Programming Assignment 1: Learning Distributed Word Representations

Version: 1.1

Changes by Version:

- (v1.1)
 - 1. (Part 1) Update calculate_log_co_occurence() to include the count for the 4th word in the sentence for diagonal entries. Remove text on needing to add 1 as it is already done in the code
 - 2. (1.5) Removed the line defining unnecessary loss variable
 - 3. (1.5) We added a gradient checker function using finite difference called <code>check_Glove_gradients()</code>. You can run the specified cell in the notebook to check your gradient implementation for both the symmetric and asymmetric models before moving forward.
 - 4. (Part 3) Fixed error with evaluate() function when calling compute loss()

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Due Date: Friday, Feb. 4, at 11:59pm

Based on an assignment by George Dahl

For CSC413/2516 in Winter 2022 with Professor Jimmy Ba and Professor Bo Wang

Submission: You must submit two files through MarkUs:

- 1. A PDF file containing your writeup, titled *a1-writeup.pdf*, which will be the PDF export of this notebook (i.e., by printing this notebook webpage as PDF). Your writeup must be typed. There will be sections in the notebook for you to write your responses. Make sure that the relevant outputs (e.g. print_gradients() outputs, plots, etc.) are included and clearly visible.
- 2. This al-code.ipynb iPython Notebook.

The programming assignments are individual work. See the Course Syllabus for detailed policies.

You should attempt all questions for this assignment. Most of them can be answered at least partially even if you were unable to finish earlier questions. If you think your computational results are incorrect, please say so; that may help you get partial credit.

The teaching assistants for this assignment are Harris Chan and Caroline Malin-Mayor. Send your email with subject "[CSC413] PA1" to mailto: csc413-2022-01-tas@cs.toronto.edu or post on Piazza with the tag pa1.

Introduction

In this assignment we will learn about word embeddings and make neural networks learn about words. We could try to match statistics about the words, or we could train a network that takes a sequence of words as input and learns to predict the word that comes next.

This assignment will ask you to implement a linear embedding and then the backpropagation computations for a neural language model and then run some experiments to analyze the learned representation. The amount of code you have to write is very short but each line will require you to think very carefully. You will need to derive the updates mathematically, and then implement them using matrix and vector operations in NumPy.

Starter code and data

First, perform the required imports for your code:

```
import collections
import pickle
import numpy as np
import os
from tqdm import tqdm
import pylab
from six.moves.urllib.request import urlretrieve
import tarfile
import sys
import itertools

TINY = 1e-30
EPS = 1e-4
nax = np.newaxis
```

If you're using colaboratory, this following script creates a folder - here we used 'CSC413/A1' - in order to download and store the data. If you're not using colaboratory, then set the path to wherever you want the contents to be stored at locally.

You can also manually download and unzip the data from [http://www.cs.toronto.edu/~jba/a1_data.tar.gz] and put them in the same folder as where you store this notebook.

Feel free to use a different way to access the files data.pk, partially_trained.pk, and raw_sentences.txt.

The file *raw_sentences.txt* contains the sentences that we will be using for this assignment. These sentences are fairly simple ones and cover a vocabulary of only 250 words (+ 1 special [MASK] token word).

```
# Setup working directory
# Change this to a local path if running locally
%mkdir -p /content/CSC413/A1/
%cd /content/CSC413/A1
# Helper functions for loading data
# adapted from
# https://github.com/fchollet/keras/blob/master/keras/datasets/cifar10.py
def get file(fname,
        origin,
        untar=False,
        extract=False,
        archive format='auto',
        cache dir='data'):
  datadir = os.path.join(cache dir)
  if not os.path.exists(datadir):
     os.makedirs(datadir)
  if untar:
     untar fpath = os.path.join(datadir, fname)
     fpath = untar fpath + '.tar.gz'
```

```
else:
        fpath = os.path.join(datadir, fname)
    print('File path: %s' % fpath)
    if not os.path.exists(fpath):
        print('Downloading data from', origin)
        error_msg = 'URL fetch failure on {}: {} -- {}'
        try:
            try:
                urlretrieve(origin, fpath)
            except URLError as e:
                raise Exception(error_msg.format(origin, e.errno, e.reason))
            except HTTPError as e:
                raise Exception(error_msg.format(origin, e.code, e.msg))
        except (Exception, KeyboardInterrupt) as e:
            if os.path.exists(fpath):
                os.remove(fpath)
            raise
    if untar:
        if not os.path.exists(untar fpath):
            print('Extracting file.')
            with tarfile.open(fpath) as archive:
                archive.extractall(datadir)
        return untar_fpath
    if extract:
        _extract_archive(fpath, datadir, archive_format)
    return fpath
    /content/CSC413/A1
# Download the dataset and partially pre-trained model
get file(fname='a1 data',
                         origin='http://www.cs.toronto.edu/~jba/a1 data.tar.gz',
                         untar=True)
```

```
drive_location = 'data'
PARTIALLY_TRAINED_MODEL = drive_location + '/' + 'partially_trained.pk'
data_location = drive_location + '/' + 'data.pk'

File path: data/al_data.tar.gz
    Downloading data from http://www.cs.toronto.edu/~jba/al_data.tar.gz
    Extracting file.
```

We have already extracted the 4-grams from this dataset and divided them into training, validation, and test sets. To inspect this data, run the following:

```
data = pickle.load(open(data location, 'rb'))
print(data['vocab'][0]) # First word in vocab is [MASK]
print(data['vocab'][1])
print(len(data['vocab'])) # Number of words in vocab
print(data['vocab']) # All the words in vocab
print(data['train inputs'][:10]) # 10 example training instances
    [MASK]
    all
    251
    ['[MASK]', 'all', 'set', 'just', 'show', 'being', 'money', 'over', 'both', 'years', 'four', 'through', 'durin
     [[ 28  26  90  144]
     [184 44 249 117]
     [183 32 76 122]
     [117 247 201 186]
     [223 190 249
     [ 42 74 26 32]
     [242 32 223 32]
     [223 32 158 144]
     [ 74 32 221 32]
     [ 42 192 91 68]]
```

Now data is a Python dict which contains the vocabulary, as well as the inputs and targets for all three splits of the data. data['vocab'] is a list of the 251 words in the dictionary; data['vocab'][0] is the word with index 0, and so on.

data['train_inputs'] is a 372,500 x 4 matrix where each row gives the indices of the 4 consecutive context words for one of the 372,500 training cases. The validation and test sets are handled analogously.

Even though you only have to modify two specific locations in the code, you may want to read through this code before starting the

→ Part 1: GloVe Word Representations (3pts)

In this section we will be implementing a simplified version of GloVe. Given a corpus with V distinct words, we define the co-occurrence matrix $X \in \mathbb{N}^{V \times V}$ with entries X_{ij} representing the frequency of the i-th word and j-th word in the corpus appearing in the same context - in our case the adjacent words. The co-occurrence matrix can be symmetric (i.e., $X_{ij} = X_{ji}$) if the order of the words do not matter, or asymmetric (i.e., $X_{ij} \neq X_{ji}$) if we wish to distinguish the counts for when i-th word appears before j-th word. GloVe aims to find a d-dimensional embedding of the words that preserves properties of the co-occurrence matrix by representing the i-th word with two d-dimensional vectors \mathbf{w}_i , $\tilde{\mathbf{w}}_i \in \mathbb{R}^d$, as well as two scalar biases b_i , $\tilde{b}_i \in \mathbb{R}$. Typically we have the dimension of the embedding d much smaller than the number of words V. This objective can be written as:

$$L(\{\mathbf{w}_i, \tilde{\mathbf{w}}_i, b_i, \tilde{b}_i\}_{i=1}^V) = \sum_{i,j=1}^V (\mathbf{w}_i^{\mathsf{T}} \tilde{\mathbf{w}}_j + b_i + \tilde{b}_j - \log X_{ij})^2$$

Note that each word is represented by two d-dimensional embedding vectors \mathbf{w}_i , $\tilde{\mathbf{w}}_i$ and two scalar biases b_i , \tilde{b}_i . When the bias terms are omitted and we tie the two embedding vectors $\mathbf{w}_i = \tilde{\mathbf{w}}_i$, then GloVe corresponds to finding a rank-d symmetric factorization of the co-occurrence matrix.

Answer the following questions:

▼ 1.1. GloVe Parameter Count [0pt]

Given the vocabulary size V and embedding dimensionality d, how many parameters does the GloVe model have? Note that each word in the vocabulary is associated with 2 embedding vectors and 2 biases.

