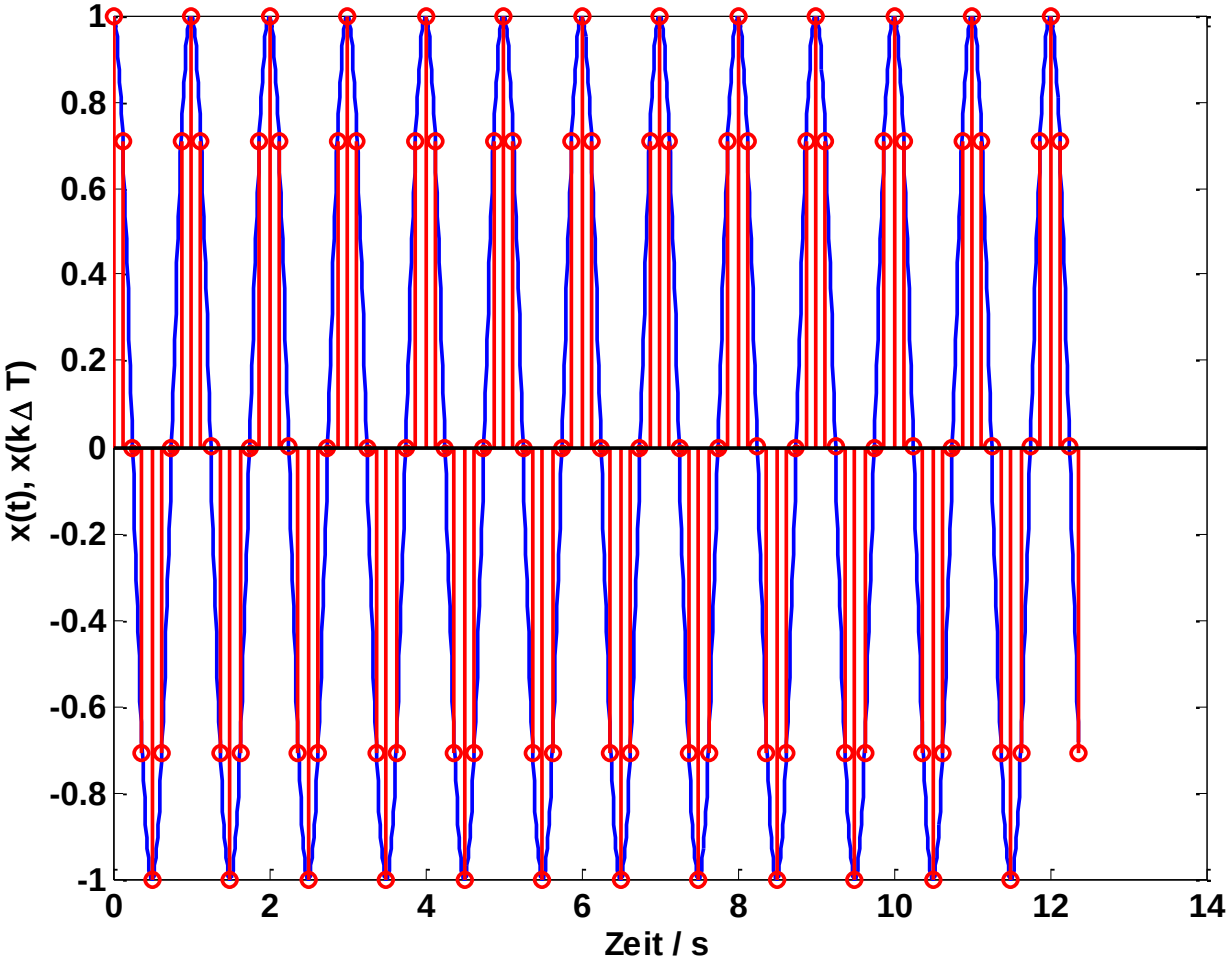
Cosinus, nicht periodisch fortsetzbar

Diskrete Fouriertransformation



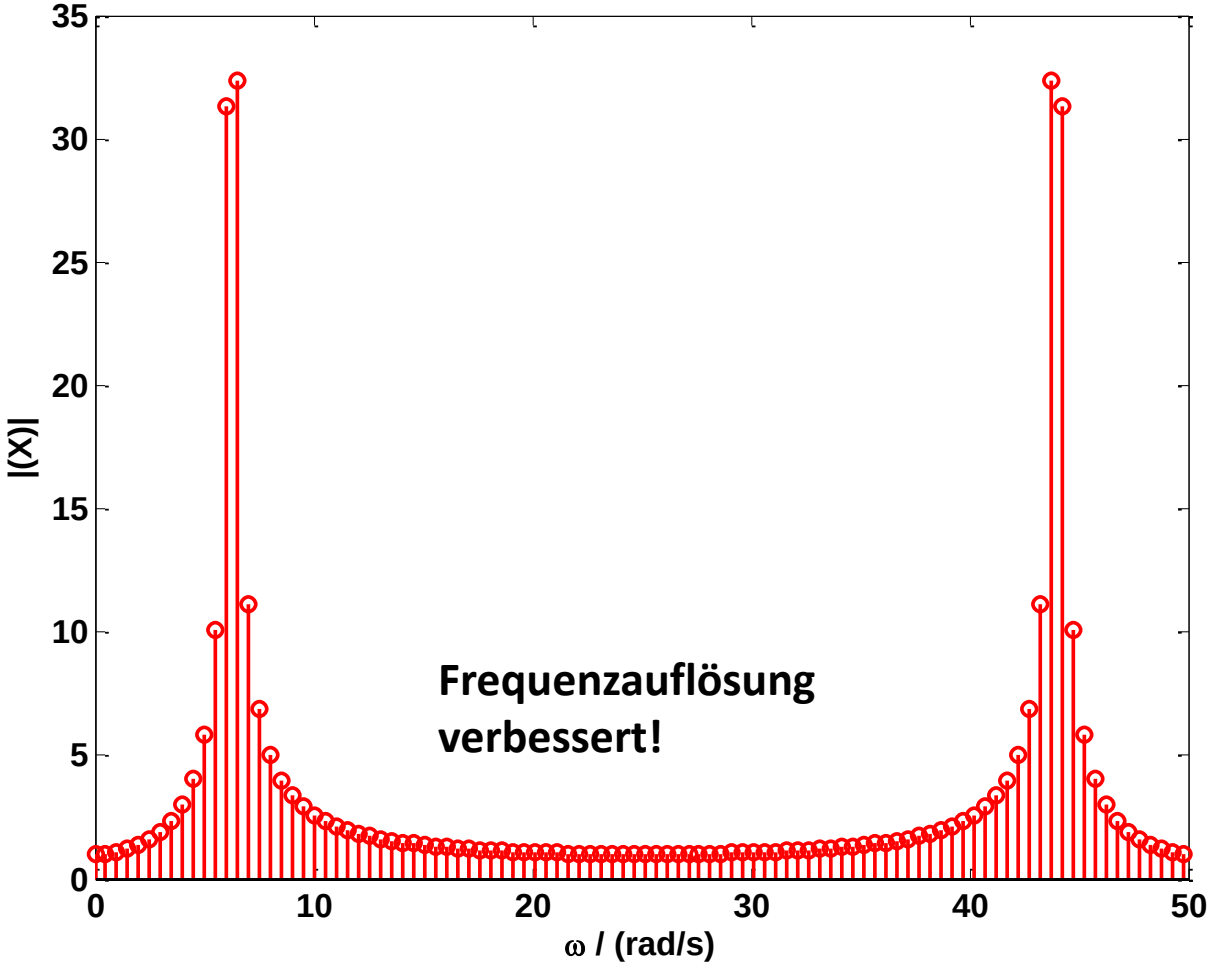
Cosinus, nicht periodisch fortsetzbar

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \quad N = 100 \quad T = NT_A = 12{,}5 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 0{,}16\pi$$



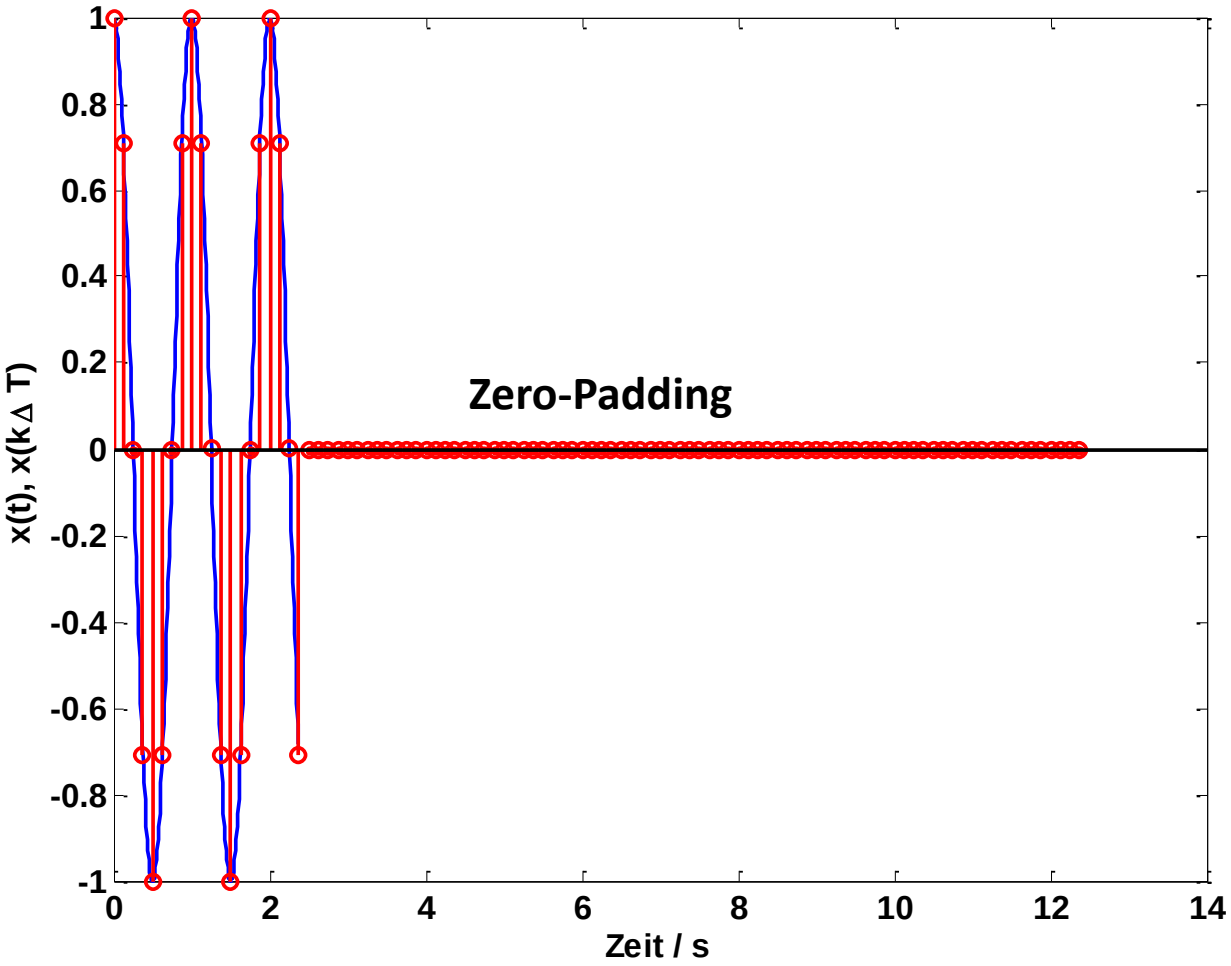
Cosinus, nicht periodisch fortsetzbar

Diskrete Fouriertransformation



Cosinus, nicht periodisch fortsetzbar

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \quad N = 100 \quad T = NT_A = 12{,}5 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 0{,}16\pi$$



