CHAPTER 7

Flow in Pipes

Consider the flow of an incompressible viscous fluid that flows in a full pipe. In the preceding chapter, efforts were made analytically to find the relationship between the velocity, pressure, etc. for this case. In this chapter, however, from a more practical and materialistic standpoint, a method of expressing the loss using an average flow velocity is stated. By extending this approach, studies will be made on how to express losses caused by a change in the cross-sectional area of a pipe, a pipe bend and a valve in addition to the frictional loss of a pipe.

Sending water by pipe has a long history. Since the time of the Roman Empire (about 1 BC), lead pipes and clay pipes have been used for the water supply system in cities as shown in Fig. 7.1.

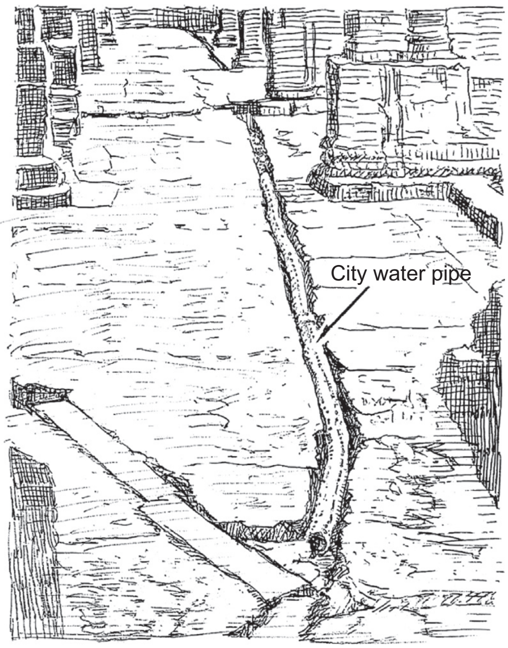

FIGURE 7.1
Lead city water pipe (Roman remains, Bath, England).

Introduction to Fluid Mechanics. http://dx.doi.org/10.1016/B978-0-08-102437-9.00007-3

7.1 FLOW IN INLET REGION

Consider a case where fluid flows from a tank into a pipe whose entrance section is fully rounded. At the entrance, the velocity distribution is roughly uniform while the static pressure head falls by $v^2/2g$ (v is the average flow velocity).

Because the velocity of a viscous fluid is zero on the wall, the fluid near the wall decelerates. Deceleration continues downstream, until eventually the boundary layers merge at the centre of the pipe. The zone shown in Fig. 7.2, between the pipe entry and the point where the velocity profile is fully developed (boundary

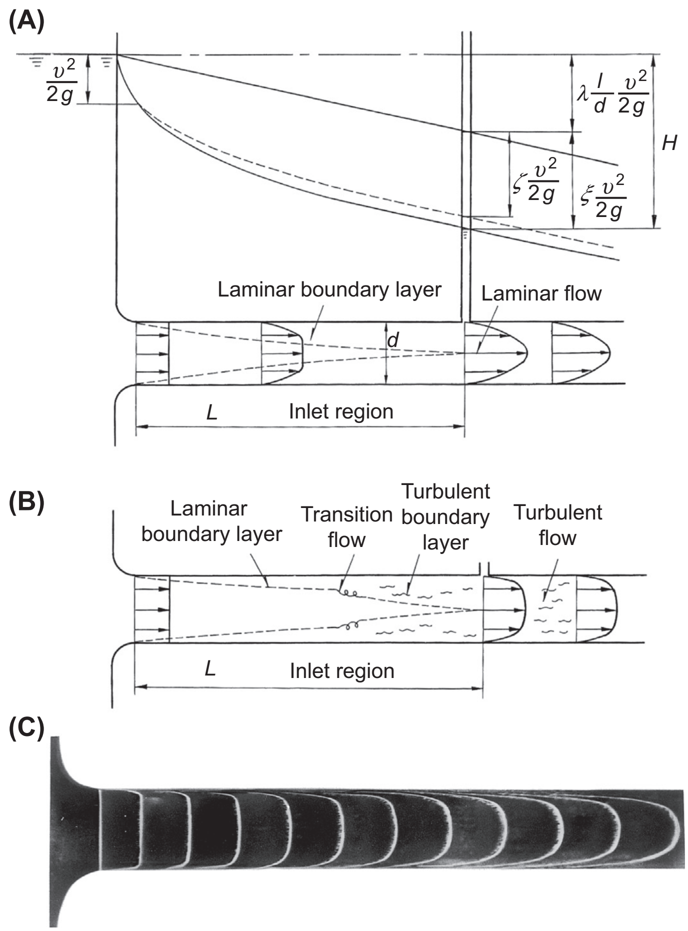

FIGURE 7.2
Flow in a circular pipe: (A) laminar flow; (B) turbulent flow; (C) laminar flow (flow visualisation using hydrogen bubble method).

layers merge), is called the inlet or entry region having a length denoted by L. Equations for L are[1]:

Laminar flow:

$$L = 0.065Re \cdot d \quad \begin{cases} \text{computation by Boussinesq} \\ \text{experiment by Nikuradse} \end{cases}$$

$L = 0.06Re \cdot d$ computation by Asao; Iwanami and Mori[1]

Turbulent flow:

$L = 0.693Re^{1/4}d$ Computation by Latzko

$L = (25{-}40)d$ Experiment by Nikuradse

Downstream of the inlet region, the static pressure in the pipe as measured by the liquid columns stemming from the pipe, shown in Fig. 7.2, drops by H below the water level in the tank, where

$$H = \lambda \frac{l}{d}\frac{v^2}{2g} + \xi \frac{v^2}{2g} \tag{7.1}$$

The length l from the inlet, $\lambda(l/d)\,(v^2/2g)$ expresses the frictional loss of head (the lost energy of fluid per unit weight). $\xi(v^2/2g)$ expresses the pressure loss equivalent to the sum of the kinetic energy stored when the velocity distribution is fully developed plus the additional frictional energy loss in excess of that in fully developed flow consumed during the change in velocity distribution.

The kinetic energy of the fluid which has attained the fully developed velocity distribution when $x = L$ is

$$E = \int_0^{d/2} 2\pi r u \frac{\rho u^2}{2}\,\mathrm{d}r \tag{7.2}$$

E is calculated by substituting the equations of the velocity distribution for laminar flow (Eq. 6.33) into u of this equation. The kinetic energy for the same flow at the average velocity is

$$E' = \frac{\pi d^2}{4} v \frac{\rho v^2}{2}$$

Putting $E/E' = \zeta$ gives $\zeta = 2$. For the case of turbulent flow, it is found by experiment to be 1.09. ζ is known as the kinetic energy correction factor.

[1] Asao, et al., Bulletin of JSME 18 (66) (1952), p. 172.

The dynamic head equivalent to this energy is

$$\frac{E}{(\pi d^2/4)v\rho g} = \zeta \frac{v^2}{2g} \tag{7.3}$$

This means that, to compensate for this increase in dynamic head when the entrance length reaches L, the dynamic head must decrease by the same amount. Furthermore, with the extra energy loss caused by the changing velocity distribution included, the value of ξ turns out to be much larger than ζ. $\xi(v^2/2g)$ expresses how much further the pressure would fall than for frictional loss in the inlet region of the pipe if a constant velocity distribution existed. With respect to the value of ξ, for laminar flow values of $\xi = 2.24$ (computation by Boussinesq), 2.16 (computation by Schiller), 2.7(experiment by Hagen) and 2.36 (experiment by Nakayama and Endo)[2] were reported, while for turbulent flow $\xi = 1.4$ (experiment by Hagen on a trumpet-like tube without entrance).

