xIBOR in transition
University of Copenhagen
Project out of Cource Scope

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1 Introduction

After the 2007-2008 Global Financial Crisis the era of single-curve term structure modelling were jeopardized due to the big spread occuring between the OIS and xIBOR rates in the market and thus the start of the era for multicurve modelling. However the xIBOR rates plunged in popularity and in 2012 the scandal of LIBOR manipulation did not help for its reputation. The final blow the the LIBOR rates where made in 2017, where it was annouced that LIBOR was to discountinue by the end of 2021. However the most important Libor, the USD LIBOR is still live until 30 June 2023. Although our focus will be on the EURO market (EURIBOR vs. €STR) - even though EURIBOR has not yet (and maybe never will) be set to discontinue.

This project is a proof of concepts on the transition from the xIBOR rates to the OIS/RFR[1] rates as the underlying floating rates for derivative contracts. Although mostly traded, we will not consider the impact of linearly interest rate derivatives, such as interest rate swaps, since the transition should have a higher impact on the non-linear derivatives such as Caplets and Caps.

As another limitation of this project, we will take on the challenge of the transition from forward-looking to backward-looking rates such as covered in [2] This is a whole theme on itself.

2 The setup

We consider a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{0\leq t\leq T^\star}, \mathbb{Q})$ Where we model the OIS rate as a classical short rate model ala Vasicek, hence:

$$dr(t) = \kappa\left[\theta - r(t)\right]dt + \sigma dW(t) \tag{1}$$

where $W(t)$ is a brownian motion under the risk-neutral $\mathbb{Q}$-measure. The OIS rate also is the bedrock for our discounting factor, which thus make our pricing consistent with standard CSA pricing of derivative contracts, ie:

$$P_D(t,T) = \mathbb{E}_t^{\mathbb{Q}}\left\{e^{-\int_t^T r_s ds}\right\} \tag{2}$$

The convenient thing with the short rate above is firstly that it is Gaussian and thus explicit computation on bond prices are possible. Given an initial short rate, $r(t)$, the price of a

[1] The terminology is a bit loose throughout the project. Whenever OIS or RFR is mentioned it shall be understood as the 'Risk-free' short term rates, which is the reference rates for discounting.

Zero-Coupon Bond in the Vasicek model is commonly known as:

$$P(t,T;r(t)) = e^{A(t,T)-B(t,T)r(t)}$$

where

$$\begin{aligned} B(t,T) &= \frac{1-e^{-\kappa(T-t)}}{\kappa} \\ A(t,T) &= \left[\theta - \frac{\sigma^2}{\kappa^2}\right]\{B(t,T)-(T-t)\} - \frac{\sigma^2}{4\kappa}B^2(t,T) \end{aligned} \tag{3}$$

We consider derivative contracts starting at some time T_0 and has paying dates at $T_1 < \cdots < T_N$, where $\tau_k^x = T_k - T_{k-1}$ is the year fraction between two payment dates and x is the tenor-structure of the contract. We will consider the most common tenor structures namely $x = \{3\text{M}, 6\text{M}, 12\text{M}\}$ and keep the time between payments fixed, whenever a tenor structure has been chosen for a given contract ie. $\tau_k^{3\text{M}} = \tau^{3\text{M}}$ for $k = 1, \ldots, N$. Another important building block upon pricing OIS intstruments, and finding the relation between the OIS and xIBOR rates are made through the simply-compounded forward rates given by:

$$F_k^x(t) = \frac{1}{\tau_k^x}\left\{\frac{P_D(t,T_{k-1})}{P_D(t,T_k)} - 1\right\} \tag{4}$$

Along the lines of [3] we consider the a model where the OIS-xIBOR spread is addative, ie:

$$\mathcal{S}_k^x(t) = L_k^x(t) - F_k^x(t) \tag{5}$$

and is modelled by:

$$S_k^x(t) = S_k^x(0) + \alpha_k^x\left[L_k^x(t) - L_k^x(0)\right] + \beta_k^x\left[\mathcal{X}_k^x(t) - \mathcal{X}_k^x(0)\right], \quad \mathcal{X}_k^x(0) = 1 \tag{6}$$

Here $\mathcal{X}_k^x(t)$ is the stochastic basis factor, which can be modelled in different ways. However we are only interested in modelling $\mathcal{X}_k^x(t)$ in a way that leads to simple and explicit computations of xIBOR contracts. Initially we model the stochastic basis as a martingale brownian motion :

$$d\mathcal{X}_k^x(t) = \eta_k^x(t)\mathcal{X}_k^x(t)dB_k^x(t) \tag{7}$$

with $dB_k^x(t)$ is a Brownian motion independent of the $\mathbb{Q}$- and T_D^k forward measure.

