

# Class 6: *Lazy* Evaluation

February 27

**what is laziness?**

$f :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$   
 $f = \dots$

*to evaluate*

$f \ 5 \ (3^{1000})$

*need to first evaluate*

$3^{1000}$

strict evaluation

$f :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$   
 $f\ x\ y = x + y$

*to evaluate*

$f\ 5\ (3^{1000})$

*need to first evaluate*

$3^{1000}$

strict evaluation

$f :: \text{Int} \rightarrow \text{Int} \rightarrow \text{Int}$   
 $f\ x\ \_ = x + 2$

*to evaluate*

$f\ 5\ (3^{1000})$

*need to first evaluate*

$3^{1000}$



strict evaluation

evaluation of function arguments is delayed as long as possible.

lazy evaluation

```
f :: Int -> Int -> Int  
f x _ = x + 2
```

*to evaluate*

```
f 5 (3^1000)
```

*don't need to evaluate*

```
3^1000
```

lazy evaluation

```
f ():  
    print "hello";  
    return 1;
```

```
g ():  
    print "bye";  
    return 2;
```

```
add (f(), g());
```

```
add (x, y):  
    return x + y;
```

side effects



```
f ():  
    print "hello";  
    return 1;
```

```
g ():  
    print "bye";  
    return 2;
```

```
add (f(), g());
```

```
add (x, y):  
    return y + x;
```

side effects

purity enables laziness.  
laziness enforces purity.

side effects

pattern matching drives evaluation

```
f1 :: Maybe a -> [Maybe a]  
f1 m = [m, m]
```

```
f2 :: Maybe a -> [a]  
f2 Nothing = []  
f2 (Just x) = [x]
```

pattern matching drives evaluation

```
f1 :: Maybe a -> [Maybe a]
f1 m = [m, m]
```

```
    f1 (safeHead [3^1000, 5])
= [safeHead [3^1000, 5],
   safeHead [3^1000, 5]]
```

pattern matching drives evaluation

```
f2 :: Maybe a -> [a]
```

```
f2 Nothing = []
```

```
f2 (Just x) = [x]
```

```
    f2 (safeHead [3^1000, 5])
```

```
= f2 (Just (3^1000))
```

```
= [3^1000]
```

pattern matching drives evaluation

**complexity**

short circuiting



$(||) :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

True     $||$      $\_ = \text{True}$

False    $||$     $b = b$

$(||!) :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

True     $||!$     $\text{True} = \text{True}$

True     $||!$     $\text{False} = \text{True}$

False    $||!$     $\text{True} = \text{True}$

False    $||!$     $\text{False} = \text{False}$

$(||)$  operator

$(||) :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$   
 $(||!) :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

True || (expensive computation)  
True ||! (expensive computation)

(||) operator

$(\mid\mid) :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$   
 $(\mid\mid!) :: \text{Bool} \rightarrow \text{Bool} \rightarrow \text{Bool}$

True  $\mid\mid$  (error “bad”)  
True  $\mid\mid!$  (error “bad”)

(&&) operator

```
if :: Bool -> a -> a -> a
if True  x _ = x
if False _ y = y
```

user-defined control structures

```
foo = if condition
      then
        let x = expensive in
        do something with x
      else
        do something else
```

where clauses

```
foo = let x = expensive in  
      if condition  
      then  
        do something with x  
      else  
        do something else
```

where clauses

```
foo = if condition
      then
        do something with x
      else
        do something else
where
  x = expensive
```

where clauses

**infinite data structures**



`repeat :: a -> [a]`

`repeat x = x : repeat x`

`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`take 2 (repeat 7)`  
`=`

infinite list: repeat

`repeat :: a -> [a]`

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`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`take 2 (repeat 7)`  
`= take 2 (7 : repeat 7)`

infinite list: repeat

`repeat :: a -> [a]`

`repeat x = x : repeat x`

`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`take 2 (7 : repeat 7)`  
`= 7 : take (2 - 1) (repeat 7)`

infinite list: repeat

`repeat :: a -> [a]`

`repeat x = x : repeat x`

`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`7 : take (2 - 1) (repeat 7)`  
`= 7 : take 1 (repeat 7)`

infinite list: repeat

`repeat :: a -> [a]`

`repeat x = x : repeat x`

`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`7 : take 1 (repeat 7)`  
`= 7 : take 1 (7 : repeat 7)`

infinite list: repeat

`repeat :: a -> [a]`

`repeat x = x : repeat x`

`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`7 : take 1 (7 : repeat 7)`  
`= 7 : 7 : take (1 - 1) (repeat 7)`

infinite list: repeat

`repeat :: a -> [a]`

`repeat x = x : repeat x`

`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`7 : 7 : take (1 - 1) (repeat 7)`  
`= 7 : 7 : take 0 (repeat 7)`

