Chapter 8: Prediction of low flows in ungauged basins - an Austrian example

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1 Introduction

This Tutorial has been developed by Gregor Laaha to illustrate the regional prediction of low flows (see, Laaha et al., 2013).

First of all load the library:

```r
library(PUBexamples)
```

Then the data:

```r
data(data4chapter8)
data4chapter8
```

```
     code          river                              station     lon     lat   elev   area cat_elev prec  temp pet runoff  DBMSNR Q95so Q95wi  Q95 SUM.AREA H.MIN H.MAX H.DIFF
1  207985           Kamp                            Rosenburg 15.6156 48.6300 261.58 1150.2      666  710  7.08 622    221 3001038  4.32  3.21 4.08    43.48  3.09  7.43   4.34
2  207993           Kamp                             Stiefern 15.6911 48.5353 217.00 1493.3      609  676  7.42 635    204 3001039  0.32  0.79 0.47    34.22  2.43  6.10   3.67
3  208041        Schmida                          Hollenstein 15.9294 48.5556 225.54  212.0      324  528  9.16 702     39 3001044  0.23  0.47 0.29    21.33  2.35  4.60   2.25
4  208058   Goellersbach                         Obermallebern 16.1589 48.4653 189.13  379.9      273  548  9.63 720     33 3001045  0.39  0.48 0.42    37.95  1.96  4.34   2.38
5  208108       Moedling                             Moedling 16.2800 48.0839 222.43   59.0      406  728  9.11 697    137 3001049  0.89  1.28 0.99     6.15  2.87  6.64   3.77
6  208447       Russbach                       Ulrichskirchen 16.4997 48.4039 173.32  131.5      255  568  9.86 726     49 3001080  0.33  0.72 0.39    12.95  1.87  3.75   1.88
7  208512     Braunaubach                          Amaliendorf 15.0647 48.8433 528.22   68.7      586  672  7.55 635    240 3001083  0.79  1.73 0.97     6.48  5.33  6.96   1.63
8  208579     Braunaubach                            Hoheneich 15.0089 48.7719 478.92  291.5      574  673  7.62 640    240 3001085  0.81  1.45 0.90    22.34  5.00  7.09   2.09
9  208611 Deutsche Thaya                           Schwarzenau 15.2592 48.7419 491.22  175.5      570  668  7.66 642    199 3001088  0.62  1.19 0.71    17.52  5.10  6.82   1.72
10 208637         Pulkau                           Zwingendorf 16.2436 48.7203 181.66  371.5      305  502  9.27 706     31 3001090  0.09  0.33 0.12    36.97  1.84  5.48   3.64
11 208678     Weidenbach                         Bad Pirawarth 16.6142 48.4322 166.59   71.0      221  567 10.03 734     56 3001094  0.73  0.83 0.75     6.89  1.88  2.98   1.10
12 208835         Pitten                                 Warth 16.1314 47.6547 372.80  277.0      830  948  7.20 628    357 3001107  2.74  4.15 3.25     6.50  3.51  9.07   5.56
13 209189           Zaya                         Niederabsdorf 16.8672 48.5758 151.27  515.6      249  551  9.87 727     43 3001135  0.52  0.76 0.57    51.49  1.59  4.73   3.14
14 210039        Rabnitz                          Piringsdorf 16.4197 47.4494 297.72  117.7      544  803  8.56 672    162 1001003  1.11  1.76 1.21    11.75  3.36  8.50   5.14
15 210054        Rabnitz           Mannersdorf an der Rabnitz 16.5328 47.4328 231.18  224.3      435  750  9.14 695    126 1001005  1.09  1.55 1.23    10.76  2.37  7.05   4.68
16 210062          Stoob                       Oberpullendorf 16.5119 47.4989 237.13  149.0      443  749  9.07 694    128 1001006  0.92  1.25 0.98    14.96  2.56  7.81   5.25
17 210088          Wulka Wulkaprodersdorf (Bundesstrasse B16) 16.5108 47.7964 161.86  220.7      295  672  9.83 727     77 1001008  0.67  0.91 0.71    22.26  1.75  7.20   5.45
18 210211       Lafnitz                           Dobersdorf 16.1372 47.0217 234.00  925.1      568  806  8.51 669    219 1001019  2.33  2.88 2.76    13.90  2.37  4.52   2.15
19 210237          Pinka                           Woppendorf 16.3933 47.2042 242.00  416.9      476  752  8.98 688    173 1001021  1.26  2.00 1.44    24.41  2.41  7.76   5.35
20 210245     Tauchenbach                       Altschlaining 16.2864 47.3139 315.78   89.2      519  754  8.83 680    144 1001022  1.63  2.19 1.83     9.04  3.28  8.34   5.06
21 210252     Tauchenbach                          Hannersdorf 16.3822 47.2292 246.96  175.4      452  722  9.13 694    124 1001023  0.72  1.21 1.06     8.51  2.64  8.27   5.63
22 210286          Strem            Guessing (Kulturzentrum) 16.3242 47.0625 209.97  316.4      297  688  9.81 725    109 1001026  0.57  0.91 0.65    31.57  2.20  4.46   2.26
23 210294          Strem                        Heiligenbrunn 16.4222 47.0336 200.13  400.4      289  680  9.84 727    116 1001027  0.46  0.71 0.63     8.41  2.01  3.34   1.33
24 210310          Guens                          Rattersdorf 16.5003 47.4108 285.24  265.3      522  800  8.77 684    176 1001028  1.45  2.28 1.72    15.16  2.99  8.80   5.81
25 210989           Raab                             Feldbach 15.8889 46.9572 275.45  689.4      576  834  8.50 668    263 6001049  3.44  3.18 3.29    19.16  2.72  5.17   2.45
26 211003        Lafnitz              Woerth an der Lafnitz 16.0872 47.2081 299.26  439.4      689  852  7.96 645    260 6001051  2.51  5.40 3.41    10.10  3.00  8.71   5.71
27 211037        Ilzbach                      Neudorf bei Ilz 15.9431 47.0828 277.85  190.1      393  772  9.33 704    187 6001054  0.94  1.56 1.09    18.95  2.87  9.49   6.62
28 211045     Feistritz                           Maierhofen 16.0086 47.0622 263.48  796.4      721  885  7.78 640    363 6001055  2.81  2.47 3.23    19.63  2.62 12.35   9.73
29 211474  Schwarzaubach                               Lipsch 15.6678 46.7542 256.42  129.5      349  834  9.56 715    249 6001097  0.53  1.00 0.62    13.10  2.60  4.86   2.26
30 211508       Gnasbach                          Fluttendorf 15.8511 46.7317 222.88  119.3      307  794  9.73 723    219 6001100  0.73  2.04 0.96    11.99  2.26  4.44   2.18
    H.MEAN M.NEIG SL.FL SL.MG SL.ST   N.GES   N.SOM   N.WIN GEOL.BM GEOL.QUA GEOL.TER GEOL.FLY GEOL.KAL GEOL.KRI GEOL.SHAL GEOL.DEEP GEOL.QUELL BONU.URB BONU.ACK BONU.DAU BONU.GRU
1  5.53800    6.2  81.3  18.5   0.2 5.84000 3.86100 1.98000    93.7      0.0      6.2      0.0      0.0      0.0       0.0       0.1          0      0.0     47.1      0.0      8.5
2  4.27404    5.5  82.9  17.1   0.0 5.17672 3.43264 1.74367    53.9      0.0      6.1      0.0      0.0      0.0       0.0      40.0          0      0.7     60.0      0.0      1.0
3  3.36733    3.3  96.8   3.2   0.0 4.81748 3.20993 1.60665    29.0      4.2     66.8      0.0      0.0      0.0       0.0       0.0          0      1.0     82.1      6.8      1.0
4  2.84673    3.8  95.6   4.4   0.0 4.92070 3.25249 1.66766     0.0      3.9     80.7      0.0      0.0      0.0      15.3       0.0          0      1.2     70.2      0.4      0.2
5  4.35600   10.2  45.8  54.2   0.0 6.43000 3.79700 2.63200     0.0      0.0     29.0      7.8     63.2      0.0       0.0       0.0          0      2.5     16.2      0.0     12.5
6  2.72600    5.0  88.9  11.1   0.0 4.93500 3.09400 1.84100     0.0      4.5     79.1     16.3      0.0      0.0       0.0       0.0          0      2.0     77.9      0.0      0.0
7  5.93400    5.4  88.3  11.7   0.0 6.29400 4.08700 2.20700   100.0      0.0      0.0      0.0      0.0      0.0       0.0       0.0          0      0.0     52.3      0.0      2.7
8  5.82148    4.2  94.1   5.9   0.0 6.27512 4.10684 2.16907    96.9      0.0      2.7      0.0      0.0      0.0       0.0       0.4          0      0.9     55.6      0.0      3.9
9  5.87100    2.8  99.4   0.6   0.0 6.15900 4.03400 2.12500    83.8      0.0     16.2      0.0      0.0      0.0       0.0       0.0          0      0.0     72.5      0.0      1.9
10 3.21085    3.9  92.7   7.3   0.0 4.67056 3.11699 1.55333    29.2      2.8     55.1      0.0      0.0      0.0       0.0      12.8          0      0.7     60.8     20.3      0.8
11 2.31900    3.4  98.5   1.5   0.0 4.92100 3.03400 1.88700     0.0      2.7     97.3      0.0      0.0      0.0       0.0       0.0          0      1.6     89.3      2.7      0.8
12 6.26800   13.2  23.7  76.0   0.4 7.96600 5.29000 2.67400     0.0     10.6     11.6      0.0      7.7     70.1       0.0       0.0          0      0.1     40.7      0.0     12.6
13 2.59527    4.0  95.9   4.1   0.0 4.85577 3.05492 1.80085     0.0      4.4     78.4      0.1      0.0      0.0      11.4       0.8          5      1.7     77.0      4.5      0.1
14 5.88500    8.3  60.5  39.5   0.0 7.37100 4.87400 2.49600     0.0      5.8     20.4      0.0      0.0     65.9       0.0       7.9          0      0.5     43.9      0.0      8.4
15 3.60400    4.8  91.5   8.6   0.0 6.69000 4.40500 2.28400     0.0      7.8     58.5      0.0      0.0     14.9       0.0      18.9          0      3.4     38.2      0.0      2.4
16 4.70100    7.4  64.6  35.4   0.0 7.01200 4.56800 2.44300     0.0     10.7     33.6      0.0      0.0     47.2       0.0       8.5          0      4.3     38.3      0.0      9.3
17 3.21219    5.9  75.9  24.1   0.0 6.30441 4.03484 2.26722     0.0     20.5     63.1      0.0      1.2     13.4       1.5       0.2          0      7.8     56.4      1.8      2.6
18 3.10100    3.9  92.3   7.7   0.0 6.83000 4.57900 2.25200     0.0      5.7     46.3      0.0      0.0      0.0      48.0       0.0          0      0.0     58.8      0.0      0.4
19 3.78307    5.0  86.4  13.6   0.0 6.80122 4.59959 2.20211     0.0      7.2     55.2      1.8      2.9      8.0      14.4      10.6          0      0.7     54.0      0.0      5.9
20 5.49500   10.4  40.4  59.6   0.0 7.31900 4.90700 2.41100     0.0     14.2     15.4     31.5     17.3     21.3       0.3       0.0          0      0.3     34.7      0.0      1.7
21 3.77400    5.6  80.9  19.1   0.0 6.80700 4.55500 2.25100     0.0      0.1     42.0     16.4     15.5      0.0      26.0       0.0          0      0.5     54.9      0.0      0.0