3 Pricing in the different models:

3.1 The OIS model

Pricing Caplets: under $\mathbb{Q}$ for $t \leq T_{k-1}$ we have:

$$Cpl(T_{k-1}, T_k, K, N; t) = \mathbb{E}_t^{\mathbb{Q}}\left\{e^{-\int_t^{T_k} r_s ds} N\left[F_k^x(t) - K\right]^+\right\}$$

With some change-of-measure trickery we are able to price the Caplet in a Black like formula as in [1]:

$$Cpl^{\text{OIS}}(T_{k-1}, T_k, K; t) = \left[P_D(t, T_{k-1})\Phi(-d + \tilde{\sigma}_k) - (1 + \tau_k^x K)P_D(t, T_k)\Phi(-d)\right] \tag{8}$$

where Φ is the CDF for the Gaussian distribution and

$$\tilde{\sigma}_k = \sigma\sqrt{\frac{1 - e^{-2\kappa(T_{k-1}-t)}}{2\kappa}}B(T_{k-1}, T_k), \quad d = \frac{1}{\tilde{\sigma}_k}\log\left\{\frac{(1 + \tau_k^x K)P_D(t, T_k)}{P_D(t, T_{k-1})}\right\} + \frac{\tilde{\sigma}_k}{2}$$

And since a Cap of a given payment structure $\mathcal{T} = \{T_1, \ldots, T_n\}$ with:

$$Cap^{\text{OIS}}(T_0, \mathcal{T}, K, N; t) = N\sum_{k=1}^{n} Cpl^{\text{OIS}}(T_{k-1}, T_k, K, N; t) \tag{9}$$

3.2 The Spread Model

In the Spread model it is not quite as trivial since we have two stochastic processes to take into account. Firstly we consider the discounted payoff of a Cap at a fixing date T_{k-1} which pays at T_k:

$$\left[L_k^x(T_{k-1}) - K\right]^+ \tau_k^x P_D(T_{k-1}, T_k)$$

since our two stochastic parameters of $r(t)$ and $\mathcal{X}_k^x(t)$ are independent we can without much endeavor find a semianalytical solution, which only makes forces us to compute one integral numerically. Recalling that a caplet is just a one-period swaption with equal paying structures for the fixed and floating leg, we are lucky that it is just a simplified version of the semi-analytical swaption pricing formula in [3]. Hence the time 0 caplet price for a given tenor structure[2] computed as:

$$\begin{aligned} Cpl^{\text{Spr}}(T_{k-1}, T_k, K; 0) &= P_D(0, T_{k-1})\mathbb{E}_0^{T_k^x}\left\{\left[L_k(T_{k-1}) - K\right]^+ \tau_k P_D(T_{k-1}, T_k)\right\} \\ &= \int_{-\infty}^{\infty} h\left\{C(r), D(r), V_{\mathcal{X}}(T_{k-1})\right\}\frac{1}{\sqrt{2\pi\nu(T_{k-1})}}e^{-\frac{(r-\mu(T_{k-1}))^2}{2\nu(T_{k-1})}}dr \end{aligned} \tag{10}$$