1.1 Answer: 4dV

▼ 1.2 Expression for the Vectorized Loss function [0.5pt]

In practice, we concatenate the V embedding vectors into matrices $\mathbf{W}, \tilde{\mathbf{W}} \in \mathbb{R}^{V \times d}$ and bias (column) vectors $\mathbf{b}, \tilde{\mathbf{b}} \in \mathbb{R}^{V}$, where V denotes the number of distinct words as described in the introduction. Rewrite the loss function L (Eq. 1) in a vectorized format in terms of $\mathbf{W}, \tilde{\mathbf{W}}, \mathbf{b}, \tilde{\mathbf{b}}, X$. You are allowed to use elementwise operations such as addition and subtraction as well as matrix operations such as the Frobenius norm and/or trace operator in your answer.

Hint: Use the all-ones column vector $\mathbf{1} = [1 \dots 1]^T \in \mathbb{R}^V$. You can assume the bias vectors are column vectors, i.e. implicitly a matrix with V rows and 1 column: $\mathbf{b}, \tilde{\mathbf{b}} \in \mathbb{R}^{V \times 1}$

1.2 Answer:
$$L = ||W\tilde{W}^{\mathsf{T}} + b\mathbf{1}^{\mathsf{T}} + 1\tilde{b}^{\mathsf{T}} - logX||_F^2$$

▼ 1.3. Expression for gradient $\frac{\partial L}{\partial \mathbf{W}}$ [0.5pt]

Write the vectorized expression for $\frac{\partial L}{\partial \mathbf{W}}$, the gradient of the loss function L with respect to the embedding matrix \mathbf{W} . The gradient should be a function of \mathbf{W} , $\tilde{\mathbf{W}}$, $\tilde{\mathbf{b}}$, $\tilde{\mathbf{b}}$, X.

Hint: Make sure that the shape of the gradient is equivalent to the shape of the matrix. You can use the all-ones vector as in the previous question.

1.3 Answer:
$$\nabla_{\mathbf{W}} L = 2(W\tilde{W}^{\mathsf{T}} + b\mathbf{1}^{\mathsf{T}} + 1\tilde{b}^{\mathsf{T}} - logX)\tilde{W}$$

▼ 1.4 Implement Vectorized Loss Function [1pt]

Implement the loss_Glove() function of GloVe.

See YOUR CODE HERE Comment below for where to complete the code

Note that you need to implement both the loss for an *asymmetric* model (from your answer in question 1.2) and the loss for a *symmetric* model which uses the same embedding matrix **W** and bias vector **b** for the first and second word in the co-occurrence, i.e.

 $\tilde{\mathbf{W}} = \mathbf{W}$ and $\tilde{\mathbf{b}} = \mathbf{b}$ in the original loss.

Hint: You may take advantage of NumPy's broadcasting feature for the bias vectors: https://numpy.org/doc/stable/user/basics.broadcasting.html

We have provided a few functions for training the embedding:

- calculate_log_co_occurence computes the log co-occurrence matrix of a given corpus
- train Glove runs momentum gradient descent to optimize the embedding
- loss Glove: TO BE IMPLEMENTED.
 - INPUT
 - $V \times d$ matrix w (collection of V embedding vectors, each d-dimensional)
 - Vxd matrix w tilde
 - V x 1 vector b (collection of V bias terms)
 - Vx1vector b_tilde
 - V x V log co-occurrence matrix.
 - OUTPUT
 - loss of the GloVe objective
- grad_GloVe: TO BE IMPLEMENTED.
 - INPUT:
 - V x d matrix w (collection of V embedding vectors, each d-dimensional), embedding for first word;
 - V x d matrix w tilde, embedding for second word;
 - V x 1 vector b (collection of V bias terms);
 - V x 1 vector b_tilde, bias for second word;
 - V x V log co-occurrence matrix.
 - OUTPUT:
 - V x d matrix grad_w containing the gradient of the loss function w.r.t. w;
 - V x d matrix grad_W_tilde containing the gradient of the loss function w.r.t. W_tilde;
 - V x 1 vector grad b which is the gradient of the loss function w.r.t. b.

■ V x 1 vector grad b tilde which is the gradient of the loss function w.r.t. b tilde.

Run the code to compute the co-occurence matrix.

```
vocab size = len(data['vocab']) # Number of vocabs
def calculate log co occurence(word data, symmetric=False):
  "Compute the log-co-occurence matrix for our data."
  log co occurence = np.zeros((vocab size, vocab size))
  for input in word data:
    # Note: the co-occurence matrix may not be symmetric
    log_co_occurence[input[0], input[1]] += 1
    log co occurence[input[1], input[2]] += 1
    log co occurence[input[2], input[3]] += 1
   # Diagonal entries are just the frequency of the word
    log co occurence[input[0], input[0]] += 1
    log co occurence[input[1], input[1]] += 1
    log co occurence[input[2], input[2]] += 1
    log_co_occurence[input[3], input[3]] += 1
    # If we want symmetric co-occurence can also increment for these.
    if symmetric:
      log_co_occurence[input[1], input[0]] += 1
      log_co_occurence[input[2], input[1]] += 1
      log_co_occurence[input[3], input[2]] += 1
  delta smoothing = 0.5 # A hyperparameter. You can play with this if you want.
  log_co_occurence += delta_smoothing # Add delta so log doesn't break on 0's.
  log co occurence = np.log(log co occurence)
  return log_co_occurence
asym log co occurence train = calculate log co occurence(data['train inputs'], symmetric=False)
asym log co occurence valid = calculate log co occurence(data['valid inputs'], symmetric=False)
```

• TO BE IMPLEMENTED: Implement the loss function. You should vectorize the computation, i.e. not loop over every word.

```
def loss GloVe(W, W tilde, b, b tilde, log co occurence):
 """ Compute the GloVe loss given the parameters of the model. When W tilde
 and b tilde are not given, then the model is symmetric (i.e. W tilde = W,
 b tilde = b).
 Args:
   W: word embedding matrix, dimension V x d where V is vocab size and d
    is the embedding dimension
   W tilde: for asymmetric GloVe model, a second word embedding matrix, with
    dimensions V x d
   b: bias vector, dimension V.
   b_tilde: for asymmetric GloVe model, a second bias vector, dimension V
   log_co_occurence: V x V log co-occurrence matrix (log X)
 Returns:
   loss: a scalar (float) for GloVe loss
 n, = log co occurence.shape
 # Symmetric Case, no W tilde and b tilde
 if W tilde is None and b tilde is None:
   # Symmetric model
   loss = np.sum((W @ W.T + b @ np.ones([1,n]) +
              np.ones([n,1])@b.T - log co occurence)**2)
   else:
   # Asymmetric model
   loss = np.sum((W @ W_tilde.T + b @ np.ones([1,n]) +
              np.ones([n,1])@b_tilde.T - log_co_occurence)**2)
   return loss
```

▼ 1.5. Implement the gradient update of GloVe. [1pt]

Implement the grad_Glove() function which computes the gradient of GloVe.

See YOUR CODE HERE Comment below for where to complete the code

Again, note that you need to implement the gradient for both the symmetric and asymmetric models.

• TO BE IMPLEMENTED: Calculate the gradient of the loss function w.r.t. the parameters W, \tilde{W} , b, and b. You should vectorize the computation, i.e. not loop over every word.

```
def grad GloVe(W, W tilde, b, b tilde, log co occurence):
  """Return the gradient of GloVe objective w.r.t its parameters
 Args:
   W: word embedding matrix, dimension V x d where V is vocab size and d
     is the embedding dimension
   W tilde: for asymmetric GloVe model, a second word embedding matrix, with
     dimensions V x d
    b: bias vector, dimension V.
   b_tilde: for asymmetric GloVe model, a second bias vector, dimension V
    log co occurence: V x V log co-occurrence matrix (log X)
  Returns:
    grad W: gradient of the loss wrt W, dimension V x d
    grad W tilde: gradient of the loss wrt W tilde, dimension V x d. Return
     None if W tilde is None.
    grad b: gradient of the loss wrt b, dimension V x 1
   grad_b_tilde: gradient of the loss wrt b, dimension V x 1. Return
     None if b_tilde is None.
  n,_ = log_co_occurence.shape
  if W_tilde is None and b_tilde is None:
   # Symmmetric case
   loss = (W @ W.T + b @ np.ones([1,n]) +
           np.ones([n,1])@b.T - 0.5*(log co occurence + log co occurence.T))
    grad W = 4 * (W.T @ loss).T
   grad b = 4 * (np.ones([1,n]) @ loss).T
```

