STA457H1/STA2202HF: Time Series Analysis
Assignment 2 - Question 4 Due data October 14, 2022

Student Name.....ID number.....

Instructions: *Show your answers in details.*

The following question is given for the graduate students only!

Q4 (2 points): Quarterly earnings per share for the Johnson & Johnson Company are given in the data file named *jj*. The earnings per share data, say y_t, covers the years from 1960 through 1980. In this problem, you are going to fit a special type of structural model, $x_t = \log(y_t)$, where $x_t = T_t + S_t + E_t$, where T_t is a trend component, S_t is a seasonal component, and E_t is error (noise) term. Note that the time t is in quarters $(1960.00, 1960.25, \cdots)$ so one unit of time is a year.

1. Fit the regression model

$$x_t = \underbrace{\beta t}_{\text{trend}} + \underbrace{\alpha_1 Q_1(t) + \alpha_2 Q_2(t) + \alpha_3 Q_3(t) + \alpha_4 Q_4(t)}_{\text{seasonal}} + \underbrace{w_t}_{\text{noise}}$$

   where $Q_i(t) = 1$ if time t corresponds to quarter $i = 1, 2, 3, 4$, and zero otherwise. The $Q_i(t)$'s are called indicator variables. We will assume for now that w_t is a Gaussian white noise sequence.

2. If the model is correct, what is the estimated average annual increase in the logged earnings per share?

3. If the model is correct, does the average logged earnings rate increase or decrease from the third quarter to the fourth quarter? And, by what percentage does it increase or decrease?

4. What happens if you include an intercept term in the model in (a)? Explain why there was a problem.

5. Graph the data, x_t, and superimpose the fitted values, say $\hat{x}_t$, on the graph. Examine the residuals, $x_t - \hat{x}_t$, and state your conclusions. Does it appear that the model fits the data well (do the residuals look white)?