Cosinus, nicht periodisch fortsetzbar

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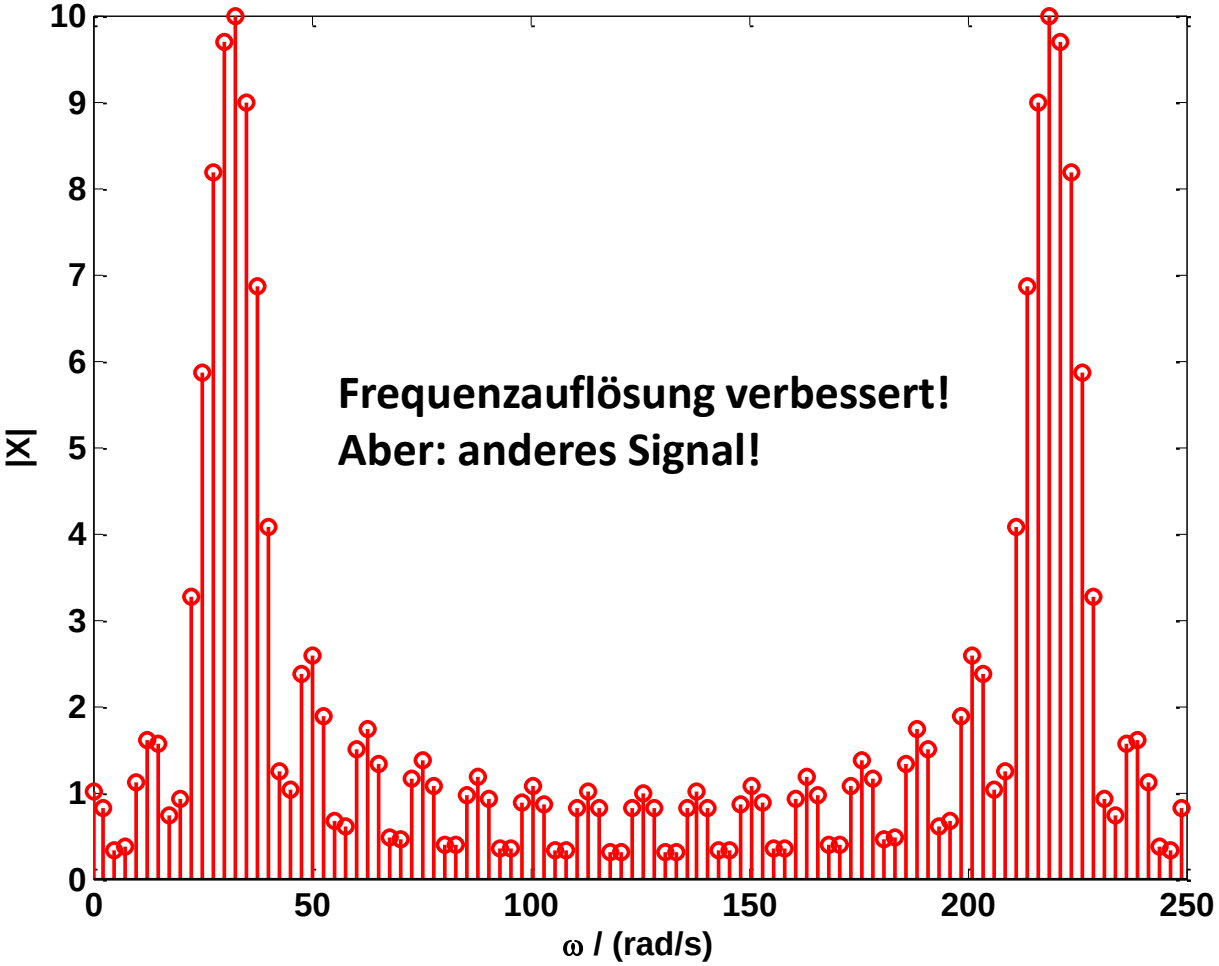
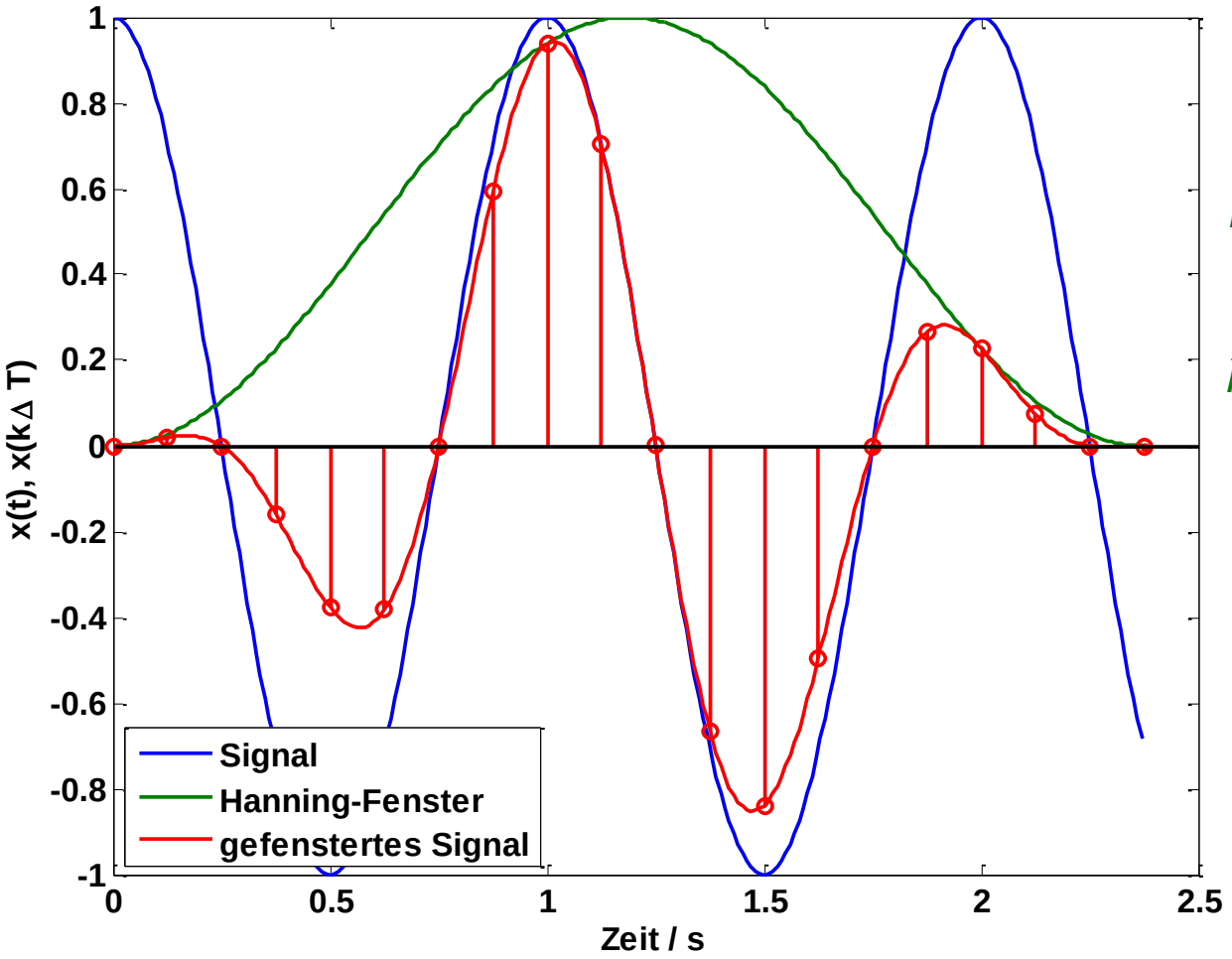