To help you debug your GloVe gradient computation, we have included a finite-difference gradien checker function defined below:

```
for name in params dict:
 if params dict[name] is None:
   continue
 dims = params dict[name].shape
 is matrix = (len(dims) == 2)
 if not is matrix:
   print()
 if params_dict[name].shape != grads_dict[name].shape:
   print('The gradient for {} should be size {} but is actually {}.'.format(
       name, params_dict[name].shape, grads_dict[name].shape))
   return
 # Run finite difference for that param
 for count in range(1000):
   if is_matrix:
        slc = np.random.randint(0, dims[0]), np.random.randint(0, dims[1])
   else:
        slc = np.random.randint(dims[0])
   params dict plus = params dict.copy()
   params dict plus[name] = params dict[name].copy()
   params dict plus[name][slc] += EPS
   obj plus = loss GloVe(params dict plus["W"],
                          params_dict_plus["W_tilde"],
                          params dict plus["b"],
                          params_dict_plus["b_tilde"],
                          log_co_occurence)
   params_dict_minus = params_dict.copy()
   params dict minus[name] = params dict[name].copy()
   params_dict_minus[name][slc] -= EPS
   obj minus = loss GloVe(params dict minus["W"],
                          params dict minus["W tilde"],
                          params dict minus["b"],
                          params dict minus["b tilde"],
                          log co occurence)
   empirical = (obj plus - obj minus) / (2. * EPS)
```

```
exact = grads_dict[name][slc]
rel = relative_error(empirical, exact)
if rel > 5e-4:
    print('The loss derivative has a relative error of {}, which is too large for param {}.'.format(rel, nam return False
print('The gradient for {} looks OK.'.format(name))
```

Run the cell below to check if your grad_Glove function passes the checker. The function will check for both the symmetric and asymmetric loss, for each of the parameter variables whether its gradient computation looks ok. The expected output is:

```
Checking asymmetric loss gradient...

The gradient for W looks OK.

The gradient for W_tilde looks OK.

The gradient for b looks OK.

The gradient for b_tilde looks OK.

Checking symmetric loss gradient...

The gradient for W looks OK.

The gradient for b looks OK.
```

Note: If you update the <code>grad_Glove</code> cell while debugging, make sure to run the <code>grad_Glove</code> cell again before re-running the cell below to check the gradient.

• **TODO**: Run this cell below to check the gradient implementation

```
np.random.seed(0)

# Store the final losses for graphing
init_variance = 0.05  # A hyperparameter. You can play with this if you want.
embedding_dim = 16

W = init_variance * np.random.normal(size=(vocab_size, embedding_dim))

W_tilde = init_variance * np.random.normal(size=(vocab_size, embedding_dim)))
b = init_variance * np.random.normal(size=(vocab_size, 1))
b_tilde = init_variance * np.random.normal(size=(vocab_size, 1))
```

```
print("Checking asymmetric loss gradient...")
check_GloVe_gradients(W, W_tilde, b, b_tilde, asym_log_co_occurence_train)
print("\nChecking symmetric loss gradient...")
check_GloVe_gradients(W, None, b, None, asym_log_co_occurence_train)

Checking asymmetric loss gradient...
The gradient for W looks OK.
The gradient for W_tilde looks OK.
The gradient for b looks OK.
The gradient for b_tilde looks OK.
Checking symmetric loss gradient...
The gradient for W looks OK.
The gradient for W looks OK.
The gradient for Dooks OK.
```

Now that you have checked taht the gradient is correct, we define the training function for the model given the initial weights and ground truth log co-occurence matrix:

```
def train GloVe(W, W tilde, b, b tilde, log co occurence train, log co occurence valid, n epochs, do print=False):
  "Traing W and b according to GloVe objective."
 n, = log_co_occurence_train.shape
  learning rate = 0.05 / n # A hyperparameter. You can play with this if you want.
  train loss list = np.zeros(n epochs)
  valid loss list = np.zeros(n epochs)
  vocab size = log co occurence train.shape[0]
  for epoch in range(n epochs):
    grad W, grad W tilde, grad b, grad b tilde = grad GloVe(W, W tilde, b, b tilde, log co occurence train)
   W = W - learning rate * grad W
    b = b - learning rate * grad b
    if not grad W tilde is None and not grad_b_tilde is None:
     W tilde = W tilde - learning_rate * grad_W_tilde
     b tilde = b tilde - learning rate * grad b tilde
   train loss, valid loss = loss GloVe(W, W tilde, b, b_tilde, log_co_occurence_train), loss_GloVe(W, W_tilde, b,
    if do print:
```

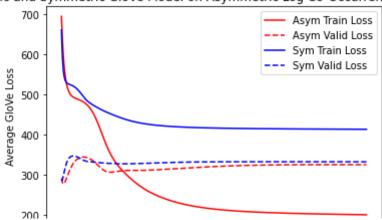
```
print(f"Average Train Loss: {train_loss / vocab_size}, Average valid loss: {valid_loss / vocab_size}, grad_n
train_loss_list[epoch] = train_loss / vocab_size
valid_loss_list[epoch] = valid_loss / vocab_size
return W, W tilde, b, b tilde, train loss list, valid loss list
```

• **TODO**: Run this cell below to run an experiment training GloVe model

```
### TODO: Run this cell ###
np.random.seed(1)
n epochs = 500 # A hyperparameter. You can play with this if you want.
# Store the final losses for graphing
do print = False # If you want to see diagnostic information during training
init variance = 0.1 # A hyperparameter. You can play with this if you want.
embedding dim = 16
W = init variance * np.random.normal(size=(vocab size, embedding dim))
W tilde = init variance * np.random.normal(size=(vocab size, embedding dim))
b = init variance * np.random.normal(size=(vocab size, 1))
b tilde = init variance * np.random.normal(size=(vocab size, 1))
# Run the training for the asymmetric and symmetric GloVe model
Asym W final, Asym W tilde final, Asym b final, Asym b tilde final, Asym train loss list, Asym valid loss list = t
Sym W final, Sym W tilde final, Sym b final, Sym b tilde final, Sym train loss list, Sym valid loss list = train G
# Plot the resulting training curve
pylab.plot(Asym train loss list, label="Asym Train Loss", color='red')
pylab.plot(Asym valid loss list, label="Asym Valid Loss", color='red', linestyle='--')
pylab.plot(Sym train loss list, label="Sym Train Loss", color='blue')
pylab.plot(Sym valid loss list, label="Sym Valid Loss", color='blue', linestyle='--')
pylab.xlabel("Iterations")
pylab.ylabel("Average GloVe Loss")
pylab.title("Asymmetric and Symmetric GloVe Model on Asymmetric Log Co-Occurrence (Emb Dim={})".format(embedding d
pylab.legend()
```

<matplotlib.legend.Legend at 0x7f2606fbca50>

Asymmetric and Symmetric GloVe Model on Asymmetric Log Co-Occurrence (Emb Dim=16)



▼ 1.6 Effects of a buggy implementation [0pt]

Suppose that during the implementation, you initialized the weight embedding matrix \mathbf{W} and $\tilde{\mathbf{W}}$ with the same initial values (i.e., $\mathbf{W} = \tilde{\mathbf{W}} = \mathbf{W}_0$).

What will happen to the values of W and \tilde{W} over the course of training. Will they stay equal to each other, or diverge from each other? Explain your answer briefly.

Hint: Consider the gradient $\frac{\partial L}{\partial \mathbf{W}}$ versus $\frac{\partial L}{\partial \tilde{\mathbf{W}}}$

1.6 Answer: **TODO: Write Part 1.6 answer here **

\checkmark 1.7. Effect of embedding dimension d [0pt]

Train the both the symmetric and asymmetric GLoVe model with varying dimensionality d by running the cell below. Comment on:

- 1. Which d leads to optimal validation performance for the asymmetric and symmetric models?
- 2. Why does / doesn't larger d always lead to better validation error?
- 3. Which model is performing better, and why?