7.2 LOSS BY PIPE FRICTION

Let us study the flow in the region where the velocity distribution is fully developed after passing through the inlet region as shown in Fig. 7.3. If a fluid is flowing in the round pipe of diameter d at the average flow velocity v, let the pressures at two points distance l apart be p_1 and p_2, respectively. The relationship between the velocity v and the head loss $h = (p_1 - p_2)/(\rho g)$ is illustrated in Fig. 7.4, where, for the laminar flow, the head loss h is proportional to the flow velocity v as can clearly be seen from Eq. (6.38). For the turbulent flow, it turns out to be proportional to $v^{1.75\sim 2}$.

The head loss is expressed by the following equation as shown in Eq. (7.1):

$$h = \lambda \frac{l}{d}\frac{v^2}{2g} \tag{7.4}$$

This equation is called the Darcy—Weisbach[3] equation, where the coefficient λ is called the friction coefficient of the pipe.

7.2.1 Laminar Flow

In this case, from Eqs (6.38) and (7.4),

$$\lambda = 64\frac{\mu}{\rho v d} = \frac{64}{Re} \tag{7.5}$$

[2] Nakayama, Endo, Bulletin of JSME 24 (145) (1958), p. 658.
[3] In place of λ, many British texts use $4f$ in this equation. Because friction factor $f = \lambda/4$, it is essential to check the definition to which a value of friction factor refers. The symbol used is not a reliable guide.

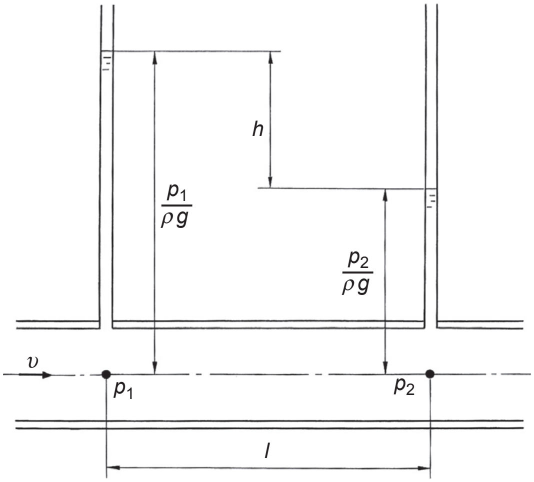

FIGURE 7.3
Pipe frictional loss.

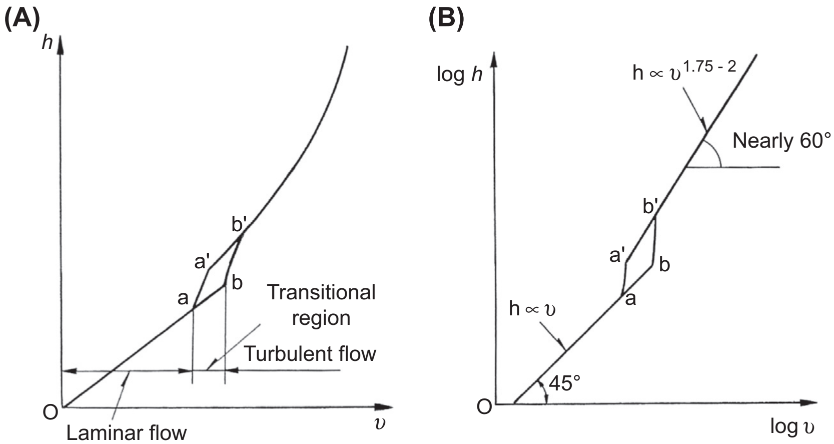

FIGURE 7.4
Relationship between flow velocity and loss head. (A) General display and (B) logarithmic display.

No effect of wall roughness is seen. The reason is probably that the flow turbulence caused by the wall face coarseness is limited to a region near the wall face because the velocity and therefore inertia are small, whereas viscous effects are large in such a laminar region.

7.2.2 Turbulent Flow

λ generally varies according to Reynolds number and the pipe wall roughness.

Smooth Circular Pipe

The roughness is inside the viscous sublayer if the height ε of wall surface roughness is

$$\varepsilon \leq 5\nu/u_* \quad \text{(fluid dynamically smooth)} \tag{7.6}$$

From Eq. (6.47) and Fig. 6.15, no effect of roughness is seen and λ varies according to Reynolds number only; thus, the pipe can be regarded as a smooth pipe.

In the case of a smooth pipe, the following equations have been developed:

Equation of Blasius

$$\lambda = 0.3164Re^{-1/4}\left(Re = 3 \times 10^3 \sim 1 \times 10^5\right) \tag{7.7}$$

Equation of Nikuradse

$$\lambda = 0.0032 + 0.221Re^{-0.237}\left(Re = 10^5 \sim 3 \times 10^6\right) \tag{7.8}$$

Equation of Karman–Nikuradse

$$\lambda = 1\Big/\left(2\log_{10}\left(Re\sqrt{\lambda}\right) - 0.8\right)^2 \left(Re = 3 \times 10^3 \sim 3 \times 10^6\right) \tag{7.9}$$

Equation of Itaya[4]

$$\lambda = \frac{0.314}{0.7 - 1.65\log_{10}Re + (\log_{10}Re)^2} \tag{7.10}$$

By combining Eq. (7.4) with Eq. (7.7), the relationship $h = cv^{1.75}$ (here c is a constant) arises, giving the relationship for turbulent flow in Fig. 7.4.

Rough Circular Pipe

From Eq. (6.52) and Fig. 6.15, where

$$\varepsilon \geq 70\nu/u_* \quad \text{(fully rough)} \tag{7.11}$$

the wall surface roughness extends into the turbulent flow region. This defines the rough pipe case where λ is determined by the roughness only and is not related to Reynolds number value.

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[4] M. Itaya, Journal of JSME 48, 332–333 (1945-2 to 12), p. 84.

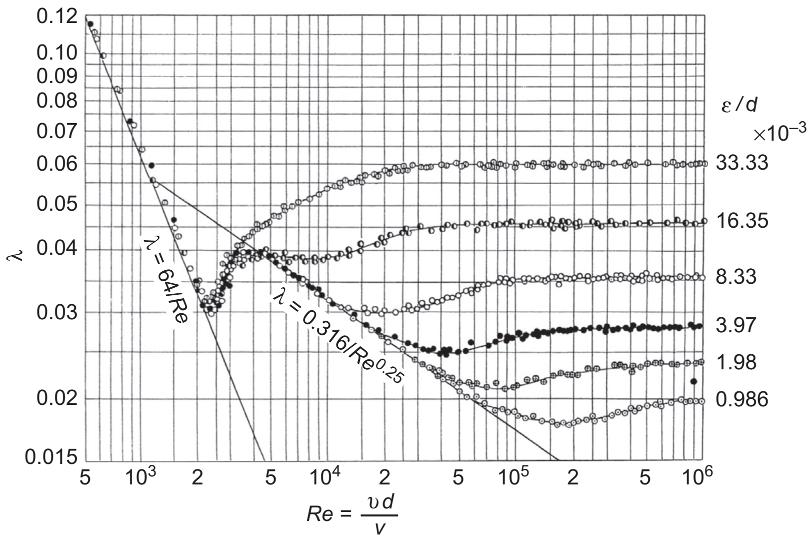

FIGURE 7.5

Friction coefficient of circular pipe roughened with sand grains.[5]

To simulate uniform roughness, Nikuradse performed an experiment in 1933 by lacquer-pasting screened sand grains of uniform diameter onto the inner wall of a tube and obtained the result shown in Fig. 7.5.[5]

According to this result, whenever $Re > 900/(\varepsilon/d)$, it turns out that

$$\lambda = \frac{1}{[1.74 - 2\log_{10}(2\varepsilon/d)]^2} \tag{7.12}$$

The velocity distribution for this case is expressed by the following equation:

$$u/u_* = 8.48 + 5.75\log_{10}(y/\varepsilon) \tag{7.13}$$

For a pipe of irregular roughness found in practice, the Moody diagram[6] shown in Fig. 7.6 is applicable. For a new commercial pipe, λ can be easily obtained from the intersection of a ε/d curve and an Re value of the Moody diagram in Fig. 7.6 using ε/d in Fig. 7.7.