Cosinus, Hanning-Fenster und Zero-Padding

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8}$$



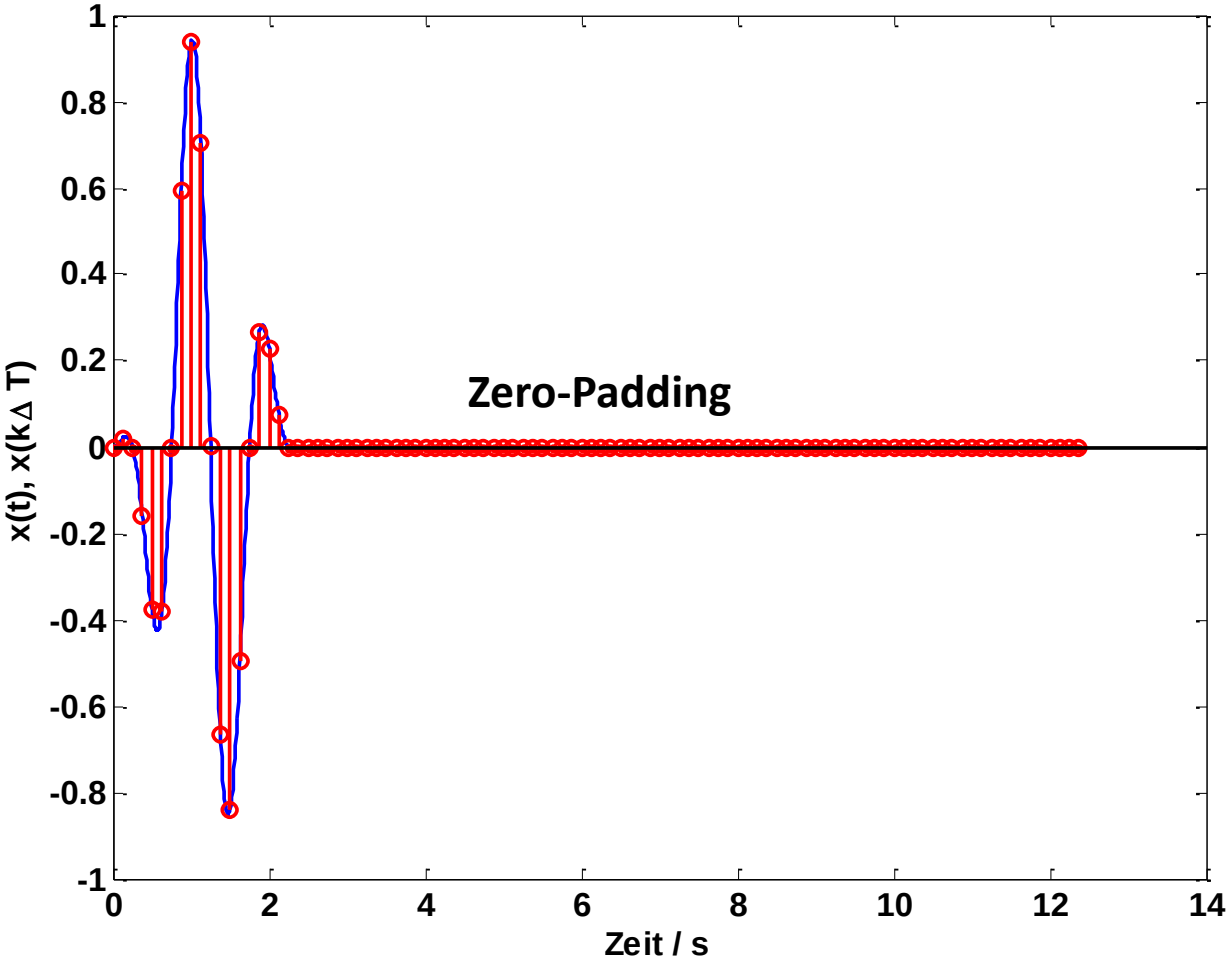
Hanning-Fenster
(nach rechts geschoben)

$$h(t) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi t}{T}\right)\right)$$

$$h(k) = \frac{1}{2}\left(1 - \cos\left(\frac{2\pi k}{N-1}\right)\right)$$

Cosinus, Hanning-Fenster und Zero-Padding

$$f(t) = \cos(2\pi t) \quad T_A = \frac{1}{8} \quad N = 100 \quad T = NT_A = 12{,}5 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = 0{,}16\pi$$