1.7 Answer: **TODO: Write Part 1.7 answer here**

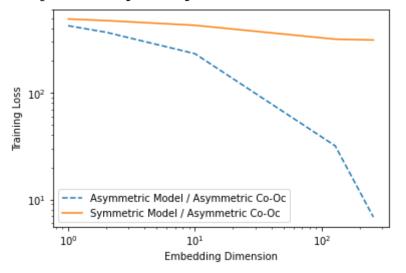
Train the GloVe model for a range of embedding dimensions

```
np.random.seed(1)
n epochs = 500 # A hyperparameter. You can play with this if you want.
embedding dims = np.array([1, 2, 10, 128, 256]) # Play with this
# Store the final losses for graphing
asymModel asymCoOc final train losses, asymModel asymCoOc final val losses = [], []
symModel asymCoOc final train losses, symModel asymCoOc final val losses = [], []
Asym W final 2d, Asym b final 2d, Asym W tilde final 2d, Asym b tilde final 2d = None, None, None
W_final_2d, b_final_2d = None, None
do print = False # If you want to see diagnostic information during training
for embedding dim in tqdm(embedding dims):
  init variance = 0.1 # A hyperparameter. You can play with this if you want.
 W = init variance * np.random.normal(size=(vocab size, embedding dim))
  W tilde = init variance * np.random.normal(size=(vocab size, embedding dim))
  b = init variance * np.random.normal(size=(vocab size, 1))
  b tilde = init variance * np.random.normal(size=(vocab size, 1))
  if do print:
    print(f"Training for embedding dimension: {embedding dim}")
  # Train Asym model on Asym Co-Oc matrix
  Asym W final, Asym W tilde final, Asym b final, Asym b tilde final, train loss list, valid loss list = train Glo
  if embedding dim == 2:
    # Save a parameter copy if we are training 2d embedding for visualization later
    Asym W final 2d = Asym W final
   Asym W tilde final 2d = Asym W tilde final
   Asym b final 2d = Asym b final
   Asym b tilde final 2d = Asym b tilde final
  asymModel_asymCoOc_final_train_losses += [train_loss_list[-1]]
  asymModel asymCoOc final val losses += [valid loss list[-1]]
  if do print:
    print(f"Final validation loss: {valid loss}")
```

Plot the training and validation losses against the embedding dimension.

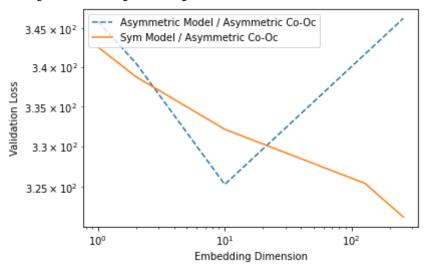
```
pylab.loglog(embedding_dims, asymModel_asymCoOc_final_train_losses, label="Asymmetric Model / Asymmetric Co-Oc", l
pylab.loglog(embedding_dims, symModel_asymCoOc_final_train_losses, label="Symmetric Model / Asymmetric Co-Oc")
pylab.xlabel("Embedding Dimension")
pylab.ylabel("Training Loss")
pylab.legend()
```

<matplotlib.legend.Legend at 0x7f2606f27210>



```
pylab.loglog(embedding_dims, asymModel_asymCoOc_final_val_losses, label="Asymmetric Model / Asymmetric Co-Oc", lin
pylab.loglog(embedding_dims, symModel_asymCoOc_final_val_losses, label="Sym Model / Asymmetric Co-Oc")
pylab.xlabel("Embedding Dimension")
pylab.ylabel("Validation Loss")
pylab.legend(loc="upper left")
```

<matplotlib.legend.Legend at 0x7f260672d690>



Part 2: Network Architecture (1pts)

See the handout for the written questions in this part.

Answer the following questions

▼ 2.1. Number of parameters in neural network model [0.5pt]

The trainable parameters of the model consist of 3 weight matrices and 2 sets of biases. What is the total number of trainable parameters in the model, as a function of V, N, D, H?

In the diagram given, which part of the model (i.e., word_embbeding_weights, embed_to_hid_weights, hid_to_output_weights, hid_bias, or output_bias) has the largest number of trainable parameters if we have the constraint that $V \gg H > D > N$? Note: The symbol \gg means ``much greater than" Explain your reasoning.

2.1 Answer:

ullet word_embbeding_weights: DN

embed_to_hid_weights: DNH

• hid_to_output_weights: HV

ullet hid_bias: H

• output_bias: V

Thus, the total number of trainable parameters in the model is DN + DNH + HV + H + V

Since $V \gg H > D > N$, hid_to_output_weights part has the largest number of trainable parameters.

▼ 2.2 Number of parameters in *n*-gram model [0.5pt]

Another method for predicting the next words is an *n*-gram model, which was mentioned in Lecture 3. If we wanted to use an n-gram model with the same context length N-1 as our network (since we mask 1 of the N words in our input), we'd need to store the counts of all possible N-grams. If we stored all the counts explicitly and suppose that we have V words in the dictionary, how many entries would this table have?

2.2 Answer: Each letter in the N-gram model has V different choices, so in total there will be V^N entries

▼ 2.3. Comparing neural network and n-gram model scaling [0pt]

How do the parameters in the neural network model scale with the number of context words N versus how the number of entries in the n-gram model scale with N? [Opt]

2.3 Answer: **TODO: Write Part 2.3 answer here**

Part 3: Training the model (3pts)

In this part, you will learn to implement and train the neural language model from Figure 1. As described in the previous section, during training, we randomly sample one of the N context words to replace with a <code>[MASK]</code> token. The goal is for the network to predict the word that was masked, at the corresponding output word position. In practice, this <code>[MASK]</code> token is assigned the index 0 in our dictionary. The weights $W^{(2)} = \text{hid_to_output_weights}$ now has the shape $NV \times H$, as the output layer has NV neurons, where the first V output units are for predicting the first word, then the next V are for predicting the second word, and so on. We call this as concatenating output units across all word positions, i.e. the (v + nV)-th column is for the word v in vocabulary for the v-th output word position. Note here that the softmax is applied in chunks of v-as well, to give a valid probability distribution over the v-th vords (For simplicity we also include the <code>[MASK]</code> token as one of the possible prediction even though we know the target should not be this token). Only the output word positions that were masked in the input are included in the cross entropy loss calculation:

$$C = -\sum_{i}^{B} \sum_{n}^{N} \sum_{v}^{V} m_{n}^{(i)} (t_{v+nV}^{(i)} \log y_{v+nV}^{(i)})$$

Where:

• $y_{v+nV}^{(i)}$ denotes the output probability prediction from the neural network for the *i*-th training example for the word v in the *n*-th output word. Denoting z as the logits output, we define the output probability y as a softmax on z over contiguous chunks of V units (see also Figure 1):

$$y_{v+nV}^{(i)} = \frac{e^{z_{v+nV}^{(i)}}}{\sum_{l}^{V} e^{z_{l+nV}^{(i)}}}$$

- $t_{v+nV}^{(i)} \in \{0,1\}$ is 1 if for the i-th training example, the word v is the n-th word in context
- $m_n^{(i)} \in \{0, 1\}$ is a mask that is set to 1 if we are predicting the n-th word position for the i-th example (because we had masked that word in the input), and 0 otherwise

There are three classes defined in this part: Params, Activations, Model. You will make changes to Model, but it may help to read through Params and Activations first.

```
class Params(object):
    """A class representing the trainable parameters of the model. This class has five fields:
           word embedding weights, a matrix of size V x D, where V is the number of words in the vocabulary
                   and D is the embedding dimension.
           embed to hid weights, a matrix of size H x ND, where H is the number of hidden units. The first D
                   columns represent connections from the embedding of the first context word, the next D columns
                   for the second context word, and so on. There are N context words.
           hid bias, a vector of length H
           hid to output weights, a matrix of size NV x H
           output bias, a vector of length NV"""
    def init (self, word embedding weights, embed to hid weights, hid to output weights,
                 hid bias, output bias):
        self.word embedding weights = word embedding weights
        self.embed to hid weights = embed to hid weights
        self.hid to output weights = hid to output weights
        self.hid bias = hid bias
        self.output bias = output bias
    def copy(self):
        return self. class (self.word embedding weights.copy(), self.embed to hid weights.copy(),
                              self.hid to_output_weights.copy(), self.hid bias.copy(), self.output bias.copy())
    @classmethod
    def zeros(cls, vocab size, context len, embedding dim, num hid):
        """A constructor which initializes all weights and biases to 0."""
       word embedding weights = np.zeros((vocab size, embedding dim))
        embed to hid weights = np.zeros((num hid, context len * embedding dim))
       hid to output weights = np.zeros((vocab size * context len, num hid))
        hid bias = np.zeros(num hid)
        output bias = np.zeros(vocab size * context len)
        return cls(word embedding weights, embed to hid weights, hid to output weights,
```

