To: **Computational Finance Course Participants**
Fr: **Jesper Andreasen**
St: **Week 3**
Dt: **14 Dec 2022**

Agenda

Class 16 Dec 2022 13:15-16 in room 4-0-05 at the Bio Center.

Bring your Windows laptop. We recommend that you try starting on the list below before class on Friday.

We give an overview over practical and theoretical properties of the theta scheme.

Discussion will be based on paper Fun with Finite Difference.

We will do an interactive session before we code our own grid runner for the Black Scholes model.

You can read Part 2 and 3 of Andreasen (2011) before class.

Learn cheat sheet.

To do list

Play with the spreadsheet 'fun with fd.xls':

Duality: tests the duality of the backward and forward roll.

Grid width: tests the impact of the width of the finite difference grid.

Transform: discusses the impact of using linear grid for a log-normal model.

Non-Smooth and smooth: the impact of smoothing the payoff.

Explicit, implicit and c-n: use the forward roll to visualise the concepts about stability.

Accuracy: tests the accuracy of the theta scheme in the Bachelier model.

Code kBlack::fdRunner() and hook up to excel.

Test convergence by making log-log plots of accuracy.

Finite Difference Cheat Sheet

Schemes	Explicit $\theta = 0$	Implicit $\theta = 1$	Crank-Nicolson $\theta = 1/2$
PDE	$0 = f_t + Af \quad , A = -r + \mu\partial_x + \frac{1}{2}\sigma^2\partial_{xx}$		
FD central	$\bar{A} = -r + \mu\delta_x + \frac{1}{2}\sigma^2\delta_{xx}$		
FD winding	$\bar{A} = -r + \mu^+\delta_x^+ + \mu^-\delta_x^- + \frac{1}{2}\sigma^2\delta_{xx}$		
Boundaries	Absorption: $\mu = \sigma = 0$, Reflection: $\mu_0 > 0, \mu_{n-1} < 0, \sigma = 0$		
Backward	$f(t_h) = [I - (1-\theta)\Delta t\bar{A}]^{-1}[I + \theta\Delta t\bar{A}]f(t_{h+1})$		
Forward	$p(t_{h+1}) = [I + \theta\Delta t\bar{A}'][I - (1-\theta)\Delta t\bar{A}']^{-1}p(t_h)$		
Grid width	$5 \cdot (\int_0^T \sigma(u, x(0))^2 du)^{1/2}$		
Transform	PDE or grid spacing: $y = \int_{x_0}^x \sigma(a)^{-1} da$		
Vanilla strikes	Mid between grid points or $O(\Delta x^2)$ accuracy loss		
Digitals	Mid between grid points or $O(\Delta x)$ accuracy loss		
Cont barriers	On grid and absorption or $O(\Delta t^{1/2})$ accuracy loss		
Von Neumann	$\Delta t \le O(\Delta x^2)$	Always	Always
$p \ge 0, \mu = 0$	$\Delta t \le O(\Delta x^2)$	Always	$\Delta t \le O(\Delta x^2)$
$p \ge 0, \mu \ne 0$	$\Delta t \le O(\Delta x^2)$	With winding	$\Delta t \le O(\Delta x^2)$
Accuracy central	$O(\Delta t + \Delta x^2)$	$O(\Delta t + \Delta x^2)$	$O(\Delta t^2 + \Delta x^2)$
Accuracy winding	$O(\Delta t + \Delta x)$	$O(\Delta t + \Delta x)$	$O(\Delta t^2 + \Delta x)$
Models	Brownian motion	Non-parametric	Parametric
Speed	TBD	TBD	TBD