Where:

$$\begin{aligned} \mathbb{E}_t^T\left\{r(T)\right\} &= \mu(T) = r(t)e^{-\kappa(T-t)} + \left[\theta\kappa - \frac{\sigma^2}{\kappa}\right]B\left(t, T\right) + \frac{\sigma^2}{2}\left[e^{-\kappa(T)-t} - e^{-2\kappa(T-t)}\right] \\ \mathbb{V}_t^T\left\{r(T)\right\} &= \nu(T) = \frac{\sigma^2}{2\kappa}\left[1 - e^{-2\kappa(T-t)}\right] \\ \mathbb{V}_t^T\left\{\log(\mathcal{X}_(T_{k-1}))\right\} &= V_{\mathcal{X}}(T_{k-1}) = \sqrt{\int_t^{T_{k-1}} \eta(s)^2 ds} \end{aligned} \tag{11}$$

[2] We supress the tenor notation on the time parameters to ease notation, since it is embedded in the difference between the start and maturity of the caplet.

and

$$
\begin{aligned}
C(r) &\triangleq \tau_k P_D\left(T_{k-1}, T_k\right)\beta_k \\
D(r) &\triangleq \left\{\tau_k K + 1 + \alpha_k - \xi_k \tau_k\right\} P_D\left(T_{k-1}, T_k\right) - (1+\alpha_k) \\
\xi_k &\triangleq L_k(0) - (1+\alpha_k) F_k(0) - \beta_k \\
h(C, D, V) &\triangleq \begin{cases} \mathrm{Bl}(C, D, V, 1), & \text{if } A, B > 0 \\ \mathrm{Bl}(-C, -D, V, -1), & \text{if } A, B < 0 \\ C - D, & \text{if } A \geq 0, B \leq 0 \\ 0, & \text{if } A \leq 0, B \geq 0 \end{cases} \\
\mathrm{Bl}(F, K, v, \psi) &\triangleq \psi F \Phi\left[\psi \frac{\log\left(F/K\right) + v^2/2}{v}\right] - \psi K \Phi\left[\frac{\log\left(F/K\right) - v^2/2}{v}\right]
\end{aligned}
\tag{12}
$$

And as above we may price a Cap with payment structure $\mathcal{T}$ and notional N:

$$
Cap^{\mathrm{Spr}}(T_0, \mathcal{T}, K, N; 0) = N \sum_{k=1}^{n} Cpl^{\mathrm{Spr}}(T_{k-1}, T_k, K; 0) \tag{13}
$$

4 Working with data

It has been possible to fetch some market data on the following EUR Data in the period 21.03.2022 to 17.03.2023: The data is denoted as i for a given date and TN for a point on the curve.:

- €STR SWAP Curve - SW_i^{TN}
- EUR SWAP Curve with 3M EURIBOR a reference - $SW3M_i^{TN}$
- EUR Basis swap: €STR Swap vs. 3M EURIBOR - $BS3M_i^{TN}$
- EUR Basis swap: 3M EURIBOR vs. 6M EURIBOR - $BS3M6M_i^{TN}$

Now the question at hand would be - what to do with this market data? Well Many things are possible, but we will merely use it for finding the depencence structure for our spread model, ie. determining α_k and β_k in (6). Furthermore we use the data initial curve fitting for the forward xIBOR curve, which in our case is the 3M and 6M EURIBOR.[3]

[3] If one would have been smart enough, one could also have fitted the initial €STR curve to a Hull-White model, so the Risk-free rate model is realistic to market data.

4.1 Determining α_k and β_k from data:

To determine α_k and β_k we find some inspiration from Section 3.1 in [3], since we set them to depend on the correlation between basis swaps and the OIS Swap rate in addition to the variance of basis swaps. Hence for a given tenor structure x we set:

$$\alpha_k^x = \alpha^x = \rho^x v^x, \quad \beta_k^x = \beta^x = \frac{1}{2}\sqrt{1-(\rho^x)^2}v^x \tag{14}$$

in [3] they set:

$$\rho_k^x = \mathrm{Corr}(S_k^x(T_{k-1}^x), F_k(T_{k-1}^x)), \quad v_k^x = \sqrt{\mathbb{V}\left(S_k^x(T_{k-1})\right)}$$

Since basis swaps are normally quoted from 1 year and up to around 30 years, we consider the correlation from year to year independent of each other. In this way we break down the correlation year by year between a Basis Swap curve and one part of the corresponding Basis swap(the €STR side of basis swap). It is important to stress that this is not quite as in [3], where they model the correlation between the basis spread and the OIS Forward rates:
Our method on the other hand measures the correlation between the €STR swap rates and the corresponding basis swap from €STR to a swap referencing 3M & 6M EURIBOR respectively