7.3 FRICTIONAL LOSS ON PIPES OTHER THAN CIRCULAR PIPES

In the case of a pipe other than a circular one (e.g., oblong or oval), how can the pressure loss be found?

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[5] J. Nikuradse, V.D.I.Forshungsheft 361 (1933).
[6] L.F. Moody, N.J. Princeton, Transactions of the ASME 66 (8) (1944), p. 671.

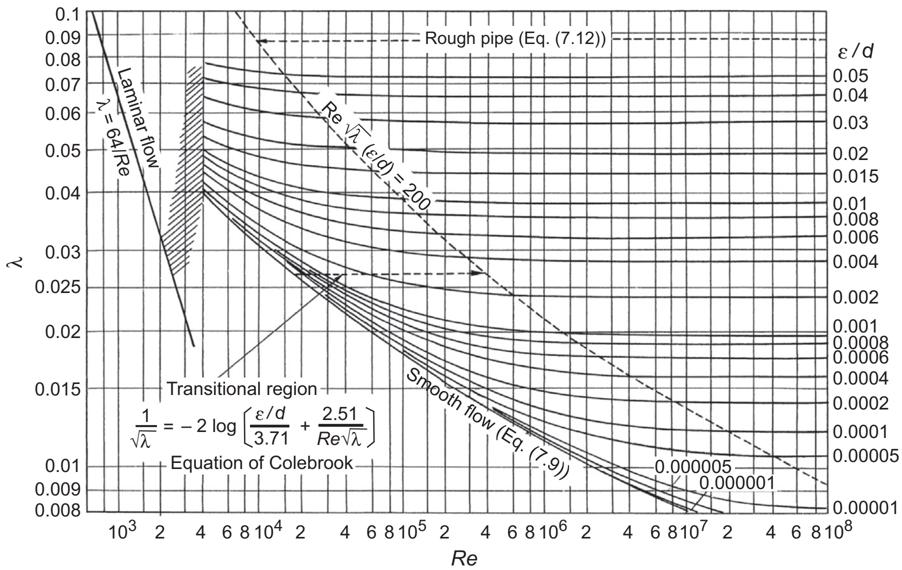

FIGURE 7.6
Moody diagram.[6]

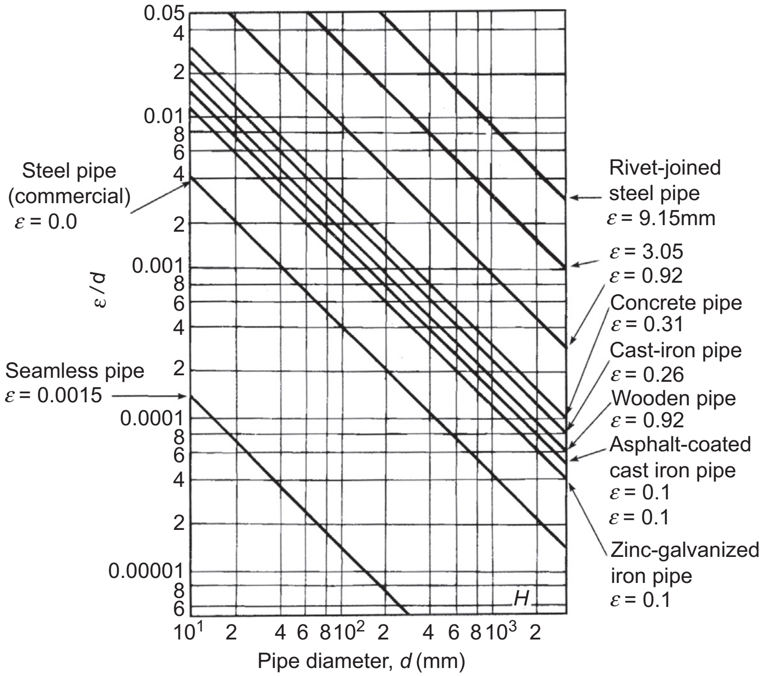

FIGURE 7.7
Roughness of commercial pipe.

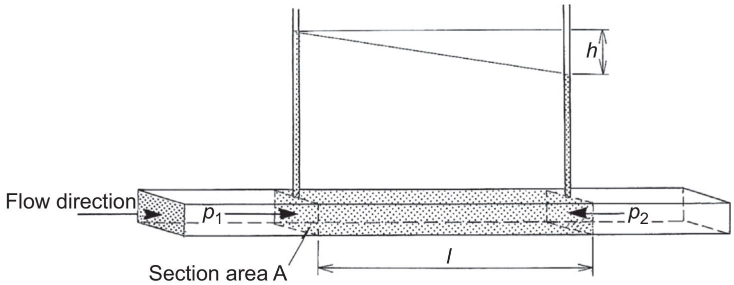

FIGURE 7.8
Flow in oblong pipe.

Where fluid flows in an oblong pipe as shown in Fig. 7.8, let the pressure drop over length l be h; the sides of the pipe be a and b, respectively; the cross-section area of the pipe be A and the wall perimeter in contact with the fluid on the section, called the length of the wetted perimeter, be s, where the shearing stress is τ_0; the shearing force acting on the pipe wall of length l is $l\tau_0 s$; and the balancing pressure force is ρghA. Then,

$$\rho ghA = \tau_0 sl \tag{7.14}$$

This equation shows that for a given pressure loss, τ_0 is determined by A/s (the ratio of the flow section area to the wetted perimeter). $A/s = m$ is called the hydraulic mean depth (see Section 8.1). In the case of a filled circular section pipe, because $A = (\pi/4)d^2$, $s = \pi d$, the relationship $m = d/4$ is obtained. So, for pipes other than a circular pipe, calculation is made using the following equation and substituting $4m$ (which is called the hydraulic diameter) as the representative size in place of d in Eq. (7.4):

$$h = \lambda \frac{l}{4m}\frac{v^2}{2g}, \quad \lambda = f(Re, \varepsilon/4m) \tag{7.15}$$

Here, assuming $Re = 4mv/\nu$, $\varepsilon/d = \varepsilon/4m$ may be found from the Moody diagram for a circular pipe. Meanwhile, $4m$ is described by following equations, respectively, for an oblong section of a by b and for coaxial pipes of inner diameter d_1 and outer diameter d_2:

$$4\frac{ab}{2(a+b)} = \frac{2ab}{a+b}, \quad 4\frac{(\pi/4)\left(d_2{}^2 - d_1{}^2\right)}{\pi(d_1 + d_2)} = d_2 - d_1 \tag{7.16}$$

7.4 VARIOUS LOSSES IN PIPE LINES[7]

In a pipe line, in addition to frictional loss, head loss is caused by additional turbulence arising when fluid flows through such components as change of area, change of direction, branching, junction, bend and valve. The head loss for such cases is generally expressed by the following equation:

$$h_s = \zeta \frac{v^2}{2g} \tag{7.17}$$

v in the above equation is the mean flow velocity on a section not affected by the section where the head loss is produced. Where the mean flow velocity changes upstream or downstream of the loss-producing section, the larger of the flow velocities is generally used.

7.4.1 Loss With Sudden Change of Area

Flow Expansion

The flow expansion loss h_s for a suddenly widening pipe becomes the following, as already shown by Eq. (5.44):

$$h_s = \frac{(v_1 - v_2)^2}{2g} = \left(1 - \frac{A_1}{A_2}\right)^2 \frac{{v_1}^2}{2g} \tag{7.18}$$

In practice, however, it becomes

$$h_s = \xi \frac{(v_1 - v_2)^2}{2g} \tag{7.19}$$

or as follows:

$$h_s = \zeta \frac{{v_1}^2}{2g} \tag{7.20}$$

$$\zeta = \xi \left(1 - \frac{A_1}{A_2}\right)^2 \tag{7.21}$$

Here, ξ is a value near 1.