hid bias, output bias)

```
@classmethod
    def random init(cls, init_wt, vocab_size, context_len, embedding_dim, num_hid):
        """A constructor which initializes weights to small random values and biases to 0."""
       word embedding weights = np.random.normal(0., init wt, size=(vocab size, embedding dim))
        embed to hid weights = np.random.normal(0., init wt, size=(num hid, context len * embedding dim))
       hid to output weights = np.random.normal(0., init wt, size=(vocab_size * context_len, num_hid))
        hid_bias = np.zeros(num_hid)
        output bias = np.zeros(vocab size * context len)
       return cls(word embedding weights, embed to hid weights, hid to output weights,
                   hid bias, output bias)
    ###### The functions below are Python's somewhat oddball way of overloading operators, so that
    ###### we can do arithmetic on Params instances. You don't need to understand this to do the assignment.
    def mul (self, a):
        return self. class (a * self.word embedding weights,
                              a * self.embed to hid weights,
                              a * self.hid to output weights,
                              a * self.hid bias,
                              a * self.output bias)
    def rmul (self, a):
        return self * a
    def add (self, other):
       return self. class _(self.word_embedding_weights + other.word_embedding_weights,
                              self.embed to hid weights + other.embed to hid weights,
                              self.hid to output weights + other.hid to output weights,
                              self.hid bias + other.hid bias,
                              self.output_bias + other.output_bias)
    def sub (self, other):
        return self + -1. * other
class Activations(object):
    """A class representing the activations of the units in the network. This class has three fields:
```

```
embedding layer, a matrix of B x ND matrix (where B is the batch size, D is the embedding dimension,
                and N is the number of input context words), representing the activations for the embedding
                layer on all the cases in a batch. The first D columns represent the embeddings for the
                first context word, and so on.
       hidden layer, a B x H matrix representing the hidden layer activations for a batch
       output layer, a B x V matrix representing the output layer activations for a batch"""
    def __init__(self, embedding layer, hidden layer, output layer):
        self.embedding layer = embedding layer
        self.hidden layer = hidden layer
       self.output layer = output layer
def get batches(inputs, batch size, shuffle=True):
    """Divide a dataset (usually the training set) into mini-batches of a given size. This is a
    'generator', i.e. something you can use in a for loop. You don't need to understand how it
   works to do the assignment."""
    if inputs.shape[0] % batch size != 0:
       raise RuntimeError('The number of data points must be a multiple of the batch size.')
    num batches = inputs.shape[0] // batch size
    if shuffle:
       idxs = np.random.permutation(inputs.shape[0])
       inputs = inputs[idxs, :]
    for m in range(num batches):
       yield inputs[m * batch_size:(m + 1) * batch_size, :]
```

In this part of the assignment, you implement a method which computes the gradient using backpropagation. To start you out, the *Model* class contains several important methods used in training:

- compute activations computes the activations of all units on a given input batch
- compute_loss_derivative computes the gradient with respect to the output logits $\frac{\partial C}{\partial z}$
- evaluate computes the average cross-entropy loss for a given set of inputs and targets

You will need to complete the implementation of two additional methods to complete the training, and print the outputs of the

3.1 Implement gradient with respect to output layer inputs [1pt]

Implement a vectorized <code>compute_loss</code> function, which computes the total cross-entropy loss on a mini-batch according to Eq. 2. Look for the <code>## YOUR CODE HERE ## comment</code> for where to complete the code. The docstring provides a description of the inputs to the function.

→ 3.2 Implement gradient with respect to parameters [1pt]

back_propagate is the function which computes the gradient of the loss with respect to model parameters using backpropagation. It uses the derivatives computed by *compute_loss_derivative*. Some parts are already filled in for you, but you need to compute the matrices of derivatives for <code>embed_to_hid_weights</code>, <code>hid_bias</code>, <code>hid_to_output_weights</code>, and <code>output_bias</code>. These matrices have the same sizes as the parameter matrices (see previous section). These matrices have the same sizes as the parameter matrices. Look for the <code>## YOUR CODE HERE ## comment</code> for where to complete the code.

In order to implement backpropagation efficiently, you need to express the computations in terms of matrix operations, rather than *for* loops. You should first work through the derivatives on pencil and paper. First, apply the chain rule to compute the derivatives with respect to individual units, weights, and biases. Next, take the formulas you've derived, and express them in matrix form. You should be able to express all of the required computations using only matrix multiplication, matrix transpose, and elementwise operations — no *for* loops! If you want inspiration, read through the code for *Model.compute_activations* and try to understand how the matrix operations correspond to the computations performed by all the units in the network.

Hint: Your implementations should also be similar to hid_to_output_weights_grad, hid_bias_grad in the same function call

```
class Model(object):
    """A class representing the language model itself. This class contains various methods used in training
    the model and visualizing the learned representations. It has two fields:
        params, a Params instance which contains the model parameters
        vocab, a list containing all the words in the dictionary; vocab[0] is the word with index
        0, and so on."""
```

```
def init (self, params, vocab):
   self.params = params
   self.vocab = vocab
   self.vocab size = len(vocab)
   self.embedding dim = self.params.word embedding weights.shape[1]
   self.embedding layer dim = self.params.embed to hid weights.shape[1]
   self.context len = self.embedding layer dim // self.embedding dim
   self.num_hid = self.params.embed_to_hid_weights.shape[0]
def copy(self):
   return self. class (self.params.copy(), self.vocab[:])
@classmethod
def random init(cls, init wt, vocab, context len, embedding dim, num hid):
    """Constructor which randomly initializes the weights to Gaussians with standard deviation init wt
   and initializes the biases to all zeros."""
   params = Params.random init(init wt, len(vocab), context len, embedding dim, num hid)
   return Model(params, vocab)
def indicator matrix(self, targets, mask zero index=True):
    """Construct a matrix where the (v + n*V)th entry of row i is 1 if the n-th target word
    for example i is v, and all other entries are 0.
    Note: if the n-th target word index is 0, this corresponds to the [MASK] token,
           and we set the entry to be 0.
    .....
   batch_size, context_len = targets.shape
   expanded targets = np.zeros((batch size, context len * len(self.vocab)))
   offset = np.repeat((np.arange(context len) * len(self.vocab))[np.newaxis, :], batch size, axis=0) # [[0, V
   targets offset = targets + offset
   for c in range(context len):
      expanded_targets[np.arange(batch_size), targets offset[:,c]] = 1.
      if mask zero index:
       # Note: Set the targets with index 0, V, 2V to be zero since it corresponds to the [MASK] token
        expanded targets[np.arange(batch size), offset[:,c]] = 0.
   return expanded targets
```

def compute_loss_derivative(self, output_activations, expanded_target_batch, target_mask):
 """Compute the gradient of cross-entropy loss wrt output logits z

For example:

$$[y_{0} \dots y_{V-1}] [y_{V}, \dots, y_{2*V-1}] [y_{2*V} \dots y_{i,3*V-1}] [y_{3*V} \dots y_{i,4*V-1}]$$

Where for column v + n*V,

$$y_{v + n*V} = e^{z_{v + n*V}} / \sum_{m=0}^{V-1} e^{z_{m + n*V}}, \text{ for } n=0,...,N-1$$

This function should return a dC / dz matrix of size [batch_size x (vocab_size * context_len)], where each row i in dC / dz has columns 0 to V-1 containing the gradient the 1st output context word from i-th training example, then columns vocab_size to 2*vocab_size - 1 for the 2nd output context word of the i-th training example, etc.

C is the loss function summed acrossed all examples as well:

$$C = -\sum_{i=1}^{n} \{i, j, n\} \text{ mask } \{i, n\} \text{ (t } \{i, j + n*V\} \text{ log y } \{i, j + n*V\}), \text{ for } j=0,...,V, \text{ and } n=0,...,N$$

where $mask_{i,n} = 1$ if the i-th training example has n-th context word as the target, otherwise $mask_{i,n} = 0$.

Args:

output_activations: A [batch_size x (context_len * vocab_size)] matrix,
 for the activations of the output layer, i.e. the y_j's.
expanded_target_batch: A [batch_size x (context_len * vocab_size)] matrix,
 where expanded_target_batch[i,n*V:(n+1)*V] is the indicator vector for
 the n-th context target word position, i.e. the (i, j + n*V) entry is 1 if the
 i'th example, the context word at position n is j, and 0 otherwise.
target_mask: A [batch_size x context_len x 1] tensor, where target_mask[i,n] = 1

if for the i'th example the n-th context word is a target position, otherwise 0

Outputs:

loss_derivative: A [batch_size x (context_len * vocab_size)] matrix,
 where loss_derivative[i,0:vocab_size] contains the gradient
 dC / dz_0 for the i-th training example gradient for 1st output
 context word, and loss_derivative[i,vocab_size:2*vocab_size] for
 the 2nd output context word of the i-th training example, etc.