Basis Spread for x = 3M:
Here we set $S_k^{3M} = BS3M^{TN}$ the correlation and standard deviation is thus:

$$\rho_{TN}^{3M} = \mathrm{Corr}\left(BS3M^{TN}, SW^{TN}\right)$$
$$v_{TN}^{3M} = \sqrt{\mathbb{V}\left(BS3M^{TN}\right)}$$

Table 1 leaves an overview of the historical yearly correlations between the €STR vs. 3M Euribor Basis swap and €STR swap rate:

	Min (at 1Y)	Max (at 25Y)	Avg.
ρ^{3M}	-0.8098	-0.3597	-0.60232

Table 1: Minimum, Maximum and average of the yearly correlations between the €STR swap rate and the €STR/3M EURIBOR basis swap

furthermore we compute the historical standard deviations, were the summary is found in Table 2.

	Min (at 30Y)	Max (at 1Y)	Avg.
v^{3M}	0.0218	0.04491	0.03316

Table 2: Standard deviations for the €STR vs. 3M Euribor basis swap.

Basis Spread for $x = 6M$:
For this we unfortunately does not have any directly observed basis spread, but we construct our own through adding the 3M vs. 6M EURIBOR basis swap to our €STR vs. 3M EURIBOR basis swap. Thus we define:

$$S_k^{6M} = S_k^{3M} + BS3M6M^{TN}$$

A method that is practioned in the industry, when directly basis swaps are not traded. As above we thus have the correlation and standard deviation:

$$\rho_{TN}^{6M} = \text{Corr}\left(BS3M^{TN} + BS3M6M^{tn}, SW^{TN}\right)$$
$$v_{TN}^{6M} = \sqrt{\mathbb{V}\left(BS3M^{TN} + BS3M6M^{TN}\right)}$$

At last we compute the historical yearly correlation and standard deviations as above. And the summary for the historical estimates can be found in Table 3 and Table 4 respectively.

	Max (at 1Y)	Min (at 25Y)	Avg
ρ^{6M}	-0.48192	-0.66764	-0.62067

Table 3: Historical correlations between our synthetic created €STR vs. 6M EURIBOR basis swap and €STR Swap rate

	Min (at 3Y)	Max (at 12Y)	Avg
v^{6M}	0.02448	0.03308	0.02868

Table 4: Historical std. dev. for synthetic created €STR vs. 6M EURIBOR basis swap

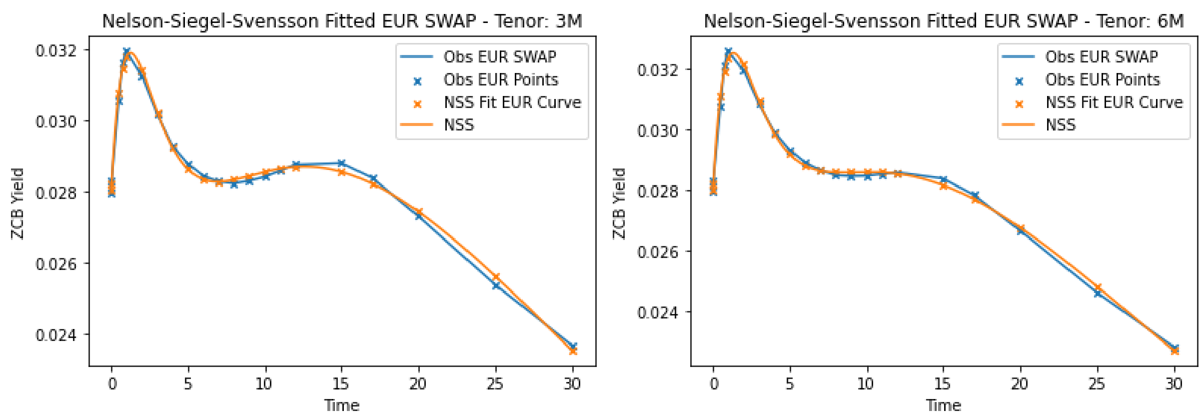
4.2 Fitting initial EURIBOR Swap curve:

For this task do not use a historical estimate but we fit the 3M and 6 EURIBOR Swap curve to a Nelson-Siegel-Svensson model (henceforth NSS) under the loosely assumption that the swap rates can be intepreted as yields and thus after we have fitted the swap curve to the NSS model we can transform is to a continous function of "Discount Factors"(Or Zero coupon bond prices), from which we can infer the initial EURIBOR forward rates.[4]. The NSS-model has the folllowing representation for the continously compounded spot rate:

$$R(0,\tau) = \beta_0 + \beta_1 \frac{1 - e^{-\tau/\lambda_1}}{\tau/\lambda_1} + \beta_2 \left\{ \frac{1 - e^{-\tau/\lambda_1}}{\tau/\lambda_1} - e^{-\tau/\lambda_1} \right\} + \beta_3 \left\{ \frac{1 - e^{-\tau/\lambda_2}}{\tau/\lambda_2} - e^{-\tau/\lambda_2} \right\}$$

[4]Obviously this is a blatantly wrong assumption. However this is done for convenience matter. As seen in Figure 1 it actually makes a surprisingly good fit.

Just as with the €STR vs. 6M EURIBOR Basis swap we have added the 3M vs. 6M EURIBOR Basis swap to the 3M EURIBOR swap curve to get an approximation of the 6M EURIBOR Swap curve. The challenge at hand is that there are more data points on the swap curve than the basis swap curve making a straight addition insufficient. missing data points on basis swap kurve have been linear interpolated/extrapolated, depending on whether the swap curve had data point surrounding the missing points. If not points have been extrapolated[5]

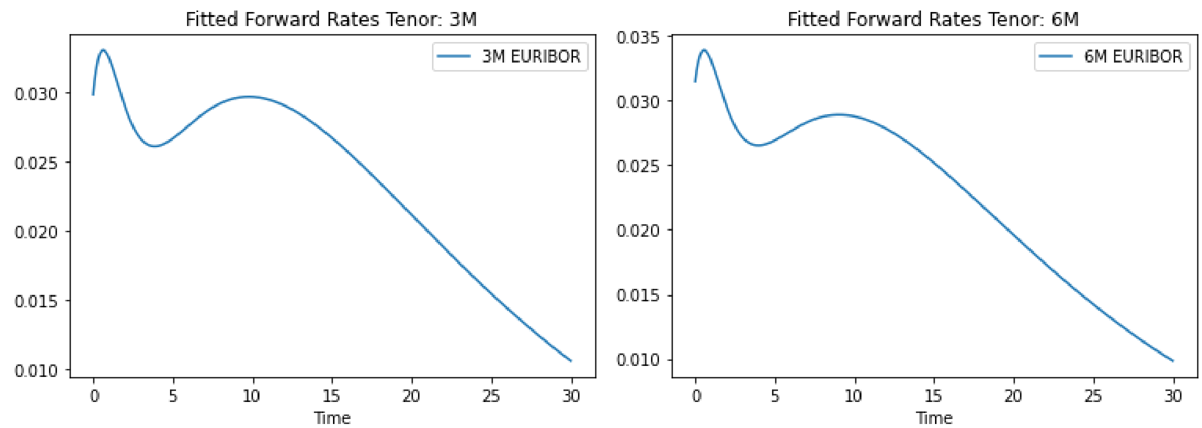


(a) 3M EURIBOR (b) 6M EURIBOR

Figure 1: Fitting of Swap curves to the NSS-model

As seen in Figure 1 the two Swap Curves is fitted alright by the NSS model, however we note that these rates indicate that the initial forward rates will decay rapidly, when we go beyond the 15Y point as seen in Figure 2.

[5]This is only the case for data points below 1Y. Hence the datapoints for the basis swap have been linear extrapolated, such that it hits 0 as the time points tends toward 0.