22 3.11818    4.6  92.6   7.4   0.0 6.63173 4.43962 2.19108     0.0     10.7     71.2      0.0      0.1      1.3      16.8       0.0          0      0.2     52.2      0.0      0.5
23 2.63700    3.8  94.1   5.9   0.0 6.30100 4.15800 2.14300     0.0      5.6     75.8      0.1      0.0      0.4      18.2       0.0          0      0.4     35.4      0.0      0.0
24 5.32000   10.0  46.1  53.9   0.0 7.16800 4.74900 2.42000     0.0      6.2     22.8     35.4      8.5      7.0      20.2       0.0          0      2.0     27.2      0.0      2.9
25 3.81100    9.1  50.1  50.0   0.0 7.67200 5.08000 2.59200     0.0      3.5     77.1      0.0      0.0      0.0      19.5       0.0          0      0.4     50.1      0.0      9.7
26 4.28987    5.9  81.3  18.7   0.0 7.26305 4.93896 2.32611     0.0     10.1     41.0      0.0      0.0     14.1      34.8       0.0          0      0.2     63.5      0.0      0.0
27 4.13700   10.9  40.2  59.3   0.4 7.45200 5.03900 2.41400     0.0      2.3     75.8      0.0      2.5      3.5      15.8       0.1          0      0.0     47.0      0.0     17.4
28 4.71400   10.7  54.9  42.7   2.4 7.36600 4.97900 2.38600     0.0      3.7     37.3      0.0      1.7     30.3      22.4       4.5          0      0.0     38.4      0.0     19.0
29 3.51700    8.1  59.6  40.4   0.0 7.99300 5.20700 2.78600     0.0     30.4     68.2      0.0      0.0      0.0       1.5       0.0          0      0.0     60.6      0.0      2.9
30 3.20800    8.9  51.7  48.3   0.0 7.78200 5.01000 2.77100     0.0     17.0     77.1      0.0      0.0      0.0       3.6       2.3          0      0.0     65.0      0.0      3.2
   BONU.WAL BONU.LOS BONU.WAS SDENS
1      43.4      0.0      1.0  8.62
2      38.3      0.0      0.0  8.77
3       9.1      0.0      0.0  6.50
4      28.0      0.0      0.0  5.20
5      68.7      0.0      0.0  6.93
6      20.0      0.0      0.0  5.67
7      44.7      0.0      0.3  9.24
8      39.0      0.0      0.5  7.99
```

```
9      25.6      0.0      0.0  7.47
10     17.3      0.0      0.0  6.70
11      5.5      0.0      0.0  3.57
12     46.6      0.0      0.0  9.15
13     16.7      0.1      0.0  5.31
14     47.2      0.0      0.0  8.81
15     56.1      0.0      0.0  7.20
16     48.1      0.0      0.0  8.39
17     31.2      0.2      0.0  6.27
18     40.8      0.0      0.0  9.15
19     39.4      0.0      0.0  8.58
20     63.3      0.0      0.0  9.52
21     44.6      0.0      0.0  9.00
22     46.9      0.0      0.3  7.53
23     64.1      0.0      0.1  8.15
24     67.9      0.0      0.0  8.66
25     39.9      0.0      0.0 10.63
26     36.3      0.0      0.0  9.50
27     35.6      0.0      0.0 10.18
28     42.2      0.2      0.3 10.49
29     36.5      0.0      0.0  9.49
30     31.8      0.0      0.0  9.53
```

```
data4chapter8[1:3,]
```

```
    code   river     station     lon     lat   elev    area cat_elev prec temp pet runoff  DBMSNR Q95so Q95wi  Q95 SUM.AREA H.MIN H.MAX H.DIFF  H.MEAN M.NEIG SL.FL SL.MG SL.ST
1 207985    Kamp   Rosenburg 15.6156 48.6300 261.58 1150.2      666  710 7.08 622    221 3001038  4.32  3.21 4.08    43.48  3.09  7.43   4.34 5.53800    6.2  81.3  18.5   0.2
2 207993    Kamp    Stiefern 15.6911 48.5353 217.00 1493.3      609  676 7.42 635    204 3001039  0.32  0.79 0.47    34.22  2.43  6.10   3.67 4.27404    5.5  82.9  17.1   0.0
3 208041 Schmida Hollenstein 15.9294 48.5556 225.54  212.0      324  528 9.16 702     39 3001044  0.23  0.47 0.29    21.33  2.35  4.60   2.25 3.36733    3.3  96.8   3.2   0.0
    N.GES   N.SOM   N.WIN GEOL.BM GEOL.QUA GEOL.TER GEOL.FLY GEOL.KAL GEOL.KRI GEOL.SHAL GEOL.DEEP GEOL.QUELL BONU.URB BONU.ACK BONU.DAU BONU.GRU BONU.WAL BONU.LOS BONU.WAS SDENS
1 5.84000 3.86100 1.98000    93.7      0.0      6.2        0        0        0         0       0.1          0      0.0     47.1      0.0      8.5     43.4        0        1  8.62
2 5.17672 3.43264 1.74367    53.9      0.0      6.1        0        0        0         0      40.0          0      0.7     60.0      0.0      1.0     38.3        0        0  8.77
3 4.81748 3.20993 1.60665    29.0      4.2     66.8        0        0        0         0       0.0          0      1.0     82.1      6.8      1.0      9.1        0        0  6.50
```

```
head(data4chapter8)
```

```
    code        river        station     lon     lat   elev    area cat_elev prec temp pet runoff  DBMSNR Q95so Q95wi  Q95 SUM.AREA H.MIN H.MAX H.DIFF  H.MEAN M.NEIG SL.FL SL.MG
1 207985         Kamp      Rosenburg 15.6156 48.6300 261.58 1150.2      666  710 7.08 622    221 3001038  4.32  3.21 4.08    43.48  3.09  7.43   4.34 5.53800    6.2  81.3  18.5
2 207993         Kamp       Stiefern 15.6911 48.5353 217.00 1493.3      609  676 7.42 635    204 3001039  0.32  0.79 0.47    34.22  2.43  6.10   3.67 4.27404    5.5  82.9  17.1
3 208041      Schmida    Hollenstein 15.9294 48.5556 225.54  212.0      324  528 9.16 702     39 3001044  0.23  0.47 0.29    21.33  2.35  4.60   2.25 3.36733    3.3  96.8   3.2
4 208058 Goellersbach  Obermallebern 16.1589 48.4653 189.13  379.9      273  548 9.63 720     33 3001045  0.39  0.48 0.42    37.95  1.96  4.34   2.38 2.84673    3.8  95.6   4.4
5 208108      Moedling       Moedling 16.2800 48.0839 222.43   59.0      406  728 9.11 697    137 3001049  0.89  1.28 0.99     6.15  2.87  6.64   3.77 4.35600   10.2  45.8  54.2
6 208447     Russbach Ulrichskirchen 16.4997 48.4039 173.32  131.5      255  568 9.86 726     49 3001080  0.33  0.72 0.39    12.95  1.87  3.75   1.88 2.72600    5.0  88.9  11.1
  SL.ST   N.GES   N.SOM   N.WIN GEOL.BM GEOL.QUA GEOL.TER GEOL.FLY GEOL.KAL GEOL.KRI GEOL.SHAL GEOL.DEEP GEOL.QUELL BONU.URB BONU.ACK BONU.DAU BONU.GRU BONU.WAL BONU.LOS BONU.WAS
1   0.2 5.84000 3.86100 1.98000    93.7      0.0      6.2      0.0      0.0        0       0.0       0.1          0      0.0     47.1      0.0      8.5     43.4        0        1
2   0.0 5.17672 3.43264 1.74367    53.9      0.0      6.1      0.0      0.0        0       0.0      40.0          0      0.7     60.0      0.0      1.0     38.3        0        0
3   0.0 4.81748 3.20993 1.60665    29.0      4.2     66.8      0.0      0.0        0       0.0       0.0          0      1.0     82.1      6.8      1.0      9.1        0        0
4   0.0 4.92070 3.25249 1.66766     0.0      3.9     80.7      0.0      0.0        0      15.3       0.0          0      1.2     70.2      0.4      0.2     28.0        0        0
5   0.0 6.43000 3.79700 2.63200     0.0      0.0     29.0      7.8     63.2        0       0.0       0.0          0      2.5     16.2      0.0     12.5     68.7        0        0
6   0.0 4.93500 3.09400 1.84100     0.0      4.5     79.1     16.3      0.0        0       0.0       0.0          0      2.0     77.9      0.0      0.0     20.0        0        0
  SDENS
1  8.62
2  8.77
3  6.50
4  5.20
5  6.93
6  5.67
```

```
str(data4chapter8)          # Shows data structure (variable types!)
```

```
'data.frame':        30 obs. of  45 variables:
 $ code      : int  207985 207993 208041 208058 208108 208447 208512 208579 208611 208637 ...
 $ river     : Factor w/ 24 levels "Braunaubach",..: 8 8 17 5 10 16 1 1 2 13 ...
 $ station   : Factor w/ 30 levels "Altschlaining",..: 22 24 11 18 15 25 2 10 23 30 ...
 $ lon       : num  15.6 15.7 15.9 16.2 16.3 ...
 $ lat       : num  48.6 48.5 48.6 48.5 48.1 ...
 $ elev      : num  262 217 226 189 222 ...
 $ area      : num  1150 1493 212 380 59 ...
 $ cat_elev  : int  666 609 324 273 406 255 586 574 570 305 ...
 $ prec      : int  710 676 528 548 728 568 672 673 668 502 ...
 $ temp      : num  7.08 7.42 9.16 9.63 9.11 9.86 7.55 7.62 7.66 9.27 ...
 $ pet       : int  622 635 702 720 697 726 635 640 642 706 ...
 $ runoff    : int  221 204 39 33 137 49 240 240 199 31 ...
 $ DBMSNR    : int  3001038 3001039 3001044 3001045 3001049 3001080 3001083 3001085 3001088 3001090 ...
 $ Q95so     : num  4.32 0.32 0.23 0.39 0.89 0.33 0.79 0.81 0.62 0.09 ...
 $ Q95wi     : num  3.21 0.79 0.47 0.48 1.28 0.72 1.73 1.45 1.19 0.33 ...
 $ Q95       : num  4.08 0.47 0.29 0.42 0.99 0.39 0.97 0.9 0.71 0.12 ...
 $ SUM.AREA  : num  43.48 34.22 21.33 37.95 6.15 ...
 $ H.MIN     : num  3.09 2.43 2.35 1.96 2.87 1.87 5.33 5 5.1 1.84 ...
 $ H.MAX     : num  7.43 6.1 4.6 4.34 6.64 3.75 6.96 7.09 6.82 5.48 ...
 $ H.DIFF    : num  4.34 3.67 2.25 2.38 3.77 1.88 1.63 2.09 1.72 3.64 ...
 $ H.MEAN    : num  5.54 4.27 3.37 2.85 4.36 ...
 $ M.NEIG    : num  6.2 5.5 3.3 3.8 10.2 5 5.4 4.2 2.8 3.9 ...
 $ SL.FL     : num  81.3 82.9 96.8 95.6 45.8 88.9 88.3 94.1 99.4 92.7 ...
 $ SL.MG     : num  18.5 17.1 3.2 4.4 54.2 11.1 11.7 5.9 0.6 7.3 ...
 $ SL.ST     : num  0.2 0 0 0 0 0 0 0 0 0 ...
 $ N.GES     : num  5.84 5.18 4.82 4.92 6.43 ...
 $ N.SOM     : num  3.86 3.43 3.21 3.25 3.8 ...
 $ N.WIN     : num  1.98 1.74 1.61 1.67 2.63 ...
 $ GEOL.BM   : num  93.7 53.9 29 0 0 0 100 96.9 83.8 29.2 ...
 $ GEOL.QUA  : num  0 0 4.2 3.9 0 4.5 0 0 0 2.8 ...
 $ GEOL.TER  : num  6.2 6.1 66.8 80.7 29 79.1 0 2.7 16.2 55.1 ...
 $ GEOL.FLY  : num  0 0 0 0 7.8 16.3 0 0 0 0 ...
 $ GEOL.KAL  : num  0 0 0 0 63.2 0 0 0 0 0 ...
 $ GEOL.KRI  : num  0 0 0 0 0 0 0 0 0 0 ...
 $ GEOL.SHAL : num  0 0 0 15.3 0 0 0 0 0 0 ...
 $ GEOL.DEEP : num  0.1 40 0 0 0 0 0 0.4 0 12.8 ...
 $ GEOL.QUELL: int  0 0 0 0 0 0 0 0 0 0 ...
 $ BONU.URB  : num  0 0.7 1 1.2 2.5 2 0 0.9 0 0.7 ...
 $ BONU.ACK  : num  47.1 60 82.1 70.2 16.2 77.9 52.3 55.6 72.5 60.8 ...
 $ BONU.DAU  : num  0 0 6.8 0.4 0 0 0 0 0 20.3 ...
 $ BONU.GRU  : num  8.5 1 1 0.2 12.5 0 2.7 3.9 1.9 0.8 ...
 $ BONU.WAL  : num  43.4 38.3 9.1 28 68.7 20 44.7 39 25.6 17.3 ...
 $ BONU.LOS  : num  0 0 0 0 0 0 0 0 0 0 ...
 $ BONU.WAS  : num  1 0 0 0 0 0 0.3 0.5 0 0 ...
 $ SDENS     : num  8.62 8.77 6.5 5.2 6.93 5.67 9.24 7.99 7.47 6.7 ...
```

```
dim(data4chapter8)          # dimension
```

```
[1] 30 45
```

```
nrow(data4chapter8)         # number of rows
```

```
[1] 30
```

```
ncol(data4chapter8)        # number of colums
```

