

Class 4: Higher-Order Patterns

February 13

```
func (\x -> length xss ...) ...
```

vs.

```
func (\x -> height ...) ...  
  where  
    height = length xss
```

Q: is the second version more performant?

A: no need to worry about optimizations for this class.
the answer in this specific case is *probably* yes, by a little.
but the answer more generally is quite interesting!

`fib 0 = 0`

`fib 1 = 1`

`fib x = fib (x - 1) + fib (x - 2)`

questions from homework

```
result1 = fib 30 + fib 30
```

```
result2 = x + x
```

```
  where
```

```
    x = fib 30
```

```
result3 = x + x
```

```
  where
```

```
    x :: Int
```

```
    x = fib 30
```

questions from homework

Q: how much does good Haskell code make use of map, filter, fold?

A: they are actually used frequently when working with lists!
many situations call for a more general or a more specific function.
but the general principle remains: break down the problem into
high-level *transformations* instead of low-level details.

Q: when are parentheses needed?

A: in general, parentheses in Haskell are used to *remove ambiguity*.
the most common case of this is to group together multiple
space-separated components into a single function argument.

ex1 (x : xs) (y : ys) = ...

ex2 = f (3 + 4) (g 3)

questions from homework

Q: how are anonymous functions used? when are they necessary?

A: anonymous functions allow us to construct a function
(often to be used as an argument to another function)
without having to give it a name

```
add3All :: [Int] -> [Int]
add3All xs = map (\x -> x + 3) xs
```

questions from homework

our first higher-order functions

`map :: (a -> b) -> [a] -> [b]`

`filter :: (a -> Bool) -> [a] -> [a]`

`foldr :: (a -> b -> b) -> b -> [a] -> b`

idiom:

Functions can be used without mentioning their arguments.

idiom:

Functions can be used without mentioning their arguments.

conceptually, this makes sense because

Functions don't always have to do. They can just be.

practically, the benefit of this is
We can reason at a higher level of abstraction.

idiom:
Functions can be used without mentioning their arguments.

conceptually, this makes sense because
Functions don't always have to do. They can just be.

technique #1:
Use function names directly.


```
absAll :: [Int] -> [Int]  
absAll xs = map (\x -> abs x) xs
```

examples

```
absAll :: [Int] -> [Int]  
absAll xs = map abs xs
```

examples

```
plus :: Int -> Int -> Int
```

```
sum :: [Int] -> Int
```

```
sum xs = foldr (\x acc -> plus x acc) 0 xs
```

examples

```
plus :: Int -> Int -> Int
```

```
sum :: [Int] -> Int
```

```
sum xs = foldr plus 0 xs
```

examples

$(+) :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$\text{sum} :: [\text{Int}] \rightarrow \text{Int}$

$\text{sum } xs = \text{foldr } (\backslash x \text{ acc} \rightarrow x + \text{acc}) \ 0 \ xs$

examples

`(+) :: Int -> Int -> Int`

`sum :: [Int] -> Int`

`sum xs = foldr (\x acc -> (+) x acc) 0 xs`

examples

`(+) :: Int -> Int -> Int`

`sum :: [Int] -> Int`

`sum xs = foldr (+) 0 xs`

examples

try locally!

```
and :: [Bool] -> Bool  
and xs = foldr (\x acc -> x && acc) True xs
```

examples


```
and :: [Bool] -> Bool  
and xs = foldr (&&) True xs
```

examples

function composition

$\text{compose} :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c)$

dot operator

`compose :: (b -> c) -> (a -> b) -> (a -> c)`
`compose f g =`

dot operator

`compose :: (b -> c) -> (a -> b) -> (a -> c)`
`compose f g = \x ->`

dot operator

`compose :: (b -> c) -> (a -> b) -> (a -> c)`
`compose f g = \x -> f (g x)`

dot operator

$(\cdot) :: (b \rightarrow c) \rightarrow (a \rightarrow b) \rightarrow (a \rightarrow c)$
 $f \cdot g = \lambda x \rightarrow f (g x)$

dot operator

technique #2:
Leverage function composition.


```
desort :: [Int] -> [Int]  
desort xs = reverse (sort xs)
```

examples

```
desort :: [Int] -> [Int]  
desort = reverse . sort
```

examples

```
oddOnly :: [Int] -> [Int]  
oddOnly xs = filter (\x -> not (even x)) xs
```

examples

```
oddOnly :: [Int] -> [Int]  
oddOnly xs = filter (not . even) xs
```

examples

try locally!

```
evenOdds :: [Int] -> Bool
evenOdds xs = even (length (oddOnly xs))
```

examples

```
evenOdds :: [Int] -> Bool  
evenOdds = even . length . oddOnly
```

examples

```

evenOdds :: [Int] -> Bool
evenOdds = even . length . oddOnly

```

$\text{Int} \rightarrow \text{Bool}$ $[\text{Int}] \rightarrow \text{Int}$ $[\text{Int}] \rightarrow [\text{Int}]$

examples

Does this type check?

```
f :: Int -> Int  
g :: Int -> Bool
```

```
f . g
```


Does this type check?

```
f :: Bool -> Int  
g :: Int -> Bool
```

```
f . g
```

Does this type check?

```
f :: Int -> Bool  
g :: Int -> Int
```

```
f . g
```

```
desort :: [Int] -> [Int]  
desort = reverse . sort
```

upon closer examination

partial application

$f :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

currying

$f :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$f :: \text{Int} \rightarrow (\text{Int} \rightarrow \text{Int})$

currying

f 3 4

currying

f 3 4

(f 3) 4

currying


```
f :: Int -> Int -> Int  
f x y = 2 * x + y
```

currying

```
f :: Int -> (Int -> Int)
f x = \y -> 2 * x + y
```

currying

```
f :: (Int -> (Int -> Int))  
f = \x -> (\y -> 2 * x + y)
```

currying

function type arrows associate to the **right**

$a \rightarrow b \rightarrow c \rightarrow d$

same as

$a \rightarrow (b \rightarrow (c \rightarrow d))$

function applications associate to the **left**

$\text{func } 3 \ 4 \ 5$

same as

$((\text{func } 3) \ 4) \ 5$

Are these the same?

