# CSC165H1 Problem Set 1

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## 1. Propositional formulas

(a)(i) The truth table for (a)

| p | q | (p $\Rightarrow$ q) | $\neg$q | (p $\Rightarrow$ q) $\Rightarrow$ $\neg$q |
|---|---|---|---|---|
| T | T | T | F | F |
| T | F | F | T | T |
| F | T | T | F | F |
| F | F | T | T | T |

(a)(ii) A logically equivalent formula for (a)

$$\begin{aligned} (p \Rightarrow q) \Rightarrow \neg q &\equiv (\neg p \vee q) \Rightarrow \neg q && (equivalence\ from\ class) \\ &\equiv \neg(\neg p \vee q) \vee \neg q && (equivalence\ from\ class) \\ &\equiv (p \wedge \neg q) \vee \neg q && (DeMorgan's\ Law) \end{aligned}$$

(b)(i) The truth table for (b)

| p | $\neg$r | q | $\neg$p | p $\Rightarrow$ $\neg$r | $\neg$p $\Rightarrow$ q | (p $\Rightarrow$ $\neg$r) $\wedge$ ($\neg$p $\Rightarrow$ q) |
|---|---|---|---|---|---|---|
| T | T | T | F | T | T | T |
| T | T | F | F | T | T | T |
| T | F | T | F | F | T | F |
| F | T | T | T | T | T | T |
| T | F | F | F | F | T | F |
| F | T | F | T | T | F | F |
| F | F | T | T | T | T | T |
| F | F | F | T | T | F | F |

(b)(ii) A logically equivalent formula for (b)

$$\begin{aligned}(p \Rightarrow \neg r) \wedge (\neg p \Rightarrow q) &\equiv (\neg p \vee \neg r) \wedge (\neg p \Rightarrow q) \quad (equivalence\ from\ class)\\ &\equiv (\neg p \vee \neg r) \wedge (p \vee q) \quad (equivalence\ from\ class)\end{aligned}$$

## 2. Fixed Points

(a) F(x): ∃x ∈ ℕ, f(x) = x

(b) L(a): P = {x ∈ ℕ|f(x) = x}, ∃a ∈ P, ∀x ∈ P, (a < x) ∨ (a = x)

(c) G(x): P = {x ∈ ℕ|f(x) = x}, ∃a ∈ P, ∀x ∈ P, (x < a) ∨ (x = a)

(d)(1) The fixed points of f are: {0, 1, 2, 3, 4, 5, 6}

(d)(2) The least fixed point of f is: 0

(d)(3) The greatest fixed point of f is: 6

## 3. Partial Orders

(a) R(x, y): ∃k ∈ ℕ, x = ky, where x,y ∈ ℕ

(b) R(a,a) = R(b,b) = R(c,c) = R(d,d) = True and all other values are False.

(c) R(a,a) = R(b,d) = R(c,d) = True and all other values are False.

$$\begin{aligned}Justify: \quad & 1.a\ is\ maximal:\ Since\ R(a,a)\ is\ True,\ so\ a\ is\ maximal.\\ & 2.not\ a\ greatest\ element:\ Since\ R(b,d)\ = R(c,d)\ = True,\ we\ know:\\ & \quad \forall b,c,d \in D,\ (b \leq d) \wedge (c \leq d),\ so\ d\ is\ also\ maximal.\\ & \quad Since\ we\ cannot\ figure\ the\ relation\ between\ a\ and\ d,\\ & \quad so\ a\ is\ not\ a\ greatest\ element.\end{aligned}$$

## 4. One-to-one functions

(a) $4 \times 4 \times 4 = 64$

(b) $4 \times 3 \times 2 = 24$

(c) $3 \times 2 \times 1 \times C_4^2 = 36$

(d) $\forall \mathrm{x} \in \mathbb{N}, \exists \mathrm{y} \in \mathbb{N}, \mathrm{R(x, y)} \wedge (\forall \mathrm{z} \in \mathbb{N}, \mathrm{R(x, z)} \Rightarrow \mathrm{z = y})$

(e) $\mathrm{Function(R)} \wedge (\forall \mathrm{y} \in \mathbb{N}, \exists \mathrm{x} \in \mathbb{N}, \mathrm{R(x, y)})$

(f) $\mathrm{Function(R)} \wedge (\forall \mathrm{x} \in \mathbb{N}, \forall \mathrm{y} \in \mathbb{N}, \mathrm{R(x, y)} \Rightarrow (\forall \mathrm{z} \in \mathbb{N}, \mathrm{R(z, y)} \Rightarrow \mathrm{z = x}))$

(g) $\mathrm{Function(R)} \wedge (\forall \mathrm{z} \in \mathbb{N}, \exists \mathrm{y} \in \mathbb{N}, \mathrm{z} < \mathrm{y} \wedge (\exists \mathrm{x} \in \mathbb{N}, \mathrm{R(x, y)}))$

(h) $\mathrm{Function(R)} \wedge (\exists \mathrm{z} \in \mathbb{N}, \forall \mathrm{y} \in \mathbb{N}, (\forall \mathrm{x} \in \mathbb{N}, \neg \mathrm{R(x, y)}) \Rightarrow \mathrm{y} \leq \mathrm{z})$