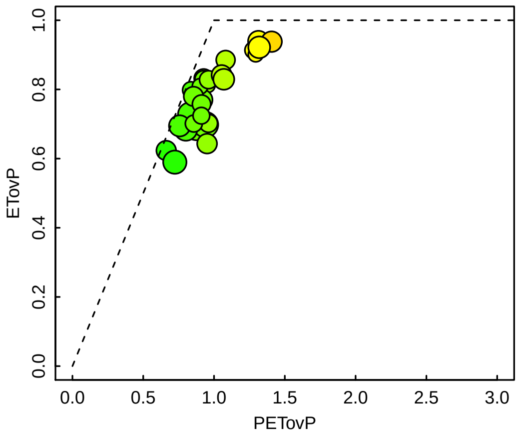
```
[1] 45
```

```
x0 <- data4chapter8[,-c(2:12)]
```

Budyko:

```
PETovP <- data4chapter8$pet/data4chapter8$prec
ETovP <- (data4chapter8$prec - data4chapter8$runoff)/data4chapter8$prec
```

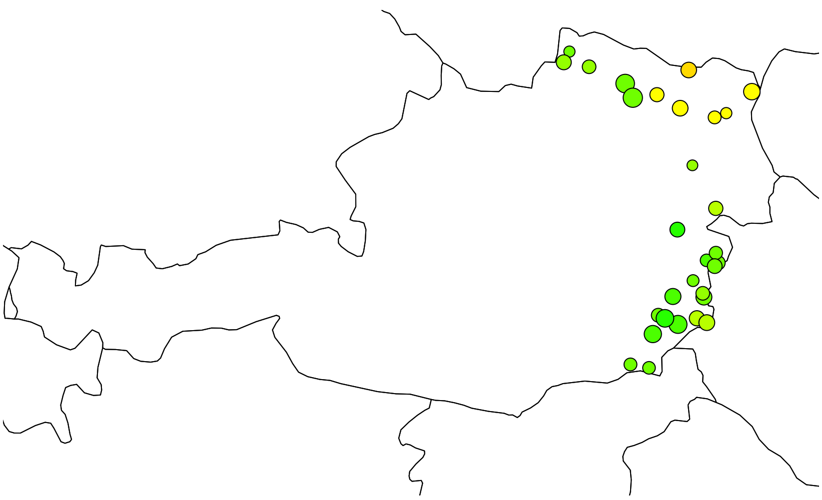
```
colori <- rev(rainbow(20, start=0, end=.45, alpha=1))
plot(PETovP, ETovP, xlim=c(0,3), ylim=c(0,1), pch=21,
     bg=colori[round(10*PETovP)],
     cex=log10(data4chapter8$area))
 segments(x0=c(0,1), y0=c(0,1), x1=c(1,4), y1=c(1,1), lty=2)
```



Plot the data on a map:

```
library(rworldmap)
newMap <- getMap(resolution="low")
```

```
plot(newMap, xlim=c(11.5, 15.5), ylim=c(46, 49))
 points(data4chapter8$lon, data4chapter8$lat, pch=21,
        bg=colori[round(10*PETovP)],
        cex=log10(data4chapter8$area))
```



2 Simple linear regression

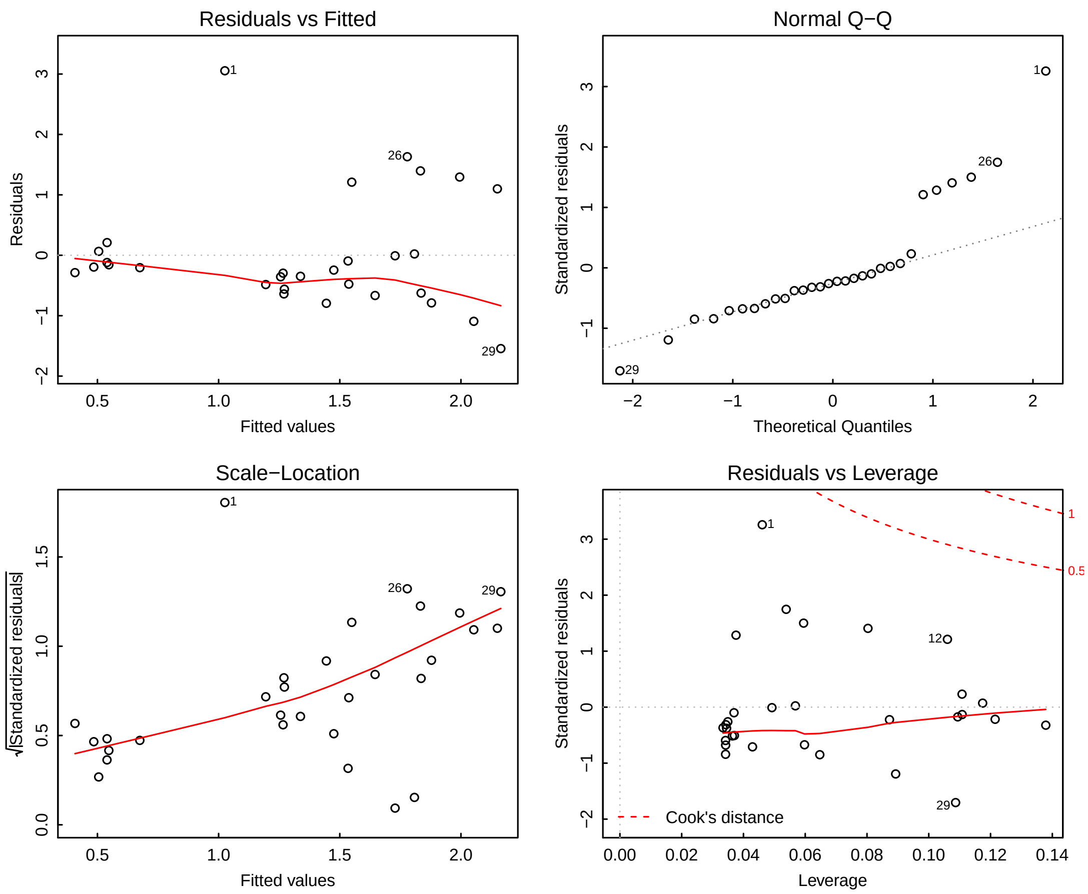
```r
x <- x0
x.lm <- lm(Q95 ~ N.GES, data=x)
summary(x.lm)
```

```
Call:
lm(formula = Q95 ~ N.GES, data = x)

Residuals:
    Min      1Q  Median      3Q     Max
-1.5445 -0.5422 -0.2251  0.0541  3.0542

Coefficients:
            Estimate Std. Error t value Pr(>|t|)
(Intercept)  -2.0631     1.1309  -1.824  0.07878 .
N.GES         0.5289     0.1727   3.062  0.00482 **
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.96 on 28 degrees of freedom
Multiple R-squared:  0.2508,	Adjusted R-squared:  0.2241
F-statistic: 9.375 on 1 and 28 DF,  p-value: 0.004817
```

```r
str(x.lm) # explore the fitted model object
```

```
List of 12
 $ coefficients : Named num [1:2] -2.063 0.529
  ..- attr(*, "names")= chr [1:2] "(Intercept)" "N.GES"
 $ residuals    : Named num [1:30] 3.054 -0.205 -0.195 -0.12 -0.348 ...
  ..- attr(*, "names")= chr [1:30] "1" "2" "3" "4" ...
 $ effects      : Named num [1:30] -7.436 2.939 -0.511 -0.446 -0.825 ...
  ..- attr(*, "names")= chr [1:30] "(Intercept)" "N.GES" "" "" ...
 $ rank         : int 2
 $ fitted.values: Named num [1:30] 1.026 0.675 0.485 0.54 1.338 ...
  ..- attr(*, "names")= chr [1:30] "1" "2" "3" "4" ...
 $ assign       : int [1:2] 0 1
 $ qr           :List of 5
  ..$ qr   : num [1:30, 1:2] -5.477 0.183 0.183 0.183 0.183 ...
  .. ..- attr(*, "dimnames")=List of 2
  .. .. ..$ : chr [1:30] "1" "2" "3" "4" ...
  .. .. ..$ : chr [1:2] "(Intercept)" "N.GES"
  .. ..- attr(*, "assign")= int [1:2] 0 1
  ..$ qraux: num [1:2] 1.18 1.21
  ..$ pivot: int [1:2] 1 2
  ..$ tol  : num 1e-07
  ..$ rank : int 2
  ..- attr(*, "class")= chr "qr"
 $ df.residual  : int 28
 $ xlevels      : Named list()
 $ call         : language lm(formula = Q95 ~ N.GES, data = x)
 $ terms        :Classes 'terms', 'formula' length 3 Q95 ~ N.GES
  .. ..- attr(*, "variables")= language list(Q95, N.GES)
  .. ..- attr(*, "factors")= int [1:2, 1] 0 1
  .. .. ..- attr(*, "dimnames")=List of 2
  .. .. .. ..$ : chr [1:2] "Q95" "N.GES"
  .. .. .. ..$ : chr "N.GES"
  .. ..- attr(*, "term.labels")= chr "N.GES"
  .. ..- attr(*, "order")= int 1
  .. ..- attr(*, "intercept")= int 1
  .. ..- attr(*, "response")= int 1
  .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  .. ..- attr(*, "predvars")= language list(Q95, N.GES)
  .. ..- attr(*, "dataClasses")= Named chr [1:2] "numeric" "numeric"
  .. .. ..- attr(*, "names")= chr [1:2] "Q95" "N.GES"
 $ model        :'data.frame':	30 obs. of  2 variables:
  ..$ Q95  : num [1:30] 4.08 0.47 0.29 0.42 0.99 0.39 0.97 0.9 0.71 0.12 ...
  ..$ N.GES: num [1:30] 5.84 5.18 4.82 4.92 6.43 ...
  ..- attr(*, "terms")=Classes 'terms', 'formula' length 3 Q95 ~ N.GES
  .. .. ..- attr(*, "variables")= language list(Q95, N.GES)
  .. .. ..- attr(*, "factors")= int [1:2, 1] 0 1
  .. .. .. ..- attr(*, "dimnames")=List of 2
  .. .. .. .. ..$ : chr [1:2] "Q95" "N.GES"
  .. .. .. .. ..$ : chr "N.GES"
  .. .. ..- attr(*, "term.labels")= chr "N.GES"
  .. .. ..- attr(*, "order")= int 1
  .. .. ..- attr(*, "intercept")= int 1
  .. .. ..- attr(*, "response")= int 1
  .. .. ..- attr(*, ".Environment")=<environment: R_GlobalEnv>
  .. .. ..- attr(*, "predvars")= language list(Q95, N.GES)
  .. .. ..- attr(*, "dataClasses")= Named chr [1:2] "numeric" "numeric"
  .. .. .. ..- attr(*, "names")= chr [1:2] "Q95" "N.GES"
 - attr(*, "class")= chr "lm"
```

```r
layout(matrix(1:4, nrow=2, byrow=TRUE))
plot(x.lm)
```