(a) 3M EURIBOR (b) 6M EURIBOR

Figure 2: Initial Forward Rate curve, $L_k^x(0)$, for $x = \{3\text{M}, 6\text{M}\}$

5 Analysing the affect of xIBOR to OIS

When comparing the Spread model in section 2 with different dependence structures with the OIS model in section 2 we want to consider different aspects namely:

- The initial price of a portfolio under the different parameterization of the spread model compared to the single curve RFR-model of (1).
- The implied volatility for different Maturities and more importantly what level of short rate volatility is needed to generate the implied volatility levels.

What we thus consider is the relative volatility change on the underlying short rate one could assume will appear when modelling and fitting ATM-volatilities. The hypothesis is that Spread models with relative large depence structure(ie. correlation, ρ^x, and spread vol, v^x) will have a relative large impact on the the calibrated short rate vol, σ, whenever they are fitted to ATM volatility for the deterministic case of the spread model, hence when $\alpha^x = \beta^x = 0$.

Firstly we note that the strike of an ATM Cap is just the initial swap rate with the floating and fixed leg paying at the same times. So for an OIS-Cap we have:

$$\bar{K} = S_{a,b}^{OIS}(0) = \frac{\sum_{k=a+1}^{b} \tau_k P_D(0,T_k) F_k(0)}{\sum_{k=a+1}^{b} \tau_k P_D(0,T_k)} = \frac{P_D(0,T_a) - P_D(0,T_b)}{\sum_{k=a+1}^{b} \tau_k P_D(0,T_k)} \tag{15}$$

and for the EURIBOR Cap it is given by:

$$\bar{K} = S_{a,b}^{Spr}(0) = \frac{\sum_{k=a+1}^{b} \tau_k P_D(0,T_k) L_k(0)}{\sum_{k=a+1}^{b} \tau_k P_D(0,T_k)} \tag{16}$$

The initial data we use for our short-rate model in (1) is:

$$\kappa = 0.06,\ \theta = 0.253,\ \sigma = 0.011,\ r(0) = 0.0279$$

and for the Stochastic basis in (6) we set $\eta = 0.5$. We consider two tenor structures namely 3M and 6M and for these we consider the dependence structures:

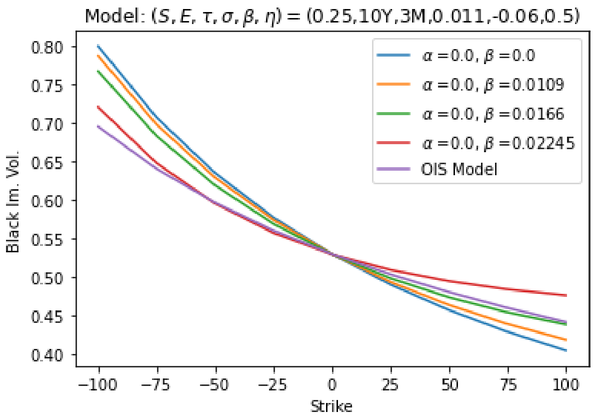
$$\rho^{3M} = \{0, -0.60232\}\,,\ v^{3M} = \{0.0, 0.0218, 0.04491, 0.03316\}$$
$$\rho^{6M} = \{0, -0.62067\}\,,\ v^{6M} = \{0.0, 0.02448, 0.03308, 0.02868\}$$

In Figure 3, Figure 4 and Table 5 we see the volatility skews, smirks and a smile generated by finding the implied volatility for the Black and Bachelier model respectively. Starting at the deterministic basis $\alpha^x = \beta^x = 0$ and calibrating σ for our other models to hit the ATM volatility for Caps of 10Y's payments starting after one quarter of year. Just as in [3] we see that our deterministic basis makes the biggest skew and that the parameterization closest to have a similar implied volatility structure as our OIS model is the model with the largest dependence parameters. However this shall not trick us - If we take a look at Table 5 we observe that it is actually the model that differs the most with our recalibrated short vol parameter, σ, being around twice to four times as small. This intuitively indicates that if the OIS model and for instance the Spread model with $\alpha^{3M} = 0.0$ and $\beta^{3M} = 0.02245$ were to have the same short rate volatility, σ the implied volatility, would be on a way higher(if not ridiculous higher) level than our OIS model. Another thing to note is that for the Black implied volatilities it merely only skew and a single smirk that is observed, where as the shapes gets a bit more funky/wild when considering the Bachelier implied volatilities. This can also be due to the implied vol of the bachelier model is quoted in nominal(Basis points), whereas Black volatilities are percentages.