$\text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$

$(\text{Int} \rightarrow \text{Int}) \rightarrow \text{Int}$

Are these the same?

$(\text{Int} \rightarrow \text{Int}) \rightarrow \text{Int} \rightarrow \text{Int}$

$(\text{Int} \rightarrow \text{Int}) \rightarrow (\text{Int} \rightarrow \text{Int})$

Does this type check?

```
f :: Int -> Int  
g :: Int -> Int
```

```
f g
```

Does this type check?

```
f :: Int -> Int -> Int  
g :: Int -> Int
```

```
f g
```


Does this type check?

```
f :: (Int -> Int) -> Int  
g :: Int -> Int
```

```
f g
```

Does this type check?

```
f :: (Int -> Int) -> (Int -> Int)
g :: Int -> Int
```

```
f g 3
```

Does this type check?

$f :: (\text{Int} \rightarrow \text{Int}) \rightarrow (\text{Int} \rightarrow \text{Int})$
 $g :: \text{Int} \rightarrow \text{Int}$

$(f\ g)\ 3$

Does this type check?

$f :: (Int \rightarrow Int) \rightarrow (Int \rightarrow Int)$

$g :: Int \rightarrow Int$

$f\ (g\ 3)$

`compose :: (b -> c) -> (a -> b) -> (a -> c)`
`compose f g = \x -> f (g x)`

currying

`compose :: (b -> c) -> (a -> b) -> a -> c`
`compose f g x = f (g x)`

currying

`plus :: Int -> Int -> Int`

`plus3 :: ???`

`plus3 = plus 3`

partial application

`plus :: Int -> Int -> Int`

`plus3 :: Int -> Int`

`plus3 = plus 3`

partial application


```
plus :: Int -> Int -> Int
```

```
plus3 :: Int -> Int  
plus3 = plus 3
```

```
result :: Int  
result = plus3 4
```

partial application

technique #3:
Leverage partial application.

```
absAll :: [Int] -> [Int]  
absAll xs = map abs xs
```

examples

```
absAll :: [Int] -> [Int]  
absAll = map abs
```

examples

try locally!

```
and :: [Bool] -> Bool  
and xs = foldr (&&) True xs
```

examples

```
and :: [Bool] -> Bool  
and = foldr (&&) True
```

examples

```
add3All :: [Int] -> [Int]
add3All xs = map (\x -> plus 3 x) xs
```

examples

```
add3All :: [Int] -> [Int]  
add3All = map (\x -> plus 3 x)
```

examples


```
add3All :: [Int] -> [Int]  
add3All = map (plus 3)
```

examples

```
add3All :: [Int] -> [Int]  
add3All = map (\x -> 3 + x)
```

examples

```
add3All :: [Int] -> [Int]  
add3All = map (3 +)
```

examples

$(\backslash x \rightarrow 3 + x)$ same as $(3 +)$
 $(\backslash x \rightarrow x + 3)$ same as $(+ 3)$

operator sections

try locally!

```
greaterThan100 :: [Int] -> [Int]  
greaterThan100 = filter (\x -> x > 100)
```

examples

```
greaterThan100 :: [Int] -> [Int]  
greaterThan100 = filter (> 100)
```

examples

All together now!

```
map :: (a -> b) -> [a] -> [b]
map f xs = foldr (\x acc -> f x : acc) [] xs
```

examples


```
map :: (a -> b) -> [a] -> [b]
map f = foldr (\x acc -> f x : acc) []
```

examples

```
map :: (a -> b) -> [a] -> [b]  
map f = foldr ((:) . f) []
```

examples

```
youngNames :: [Person] -> [String]
youngNames xs =
    map name (filter (\x -> age x <= 18) xs)
```

```
youngNames :: [Person] -> [String]
youngNames =
    map name . filter (\x -> age x <= 18)
```

```
youngNames :: [Person] -> [String]
youngNames =
    map name . filter ((<= 18) . age)
```

wholemeal programming

```
func :: [Int] -> Int
func [] = 1
func (x : xs)
  | x > 3      = (x + 7) * func xs
  | otherwise = func xs
```

```
func :: [Int] -> Int
func [] = 1
func (x : xs)
  | x > 3      = (x + 7) * func xs
  | otherwise = func xs
```



```
func :: [Int] -> Int
func [] = 1
func (x : xs)
  | x > 3      = (x + 7) * func xs
  | otherwise = func xs
```

```
func :: [Int] -> Int
func [] = 1
func (x : xs)
    | x > 3      = (x + 7) * func xs
    | otherwise = func xs
```

```
func :: [Int] -> Int  
func = product . map (+ 7) . filter (> 3)
```