3 Stepwise regression

First create fomula object, to serve as a scope of model fitting (all potential predictors).

```
x.form <- formula(~ H.MIN + H.MAX + H.DIFF + H.MEAN + M.NEIG + SL.FL + SL.MG + SL.ST +
                    N.GES + N.SOM + N.WIN + GEOL.BM + GEOL.QUA + GEOL.TER + GEOL.FLY + GEOL.KAL +
                    GEOL.KRI + GEOL.SHAL + GEOL.DEEP + GEOL.QUELL + BONU.URB + BONU.ACK + BONU.DAU +
                    BONU.GRU + BONU.WAL + BONU.LOS + BONU.WAS + SDENS)
# better automatic:
x.form <- as.formula(paste("~", paste(names(x)[-(1:6)], collapse="+")))
```

Add one additional variable with highest explicative value:

```
add1(x.lm, scope=x.form)
```

```
Single term additions

Model:
Q95 ~ N.GES
           Df Sum of Sq    RSS     AIC
<none>                  25.804 -0.5204
H.MIN       1    0.0723 25.731  1.3954
H.MAX       1    2.5418 23.262 -1.6315
H.DIFF      1    2.2560 23.548 -1.2651
H.MEAN      1    1.6324 24.171 -0.4810
M.NEIG      1    0.7097 25.094  0.6429
SL.FL       1    0.1427 25.661  1.3132
SL.MG       1    0.1105 25.693  1.3509
SL.ST       1    3.0734 22.730 -2.3251
N.SOM       1    2.0601 23.744 -1.0166
N.WIN       1    2.0292 23.774 -0.9775
GEOL.BM     1    0.8388 24.965  0.4882
GEOL.QUA    1    3.4904 22.313 -2.8804
GEOL.TER    1    1.8171 23.987 -0.7111
GEOL.FLY    1    0.0693 25.734  1.3989
GEOL.KAL    1    0.1086 25.695  1.3530
GEOL.KRI    1    0.3474 25.456  1.0729
GEOL.SHAL   1    2.9767 22.827 -2.1977
GEOL.DEEP   1    0.3058 25.498  1.1219
GEOL.QUELL  1    0.0048 25.799  1.4740
BONU.URB    1    1.2721 24.532 -0.0371
BONU.ACK    1    0.1356 25.668  1.3215
BONU.DAU    1    0.1649 25.639  1.2873
BONU.GRU    1    1.9892 23.814 -0.9272
BONU.WAL    1    0.0083 25.795  1.4700
BONU.LOS    1    0.3691 25.434  1.0473
BONU.WAS    1    6.4173 19.386 -7.0989
SDENS       1    2.5644 23.239 -1.6606
```

Drop one predictor included in the model with least explicative value:

```
drop1(x.lm)
```

```
Single term deletions

Model:
Q95 ~ N.GES
       Df Sum of Sq    RSS     AIC
<none>              25.804 -0.5204
N.GES   1    8.6401 34.444  6.1439
```