11 11 11

```
# Reshape output activations and expanded target batch and use broadcasting
   output activations reshape = output activations.reshape(-1, self.context len, len(self.vocab))
   expanded target batch reshape = expanded target batch.reshape(-1, self.context len, len(self.vocab))
   gradient masked reshape = target mask * (output activations reshape - expanded target batch reshape)
   gradient masked = gradient masked reshape.reshape(-1, self.context len * len(self.vocab))
   return gradient masked
def compute loss(self, output activations, expanded target batch, target mask):
    """Compute the total cross entropy loss over a mini-batch.
   Args:
     output_activations: [batch_size x (context_len * vocab_size)] matrix,
           for the activations of the output layer, i.e. the y j's.
     expanded target batch: [batch size (context len * vocab size)] matrix,
           where expanded target batch[i,n*V:(n+1)*V] is the indicator vector for
           the n-th context target word position, i.e. the (i, j + n*V) entry is 1 if the
           i'th example, the context word at position n is j, and 0 otherwise. matrix obtained
     target mask: A [batch size x context len x 1] tensor, where target mask[i,n,0] = 1
           if for the i'th example the n-th context word is a target position, otherwise 0
   Returns:
     loss: a scalar for the total cross entropy loss over the batch,
           defined in Part 3
                              ##############################
   target mask repeat = np.repeat(np.squeeze(target mask, axis=-1),
                                len(self.vocab), axis=-1)
   loss = -np.sum(target mask repeat * expanded target batch *
                 np.log(output_activations + TINY))
   return loss
def compute activations(self, inputs):
   """Compute the activations on a batch given the inputs. Returns an Activations instance.
   You should try to read and understand this function, since this will give you clues for
   how to implement back propagate."""
```

```
batch size = inputs.shape[0]
   if inputs.shape[1] != self.context len:
        raise RuntimeError('Dimension of the input vectors should be {}, but is instead {}'.format(
            self.context len, inputs.shape[1]))
   # Embedding layer
   # Look up the input word indices in the word embedding weights matrix
   embedding layer_state = self.params.word_embedding_weights[inputs.reshape([-1]), :].reshape([batch_size, s
   # Hidden layer
   inputs to hid = np.dot(embedding layer state, self.params.embed to hid weights.T) + \
                    self.params.hid bias
   # Apply logistic activation function
   hidden layer state = 1. / (1. + np.exp(-inputs to hid))
   # Output layer
   inputs to softmax = np.dot(hidden layer state, self.params.hid to output weights.T) + \
                        self.params.output bias
   # Subtract maximum.
   # Remember that adding or subtracting the same constant from each input to a
   # softmax unit does not affect the outputs. So subtract the maximum to
   # make all inputs <= 0. This prevents overflows when computing their exponents.
   inputs to softmax -= inputs to softmax.max(1).reshape((-1, 1))
   # Take softmax along each V chunks in the output layer
   output_layer_state = np.exp(inputs_to_softmax)
   output_layer_state_shape = output_layer_state.shape
   output_layer_state = output_layer_state.reshape((-1, self.context_len, len(self.vocab)))
   output_layer_state /= output_layer_state.sum(axis=-1, keepdims=True) # Softmax along vocab of each target
   output layer state = output layer state.reshape(output layer state shape) # Flatten back to 2D matrix
   return Activations(embedding layer state, hidden layer state, output layer state)
def back propagate(self, input batch, activations, loss derivative):
    """Compute the gradient of the loss function with respect to the trainable parameters
   of the model.
```

Part of this function is already completed, but you need to fill in the derivative

computations for hid_to_output_weights_grad, output_bias_grad, embed_to_hid_weights_grad, and hid_bias_grad. See the documentation for the Params class for a description of what these matrices represent.

```
Args:
```

```
input_batch: A [batch_size x context_length] matrix containing the
   indices of the context words
activations: an Activations object representing the output of
   Model.compute_activations
loss_derivative: A [batch_size x (context_len * vocab_size)] matrix,
   where loss_derivative[i,0:vocab_size] contains the gradient
   dC / dz_0 for the i-th training example gradient for 1st output
   context word, and loss_derivative[i,vocab_size:2*vocab_size] for
   the 2nd output context word of the i-th training example, etc.
   Obtained from calling compute_loss_derivative()
```

Returns:

The matrix of derivatives for the embedding layer

hid bias grad = hid deriv.sum(0)

```
embed deriv = np.dot(hid deriv, self.params.embed to hid weights)
   # Word Embedding Weights gradient
   word embedding weights grad = np.dot(self.indicator matrix(input batch.reshape([-1,1]), mask zero index=Fa
                                             embed deriv.reshape([-1, self.embedding dim]))
   return Params (word embedding weights grad, embed to hid weights grad, hid to output weights grad,
                  hid bias grad, output bias grad)
def sample input mask(self, batch size):
    """Samples a binary mask for the inputs of size batch_size x context_len
   For each row, at most one element will be 1.
   mask idx = np.random.randint(self.context len, size=(batch size,))
   mask = np.zeros((batch size, self.context len), dtype=np.int)# Convert to one hot B x N, B batch size, N c
   mask[np.arange(batch size), mask idx] = 1
   return mask
def evaluate(self, inputs, batch size=100):
    """Compute the average cross-entropy over a dataset.
        inputs: matrix of shape D x N"""
   ndata = inputs.shape[0]
   total = 0.
   for input_batch in get_batches(inputs, batch_size):
        mask = self.sample_input_mask(batch_size)
        input batch masked = input batch * (1 - mask)
        activations = self.compute_activations(input batch masked)
        expanded target batch = self.indicator matrix(input batch)
        target_mask = np.expand_dims(mask, axis=2)
        cross entropy = self.compute loss(activations.output layer, expanded target batch, target mask)
        total += cross entropy
   return total / float(ndata)
def display_nearest_words(self, word, k=10):
    """List the k words nearest to a given word, along with their distances."""
```

```
if word not in self.vocab:
       print('Word "{}" not in vocabulary.'.format(word))
       return
   # Compute distance to every other word.
   idx = self.vocab.index(word)
   word rep = self.params.word embedding weights[idx, :]
   diff = self.params.word embedding weights - word_rep.reshape((1, -1))
   distance = np.sqrt(np.sum(diff ** 2, axis=1))
   # Sort by distance.
   order = np.argsort(distance)
   order = order[1:1 + k] # The nearest word is the query word itself, skip that.
   for i in order:
       print('{}: {}'.format(self.vocab[i], distance[i]))
def word distance(self, word1, word2):
    """Compute the distance between the vector representations of two words."""
   if word1 not in self.vocab:
       raise RuntimeError('Word "{}" not in vocabulary.'.format(word1))
   if word2 not in self.vocab:
       raise RuntimeError('Word "{}" not in vocabulary.'.format(word2))
   idx1, idx2 = self.vocab.index(word1), self.vocab.index(word2)
   word_rep1 = self.params.word_embedding_weights[idx1, :]
   word rep2 = self.params.word embedding weights[idx2, :]
   diff = word rep1 - word rep2
   return np.sqrt(np.sum(diff ** 2))
```

→ 3.3 Print the gradients [1pt]