infinite list: repeat

`repeat :: a -> [a]`

`repeat x = x : repeat x`

`take :: Int -> [a] -> [a]`

`take n _ | n <= 0 = []`

`take _ [] = []`

`take n (x : xs) = x : take (n - 1) xs`

`7 : 7 : take (1 - 1) (repeat 7)`  
`= 7 : 7 : []`

infinite list: repeat



```
nats :: [Int]
nats = 0 : map (+ 1) nats
```

infinite list: natural numbers

```
take 2 nats
= take 2 (0 : map (+ 1) nats)
= 0 : take 1 (map (+ 1) nats)
= 0 : take 1 (map (+ 1) (0 : map (+ 1) nats))
= 0 : take 1 (1 : map (+ 1) (map (+ 1) nats))
= 0 : 1 : take 0 (map (+ 1) (map (+ 1) nats))
= 0 : 1 : []
```

infinite list: natural numbers

```
upTo10 :: [Int]
upTo10 = [0..9]
```

infinite list: natural numbers

```
upTo10 :: [Int]
upTo10 = [0..9]
```

```
nats :: [Int]
nats = [0..]
```

infinite list: natural numbers

```
fib :: Int -> Int
fib 0 = 1
fib 1 = 1
fib n = fib (n - 1) + fib (n - 2)
```

infinite list: fibonacci numbers

```
fibs :: [Int]
fibs = 1 : 1 : zipWith (+) fibs (tail fibs)
```

```
fib :: Int -> Int
fib n = fibs !! n
```

infinite list: fibonacci numbers

```
data Stream a = Cons a (Stream a)
```

streams

```
streamToList :: Stream a -> [a]  
streamToList (Cons x xs) = x : streamToList xs
```

```
streamTake :: Int -> Stream a -> [a]  
streamTake n = take n . streamToList
```



*(exercise: streamlterate)*

function reuse

`any :: (a -> Bool) -> [a] -> Bool`

`any f [] = False`

`any f (x : xs) = f x || any f xs`

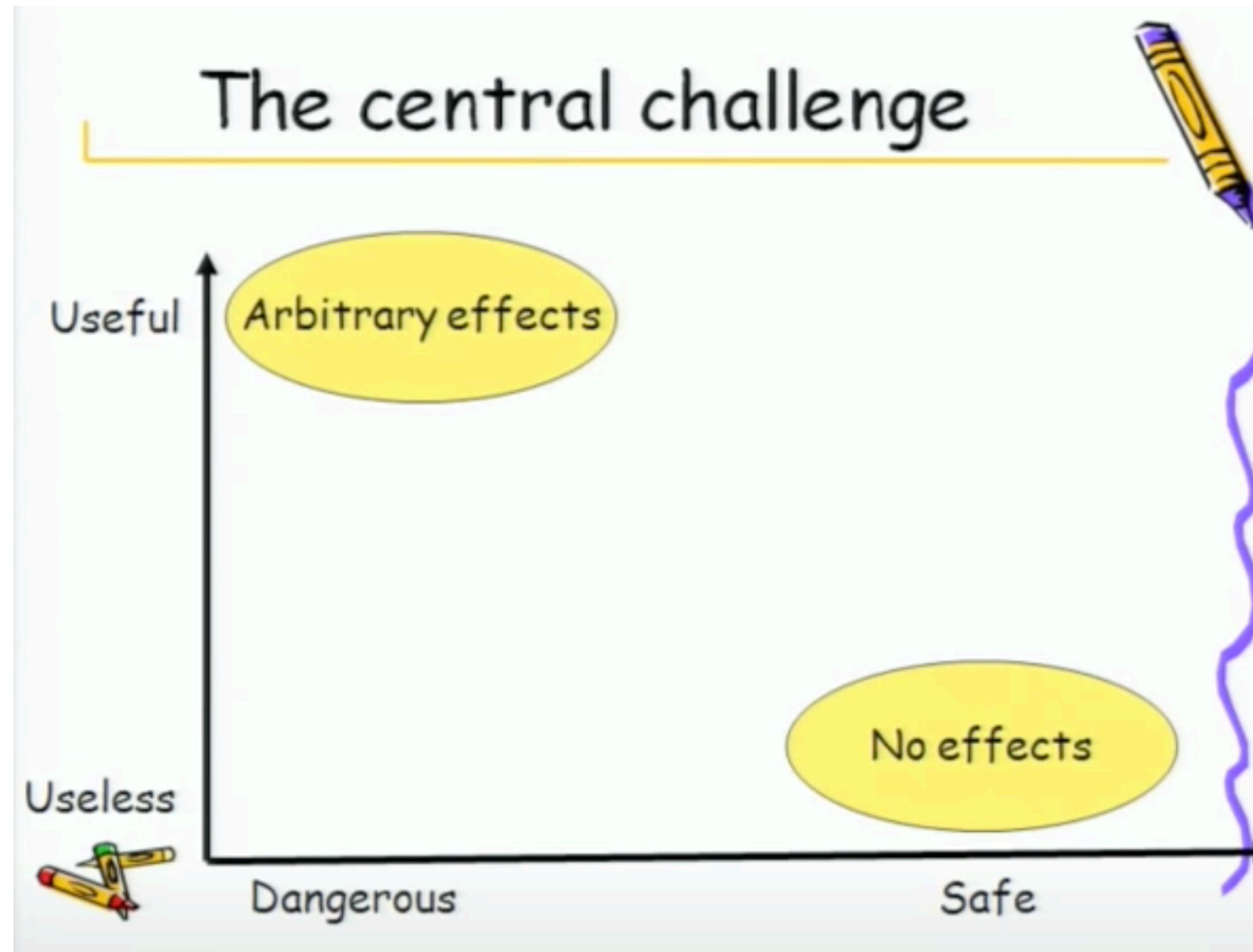
any

```
or :: [Bool] -> Bool  
or = foldr (||) False
```

```
any :: (a -> Bool) -> [a] -> Bool  
any f = or . map f
```