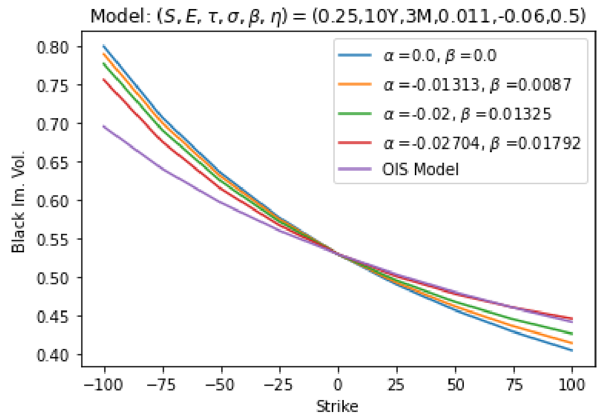
Now turning our attention to Table 6, Table 7 and Table 8. Here σ is not calibrated to match any implied volatilities.[6] We see that the prices actually follows quite nicely as maturity increases(although one would wonder whether the prices are rather large - The same could one think about the implied volatilities above). None of the parameterizations that dramatically increase or decrease. The dependence structures that has the highest prices are also the ones who differed the most in the σ calibration from above. Although it is remarkable that the prices for all spread models are so much higher than the OIS model, consequently at least twice as large. This could be due to the simplistic OIS model that is not initially fitted to

[6] One could try to do so to see if that would have an interesting effect.

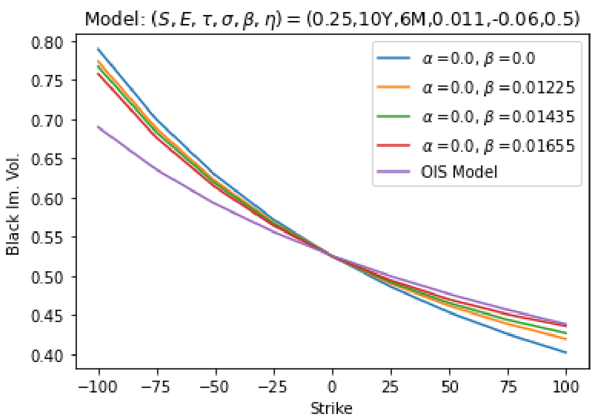
data(and might have a unrealistic forward curve) and is 'internally priced', since $F_k(t)$ are generated from the discount factors.