At the outlet of the pipe as shown in Fig. 7.9, because $v_2 \approx 0$, Eq. (7.19) becomes

$$h_s = \xi \frac{{v_1}^2}{2g} \tag{7.22}$$

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[7] Technical literature, Fluid resistance in pipe lines and fluid conduit, The Japanese Society of Mechanical Engineers (1979).

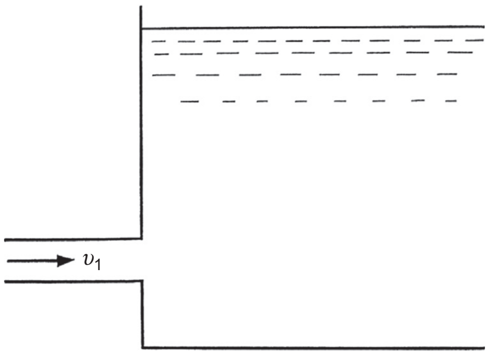

FIGURE 7.9
Outlet of pipeline.

Flow Contraction

Because of inertia, section 1 (section area A_1) of the fluid shrinks to section 2 (section area A_c) and then widens to section 3 (section area A_2). The loss when the flow is accelerated is extremely small, followed by a head loss similar to that in the case of sudden expansion as shown in Fig. 7.10. Like Eq. (7.18), it is expressed by

$$h_s = \frac{(v_c - v_2)^2}{2g} = \left(\frac{A_2}{A_c} - 1\right)^2 \frac{{v_2}^2}{2g} = \left(\frac{1}{C_c} - 1\right)^2 \frac{{v_2}^2}{2g} \tag{7.23}$$

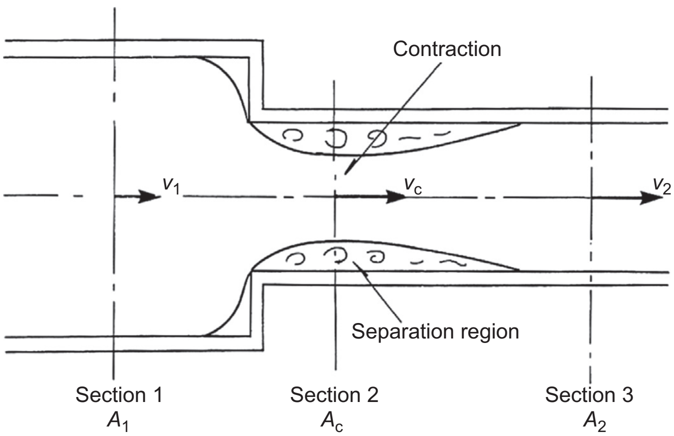

FIGURE 7.10
Sudden contraction pipe.

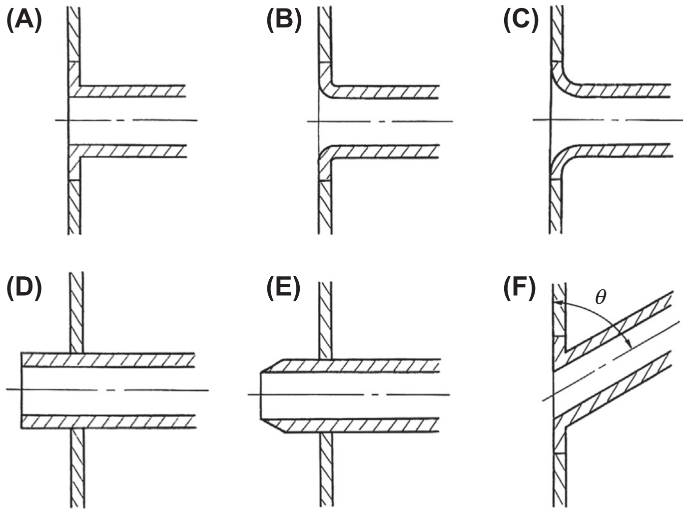

FIGURE 7.11
Inlet shape and loss factor (A) $\zeta = 0.50$; (B) $\zeta = 0.25$; (C) $\zeta = 0.06–0.005$; (D) $\zeta = 0.56$; (E) $\zeta = 3.0–1.3$ and (F) $\zeta = 0.5 + 0.3\cos\theta + 0.2\cos^2\theta$.

Here $C_c = A_c/A_2$ is a contraction coefficient. For example, when $A_2/A_1 = 0.1$, $C_c = 0.61$.[8]

Inlet of Pipe Line

As shown in Fig. 7.11, the loss of head in the case where fluid enters from a large vessel is expressed by the following equation:

$$h_s = \zeta\frac{v^2}{2g} \tag{7.24}$$

In this case, however, ζ is the inlet loss factor and v is the mean flow velocity in the pipe. The value of ζ will be the value as show in Fig. 7.11.[9]

Throttle

A device which decreases the flow area, bringing about the extra resistance in a pipe, is generally called a throttle. There are three kinds of throttle, i.e., choke, orifice and nozzle. If the length of the narrow section is long compared with its diameter, the throttle is called a choke. Because the orifice is explained in Sections 5.2.2 and 11.2.2, and a nozzle is dealt with in Section 11.2.2, only the choke will be explained here.

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[8] H. Richter, Rohrhydraulik, third Aufl., Springer, (1985), p. 172.
[9] J. Weisbach, Ingenieur-und Machienen-Mechanik, 1, (1896), p. 1003.

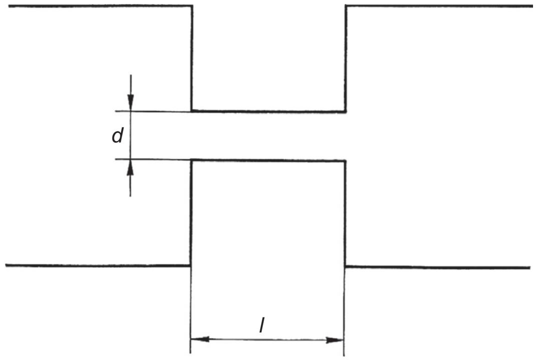

FIGURE 7.12
Choke.

The coefficient of discharge C in Fig. 7.12 can be expressed as follows, as Eq. (5.25), where the difference between the pressure upstream and downstream of the throttle is Δp:

$$Q = C\frac{\pi d^2}{4}\sqrt{\frac{2\Delta p}{\rho}} \tag{7.25}$$

and C is expressed as a function of the choke number $\sigma = Q/(\nu l)$, C is as shown in Fig. 7.13, and is expressed by the following equations[10] if the entrance is not rounded:

$$C = \frac{1}{1.16 + 6.25\sigma^{-0.61}} \tag{7.26}$$

and if the entrance is rounded:

$$C = \frac{1}{1 + 5.3/\sqrt{\sigma}} \tag{7.27}$$

7.4.2 Loss With Gradual Change of Area

Divergent Pipe or Diffuser

The head loss for a divergent pipe as shown in Fig. 7.14 is expressed in the same manner as Eq. (7.19) for a suddenly widening pipe:

$$h_s = \xi\frac{(v_1 - v_2)^2}{2g} \tag{7.28}$$

[10] Hibi, et al. Journal of the Japan Hydraulics & Pneumatics Society 2, (1971), p. 72.

