Úkol 2

Ptáci jsou na 1000 až 8000 Hz

dolní propust ⇒ propustí nižší

↳ hodnota trvalení = 10 852 (= nová hodnota 10 88h)

| Parametr | Hodnota |
|---|---|
| Řád | 2 |
| Tlumení | na mezi aperiodicity |
| Potlačení rušivých složek | alespoň 20 dB[1] |
| Zesílení v propustném pásmu | přibližně 0 dB |

LPF

zeslabuje vyšší a propouští nižší

↳ v čitateli $p^0$ ⇒ $p \to 0$ ⇒ $k = 1$ (0 dB)

$$\boxed{\omega_0 = \frac{1}{T}}$$

$\xi = 1$, $k = 1$

$$F(p) = \frac{k\,\omega_0^2}{p^2 + 2\xi\omega_0 p + \omega_0^2}$$

$$\boxed{F(p) = \frac{k}{(Tp+1)^2} \quad \text{pro } \xi = 1}$$

$k = 1$ (0 dB)

$$F(p) = \frac{\omega_0^2}{p^2 + 2p\omega_0 + \omega_0^2} = \frac{\omega_0^2}{(p + \omega_0)^2}$$

moje přenos. funkce!

Chceme aby signál byl bez změny $(p = 0)$ ⇒ aby byl 1

bez změny ⇒ $\frac{\omega_0^2}{0 + \omega_0^2} = \underline{\underline{1}}$ ⇒ proto v čitateli $\omega_0^2$ Ondra

$20 \log_{10} k = -20$

$\log_{10} k = -1$

$\underbrace{10^{-1} = k}$

$k = \frac{1}{10} = 0{,}1$

$|w_0 + jw| = \sqrt{w_0^2 + jw^2}$

$|(w_0 + jw)^2| = \left(\sqrt{w_0^2 + jw^2}\right)^2 = w_0^2 + jw^2$

vždy kladné

$$F(jw) = \left| \frac{w_0^2}{(jw + w_0)^2} \right| = \frac{|w_0^2|}{|(jw + w_0)^2|} = \frac{w_0^2}{\underline{w_0^2 + jw^2}}$$

pro vykreslení Bode!

$\underset{\sim\sim\sim}{w = 0:}$

$\frac{w_0^2}{w_0^2} = 1 \; (0 \, dB)$

$\underset{\sim\sim\sim}{w \to \infty:}$

$F(jw) = \to 0$

$\underset{\sim\sim\sim}{w = w_0:}$

$\frac{w_0^2}{2w_0^2} = \frac{1}{2}$

# DISKRETIZACE (úl. 3)

cílem navzorkovat a dostat se z $F(p)$ na $F(z)$

$$F(p) = \frac{w_0^2}{(p+w_0)^2}$$

$$F(z) = (1 - z^{-1}) \cdot Z\left\{ \mathcal{L}^{-1}\left\{ \frac{1}{p} \cdot F(p) \right\} \right\}$$

$$y(t) = \mathcal{L}^{-1}\left\{ \frac{1}{p} \cdot \frac{w_0^2}{(p+w_0)^2} \right\} = \mathcal{L}^{-1}\left\{ \frac{w_0^2}{p(p+w_0)^2} \right\} = \mathcal{L}^{-1}\left\{ \frac{A}{p} + \frac{B}{p+w_0} + \frac{C}{(p+w_0)^2} \right\}$$

$$\frac{w_0^2}{p(p+w_0)^2} = \frac{A}{p} + \frac{B}{p+w_0} + \frac{C}{(p+w_0)^2}$$

$$w_0^2 = A(p+w_0)^2 + Bp(p+w_0) + Cp$$

$$w_0^2 = Ap^2 + 2Apw_0 + Aw_0^2 + Bp^2 + Bpw_0 + Cp$$

$p^2$: $0 = A + B \Rightarrow A = -B \Rightarrow B = -A$

$p^1$: $0 = 2Aw_0 + Bw_0 + C \Rightarrow 0 = -2Bw_0 + Bw_0 + C \Rightarrow$

$\Rightarrow C = Bw_0$

$p^0$: $w_0^2 = Aw_0^2$

$$A = 1 \qquad B = -1 \qquad C = -w_0$$

vytknu

$$\mathcal{L}^{-1}\left\{ \frac{1}{p} - \frac{1}{p+w_0} - \frac{w_0}{\underbrace{(p+w_0)^2}_{\frac{1}{(p+w_0)^2}}} \right\} = \sigma(t) - e^{-w_0 t} - w_0 t\, e^{-w_0 t}, \quad t \geq 0$$