Cosinus, Hanning-Fenster und Zero-Padding

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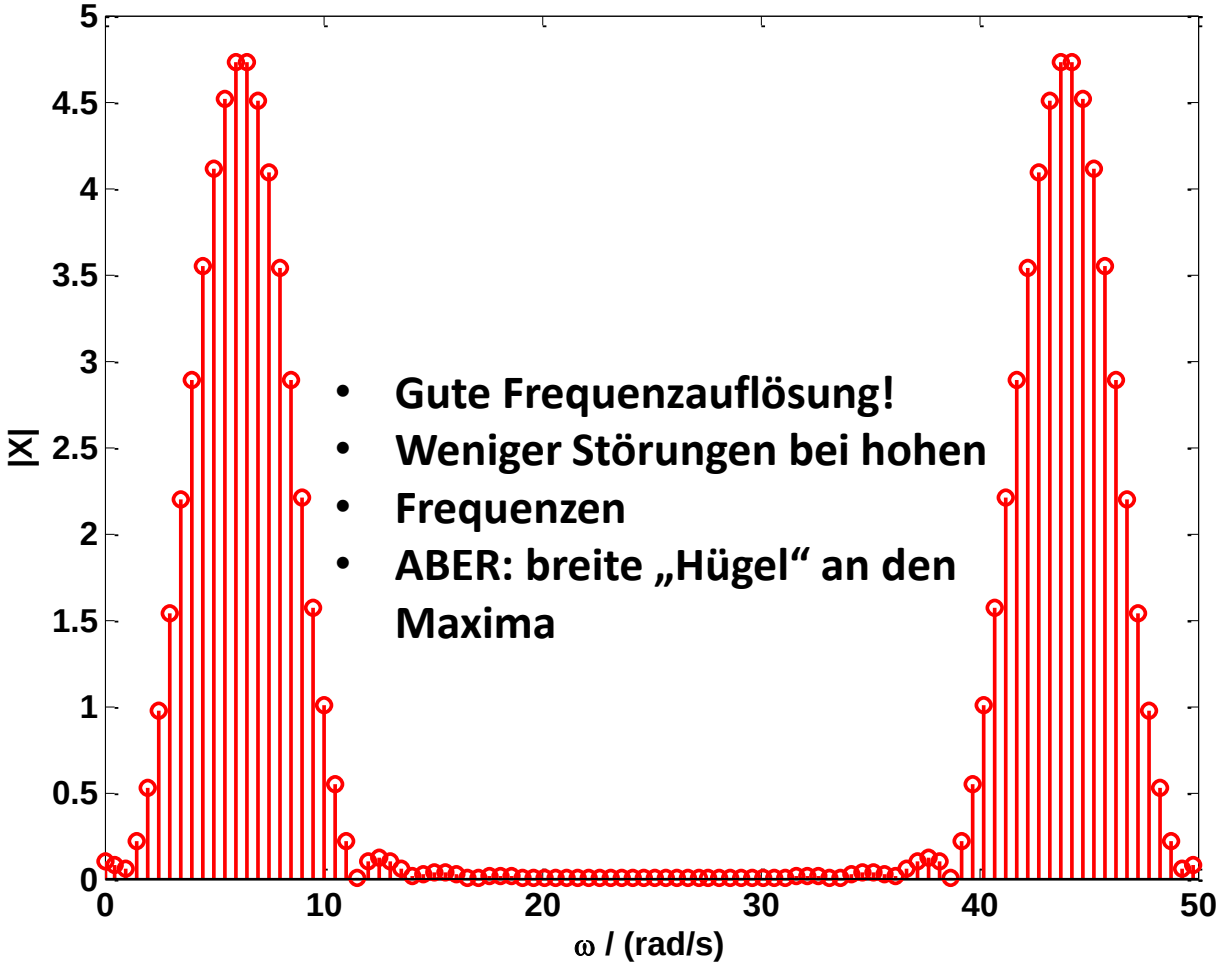
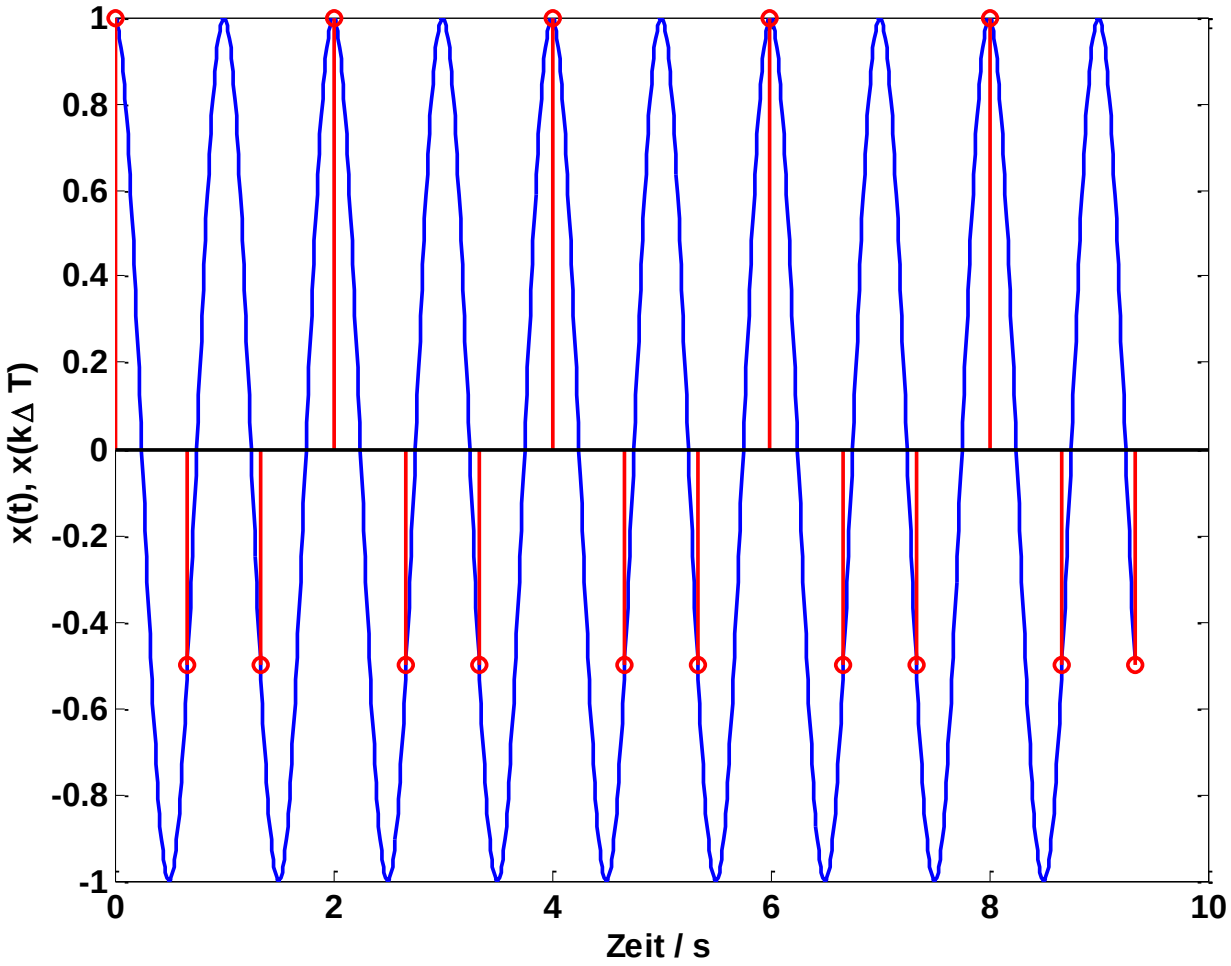