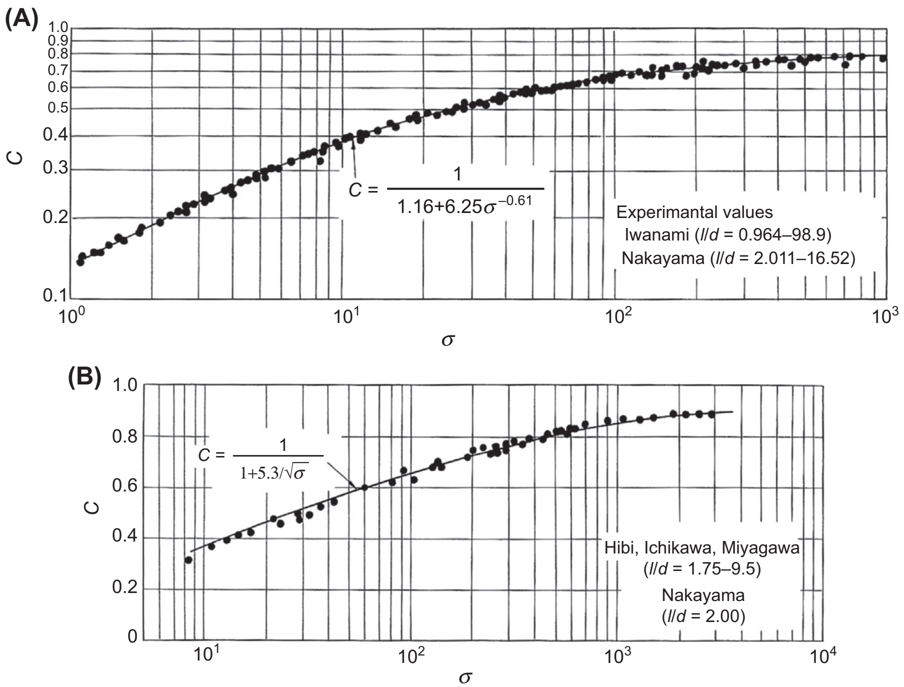

FIGURE 7.13
Coefficient of discharge for cylindrical chokes: (A) entrance not rounded; (B) entrance rounded.[10]

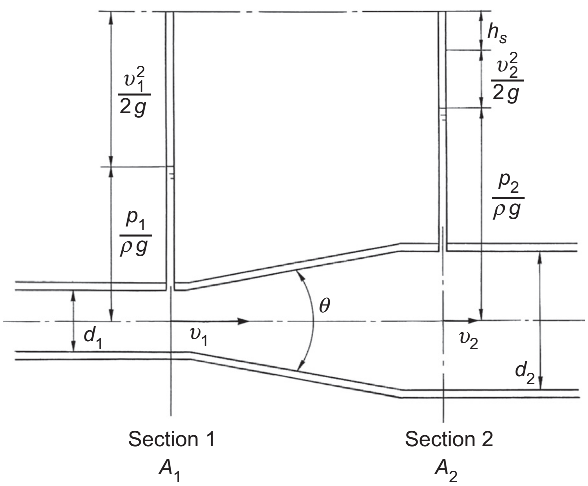

FIGURE 7.14
Divergent flow.

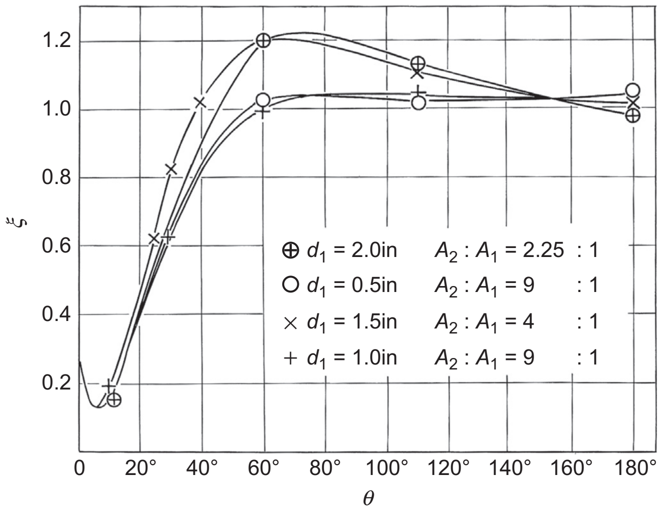

FIGURE 7.15
Loss factor for divergent pipes.[11,12]

The value of ξ for circular divergent pipes is shown in Fig. 7.15.[11,12] The value of ξ varies according to θ. For a circular section, $\xi = 0.135$ (minimum) when $\theta \approx 5°30'$; for the rectangular section, $\xi = 0.145$ (minimum) when $\theta \approx 6$ degrees; and $\xi \approx 1$ (almost constant) whenever $\theta = 50-60$ degrees or more.

For a two-dimensional duct, if θ is small the fluid flow attaches to one of the side walls as a result of a wall attachment phenomenon (the wall effect).[13] In the case of a circular pipe, when θ becomes larger than the angle which gives the minimum value of ξ, the flow separates midway as shown in Fig. 7.16. Because of the turbulence accompanying such a separation of flow, the loss of head suddenly increases. This phenomenon is visualised in Fig. 7.17.

A divergent pipe is also used as a diffuser to convert kinetic energy into pressure energy. In the case of Fig. 7.14, the following equation is obtained by applying Bernoulli's principle:

$$\frac{p_1}{\rho g} + \frac{{v_1}^2}{2g} = \frac{p_2}{\rho g} + \frac{{v_2}^2}{2g} + h_s$$

[11] A.H. Gibson, Hydraulics, Constable, London, (1952), p. 91.
[12] T. Uematsu, Bulletin of JSME 2.7, (1936), p. 254.
[13] Because such a phenomenon where fluid flows attached to the wall face was found in 1932 by H. Coanda, it is called the Coanda effect after him. The effect is the basic principle of the technology of fluidics.

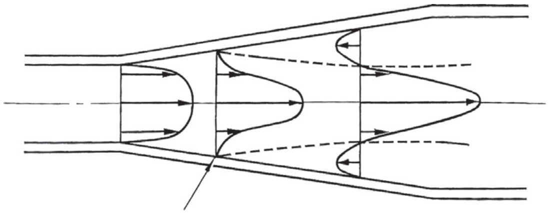

FIGURE 7.16
Velocity distribution in a divergent pipe.

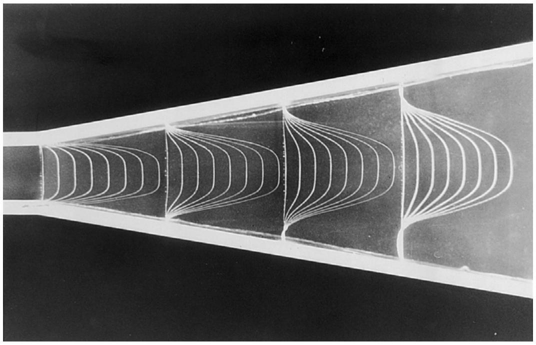

FIGURE 7.17
Separation occurring in a divergent pipe (hydrogen bubble method), in water; inlet velocity = 6 cm/s, *Re* (inlet port) = 900, divergent angle = 20 degrees.

Therefore

$$\frac{p_2 - p_1}{\rho g} = \frac{{v_1}^2 - {v_2}^2}{2g} - h_s \tag{7.29}$$

Putting $p_{2\text{th}}$ for p_2 for the case where there is no loss,

$$\therefore \frac{p_{2\text{th}} - p_1}{\rho g} = \frac{{v_1}^2 - {v_2}^2}{2g} \tag{7.30}$$

The pressure recovery efficiency η for a diffuser is therefore

$$\eta = \frac{p_2 - p_1}{p_{2\text{th}} - p_1} = 1 - \frac{h_s}{({v_1}^2 - {v_2}^2)/2g} \tag{7.31}$$

Substituting in Eq. (7.28), the above equation becomes

$$\eta = 1 - \xi\frac{v_1 - v_2}{v_1 + v_2} = 1 - \xi\frac{1 - A_1/A_2}{1 + A_1/A_2} \tag{7.32}$$

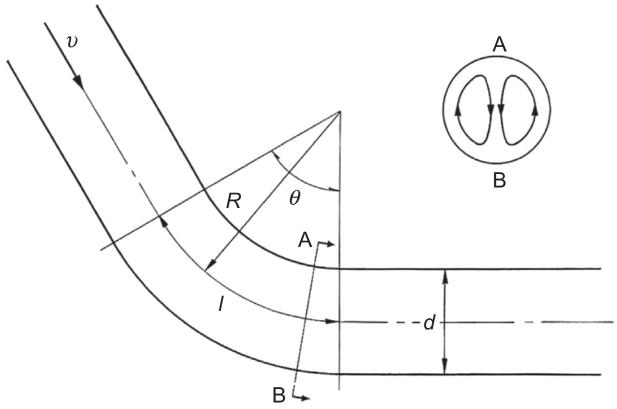

FIGURE 7.18
Bend.

