Mathematik 3

Euler-Gleichung

Wintersemester 2013/14

Euler-Gleichung

Bekannte Reihenentwicklungen

$$e^x = 1 + \frac{x^1}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \frac{x^6}{6!} + \frac{x^7}{7!} + \cdots$$

$$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots \qquad \cos(x) = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

Reihenentwicklung von e^{jx}

$$e^{jx} = 1 + \frac{jx}{1!} + \frac{(jx)^2}{2!} + \frac{(jx)^3}{3!} + \frac{(jx)^4}{4!} + \frac{(jx)^5}{5!} + \frac{(jx)^6}{6!} + \frac{(jx)^7}{7!} + \cdots$$

$$= 1 + j\frac{x}{1!} - \frac{x^2}{2!} - j\frac{x^3}{3!} + \frac{x^4}{4!} + j\frac{x^5}{5!} - \frac{x^6}{6!} - j\frac{x^7}{7!} + \cdots$$

$$= \underbrace{\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots\right)}_{\cos(x)} + j\underbrace{\left(x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots\right)}_{\sin(x)} = \cos(x) + j\sin(x)$$

Darstellung von sin und cos durch komplexe Exponentialfunktion

Euler-Gleichung

$$\boxed{e^{jx} = \cos(x) + j\sin(x)}$$

Cosinus

$$
\begin{aligned}
& e^{jx} = \cos(x) + j\sin(x) \\
+ \quad & e^{-jx} = \cos(x) - j\sin(x) \\
\hline
& e^{jx} + e^{-jx} = 2\cos(x)
\end{aligned}
$$

$$\boxed{\cos(x) = \frac{1}{2}\left(e^{jx} + e^{-jx}\right)}$$

Sinus

$$
\begin{aligned}
& e^{jx} = \cos(x) + j\sin(x) \\
- \quad & e^{-jx} = \cos(x) - j\sin(x) \\
\hline
& e^{jx} - e^{-jx} = 2j\sin(x)
\end{aligned}
$$

$$\boxed{\sin(x) = \frac{1}{2j}\left(e^{jx} - e^{-jx}\right)}$$