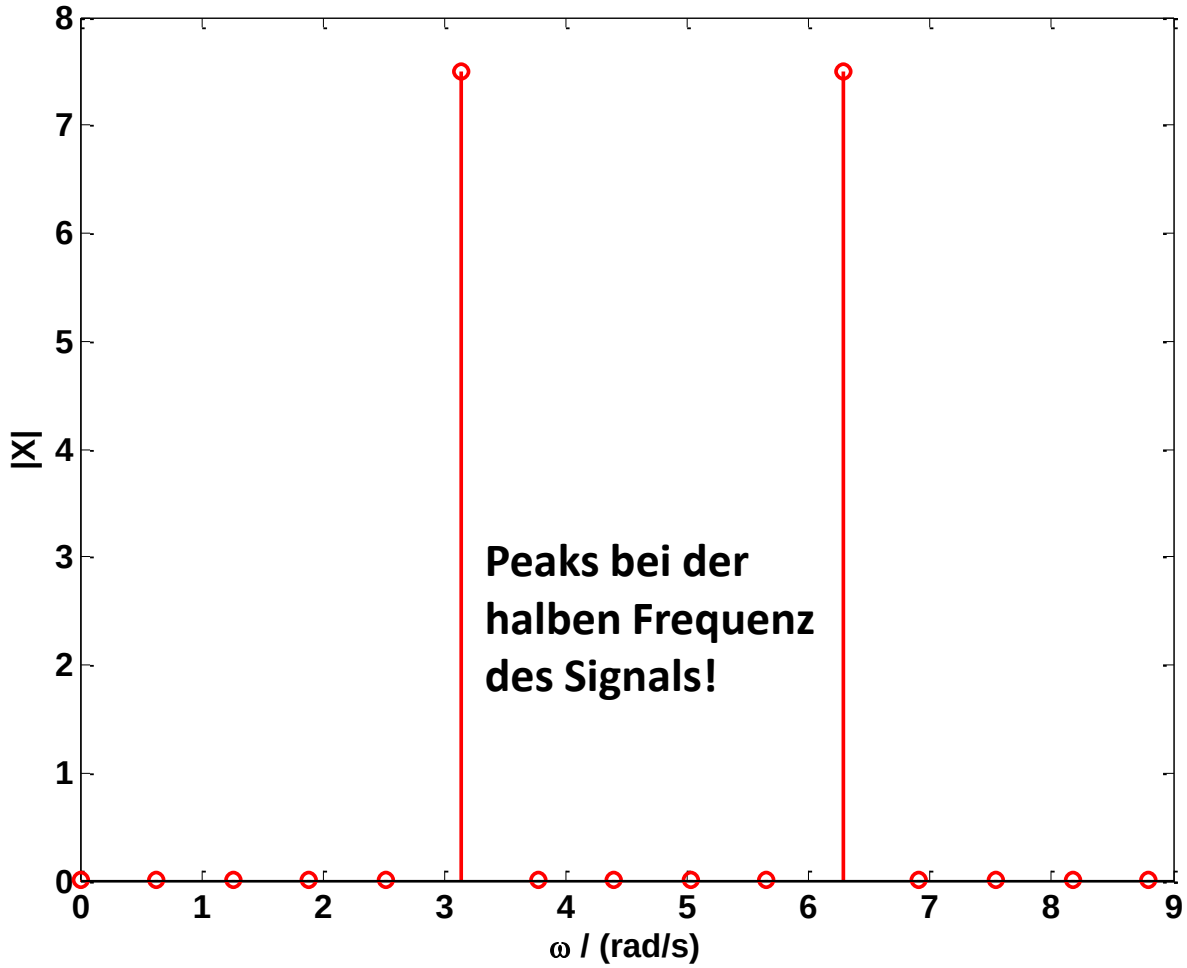
Cosinus, Aliasing

$$f(t) = \cos(2\pi t) \quad T_A = \frac{2}{3} \qquad N = 15 \quad T = NT_A = 10 \quad \Rightarrow \quad \omega_0 = \frac{2\pi}{T} = \frac{\pi}{5}$$