Convergent Pipe

In the case where a pipe section gradually becomes smaller, because the pressure decreases in the direction of flow, the flow runs freely without extra turbulence. Therefore, losses other than the pipe friction are normally negligible.

7.4.3 Loss Whenever the Flow Direction Changes

Bend

The gently curving part of a pipe shown in Fig. 7.18 is referred to as a pipe bend. In a bend, in addition to the head loss caused by pipe friction, a loss caused by the change in flow direction is also produced. The total head loss h_b is expressed by the following equation:

$$h_b = \zeta_b \frac{v^2}{2g} = \left(\zeta + \lambda \frac{l}{d}\right) \frac{v^2}{2g} \tag{7.33}$$

Here, ζ_b is the total loss factor and ζ is the loss factor caused by the bend effect. The values of ζ are shown in Table 7.1.[14,15]

In a bend, secondary flow is produced as shown in the figure because of the introduction of the centrifugal force, and the loss increases. If guide blades are fixed in the bend section, the head loss can be very small.

[14] A. Hoffman, Mitt. Hydr. Inst. T. H. Munchen, 3, (1929), 45.
[15] R. Wasielewski, Mitt. Hydr. Inst. T. H. Munchen, 5, (1932), 66.

Table 7.1 Loss Factor ζ for Bends (Smooth Wall $Re = 225{,}000$, Coarse Wall Face $Re = 146{,}000$)[14,15]

Wall Face	θ (degrees)	$R/d = 1$	2	3	4	5
Smooth	15	0.03	0.03	0.03	0.03	0.03
	22.5	0.045	0.045	0.045	0.045	0.045
	45	0.14	0.14	0.08	0.08	0.07
	60	0.19	0.12	0.095	0.085	0.07
	90	0.21	0.135	0.10	0.085	0.105
Coarse	90	0.51	0.51	0.23	0.18	0.20

Elbow

As shown in Fig. 7.19, the section where the pipe curves sharply is called an elbow. The head loss h_b is given in the same form as Eq. (7.33). Because the flow separates from the wall in the curving part, the loss is larger than in the case of a bend. Table 7.2 shows values of ζ for elbows.[16]

7.4.4 Branch Pipe and Junction Pipe

Pipe Branch

As shown in Fig. 7.20, a pipe dividing into separate pipes is called a pipe branch. Putting h_{s1} as the head loss produced when the flow runs from pipe

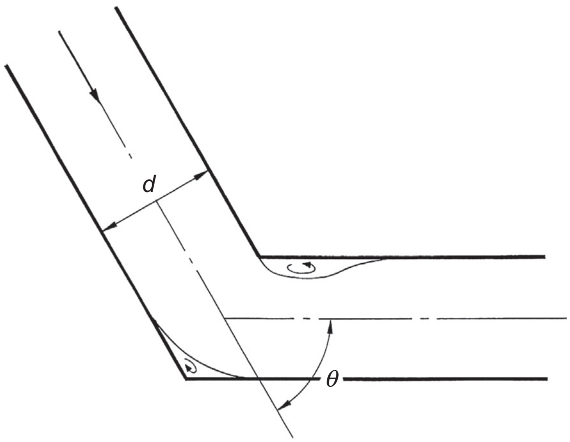

FIGURE 7.19
Elbow.

[16] H. Kirchbach, W. Schubart, Mitt. Hydr. Inst. T. H. Munchen, 2, (1928), 72; 3 (1929), p. 121.

Table 7.2 Loss Factor ζ for Elbows[16]

	θ degrees	5 degrees	10 degrees	15 degrees	22.5 degrees	30 degrees	45 degrees	60 degrees	90 degrees
ζ	Smooth	0.016	0.034	0.042	0.066	0.130	0.236	0.471	1.129
	Coarse	0.024	0.044	0.062	0.154	0.165	0.320	0.687	1.265

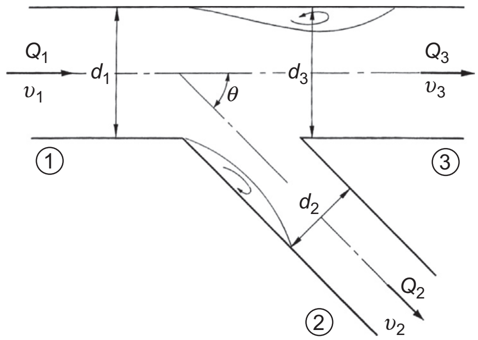

FIGURE 7.20
Pipe branch.

① to pipe ③, and h_{s2} as the head loss produced when the flow runs from pipe ① to pipe ②, these are, respectively, expressed as follows:

$$h_{s1} = \zeta_1 \frac{{v_1}^2}{2g}, \quad h_{s2} = \zeta_2 \frac{{v_1}^2}{2g} \tag{7.34}$$

Because the loss factors ζ_1, ζ_2 vary according to the branch angle θ, diameter ratio d_1/d_2 or d_1/d_3 and the discharge ratio Q_1/Q_2 or Q_1/Q_3, experiments were performed for various combinations. Such results were summarised.[17,18]

Pipe Junction

As shown in Fig. 7.21, two pipe branches converging into one is called a pipe junction. Putting h_{s1} as the head loss when the flow runs from pipe ① to pipe ③ and h_{s2} as the head loss when the flow runs from pipe ② to pipe ③, these are expressed as follows:

$$h_{s1} = \zeta_1 \frac{{v_3}^2}{2g}, \quad h_{s2} = \zeta_2 \frac{{v_3}^2}{2g} \tag{7.35}$$

Values of ζ_1 and ζ_1 are similar to the case of pipe branch.[17,18]

[17] G. Vogel, Mitt. Hydr. Inst. T. H. Munchen, 1 (1926), 75; 2, (1928), p. 61.
[18] F. Petermann, Mitt. Hydr. Inst. T. H. Munchen, 3 (1929), p. 98.

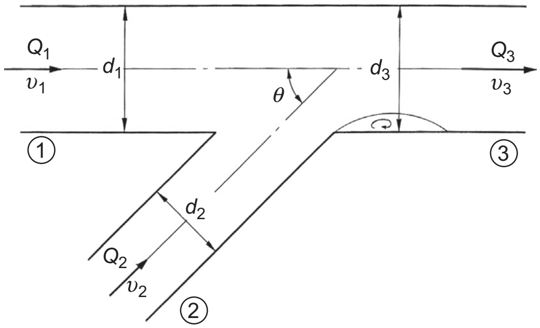

FIGURE 7.21
Pipe junction.

7.4.5 Valve and Cock

Head loss in valves is brought about by changes in their section areas and is expressed by Eq. (7.17) provided that v indicates the mean flow velocity at the point not affected by the valve.

Gate Valve

The valve as shown in Fig. 7.22 is called a gate valve. Putting d as the diameter and d' as the valve opening, ζ varies according to d'/d. Table 7.3 shows values of ζ for a l inch (2.54 cm) nominal diameter valve.[19]

Globe Valve

Table 7.4 shows values of ζ for the globe valve shown in Fig. 7.23, at various openings.[20]

Butterfly Valve

Table 7.5 shows values of ζ for a butterfly valve[21] shown in Fig. 7.24. As the inclination angle θ of the valve plate increases, the section area immediately downstream of the valve suddenly increases, bringing about an increased value of ζ.

