STA457H1: Time Series Analysis

Assignment 1 **Due data September 30, 2022**

Student Name..ID number..

Instructions: *Show your answers in details.*

Q1 (2 points): Consider the autoregression model

$$x_t = -.9x_{t-2} + w_t, \quad w_t \underset{\sim}{iid} \mathcal{N}(0,1)$$

1. Generate $n = 200$ observations from this process.
2. Apply the moving average filter $\nu_t = (x_t + x_{t-1} + x_{t-2} + x_{t-3})/4$ to the data you generated.
3. Plot x_t as a line and superimpose ν_t as a dashed line. Comment!

Q2 (2 points): Consider the random walk with drift model

$$x_t = \delta + x_{t-1} + w_t$$

for $t = 1, 2, \cdots$, with $x_0 = 0$, where w_t is white noise with variance σ_w^2.

1. Show that the model can be written as $x_t = \delta t + \sum_{j=1}^{t} w_j$.
2. Find the mean function and the autocovariance function of x_t.
3. Show that x_t is not stationary.
4. Show that $\rho_x(t-1, t) = \sqrt{(t-1)/t}$ as $t \to \infty$. What is the implication of this result?
5. Suggest a transformation to make the series stationary, and prove that the transformed series is stationary.

Q3 (2 points): For the following yearly time series (10 years).

(t, x_t) : $(1, 24)$, $(2, 20)$, $(3, 25)$, $(4, 31)$, $(5, 30)$, $(6, 32)$, $(7, 37)$, $(8, 33)$, $(9, 40)$, $(10, 38)$

1. Compute the sample autocorrelation function, $\hat{\rho}(h)$, at lags $h = 0, 1, 2,$ and 3.
2. Test the null hypothesis that the theoretical autocorrelation at lag $h = 1$ equals zero.
3. Use R and redo parts (1) and (2).

Q4 (2 points):

1. Simulate a series of $n = 500$ observations from the process $x_t = \frac{1}{3}(w_{t-1} + w_t + w_{t+1})$, where w_t is Gaussian white noise series and compute the sample ACF, $\hat{\rho}(h)$, to lag 20. Compare the sample ACF you obtain to the actual ACF, $\rho(h)$.
2. Repeat part (1) using only $n = 50$. How does changing n affect the results?

Q5 (2 points): Consider the process: $x_t = \cos\left[2\pi(\frac{t}{12} + \phi)\right]$, where ϕ is a random variable with a Uniform distribution $(0, 1)$, and $t = 0, \pm1, \pm2$. Show that this process is weak stationary and find its autocorrelation function.