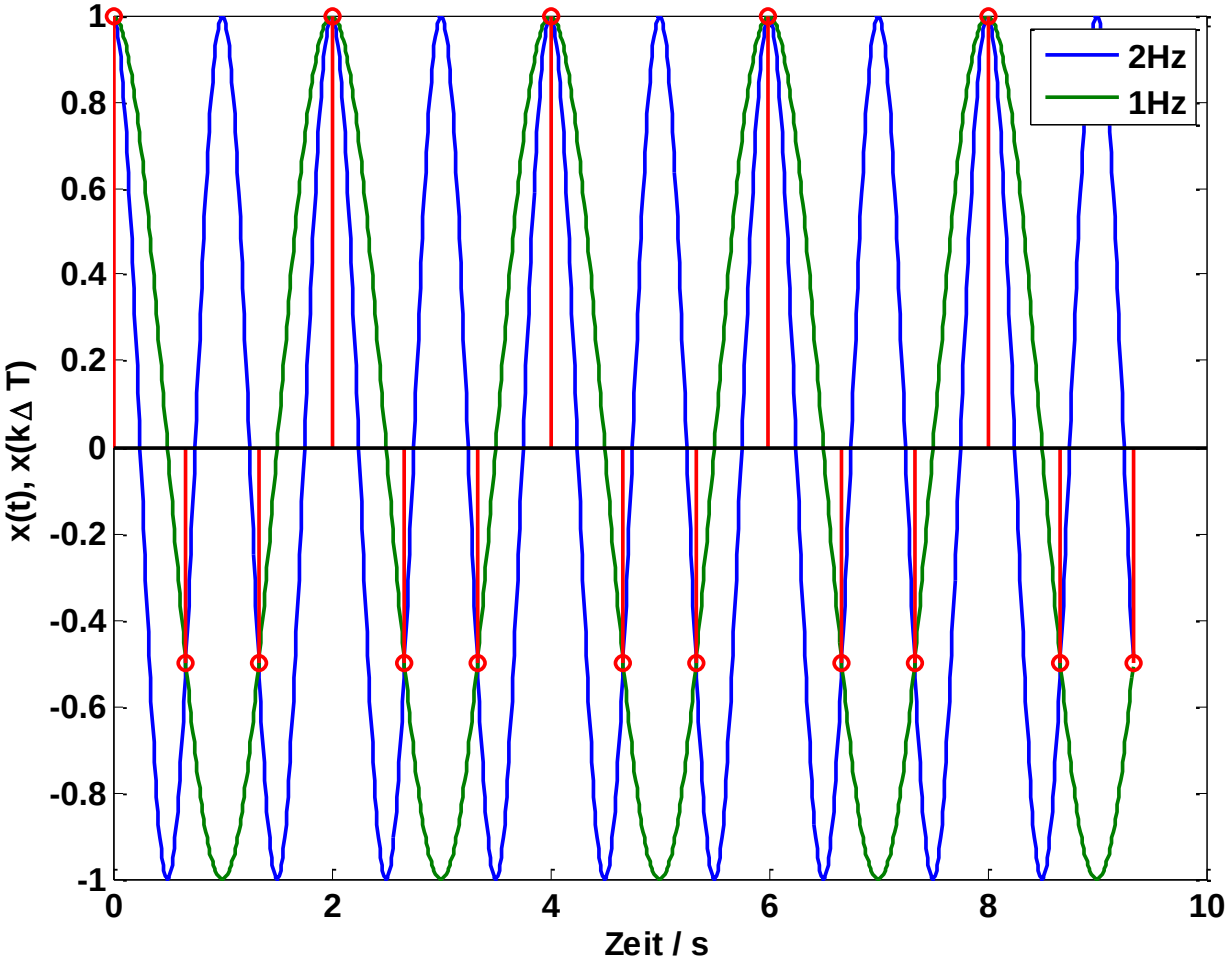
Cosinus, Aliasing

Diskrete Fouriertransformation



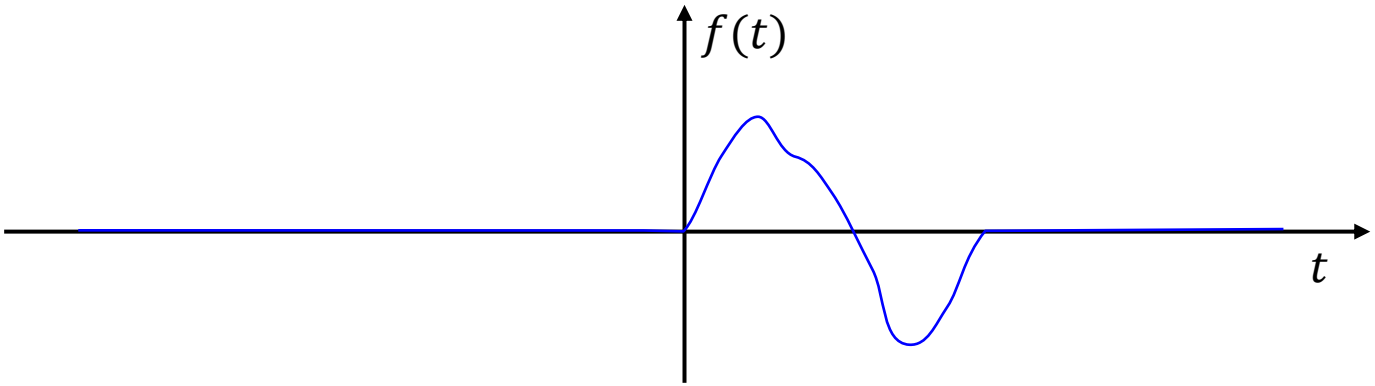
Cosinus, Aliasing

$f(t) = \cos(2\pi t)$ $f(t) = \cos(\pi t)$ $T_A = \frac{2}{3}$

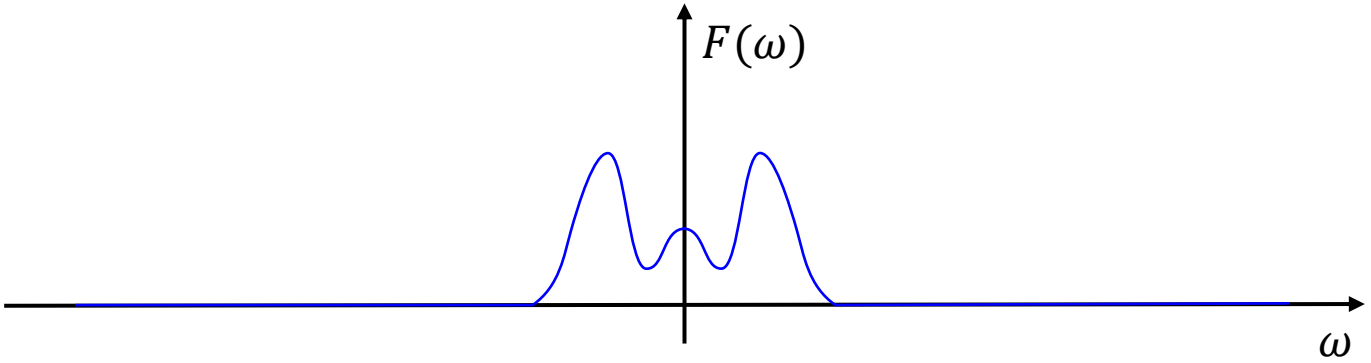


Konzept Diskrete Fouriertransformation

Kontinuierliche Fouriertransformation



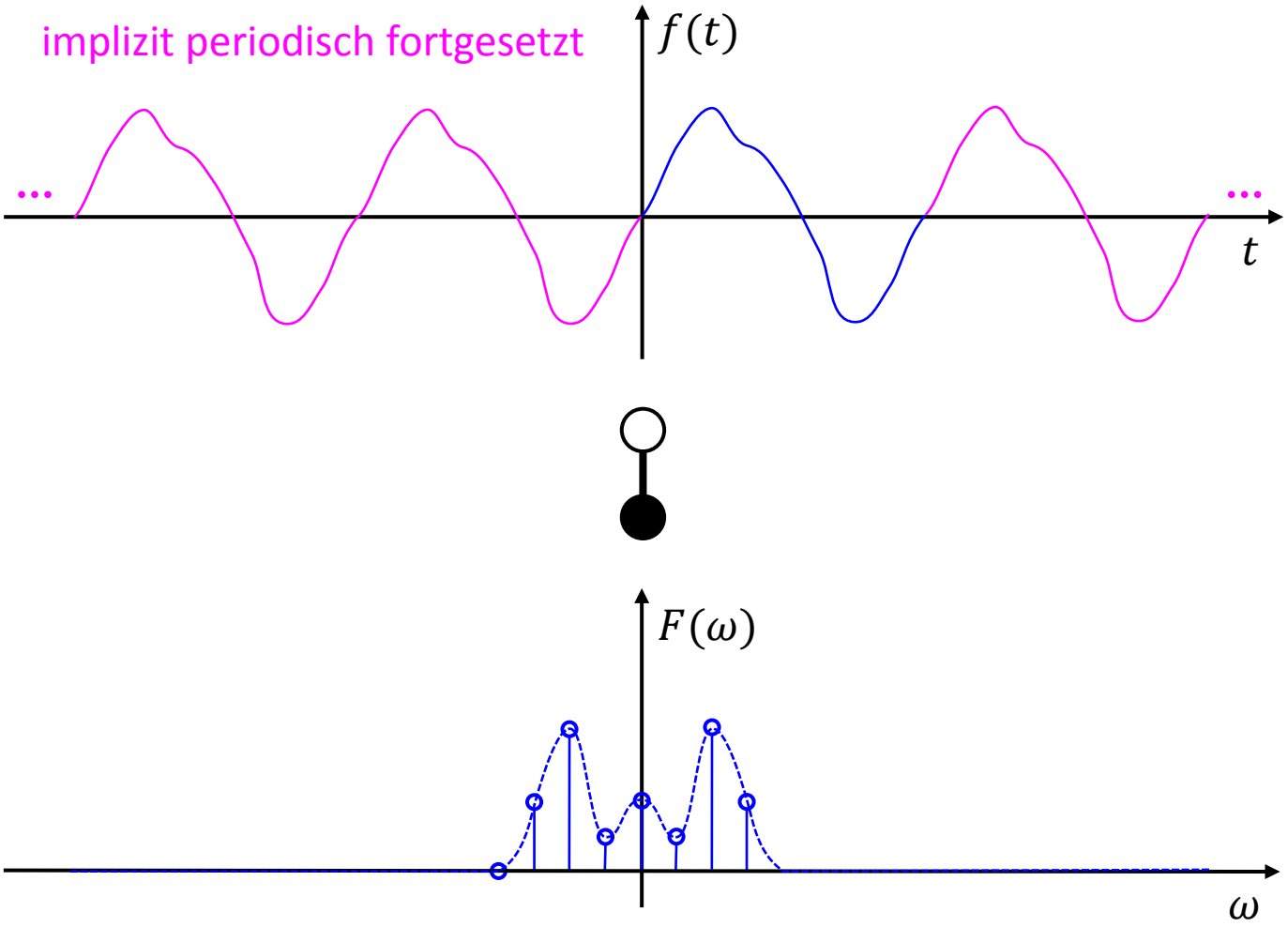