Now atomatically - stepwise regression with `step`:

```
x <- x0
x.lm0 <- lm(Q95 ~ N.GES, data=x)
x.lm1 <- step(x.lm0, scope=x.form)
```

```
Start:  AIC=-0.52
Q95 ~ N.GES

             Df Sum of Sq    RSS     AIC
+ BONU.WAS    1    6.4173 19.386 -7.0989
+ GEOL.QUA    1    3.4904 22.313 -2.8804
+ SL.ST       1    3.0734 22.730 -2.3251
+ GEOL.SHAL   1    2.9767 22.827 -2.1977
+ SDENS       1    2.5644 23.239 -1.6606
+ H.MAX       1    2.5418 23.262 -1.6315
+ H.DIFF      1    2.2560 23.548 -1.2651
+ N.SOM       1    2.0601 23.744 -1.0166
+ N.WIN       1    2.0292 23.774 -0.9775
+ BONU.GRU    1    1.9892 23.814 -0.9272
+ GEOL.TER    1    1.8171 23.987 -0.7111
<none>                    25.804 -0.5204
+ H.MEAN      1    1.6324 24.171 -0.4810
+ BONU.URB    1    1.2721 24.532 -0.0371
+ GEOL.BM     1    0.8388 24.965  0.4882
+ M.NEIG      1    0.7097 25.094  0.6429
+ BONU.LOS    1    0.3691 25.435  1.0473
+ GEOL.KRI    1    0.3474 25.456  1.0729
+ GEOL.DEEP   1    0.3058 25.498  1.1219
+ BONU.DAU    1    0.1649 25.639  1.2873
+ SL.FL       1    0.1427 25.661  1.3132
+ BONU.ACK    1    0.1356 25.668  1.3215
+ SL.MG       1    0.1105 25.693  1.3509
+ GEOL.KAL    1    0.1086 25.695  1.3530
+ H.MIN       1    0.0723 25.731  1.3954
+ GEOL.FLY    1    0.0693 25.734  1.3989
+ BONU.WAL    1    0.0083 25.795  1.4700
+ GEOL.QUELL  1    0.0048 25.799  1.4740
- N.GES       1    8.6401 34.444  6.1439

Step:  AIC=-7.1
Q95 ~ N.GES + BONU.WAS

             Df Sum of Sq    RSS      AIC
+ GEOL.SHAL   1    3.9512 15.435 -11.9365
+ H.DIFF      1    1.9431 17.443  -8.2674
+ GEOL.QUA    1    1.5594 17.827  -7.6146
+ SL.ST       1    1.3085 18.078  -7.1954
+ N.SOM       1    1.2926 18.094  -7.1691
+ N.WIN       1    1.2775 18.109  -7.1440
<none>                    19.386  -7.0989
+ H.MAX       1    1.1171 18.269  -6.8794
+ GEOL.BM     1    0.9196 18.467  -6.5568
+ M.NEIG      1    0.8442 18.542  -6.4346
+ GEOL.KRI    1    0.7492 18.637  -6.2813
+ SDENS       1    0.6645 18.722  -6.1453
+ BONU.GRU    1    0.6308 18.756  -6.0913
+ SL.FL       1    0.5889 18.797  -6.0243
+ SL.MG       1    0.5531 18.833  -5.9673
+ H.MIN       1    0.5058 18.881  -5.8920
+ BONU.URB    1    0.3049 19.081  -5.5745
+ BONU.LOS    1    0.2192 19.167  -5.4400
+ BONU.WAL    1    0.1733 19.213  -5.3682
+ GEOL.QUELL  1    0.1055 19.281  -5.2625
+ GEOL.TER    1    0.0989 19.287  -5.2523
+ H.MEAN      1    0.0555 19.331  -5.1849
+ BONU.ACK    1    0.0308 19.355  -5.1466
+ GEOL.DEEP   1    0.0229 19.363  -5.1343
+ GEOL.FLY    1    0.0158 19.370  -5.1234
+ BONU.DAU    1    0.0062 19.380  -5.1085
+ GEOL.KAL    1    0.0006 19.386  -5.0998
- BONU.WAS    1    6.4173 25.804  -0.5204
- N.GES       1    9.7287 29.115   3.1017

Step:  AIC=-11.94
Q95 ~ N.GES + BONU.WAS + GEOL.SHAL

             Df Sum of Sq    RSS      AIC
+ M.NEIG      1    2.8725 12.563 -16.1140
+ SL.FL       1    2.7642 12.671 -15.8566
+ SL.MG       1    2.7517 12.683 -15.8270
+ GEOL.KRI    1    2.6955 12.740 -15.6943
+ BONU.GRU    1    1.4617 13.973 -12.9212
+ H.DIFF      1    1.4330 14.002 -12.8595
+ H.MEAN      1    1.3509 14.084 -12.6841
+ H.MAX       1    1.2433 14.192 -12.4559
<none>                    15.435 -11.9365
+ GEOL.TER    1    0.6925 14.743 -11.3136
+ SL.ST       1    0.6868 14.748 -11.3020
+ GEOL.QUA    1    0.5718 14.863 -11.0690
+ BONU.WAL    1    0.2020 15.233 -10.3316
+ GEOL.BM     1    0.1059 15.329 -10.1431
+ SDENS       1    0.0929 15.342 -10.1176
+ BONU.LOS    1    0.0898 15.345 -10.1115
+ GEOL.DEEP   1    0.0711 15.364 -10.0749
+ N.SOM       1    0.0701 15.365 -10.0731
+ N.WIN       1    0.0697 15.366 -10.0723
+ BONU.DAU    1    0.0660 15.369 -10.0650
+ GEOL.KAL    1    0.0605 15.375 -10.0543
+ H.MIN       1    0.0265 15.409  -9.9880
+ GEOL.QUELL  1    0.0099 15.425  -9.9558
```

```
+ GEOL.FLY    1    0.0093 15.426  -9.9545
+ BONU.URB    1    0.0012 15.434  -9.9388
+ BONU.ACK    1    0.0009 15.434  -9.9383
- GEOL.SHAL   1    3.9512 19.386  -7.0989
- N.GES       1    6.3218 21.757  -3.6380
- BONU.WAS    1    7.3918 22.827  -2.1977

Step:  AIC=-16.11
Q95 ~ N.GES + BONU.WAS + GEOL.SHAL + M.NEIG

             Df Sum of Sq    RSS      AIC
+ GEOL.KRI    1    1.6050 10.958 -18.2148
- N.GES       1    0.1320 12.695 -17.8005
<none>                    12.563 -16.1140
+ H.MEAN      1    0.7616 11.801 -15.9902
+ BONU.WAL    1    0.5766 11.986 -15.5235
+ BONU.ACK    1    0.5029 12.060 -15.3397
+ GEOL.TER    1    0.4216 12.141 -15.1380
+ GEOL.FLY    1    0.3032 12.259 -14.8469
+ GEOL.KAL    1    0.2998 12.263 -14.8386
+ N.SOM       1    0.2268 12.336 -14.6606
+ N.WIN       1    0.2244 12.338 -14.6548
+ H.MAX       1    0.1931 12.370 -14.5787
+ H.DIFF      1    0.1242 12.438 -14.4120