To make your life easier, we have provided the routine <code>check_gradients</code>, which checks your gradients using finite differences. You should make sure this check passes before continuing with the assignment. Once <code>check_gradients()</code> passes, call <code>print gradients()</code> and include its output in your write-up.

```
def relative_error(a, b):
    return np.abs(a - b) / (np.abs(a) + np.abs(b))
def check output derivatives (model, input batch, target batch, mask):
    def softmax(z):
        z = z.copy()
        z = z.max(-1, keepdims=True)
        y = np.exp(z)
        y /= y.sum(-1, keepdims=True)
        return y
    batch_size = input_batch.shape[0]
    z = np.random.normal(size=(batch_size, model.context_len, model.vocab_size))
    y = softmax(z).reshape((batch_size, model.context_len * model.vocab_size))
    z = z.reshape((batch_size, model.context_len * model.vocab_size))
    expanded_target_batch = model.indicator_matrix(target_batch)
    target_mask = np.expand_dims(mask, axis=2)
    loss derivative = model.compute loss derivative(y, expanded target batch, target mask)
    if loss derivative is None:
        print('Loss derivative not implemented yet.')
        return False
    if loss derivative.shape != (batch size, model.vocab size * model.context len):
        print('Loss derivative should be size {} but is actually {}.'.format(
            (batch size, model.vocab size), loss derivative.shape))
        return False
    def obj(z):
        z = z.reshape((-1, model.context len, model.vocab size))
       y = softmax(z).reshape((batch size, model.context len * model.vocab size))
        return model.compute loss(y, expanded target batch, target mask)
    for count in range(1000):
        i, j = np.random.randint(0, loss derivative.shape[0]), np.random.randint(0, loss derivative.shape[1])
```

```
z plus = z.copy()
        z plus[i, j] += EPS
       obj plus = obj(z plus)
        z \min s = z.copy()
       z minus[i, j] -= EPS
       obj_minus = obj(z_minus)
       empirical = (obj_plus - obj_minus) / (2. * EPS)
       rel = relative error(empirical, loss derivative[i, j])
       if rel > 1e-4:
            print('The loss derivative has a relative error of {}, which is too large.'.format(rel))
            return False
    print('The loss derivative looks OK.')
    return True
def check param gradient(model, param name, input batch, target batch, mask):
    activations = model.compute activations(input batch)
    expanded target batch = model.indicator matrix(target batch)
    target mask = np.expand dims(mask, axis=2)
    loss derivative = model.compute loss derivative(activations.output layer, expanded target batch, target mask)
    param gradient = model.back propagate(input batch, activations, loss derivative)
    def obj(model):
        activations = model.compute_activations(input_batch)
       return model.compute loss(activations.output layer, expanded target batch, target mask)
    dims = getattr(model.params, param name).shape
    is matrix = (len(dims) == 2)
    if getattr(param gradient, param name).shape != dims:
       print('The gradient for {} should be size {} but is actually {}.'.format(
            param name, dims, getattr(param gradient, param name).shape))
       return
    for count in range(1000):
```

```
if is matrix:
            slc = np.random.randint(0, dims[0]), np.random.randint(0, dims[1])
        else:
            slc = np.random.randint(dims[0])
        model plus = model.copy()
        getattr(model plus.params, param name)[slc] += EPS
       obj_plus = obj(model_plus)
        model minus = model.copy()
        getattr(model_minus.params, param_name)[slc] -= EPS
       obj_minus = obj(model_minus)
        empirical = (obj plus - obj minus) / (2. * EPS)
       exact = getattr(param_gradient, param_name)[slc]
       rel = relative_error(empirical, exact)
        if rel > 5e-4:
            print('The loss derivative has a relative error of {}, which is too large for param {}.'.format(rel, p
            return False
    print('The gradient for {} looks OK.'.format(param name))
def load partially trained model():
    obj = pickle.load(open(PARTIALLY TRAINED MODEL, 'rb'))
    params = Params(obj['word embedding weights'], obj['embed to hid weights'],
                                   obj['hid to output weights'], obj['hid bias'],
                                   obj['output bias'])
    vocab = obj['vocab']
    return Model(params, vocab)
def check gradients():
    """Check the computed gradients using finite differences."""
    np.random.seed(0)
    np.seterr(all='ignore') # suppress a warning which is harmless
    model = load partially trained model()
```

```
data_obj = pickle.load(open(data_location, 'rb'))
    train inputs = data obj['train inputs']
    input batch = train inputs[:100, :]
    mask = model.sample input mask(input batch.shape[0])
    input batch masked = input batch * (1 - mask)
    if not check output derivatives (model, input batch masked, input batch, mask):
        return
    for param name in ['word embedding weights', 'embed to hid weights', 'hid to output weights',
                       'hid bias', 'output bias']:
        check param gradient(model, param name, input batch masked, input batch, mask)
def print gradients():
    """Print out certain derivatives for grading."""
    np.random.seed(0)
    model = load partially trained model()
    data obj = pickle.load(open(data location, 'rb'))
    train inputs = data obj['train inputs']
    input batch = train inputs[:100, :]
    mask = model.sample input mask(input batch.shape[0])
    input_batch_masked = input_batch * (1 - mask)
    activations = model.compute activations(input batch masked)
    expanded_target batch = model.indicator_matrix(input_batch)
    target_mask = np.expand_dims(mask, axis=2)
    loss derivative = model.compute loss derivative(activations.output layer, expanded target batch, target mask)
    param gradient = model.back propagate(input batch, activations, loss derivative)
    print('loss_derivative[46, 785]', loss_derivative[46, 785])
    print('loss derivative[46, 766]', loss derivative[46, 766])
    print('loss derivative[5, 42]', loss derivative[5, 42])
    print('loss derivative[5, 31]', loss derivative[5, 31])
    print()
    print('param_gradient.word_embedding_weights[27, 2]', param_gradient.word_embedding_weights[27, 2])
    print('param gradient.word embedding weights[43, 3]', param gradient.word embedding weights[43, 3])
    print('param gradient.word embedding weights[22, 4]', param gradient.word embedding weights[22, 4])
```

```
print('param gradient.word embedding weights[2, 5]', param gradient.word embedding weights[2, 5])
    print()
    print('param gradient.embed to hid weights[10, 2]', param gradient.embed to hid weights[10, 2])
    print('param gradient.embed to hid weights[15, 3]', param gradient.embed to hid weights[15, 3])
    print('param gradient.embed to hid weights[30, 9]', param gradient.embed to hid weights[30, 9])
    print('param gradient.embed to hid weights[35, 21]', param gradient.embed to hid weights[35, 21])
    print()
    print('param gradient.hid bias[10]', param gradient.hid bias[10])
    print('param gradient.hid bias[20]', param gradient.hid bias[20])
    print()
    print('param gradient.output bias[0]', param gradient.output bias[0])
    print('param gradient.output bias[1]', param gradient.output bias[1])
    print('param gradient.output bias[2]', param gradient.output bias[2])
    print('param gradient.output bias[3]', param gradient.output bias[3])
# Run this to check if your implement gradients matches the finite difference within tolerance
# Note: this may take a few minutes to go through all the checks
check gradients()
    The loss derivative looks OK.
    The gradient for word embedding weights looks OK.
    The gradient for embed to hid weights looks OK.
    The gradient for hid to output weights looks OK.
    The gradient for hid_bias looks OK.
    The gradient for output bias looks OK.
# Run this to print out the gradients
print_gradients()
    loss derivative[46, 785] 0.7137561447745507
    loss derivative[46, 766] -0.9661570033238931
    loss derivative[5, 42] -0.0
    loss derivative[5, 31] 0.0
    param gradient.word embedding weights[27, 2] 0.0
    param gradient.word embedding weights[43, 3] 0.011596892511489458
    param gradient.word embedding weights[22, 4] -0.0222670623817297
    param gradient.word embedding weights[2, 5] 0.0
```

```
param_gradient.embed_to_hid_weights[10, 2] 0.3793257091930164
param_gradient.embed_to_hid_weights[15, 3] 0.01604516132110917
param_gradient.embed_to_hid_weights[30, 9] -0.4312854367997419
param_gradient.embed_to_hid_weights[35, 21] 0.06679896665436337

param_gradient.hid_bias[10] 0.023428803123345148
param_gradient.hid_bias[20] -0.024370452378874197

param_gradient.output_bias[0] 0.000970106146902794
param_gradient.output_bias[1] 0.16868946274763222
param_gradient.output_bias[2] 0.0051664774143909235
param_gradient.output_bias[3] 0.15096226471814364
```

▼ 3.4 Run model training [0pt]

Once you've implemented the gradient computation, you'll need to train the model. The function *train* implements the main training procedure. It takes two arguments:

- embedding_dim: The number of dimensions in the distributed representation.
- num hid: The number of hidden units

As the model trains, the script prints out some numbers that tell you how well the training is going. It shows:

- The cross entropy on the last 100 mini-batches of the training set. This is shown after every 100 mini-batches.
- The cross entropy on the entire validation set every 1000 mini-batches of training.

At the end of training, this function shows the cross entropies on the training, validation and test sets. It will return a *Model* instance.

'show training CE after': 100, # measure training error after this many mini-batches

