Modelling Uncertainty

SC03 Group 1

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1 Question 1

p : probability of going up by 1 unit
1 - p : probability of going down by 1 unit

p($X_i = 1$) = p , p(X_i = -1) = q
q = 1 - p
$S_n = \sum_{i=1}^{n} X_i$

1(a)

Largest value of S_n : all values of $X_i = 1$
$\sum_{i=1}^{n} 1$ = n

Smallest value of S_n : all values of X_i = - 1
$\sum_{i=1}^{n} -1$ = -n

1(b)

P($S_n = 0$) as a function of n when :
Let Y $\sim$ Binomial (n,p) where Y is the no. of days when
X_i is 1

For $S_n = 0$, while n is even :
Equal no. of days when X_i = -1
Y = n/2

While n is odd : Impossible for Sn = 0
P ($S_n = 0$ | n is an odd number) = 0
P ($S_n = 0$ | n is an even number) = $\binom{N}{k}$

1(c)

P (S_n = 2m + 1) where m is group number (1) , $S_n = 2(1) + 1 = 3$
P ($S_n = 3$ | even n) = 0
For $S_n = 3$, 3 + (n -3)/2 cases of $X_i = 1$
(n-3)/2 cases of X_i = -1

P ($S_n = 3$ | odd n) = $\binom{n}{(n-3)/2} p^{3+(n-3)/2} p^{(n-3)/2}$
= $\binom{n}{(n-3)/2} p^{3+(n-3)/2} p^{(n-3)/2}$

$$P(S_n = 3) = \left\{ \begin{array}{ll} 0 & if\ n\ is\ odd \\ \binom{n}{(n-3)/2)} p^{3+(n-3)/2} p^{(n-3)/2} & if\ n\ is\ even \end{array} \right\}$$

1(d)

$X_i \sim$ Bernoulli (p)
Let Y $\sim$ binomial (n,p), where Y is the no. of days where $X_i = 1$
S_n = Y - (n - Y)
S_n = 2Y - n

1(e)

E[Y] = np
E[S_n] = E[2Y - n] = 2E[Y] - n = 2np - n
E[S_n] = (2p - 1)n

$$\text{Var}[S_n] = \text{Var}[2Y - n] = \text{Var}[2Y] - \text{Var}[n] = 4\text{Var}[Y] - \text{Var}[n]$$
$$\text{Var}[n] = 0 \text{ , n =/= Random Variable}$$
$$4\text{Var}[Y] - \text{Var}[n] = 4\text{Var}[Y]$$
$$\text{Var}[Y] = np(1-p) = npq \text{ , Variance of Binomial Distribution}$$
$$\text{Var}[S_n] = 4\text{Var}[y] = 4npq$$

1(f) S_n exists as a binomial distribution as it is only a scalar and stretching translation of a binomial distribution. When n is large, we can deduce that it carries the same mean and variance.

$$\mu = \text{E}[S_n] = n(2p-1) \text{ ; } \sigma^2 = 4npq$$
$$S_n \sim \text{normal}(n(2p-1) \text{ , } 4npq)$$

2 Question 2

NEEDS UPDATING

$$\text{P}(X_i = 1) = p \text{ , P}(X_j = -1) = q \text{ , where } 0 \text{ ¡ p ¡ } 1$$

S_n refers to net amount of investment units e.g for step 3, i = 2

$$\text{hence, max}S_n = n$$
$$\text{P(max}S_n) = p^n$$
$$\text{min}S_n = -n$$
$$\text{P(min}S_n = q_n$$

2(a)

$$N = k + S_n, \text{ where N = 3 and k = 1 (given) =¿ } S_n = 2$$
$$\text{hence win condition : no. of p = 2 + no. of q}$$

$$\text{Probability of winning , P(win)} = p^2 + pqp^2 + pqpqp^2 + ...$$
$$= p^2 + pqp^2 + p^2q^2p^2 + p^3q^3p^2 + ...$$
$$= p^2(1 + pq + (pq)^2 + (pq)^3 + ...)$$
$$= p^2/(1 - pq)$$
$$= p^2 \text{ / } (p^2 - p + 1 \text{)}$$
$$= \text{no. of p - no. of q}$$

2(b)

$$\text{Let B = probability of } (X_i = 1) \text{ , UPDATING}$$
$$\text{A = probability of winning with k}$$
$$\text{Law of probability : P(B) = P(B}\sim\text{A)P(A) + Updating}$$
$$\text{Whole question needs updating}$$

2(c)

$$P_k = p^(1 - Q) \text{ CONTINUE FROM HERE}$$

We can also place code directly into latex without importing it from a file:

```
print("Hi, I'm Python 3!")
```

2.1 First subsection

We can create and format mathematical expressions like so:

$$x' = x \cdot scos\theta - y \cdot ssin\theta + t_x$$
$$y' = x \cdot ssin\theta + y \cdot scos\theta + t_y$$

We can also make a nice list:

1. I am the first thing in the list
2. I am the second thing in the list

We can inline mathematical expressions such as this one "$4\sigma_0$" using the "$" sign. We can make mathematical expressions that occupy their own line, like this:

$$u = (x - x_0)\frac{1}{4\sigma_0}cos\theta_0 - (y - y_0)\frac{1}{4\sigma_0}sin\theta_0 + 4 = (0 - 16)\frac{1}{4} - 0 + 4$$

2.2 Second subsection

We can also make tables and charts using the array type like so:

$$\phi = \left\{ \begin{array}{ll} \theta_0 + \theta_{pt} & if\ \theta_0 + \theta_{pt} \in [0, 2\pi) \\ \theta_0 + \theta_{pt} + 2\pi & if\ \theta_0 + \theta_{pt} < 0 \\ \theta_0 + \theta_{pt} - 2\pi & if\ \theta_0 + \theta_{pt} \geq 2\pi \end{array} \right\}$$

I can start an enumerated list of items here...

1. One thing
2. Another thing

And then...

3 Second section

...I can continue it here!

3. Yet more stuff
4. Some other things

Inserting figures is also relatively easy to do:

Figure 1: I can embed images too

We can make a table with centered elements:

$$\begin{bmatrix} 1.1754 & -0.8334 & 193.4191 \\ 0.2062 & 1.0380 & -141.0333 \\ -0.0008 & 0.0007 & 1.0000 \end{bmatrix}$$

There you go! That should be enough to get you started on LaTeX!