+ GEOL.QUA    1    0.1138 12.449 -14.3870
+ GEOL.DEEP   1    0.0991 12.464 -14.3517
+ GEOL.BM     1    0.0830 12.480 -14.3130
+ H.MIN       1    0.0536 12.509 -14.2422
+ BONU.GRU    1    0.0218 12.541 -14.1661
+ BONU.DAU    1    0.0105 12.552 -14.1390
+ GEOL.QUELL  1    0.0089 12.554 -14.1352
+ BONU.LOS    1    0.0043 12.558 -14.1243
+ SL.ST       1    0.0031 12.560 -14.1214
+ SL.MG       1    0.0021 12.561 -14.1190
+ SL.FL       1    0.0009 12.562 -14.1162
+ BONU.URB    1    0.0004 12.562 -14.1151
+ SDENS       1    0.0004 12.562 -14.1150
- M.NEIG      1    2.8725 15.435 -11.9365
- GEOL.SHAL   1    5.9794 18.542  -6.4346
- BONU.WAS    1    7.9926 20.555  -3.3423

Step:  AIC=-18.21
Q95 ~ N.GES + BONU.WAS + GEOL.SHAL + M.NEIG + GEOL.KRI

             Df Sum of Sq    RSS      AIC
- N.GES       1    0.0070 10.965 -20.1958
<none>                    10.958 -18.2148
+ BONU.ACK    1    0.6619 10.296 -18.0840
+ BONU.WAL    1    0.5727 10.385 -17.8253
+ GEOL.BM     1    0.2743 10.683 -16.9753
+ SL.MG       1    0.2000 10.758 -16.7675
+ H.MEAN      1    0.1922 10.765 -16.7457
+ SL.FL       1    0.1845 10.773 -16.7243
+ GEOL.TER    1    0.0817 10.876 -16.4394
+ GEOL.FLY    1    0.0759 10.882 -16.4234
+ BONU.LOS    1    0.0687 10.889 -16.4036
+ SL.ST       1    0.0635 10.894 -16.3891
+ GEOL.QUA    1    0.0498 10.908 -16.3516
+ H.MIN       1    0.0481 10.910 -16.3467
+ H.DIFF      1    0.0460 10.912 -16.3410
+ SDENS       1    0.0437 10.914 -16.3346
+ GEOL.KAL    1    0.0346 10.923 -16.3097
+ GEOL.QUELL  1    0.0245 10.933 -16.2820
+ GEOL.DEEP   1    0.0233 10.934 -16.2788
+ BONU.URB    1    0.0224 10.935 -16.2761
+ BONU.GRU    1    0.0135 10.944 -16.2517
+ H.MAX       1    0.0065 10.951 -16.2326
+ N.SOM       1    0.0045 10.953 -16.2273
+ N.WIN       1    0.0038 10.954 -16.2251
+ BONU.DAU    1    0.0036 10.954 -16.2248
- GEOL.KRI    1    1.6050 12.563 -16.1140
- M.NEIG      1    1.7820 12.740 -15.6943
- GEOL.SHAL   1    7.2308 18.188  -5.0124
- BONU.WAS    1    8.8406 19.798  -2.4682

Step:  AIC=-20.2
Q95 ~ BONU.WAS + GEOL.SHAL + M.NEIG + GEOL.KRI

             Df Sum of Sq    RSS      AIC
<none>                    10.965 -20.1958
+ BONU.ACK    1    0.6214 10.343 -19.9462
+ BONU.WAL    1    0.4794 10.485 -19.5370
+ GEOL.BM     1    0.2811 10.684 -18.9750
+ SL.MG       1    0.2009 10.764 -18.7506
+ H.MEAN      1    0.1985 10.766 -18.7438
+ SL.FL       1    0.1867 10.778 -18.7109
+ GEOL.TER    1    0.0847 10.880 -18.4283
+ GEOL.FLY    1    0.0816 10.883 -18.4200
+ BONU.LOS    1    0.0750 10.890 -18.4017
+ SL.ST       1    0.0703 10.894 -18.3888
+ H.MIN       1    0.0548 10.910 -18.3462
+ H.DIFF      1    0.0521 10.912 -18.3386
+ SDENS       1    0.0461 10.918 -18.3222
+ GEOL.KAL    1    0.0396 10.925 -18.3044
+ GEOL.QUELL  1    0.0304 10.934 -18.2790
+ BONU.URB    1    0.0246 10.940 -18.2631
+ GEOL.QUA    1    0.0215 10.943 -18.2547
+ GEOL.DEEP   1    0.0204 10.944 -18.2516
+ BONU.GRU    1    0.0159 10.949 -18.2394
+ N.SOM       1    0.0092 10.955 -18.2209
+ N.GES       1    0.0070 10.958 -18.2148
+ H.MAX       1    0.0063 10.958 -18.2131
+ N.WIN       1    0.0024 10.962 -18.2024
+ BONU.DAU    1    0.0004 10.964 -18.1970
- GEOL.KRI    1    1.7301 12.695 -17.8005
- M.NEIG      1    3.4587 14.423 -13.9705
- BONU.WAS    1    8.8869 19.852  -4.3875
- GEOL.SHAL   1    9.2632 20.228  -3.8242
```

```
summary(x.lm1)
```

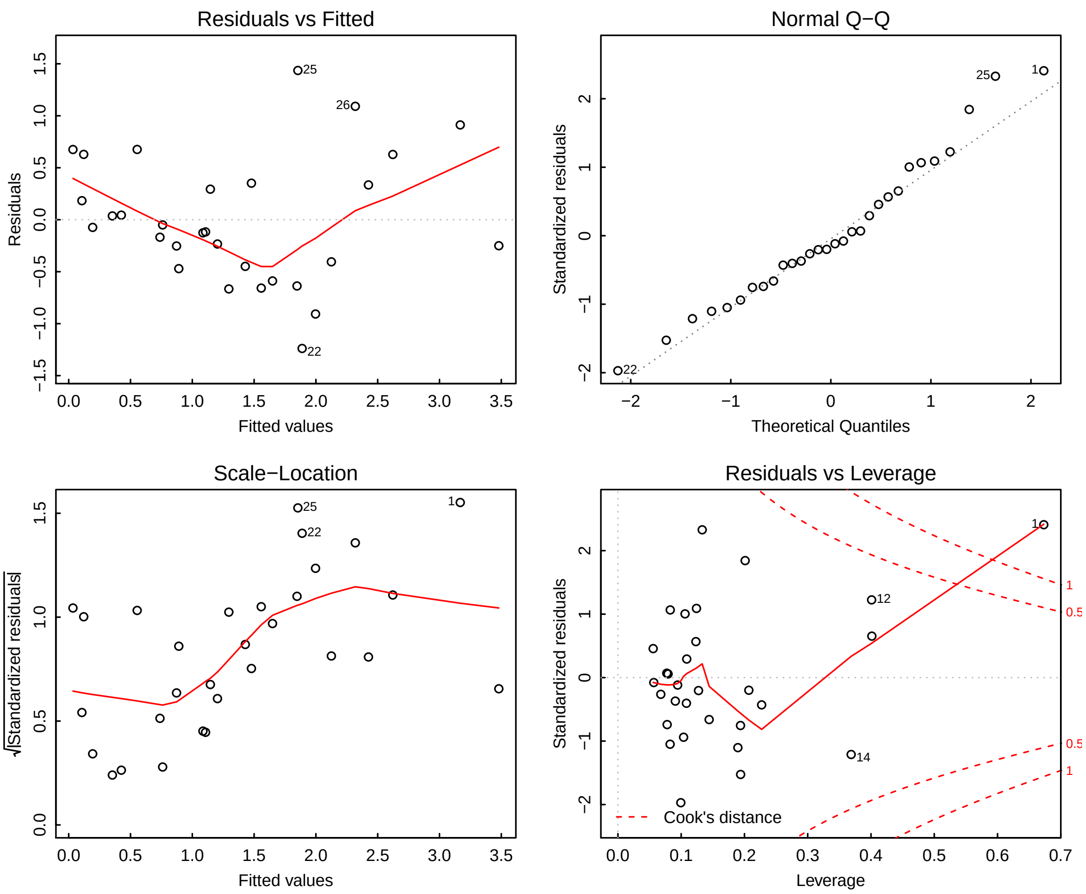
```
Call:
lm(formula = Q95 ~ BONU.WAS + GEOL.SHAL + M.NEIG + GEOL.KRI,
    data = x)

Residuals:
     Min       1Q   Median       3Q      Max
-1.23854 -0.43772 -0.09556  0.34741  1.43576
```

```
Coefficients:
             Estimate Std. Error t value Pr(>|t|)
(Intercept) -0.371534   0.341822  -1.087 0.287439
BONU.WAS     2.640617   0.586619   4.501 0.000136 ***
GEOL.SHAL    0.046486   0.010115   4.596 0.000106 ***
M.NEIG       0.144978   0.051626   2.808 0.009524 **
GEOL.KRI     0.015409   0.007758   1.986 0.058088 .
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1

Residual standard error: 0.6623 on 25 degrees of freedom
Multiple R-squared:  0.6817,	Adjusted R-squared:  0.6307
F-statistic: 13.38 on 4 and 25 DF,  p-value: 5.817e-06
```

```
anova(x.lm1)    # also possible to have an anova-like F-test of predictor significance
```

```
Analysis of Variance Table

Response: Q95
          Df  Sum Sq Mean Sq F value    Pr(>F)
BONU.WAS   1  5.3288  5.3288 12.1499 0.0018301 **
GEOL.SHAL  1  7.3581  7.3581 16.7769 0.0003871 ***
M.NEIG     1  9.0623  9.0623 20.6626 0.0001210 ***
GEOL.KRI   1  1.7301  1.7301  3.9447 0.0580884 .
Residuals 25 10.9646  0.4386
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
layout(matrix(1:4, nrow=2, byrow=TRUE))
plot(x.lm1)
```