'show validation CE after': 1000, # measure validation error after this many mini-batc } def find occurrences(word1, word2, word3): """Lists all the words that followed a given tri-gram in the training set and the number of times each one followed it.""" # cache the data so we don't keep reloading global _train_inputs, _train_targets, _vocab if _train_inputs is None: data_obj = pickle.load(open(data_location, 'rb')) vocab = data_obj['vocab'] _train_inputs, _train_targets = data_obj['train_inputs'], data_obj['train_targets'] if word1 not in vocab: raise RuntimeError('Word "{}" not in vocabulary.'.format(word1)) if word2 not in vocab: raise RuntimeError('Word "{}" not in vocabulary.'.format(word2)) if word3 not in vocab: raise RuntimeError('Word "{}" not in vocabulary.'.format(word3)) idx1, idx2, idx3 = vocab.index(word1), vocab.index(word2), vocab.index(word3) idxs = np.array([idx1, idx2, idx3])matches = np.all(train inputs == idxs.reshape((1, -1)), 1) if np.any(matches): counts = collections.defaultdict(int) for m in np.where(matches)[0]: counts[_vocab[_train_targets[m]]] += 1 word counts = sorted(list(counts.items()), key=lambda t: t[1], reverse=True) print('The tri-gram "{} {} {}" was followed by the following words in the training set:'.format(word1, word2, word3)) for word, count in word counts: if count > 1: print(' {} ({} times)'.format(word, count))

```
else:
                print('
                          {} (1 time)'.format(word))
    else:
       print('The tri-gram "{} {} {}" did not occur in the training set.'.format(word1, word2, word3))
def train(embedding dim, num hid, config=DEFAULT TRAINING CONFIG):
    """This is the main training routine for the language model. It takes two parameters:
        embedding dim, the dimension of the embedding space
       num hid, the number of hidden units."""
    # For reproducibility
    np.random.seed(123)
    # Load the data
    data obj = pickle.load(open(data_location, 'rb'))
    vocab = data obj['vocab']
    train inputs = data obj['train inputs']
    valid inputs = data obj['valid inputs']
    test inputs = data obj['test inputs']
    # Randomly initialize the trainable parameters
   model = Model.random init(config['init wt'], vocab, config['context len'], embedding dim, num hid)
    # Variables used for early stopping
    best_valid_CE = np.infty
    end training = False
    # Initialize the momentum vector to all zeros
    delta = Params.zeros(len(vocab), config['context len'], embedding dim, num hid)
    this_chunk_CE = 0.
    batch count = 0
    for epoch in range(1, config['epochs'] + 1):
        if end training:
            break
        print()
        print('Epoch', epoch)
```

```
for m, (input batch) in enumerate(get batches(train inputs, config['batch size'])):
    batch count += 1
    # For each example (row in input batch), select one word to mask out
    mask = model.sample input mask(config['batch size'])
    input batch masked = input batch * (1 - mask) # We only zero out one word per row
    # Forward propagate
    activations = model.compute activations(input batch masked)
    # Compute loss derivative
    expanded_target_batch = model.indicator_matrix(input_batch)
    loss derivative = model.compute loss derivative(activations.output layer, expanded target batch, mask[
    loss derivative /= config['batch_size']
    # Measure loss function
    cross entropy = model.compute loss(activations.output layer, expanded target batch, np.expand dims(mas
    this chunk CE += cross entropy
    if batch count % config['show training CE after'] == 0:
        print('Batch {} Train CE {:1.3f}'.format(
            batch count, this chunk CE / config['show training CE after']))
        this chunk CE = 0.
    # Backpropagate
    loss gradient = model.back propagate(input batch, activations, loss derivative)
    # Update the momentum vector and model parameters
    delta = config['momentum'] * delta + loss gradient
    model.params -= config['learning rate'] * delta
    # Validate
    if batch count % config['show validation CE after'] == 0:
        print('Running validation...')
       cross entropy = model.evaluate(valid inputs)
        print('Validation cross-entropy: {:1.3f}'.format(cross entropy))
        if cross entropy > best valid CE:
            print('Validation error increasing! Training stopped.')
```

Run the training.

```
embedding_dim = 16
num_hid = 128
trained model = train(embedding_dim, num_hid)
```

To convince us that you have correctly implemented the gradient computations, please include the following with your assignment submission:

- You will submit al-code.ipynb through MarkUs. You do not need to modify any of the code except the parts we asked you to implement.
- In your writeup, include the output of the function <code>print_gradients</code>. This prints out part of the gradients for a partially trained network which we have provided, and we will check them against the correct outputs. **Important:** make sure to give the output of <code>print_gradients</code>, **not** <code>check_gradients</code>.

→ Part 4: Bias in Word Embeddings (2pts)

Unfortunately, stereotypes and prejudices are often reflected in the outputs of natural language processing algorithms. For example, Google Translate is more likely to translate a non-English sentence to "He is a doctor" than "She is a doctor when the sentence is ambiguous. In this section, you will explore how bias enters natural language processing algorithms by implementing and analyzing a popular method for measuring bias in word embeddings.

Note: In AI and machine learning, **bias** generally refers to prior information, a necessary prerequisite for intelligent action. However, bias can be problematic when it is derived from aspects of human culture known to lead to harmful behaviour, such as stereotypes and prejudices.

4.1 WEAT method for detecting bias [1pt]

Word embedding models such as GloVe attempt to learn a vector space where semantically similar words are clustered close together. However, they have been shown to learn problematic associations, e.g. by embedding "man" more closely to "doctor" than "woman" (and vice versa for "nurse"). To detect such biases in word embeddings, "Semantics derived automatically from language corpora contain human-like biases" introduced the Word Embedding Association Test (WEAT). The WEAT test measures whether two target word sets (e.g., {programmer, engineer, scientist, ...} and {nurse, teacher, librarian, ...}) have the same relative association to two attribute word sets (e.g., man, male, ... and woman, female ...).

There is an excellent blog on bias in word embeddings and the WEAT test here.

In the following section, you will run a WEAT test for a given set of target and attribute words. Specifically, you must implement the function weat_association_score and then run the remaining cells to compute the p-value and effect size. Before you begin, make sure you understand the formal definition of the WEAT test given in section 4.1 of the handout.

Run the following cell to download pretrained GloVe embeddings.

```
import gensim.downloader as api
glove = api.load("glove-wiki-gigaword-50")
```

Before proceeding, you should familiarize yourself with the similarity method, which computes the cosine similarity between two words. You will need this method to implement weat association score. Some examples are given below.

Can you spot the gender bias between occupations in the examples below?

```
print(glove.similarity("man", "scientist"))
print(glove.similarity("man", "nurse"))
print(glove.similarity("woman", "scientist"))
print(glove.similarity("woman", "nurse"))

0.49226817
0.5718704
0.43883628
0.715502
```

Below, we define our target words (occupations) and attribute words (A and B). Our target words consist of occupations, and our attribute words are *gendered*. We will use the WEAT test to determine if the word embeddings contain gender biases for certain occupations.

```
# Target words (occupations)
occupations = ["programmer", "engineer", "scientist", "nurse", "teacher", "librarian"]
# Two sets of gendered attribute words, A and B
A = ["man", "male", "he", "boyish"]
B = ["woman", "female", "she", "girlish"]
```

• **TODO**: Implement the following function, weat_association_score which computes the association of a word w with the attribute:

$$s(w, A, B) = \operatorname{mean}_{a \in A} \cos(w, a) - \operatorname{mean}_{b \in B} \cos(w, b)$$

Use the following code to check your implementation:

```
np.isclose(weat_association_score("programmer", A, B, glove), 0.019615129)
True
```

Now, compute the WEAT association score for each element of occupations and the attribute sets A and B. Include the printed out association scores in your pdf.

-0.024141337256878614

4.2 Reasons for bias in word embeddings [0pt]

Based on these WEAT association scores, do the pretrained word embeddings associate certain occuptations with one gender more than another? What might cause word embedding models to learn certain stereotypes and prejudices? How might this be a problem in downstream applications?

4.2 Answer: **TODO: Write Part 4.2 answer here**

▼ 4.3 Analyzing WEAT [1pt]

While WEAT makes intuitive sense by asserting that closeness in the embedding space indicates greater similarity, more recent work (<u>Ethayarajh et al. [2019]</u>) has further analyzed the mathematical assertions and found some flaws with this method. Analyzing edge cases is a good way to find logical inconsistencies with any algorithm, and WEAT in particular can behave strangely when A and B contain just one word each.

▼ 4.3.1 1-word subsets [0.5 pts]

Find 1-word subsets of the original A and B that reverse the sign of the association score for at least some of the occupations

- -0.1547197625041008
- -0.41798314079642296
- -0.3417777419090271
- -0.5368140190839767
- -0.45463715493679047
- -0.3014984279870987

▼ 4.3.2 How word frequency affects embedding similarity [0.5 pts]

Consider the fact that the squared norm of a word embedding is linear in the log probability of the word in the training corpus. In other words, the more common a word is in the training corpus, the larger the norm of its word embedding. (See handout for more thorough description)

Briefly explain how this fact might contribute to the results from the previous section when using different attribute words. Provide your answers in no more than three sentences.

Hint 2: The paper cited above is a great resource if you are stuck.

4.3 Answer: Since the WEAT users could determine composition of the attribute word sets, so they could cherry-pick the attribute words to acheive their desired test outcome, which means WEAT is not a very stable test for word embendding ossociation measurement. (Not sure about this question actually)

▼ 4.3.3 Relative association between two sets of target words [0 pts]

In the original WEAT paper, the authors do not examine the association of individual words with attributes, but rather compare the relative association of two sets of target words. For example, are insect words more associated with positive attributes or negative attributes than flower words.

Formally, let X and Y be two sets of target words of equal size. The WEAT test statistic is given by:

$$s(X, Y, A, B) = \sum_{x \in X} s(x, A, B) - \sum_{y \in Y} s(y, A, B)$$

Will the same technique from the previous section work to manipulate this test statistic as well? Provide your answer in no more than 3 sentences.

4.3.3 Answer: TODO: Write 4.3.3 answer here

What you have to submit

Refer to the handout for the checklist

✓ 0s completed at 8:25 PM

Could not connect to the reCAPTCHA service. Please check your internet connection and reload to get a reCAPTCHA challenge.

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