Nichtperiodisches Signal



Kontinuierliches Spektrum

Konzept Diskrete Fouriertransformation

Abtastung im Frequenzbereich

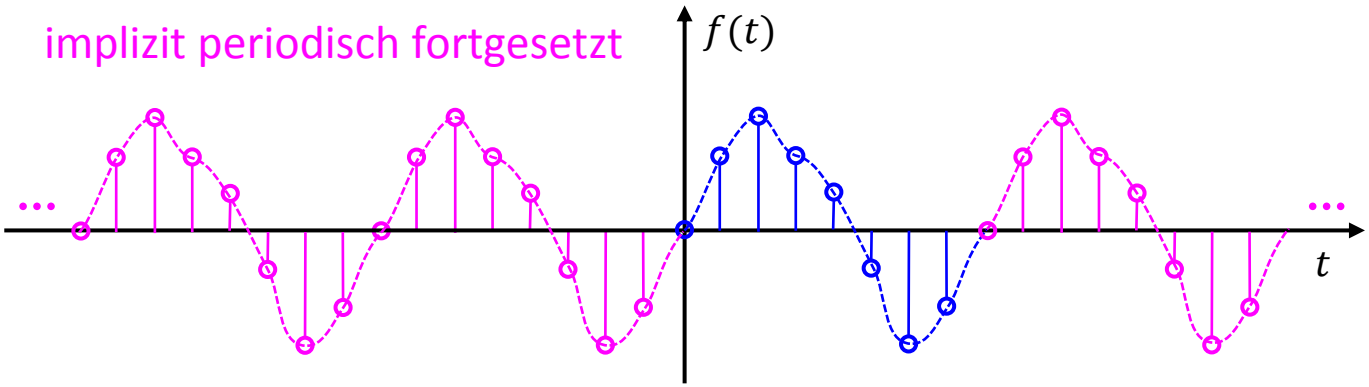


Periodisches Signal

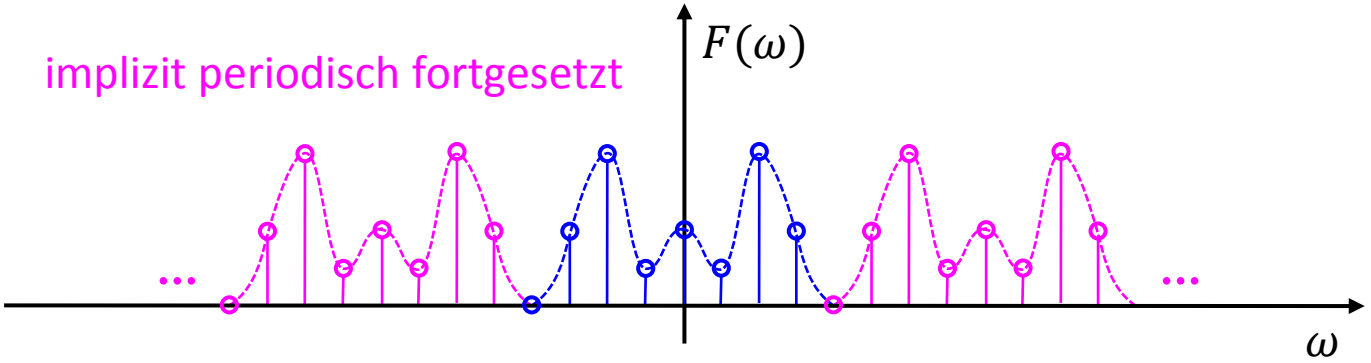
Diskretes Spektrum

Konzept Diskrete Fouriertransformation

Abtastung im Zeitbereich



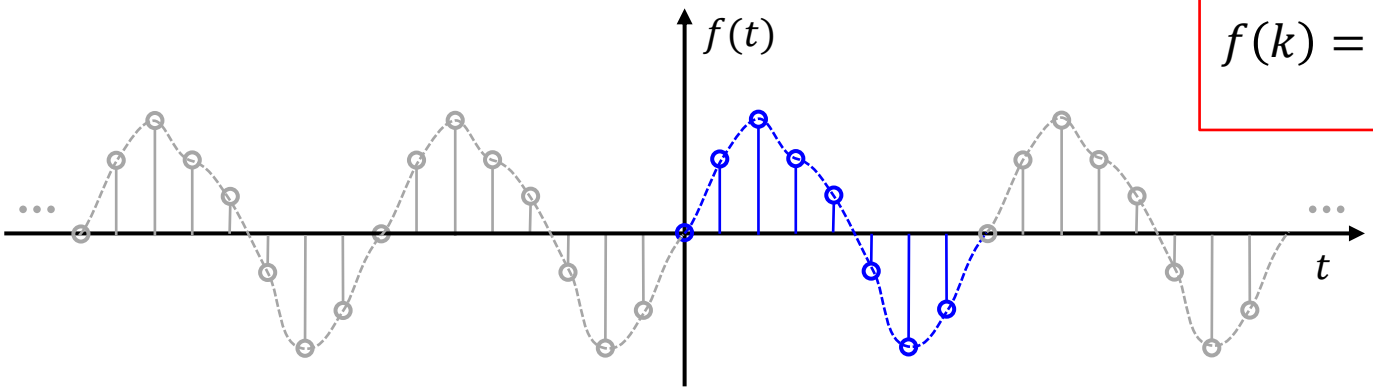