Again stepwise - after eliminating outliers:

1. Detect outliers based on Cook's distance (in diagnosis plots)
2. Now eliminate outliers manually

```
x <- x0[-c(12),]
dim(x)
```

```
[1] 29 34
```

3. Then: back to start! (model fitting...)

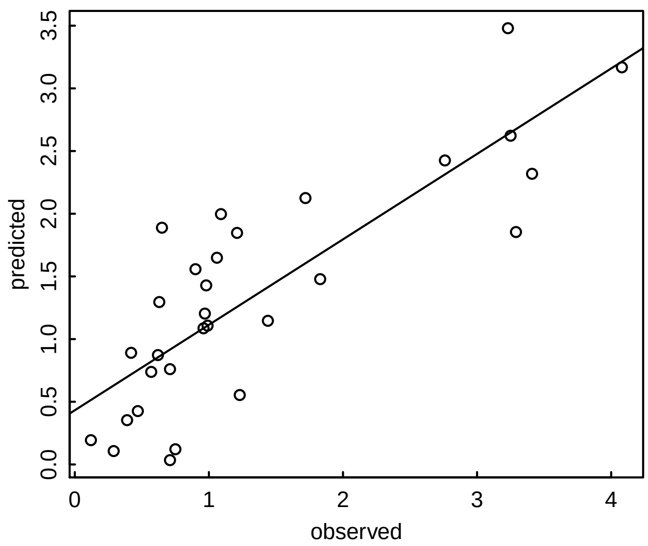
4 More plotting

A) show model performance in a scatter plot (predicted vs. observed values)

```
attributes(x.lm1)
```

```
$names
 [1] "coefficients"  "residuals"     "effects"       "rank"          "fitted.values" "assign"        "qr"            "df.residual"   "xlevels"       "call"          "terms"
[12] "model"         "anova"

$class
[1] "lm"
```

```
plot(x.lm1$model$Q95 , x.lm1$fitted.values, xlab="observed", ylab="predicted")
 abline(lsfit(x.lm1$model$Q95, x.lm1$fitted.values))
```



B) What is the contribution of each predictor to the model estimate? => templot ... plots each regression term (`BETAj*xj`) against its predictor `xj`

```
termplot(x.lm1, partial=TRUE)
# Gregor!! this gives me the following error:
# Error in xy.coords(x, y) : 'x' and 'y' lengths differ
```

5 Robust regression

This is alternative to manual outlier detection.

```
library(robustbase)
```

```
x.lm1$call
```

```
lm(formula = Q95 ~ BONU.WAS + GEOL.SHAL + M.NEIG + GEOL.KRI,
    data = x)
```

```
x.lts <- ltsReg(formula = Q95 ~ GEOL.SHAL + GEOL.KRI + M.NEIG + H.MIN, data=x, alpha=0.9)
summary(x.lts)
```

```
Call:
ltsReg.formula(formula = Q95 ~ GEOL.SHAL + GEOL.KRI + M.NEIG +
    H.MIN, data = x, alpha = 0.9)

Residuals (from reweighted LS):
    Min      1Q  Median      3Q     Max
-0.6997 -0.2848  0.0000  0.1181  0.9294

Coefficients:
           Estimate Std. Error t value Pr(>|t|)
Intercept -0.523924   0.360788  -1.452   0.1606
```

```
GEOL.SHAL  0.048783   0.007447   6.551 1.37e-06 ***
GEOL.KRI   0.014718   0.006341   2.321   0.0299 *
M.NEIG     0.083646   0.039235   2.132   0.0444 *
H.MIN      0.201944   0.096231   2.099   0.0476 *
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1

Residual standard error: 0.4741 on 22 degrees of freedom
Multiple R-Squared: 0.7273,      Adjusted R-squared: 0.6777
F-statistic: 14.67 on 4 and 22 DF,  p-value: 5.592e-06
```

```
attributes(x.lts)
```

```
$names
 [1] "alpha"            "raw.weights"      "best"             "raw.coefficients" "quan"             "raw.scale"        "raw.resid"        "coefficients"     "scale"
[10] "resid"            "crit"             "rsquared"         "method"           "intercept"        "RD"               "lts.wt"           "residuals"        "fitted.values"
[19] "Y"                "X"                "raw.cnp2"         "cnp2"             "call"             "xlevels"          "terms"            "model"

$class
[1] "lts"
```

```
add1(x.lts, scope=x.form)
# Gregor!! this gives me the following error:
# Error:  chunk 23
# Error in UseMethod("extractAIC") :
#   no applicable method for 'extractAIC' applied to an object of class "lts"
```

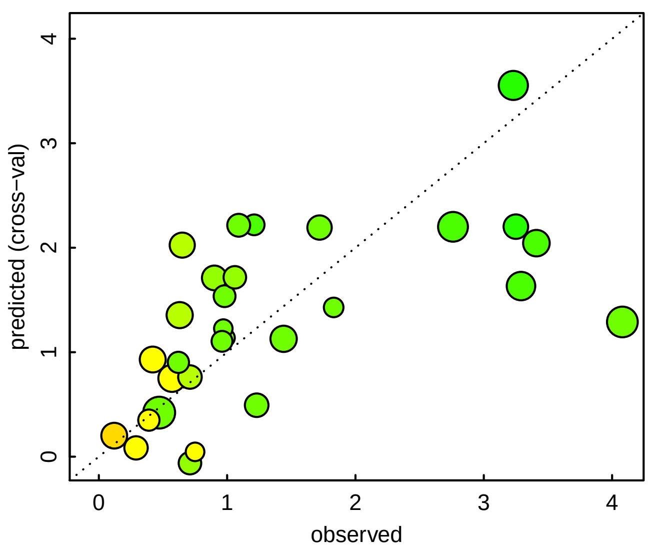
```
plot(x.lts)
```

6 Jackknife cross-validation

We keep the identified model structure and remove one site at the time, which is in turn used for validation:

```
obsQ95 <- x0$Q95
 names(obsQ95) <- x0$code
predQ95cv <- rep(NA, length(x0$Q95))
 names(predQ95cv) <- x0$code
for (i in 1:length(x0$code)) {
 Target.code <- x0$code[i]  # Target Site
 # Target site characteristics
 Target.x0 <- x0[which(x0$code == Target.code),]
 # Regional sample without Target site
 Reg.x0 <- x0[-which(x0$code == Target.code),]
 # Regression
 x.lmReg <- lm(formula=Q95 ~ BONU.WAS + GEOL.SHAL + M.NEIG + GEOL.KRI, data=Reg.x0)
 # Predict at target
 predQ95cv[i] <- predict(object=x.lmReg, newdata=Target.x0)
}
```

```
plot(obsQ95, predQ95cv, xlab="observed", ylab="predicted (cross-val)", pch=21,
     bg=colori[round(10*PETovP)],
     cex=log10(data4chapter8$area),
     xlim=range(c(obsQ95,predQ95cv)), ylim=range(c(obsQ95,predQ95cv)))
 abline(0, 1, lty=3)
```