$$\overset{y(t)}{\overset{\shortparallel}{h(t)}} = \sigma(t) - e^{-w_0 t} - w_0 t\, e^{-a_0 t}, \quad t \geq 0$$

$$\Downarrow \quad \Big|_{t = kT_s}$$

$$h(kT_s) = 1 - e^{-w_0 kT_s} - w_0 kT_s\, e^{-w_0 kT_s}$$

$$\Downarrow$$

$$y(k)$$

$$F_e(z) = (1 - z^{-1})\, Z\left\{ 1 - e^{-w_0 kT_s} - w_0 kT_s\, e^{-w_0 kT_s} \right\} \qquad k \geq 0$$

$$Z\{1\} = \quad {=\sigma(k)} \quad Z\{\sigma(k)\} = \frac{z}{z-1}$$

$$Z\{e^{-akT_s}\} = Z\{[\underbrace{e^{-aT_s}}_{a}]^k\} = Z\{a^k\} = \frac{z}{z-a} = \frac{z}{z - e^{-aT_s}}$$

$$w_0 Z\{kT_s\, e^{-akT_s}\} = w_0 T_s \cdot Z\{k \cdot a^k\} = \frac{a z}{(z-a)^2}$$

$$a = e^{-aT_s}$$

$$[e^{-aT_s}]^2$$

$$\hookrightarrow w_0 T_s \cdot \frac{e^{-aT_s} \cdot z}{(z - e^{-aT_s})^2}$$

$$F_e(z) = \underbrace{(1 - z^{-1})}_{\frac{\cancel{z-1}}{\cancel{z}}} \cdot \left( \frac{\cancel{z}}{\cancel{z-1}} - \frac{\cancel{z}}{z - e^{-aT_s}} - \frac{w_0 T_s e^{-aT_s} \cdot \cancel{z}}{\left(z - e^{-aT_s}\right)^2} \right)$$

Pozn. nekonečné

$$\Rightarrow = 1 - \frac{z-1}{\left(z - e^{-aT_s}\right)} - \frac{w_0 T_s e^{-aT_s} (z-1)}{\left(z - e^{-aT_s}\right)^2}$$

$$E = e^{-aT_s}$$

$$= \frac{\left(z^2 - 2zE + E^2\right) - \left(z^2 - zE - z + E\right) - \left(w_0 T_s E z - w_0 T E\right)}{(z - E)^2}$$

$z^2: 1 - 1 = 0$

$z^1: -2E + E + 1 - w_0 T_s E = 1 - E - w_0 T_s E$

$z^0: E^2 - E + w_0 T E$

$$F_e(z) = \frac{z\left(1 - E - w_0 T_s E\right) + E^2 - E + w_0 T E}{(z - E)^2}$$

$$E = e^{-aT_s}$$

# 4. filter

$$F_e(z) = \frac{z(1 - E - w_0 T_s E) + E^2 - E + w_0 T E}{z^2 - 2zE + E^2}$$

$$b_1 = 1 - E - w_0 T_s E$$

$$b_0 = E^2 - E + w_0 T_s E$$

$$a_1 = -2E$$

$$a_0 = E^2$$

$$F_e(z) = \frac{Y(z)}{U(z)} = \frac{b_1 z + b_0}{1 z^2 + a_1 z + a_0} \quad \Big/ \ \frac{z^{-2}}{z^{-2}}$$

$$= \frac{b_1 z^{-1} + b_0 z^{-2}}{1 z^0 + a_1 z^{-1} + a_0 z^{-2}} = \frac{Y(z)}{U(z)}$$

$$U(z)\left(b_1 z^{-1} + b_0 z^{-2}\right) = Y(z)\left(1 + a_1 z^{-1} + a_0 z^{-2}\right)$$

prohodíme

$$Y(z) + Y(z)\, a_1 z^{-1} + Y(z) \cdot a_0 z^{-2} = U(z)\, b_1 z^{-1} + U(z)\, b_0 z^{-2}$$

$$\downarrow$$

$$y(k) + y(k-1)\, a_1 + y(k-2)\, a_0 = b_1 u(k-1) + b_0 u(k-2)$$

$$y(k) = b_1 u(k-1) + b_0 u(k-2) - a_1 y(k-1) - a_0 y(k-2)$$