Periodisches diskretes Signal



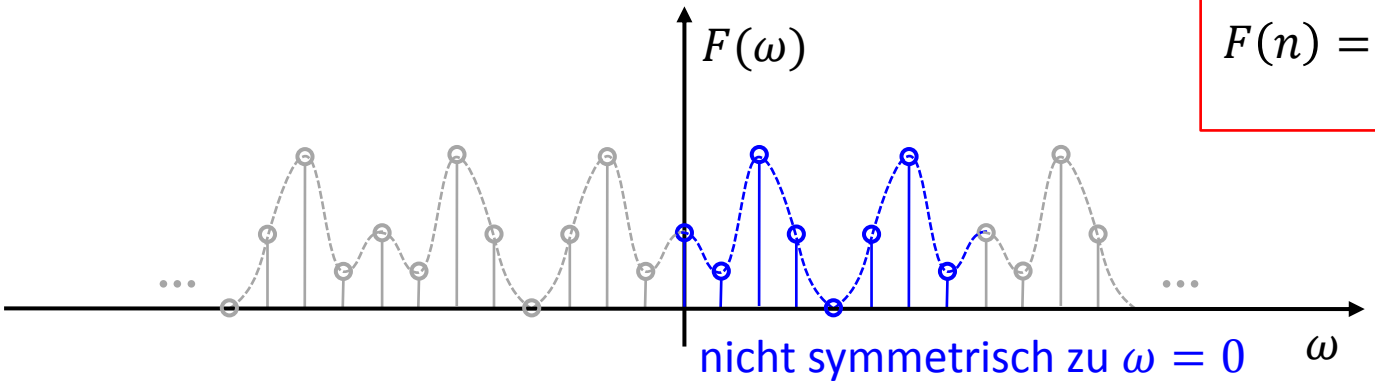
Periodisches diskretes Spektrum

Konzept Diskrete Fouriertransformation

Diskrete Fouriertransformation



$$f(k) = \frac{1}{N} \sum_{n=0}^{N-1} F(n) \cdot e^{j\frac{2\pi}{N}kn}$$



$$F(n) = \sum_{k=0}^{N-1} f(k) \cdot e^{-j\frac{2\pi}{N}kn}$$

$$\omega_0 = \frac{2\pi}{N\,T_A}$$