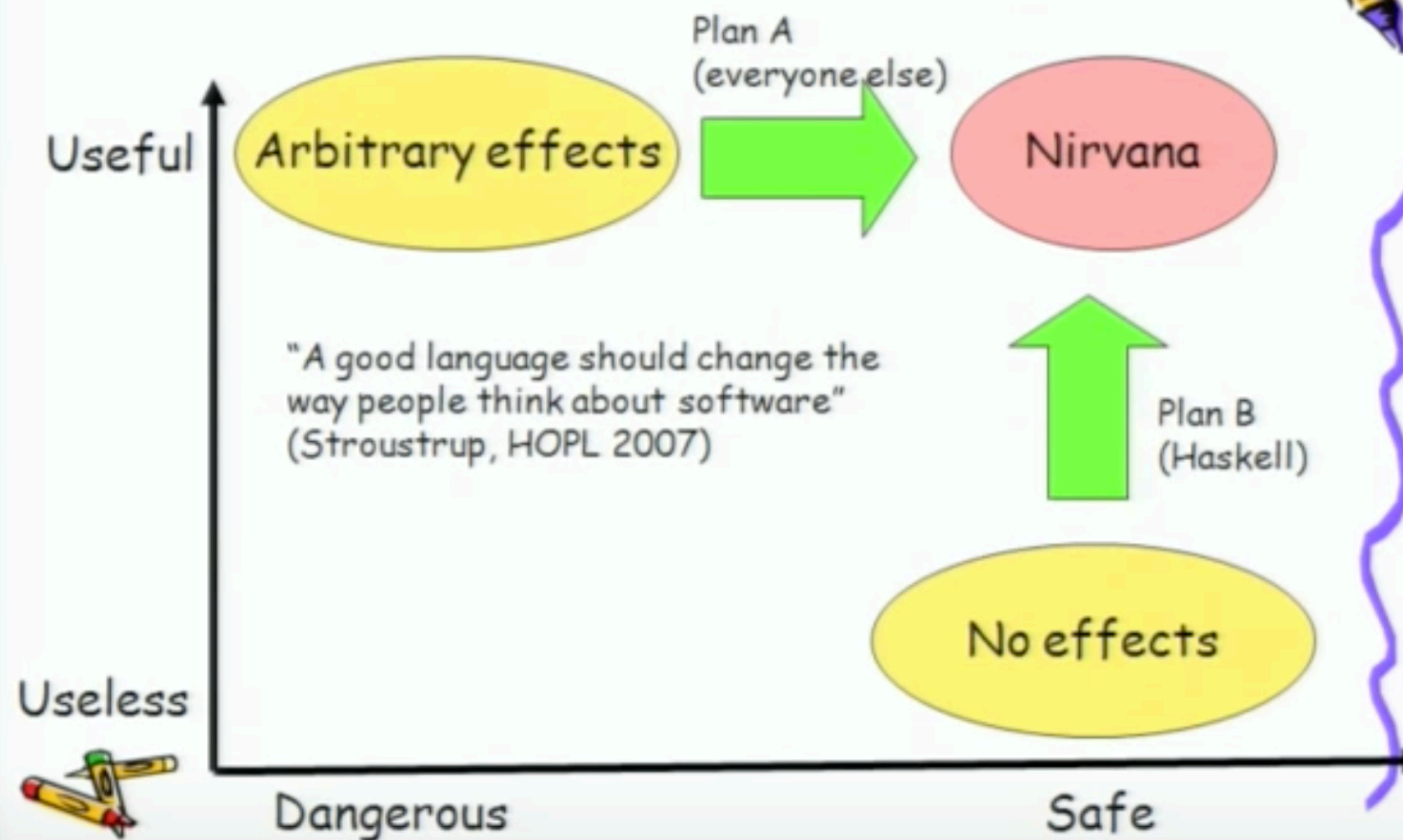
any

big picture



“a history of Haskell: being lazy with class”

# The challenge of effects



“a history of Haskell: being lazy with class”