7 Compare to the PUB book assessment

In the Level 2 Assessment of the PUB book (Blöschl et al., 2013) in Chapter 8 the normalised error and the absolute normalised error in the estimation of Q95 is calculated.

```
NE <- (predQ95cv - obsQ95)/obsQ95
ANE <- abs(NE)
tabella <- data.frame(data4chapter8[,c("code", "area", "temp", "cat_elev")], aridity=PETovP, NE=round(NE, 3), ANE=round(ANE, 3))
 tabella
```

```
     code   area  temp cat_elev   aridity     NE   ANE
1  207985 1150.2  7.08      666 0.8760563 -0.684 0.684
2  207993 1493.3  7.42      609 0.9393491 -0.102 0.102
3  208041  212.0  9.16      324 1.3295455 -0.708 0.708
4  208058  379.9  9.63      273 1.3138686  1.215 1.215
5  208108   59.0  9.11      406 0.9574176  0.149 0.149
6  208447  131.5  9.86      255 1.2781690 -0.102 0.102
7  208512   68.7  7.55      586 0.9449405  0.265 0.265
8  208579  291.5  7.62      574 0.9509658  0.902 0.902
9  208611  175.5  7.66      570 0.9610778 -1.086 1.086
10 208637  371.5  9.27      305 1.4063745  0.680 0.680
11 208678   71.0 10.03      221 1.2945326 -0.938 0.938
12 208835  277.0  7.20      830 0.6624473 -0.322 0.322
13 209189  515.6  9.87      249 1.3194192  0.317 0.317
14 210039  117.7  8.56      544 0.8368618  0.834 0.834
15 210054  224.3  9.14      435 0.9266667 -0.599 0.599
16 210062  149.0  9.07      443 0.9265688  0.568 0.568
17 210088  220.7  9.83      295 1.0818452  0.075 0.075
18 210211  925.1  8.51      568 0.8300248 -0.203 0.203
19 210237  416.9  8.98      476 0.9148936 -0.216 0.216
20 210245   89.2  8.83      519 0.9018568 -0.219 0.219
21 210252  175.4  9.13      452 0.9612188  0.620 0.620
22 210286  316.4  9.81      297 1.0537791  2.116 2.116
23 210294  400.4  9.84      289 1.0691176  1.152 1.152
24 210310  265.3  8.77      522 0.8550000  0.275 0.275
25 210989  689.4  8.50      576 0.8009592 -0.503 0.503
26 211003  439.4  7.96      689 0.7570423 -0.401 0.401
27 211037  190.1  9.33      393 0.9119171  1.032 1.032
28 211045  796.4  7.78      721 0.7231638  0.100 0.100
29 211474  129.5  9.56      349 0.8573141  0.457 0.457
30 211508  119.3  9.73      307 0.9105793  0.151 0.151
```

```
aridity_class <- cut(tabella$aridity, breaks=c(-Inf,0.4,0.6,0.8,1,2,Inf))
temp_class <- cut(tabella$temp,  breaks=c(-Inf,3,6,8,10,12,Inf))
elev_class <- cut(tabella$cat_elev, breaks=c(0,300,600,900,1200,1500,Inf))
area_class <- cut(tabella$area, breaks=c(0,50,100,500,1000,5000,Inf))
```

Notice that the method used here is a global regression.

```
add_points <- function(performance="ANE", variable="area", classes, table) {
 # to add points in a nice way
 for (j in 1:length(levels(classes))) {
  dummy <- table[as.numeric(classes) == j,]
  perf <- dummy[, performance]
  stratif <- dummy[, variable]
  if (length(stratif) > 0) {
   if (length(stratif) == 1) {
    points(j, perf, pch=21,
           bg=colori[round(10*dummy$aridity)],
           cex=log10(dummy$area))
   } else {
      points(j + 0.1*(stratif - mean(stratif))/sd(stratif),
             perf, pch=21,
             bg=colori[round(10*dummy$aridity)],
             cex=log10(dummy$area))
   }
  }
 }
}
```

Fig 8.19 at page 185 of the book:

```
layout(matrix(1:4, nrow=1, byrow=TRUE))
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="ANE",
                     characteristic="Aridity", ylim=c(2,0),
                     main="Global_regr")
 add_points(performance="ANE", variable="aridity", classes=aridity_class, table=tabella)
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="ANE",
                     characteristic="MAT", ylim=c(2,0))
 add_points(performance="ANE", variable="temp", classes=temp_class, table=tabella)
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="ANE",
                     characteristic="Elevation", ylim=c(2,0))
 add_points(performance="ANE", variable="cat_elev", classes=elev_class, table=tabella)
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="ANE",
                     characteristic="Area", ylim=c(2,0))
 add_points(performance="ANE", variable="area", classes=area_class, table=tabella)
```

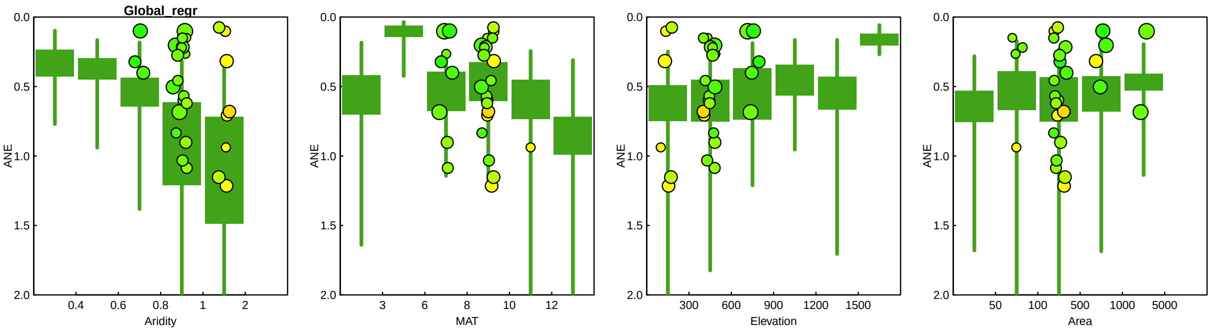
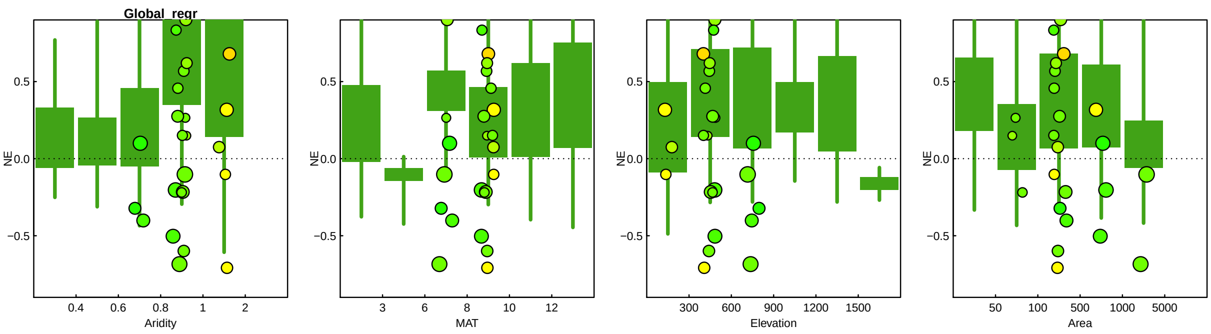


Fig 8.20 at page 186 of the book:

```
layout(matrix(1:4, nrow=1, byrow=TRUE))
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="NE",
                     characteristic="Aridity", ylim=c(-.9,.9),
                     main="Global_regr"); abline(h=0, lty=3)
 add_points(performance="NE", variable="aridity", classes=aridity_class, table=tabella)
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="NE",
                     characteristic="MAT", ylim=c(-.9,.9)); abline(h=0, lty=3)
 add_points(performance="NE", variable="temp", classes=temp_class, table=tabella)
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="NE",
                     characteristic="Elevation", ylim=c(-.9,.9)); abline(h=0, lty=3)
 add_points(performance="NE", variable="cat_elev", classes=elev_class, table=tabella)
plotPUBfiguresLevel2(chapter=8, method="Global_regr", performance="NE",
                     characteristic="Area", ylim=c(-.9,.9)); abline(h=0, lty=3)
 add_points(performance="NE", variable="area", classes=area_class, table=tabella)
```



References

Blöschl, G., Sivapalan, M., Wagener, T., Viglione, A. and Savenije, H. (2013) *Runoff Prediction in Ungauged Basins: Synthesis Across Processes, Places and Scales*, University Press, Cambridge, 484 pages, ISBN:9781107028180.

Laaha, G., Demuth, S., Hisdal, H., Kroll, C.N., van Lanen, H.A.J. Nester, T., Rogger, M., Sauquet, E., Tallaksen, L.M., Woods R.A. and Young A. (2013). Prediction of low flows in ungauged basins. In *Runoff Prediction in Ungauged Basins: Synthesis Across Processes, Places and Scales*, University Press, Cambridge, 163-188, ISBN:9781107028180.