

Dissertation title:

Branching random walks with selection

Dissertation supervisor:

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Description of proposal:

A branching random walk with selection is a discrete time Markov process which evolves as follows: at all times, the process consists of N particles on the real line whose positions we denote by X_1, X_2, \dots, X_N . Each particle then produce offspring particles in the next generation whose number and positions relative to their parent are given by an independent copy of a certain point measure. A typical exemple for this reproduction-displacement mechanism would be to draw an independent random variable K for the number of offsprings and then, conditionally on K to pick i.i.d. displacements with some distribution ρ . To keep the population size constant equal to N , we then select the N rightmost particles produced to form the next generation.

If we interpret the position of particles as fitness of individuals, this is a model of a population evolving under selection.

- in [1], the case where the displacements have a thin tailed is studied. The main results are the study of the correction term for the speed of the system.
- In [4], the continuous time analogue of the N branching Brownian motion is studied and a rich and detailed picture of the behaviour of the system is given.
- In [2], the case of a N branching random walk with jumps of regular varying tails. They are again able to give a very precise estimate of the speed of the cloud of particles depending on N .
- The recent paper [5] study a variant in which the offsprings positions form a shifted exponential Poisson point process and focuses on the question of the genealogy of the process,

Possible avenues of investigation:

A natural question (and possibility for future research) is to investigate what happens in between the two scenarios of light-tailed displacement distributions (satisfying an exponential moment assumption) as in [1] and the polynomial tails considered in [2]. This seems to be a delicate and interesting question. Even the heuristic picture is not clear. For example, it is not clear whether for all subexponential displacement distributions the advance of the cloud of particles arises from jumps of single particles as is the case for the regularly varying tails considered in [2]. One would expect this behavior from the large deviation behavior of general random walks with subexponential displacement distributions, but the interaction between the particles might cause other effects.

This question is related to the genealogy of the model. In general for thin tailed displacement it is conjectured that the genealogy is given by the so called Bolthausen-Sznitman coalescent. The only case where this is fully understood is the so-called Poisson with exponential intensity one [3].

In the regularly varying case, it is believed (but the question is open) that the genealogy is described by the star-shaped coalescent (i.e. the coalescent where all particles coagulate to a single block). Does this coalescent describe the genealogy of N -BRW for every subexponential displacement distribution?

Recently, a variation of the Poisson with exponential intensity case was studied in [?] (using results from [6]), and a richer family of genealogical behaviours was obtained, namely the so-called *Beta-coalescents*. The authors of this work conjecture that this result should also hold true for the N -branching Ornstein-Uhlenbeck process, a variant of the N -branching Brownian motion where the movement is a diffusion process called an Ornstein-Uhlenbeck process. This could be checked through simulations and investigated.

Pre-requisite knowledge:

Essential: material from probability course such as A8 probability (<https://courses.maths.ox.ac.uk/node/37706>), B8.1 Martingales through Measure Theory (<https://courses.maths.ox.ac.uk/node/36505>), SB3a Applied Probability (<https://courses.maths.ox.ac.uk/node/36568>), B8.2 Continuous Martingales and Stochastic Calculus (<https://courses.maths.ox.ac.uk/node/36517>)

Useful: C8.1 Stochastic Differential Equations (<https://courses.maths.ox.ac.uk/node/37062>), C8.2 Stochastic Analysis and PDEs (<https://courses.maths.ox.ac.uk/node/37070>)

References

- [1] Jean Bérard and Jean-Baptiste Gouéré. Brunet-Derrida behavior of branching-selection particle systems on the line. *Communications in Mathematical Physics*, 298(2):323?342, June 2010.
- [2] Jean Bérard and Pascal Maillard. The limiting process of N -particle branching random walk with polynomial tails. *Electronic Journal of Probability*, 19:no. 22, 1?17, 2014.
- [3] Brunet, É., Derrida, B., Mueller, A.H. and Munier, S., 2007. Effect of selection on ancestry: an exactly soluble case and its phenomenological generalization. *Physical Review E*, 76(4), p.041104.
- [4] Pascal Maillard. Speed and fluctuations of N -particle branching Brownian motion with spatial selection. *Probability Theory and Related Fields* 166:no 3?4, pp 1061?1173, 2016
- [5] Aser Cortines and Bastien Mallein. The genealogy of an exactly solvable Ornstein-Uhlenbeck type branching process with selection *Preprint* arXiv: <https://arxiv.org/abs/1802.01132>.
- [6] Jason Schweinsberg. Coalescent processes obtained from supercritical Galton-Watson processes. *Stochastic Process. Appl.* 106 (2003), 107-139