For a circular butterfly valve, when $\theta = 0$ degree, the value of ζ is

$$\zeta \approx \frac{t}{d} \tag{7.36}$$

where t and d are thickness and diameter of the valve plate, respectively.

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[19] Corp, C.I., Bulletin of the University of Wisconsin, Engineering Series, vol. 9-1, (1922), p. 1.
[20] I. Oki, Suirikigaku (Hydraulics), Iwanami, Tokyo, (1942), p. 344.
[21] J. Weisbach, Ingenieur-und Meschienen-Mechanik, 1, (1896), p. 1050.

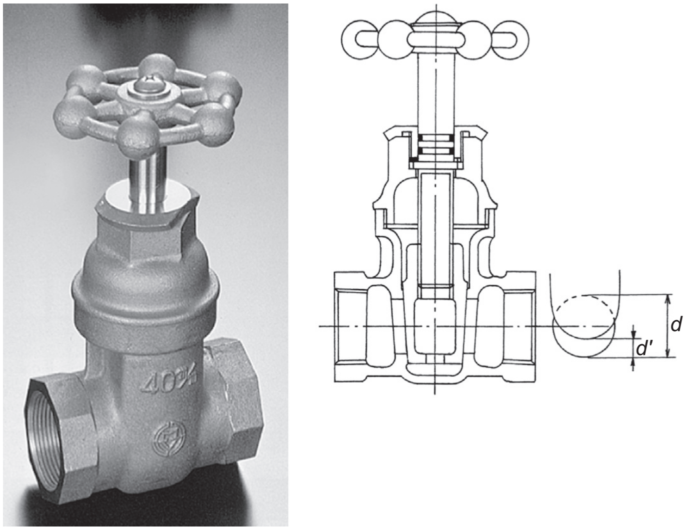


FIGURE 7.22
Gate valve.

Table 7.3 Values for ζ for 1-inch Gate Valves ($d = 25.5$ mm)[19]

d'/d	1/8	1/4	3/8	1/2	3/4	1
ζ	211	40.3	10.15	3.54	0.882	0.233

Table 7.4 Values of ζ for 1-inch Screw-in Globe Valves ($d = 25.5$ mm)[20]

l/d	1/4	1/2	3/4	1
ζ	16.3	10.3	7.68	6.09

Cock

Table 7.6 shows values[21] of ζ for a cock shown in Fig. 7.25. For cocks, too, as angle θ increases, large changes in section area of flow are brought about, increasing the value of ζ.

Other Valves

Values[22] of ζ for various valves are shown on Table 7.7.

[22] F.D. Yeaple, Hydraulic and Pneumatic Power Control, McGraw-Hill, New York, (1966), p. 89.

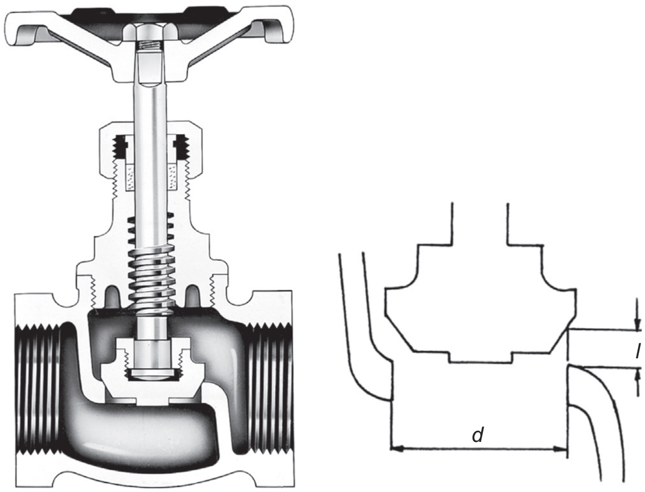


FIGURE 7.23
Globe valve.

Table 7.5 Values of ζ for Circular Butterfly Valves[21]

θ degrees	10 degrees	20 degrees	30 degrees	50 degrees	70 degrees
ζ	0.52	1.54	3.91	32.6	751

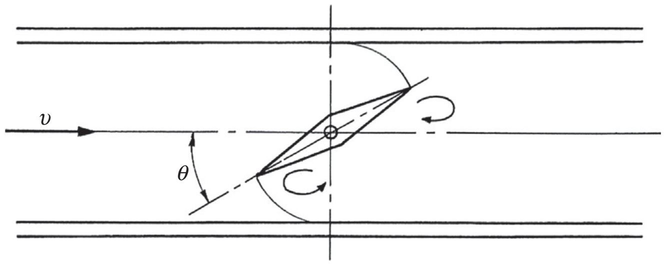


FIGURE 7.24
Butterfly valve.

Table 7.6 Values of ζ for Cocks[21]

θ degrees	10 degrees	30 degrees	50 degrees	60 degrees
ζ	0.29	5.47	52.6	206

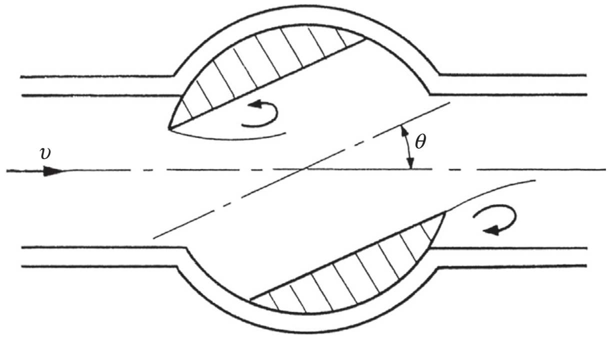


FIGURE 7.25
Cock.

7.4.6 Total Loss Along a Pipe Line

For a pipe with flow velocity v, inner diameter d and length l, the total loss from pipe entrance to exit is

$$h = \left(\lambda\frac{l}{d} + \sum\zeta\right)\frac{v^2}{2g} \tag{7.37}$$

The first term on the right expresses the total loss by friction, while $\Sigma\zeta(v^2/2g)$ represents the sum of the loss heads at such sections as the entrance, bends and valves. Whenever a pipe line consists of pipes of different diameters, it is necessary to use the appropriate valve for the flow velocity for each pipe.

When two tanks with a water-level differential h are connected by a pipe line, the exit velocity energy is generally lost. Therefore,

$$h = \left(\lambda\frac{l}{d} + \sum\zeta + 1\right)\frac{v^2}{2g} \tag{7.38}$$

However, when the pipe line is long such that $l/d > 2000$ and it has no valves of small opening etc., losses other than frictional loss may be neglected. Conversely, if h is known, the flow velocity could be obtained from Eq (7.37) or (7.38).

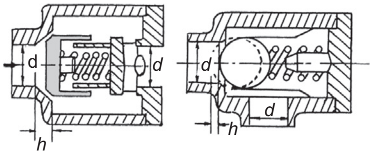
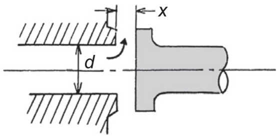
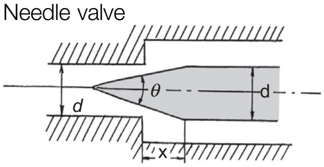
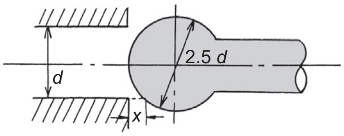
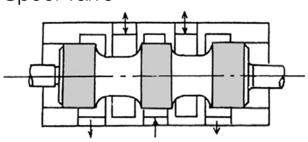
In general, for urban water pipes, $v = 1.0-1.5$ m/s is typical for long pipe runs, while up to approximately 2.5 m/s is typical for short pipe runs. For the head-race of a hydraulic power plant, 2−5 m/s is the usual range.

7.5 PUMPING TO HIGHER LEVELS

A pump can deliver to high levels because it gives energy to the water as shown in Fig. 7.26. The head H across the pump is called the total head. The differential height H_a between two water levels is called the actual head and

$$H = H_a + h \tag{7.39}$$

where h is the sum of h_s and h_d expressing the total loss.

<table>
<tr><th colspan="8">**Table 7.7** Loss Factor for Various Valves[22]</th></tr>
<tr><th>Valve</th><th colspan="7">Loss Coefficients, ζ</th></tr>
<tr><td rowspan="2">Relief valve
</td><td>h/d</td><td>0.05</td><td>0.1</td><td>0.15</td><td>0.2</td><td>0.25</td><td>0.3</td></tr>
<tr><td>ζ</td><td>3.35</td><td>2.85</td><td>2.4</td><td>2.4</td><td>1.7</td><td>1.35</td></tr>
<tr><td>Disc valve
</td><td colspan="7">Throttle area $a = \pi dx$
Section area of valve seat hole $A = \pi d^2/4$
When $x = d/4$, $a = A$
Loss coefficient $\zeta = 1.3 + 0.2(A/a)^2$</td></tr>
<tr><td>Needle valve
</td><td colspan="7">$a = \pi\left(dx \tan(\theta/2) - x^2 \tan^2(\theta/2)\right)$
$Ax = 0$ when $x = 0$
$\zeta = 0.5 + 0.15(A/a)^2$</td></tr>
<tr><td>Ball valve
</td><td colspan="7">$a \approx 0.75\pi dx$
$\zeta = 0.5 + 0.15(A/a)^2$</td></tr>
<tr><td>Spool valve
</td><td colspan="7">At full open position
$\zeta = 3–5.5$</td></tr>
</table>

The volume of water that passes through a pump in a unit time is called the pump discharge. Because the energy that a pump gives water in a unit time is H per unit weight, the energy L_w given to water per unit time is

$$L_w = \rho g Q H \tag{7.40}$$

The energy L_w as shown above is sometimes known as the water horsepower.

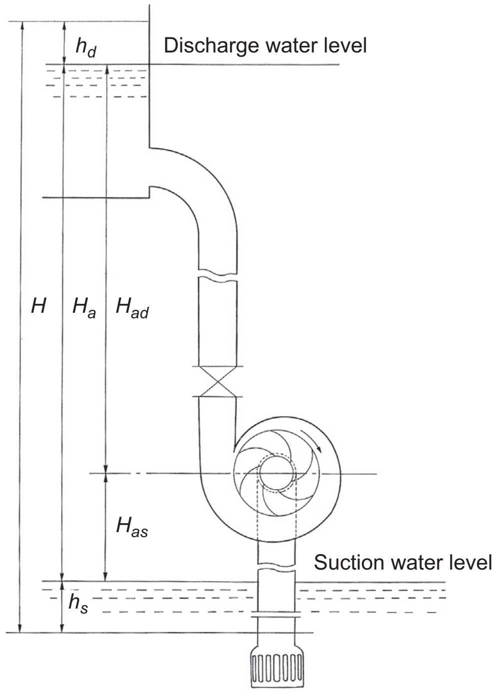


FIGURE 7.26
Storage pump: H, total head; H_a, actual head; H_{as}, suction head; H_{ad}, discharge head; h_s, losses on suction; h_d, losses on discharge side.

The power L_s needed by a pump is called the shaft horsepower.

$$\frac{L_w}{L_s} = \eta \tag{7.41}$$

where η is the efficiency of the pump. Because the energy supplied to a pump is not all transmitted to the water as a result of the energy loss within the pump, it turns out that $\eta < 1$.

As shown in Fig. 7.27, the curve that expresses the relationship between the pump discharge Q and the head H is called the characteristic curve or head curve. In general, the head loss h is proportional to the square of the mean flow velocity in the pipe, and therefore to the square of the pump discharge and is called the resistance curve. It must be summed with H_a to give the pump load curve. The pump discharge is given, as shown in Fig. 7.27, by the intersecting point of the head curve and this load curve.

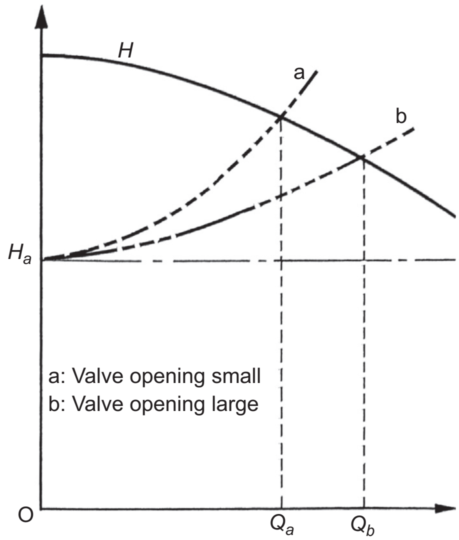


FIGURE 7.27
Total head and load curve of pump.

7.6 PROBLEMS

1. Verify that the kinetic energy for laminar flow in a circular pipe with a fully developed velocity distribution is twice that with uniform velocity.
2. What is the relationship between the flow velocity and the pressure loss in a circular pipe?
3. For laminar flow in a circular pipe, verify that the pipe frictional coefficient can be expressed by the following equation:

$$\lambda = \frac{64}{Re}$$

4. For turbulent flow in a circular pipe, show that, assuming the pipe frictional coefficient is subject to $\lambda = 0.3164\, Re^{-1/4}$, the pressure loss is proportional to a power of 1.75 of the mean flow velocity.
5. For flow in a circular pipe, with constant pipe friction coefficient, show that the frictional head loss is inversely proportional to the fifth power of the pipe diameter. Also, if the diameter is measured with α% error, what would be the percentage error in head loss?
6. How much head loss will be produced by sending 0.5 m^3/min of water a distance of 2000 m using commercial steel pipes of diameter 50 mm? Also, what would be the head loss if the diameter is 100 mm? The water temperature is assumed to be 20°C.

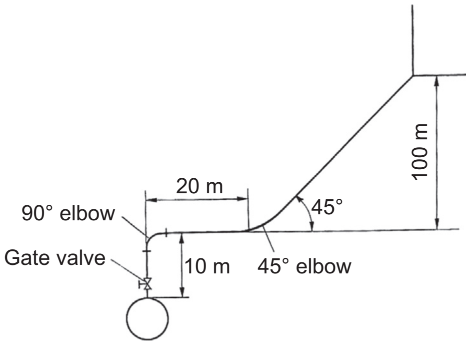


FIGURE 7.28
System for Exercise 7.

7. What is the necessary shaft horsepower to send 1 m^3/min of water through a conduit 100 mm in diameter as shown in Fig. 7.28? Assume pump efficiency $\eta = 80\%$, loss coefficient of sluice valve $\zeta_v = 0.175$, of 90 degrees elbow $\zeta_{90} = 1.265$, of 45 degrees elbow $\zeta_{45} = 0.320$, and pipe frictional coefficient $\lambda = 0.026$.
8. A flow of 0.6 m^3/s of air discharges through a square duct of sides 20 cm. What is the pressure loss if the duct length is 50 m? Assume an air temperature of 20°C, standard atmospheric pressure, and smooth walls for the duct.
9. Water flows through a sudden expansion where a circular pipe of 40 mm diameter is directly connected to one of 80 mm. If the discharge is 0.08 mm^3/min, find the expansion loss.
10. There is a pipeline connecting a circular tube with a diameter of 40 mm to a circular tube with a diameter of 80 mm via a diffuser spreading angle of 10 degree. If a flow rate of 0.3 m^3/min is delivered to this pipe, calculate the loss head hs and the pressure recovery rate η of the spread tube.