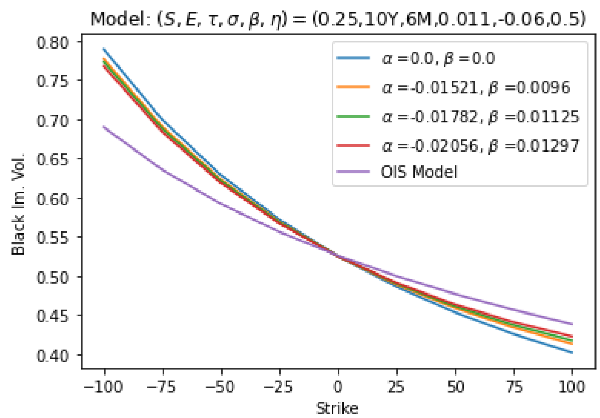
(a) $x = 3\text{M}$, $\rho = 0.0$



(b) $x = 3\text{M}$, $\rho = -0.6023$

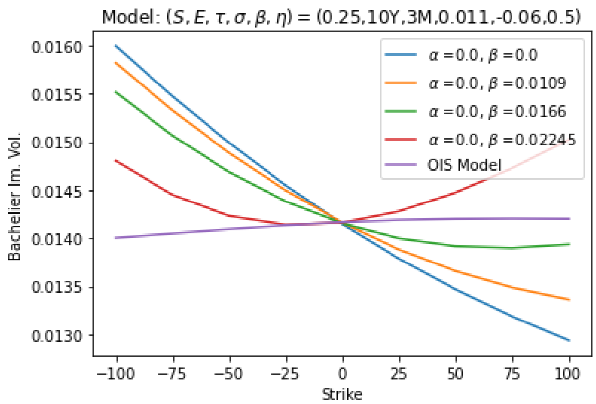


(c) $x = 6\text{M}$, $\rho = 0.0$

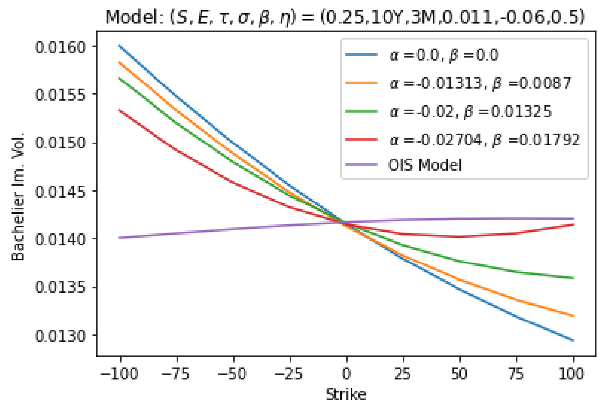


(d) $x = 6\text{M}$, $\rho = -0.6210$

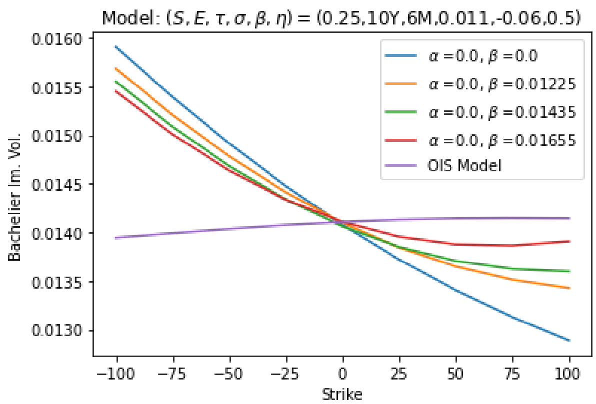
Figure 3: Black Implied Volatilities



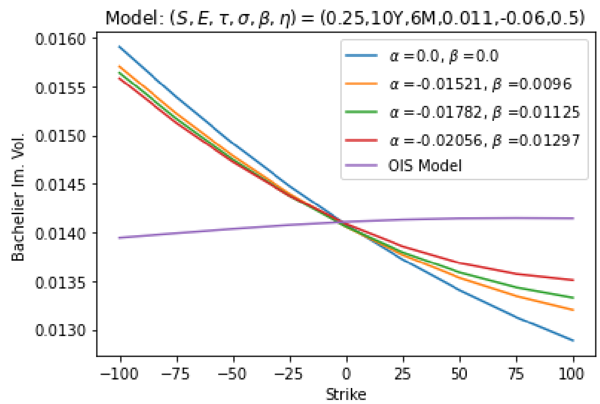
(a) $x = 3\text{M}$, $\rho = 0.0$



(b) $x = 3\text{M}$, $\rho = -0.6023$



(c) $x = 6\text{M}$, $\rho = 0.0$



(d) $x = 6\text{M}$, $\rho = -0.6210$

Figure 4: Bachelier Implied Volatilities

Tenor	α^x	β^x	Bachelier	Black
3M	0.0	0.0	0.011	0.011
		0.00109	0.0091437	0.00912656
		0.0166	0.0071435	0.00713012
		0.02245	0.0036610	0.0036096
	-0.01313	0.0087	0.009900	0.0099343
	-0.02	0.01325	0.008724	0.008715
	-0.02704	0.01792	0.0069249	0.0069454
	OIS		0.0166375	0.0146781
6M	0.0	0.0	0.011	0.011
		0.01225	0.008731	0.008705
		0.01435	0.007967	0.007998
		0.01655	0.007170	0.007129
	-0.01521	0.0096	0.00969375	0.009719531
	-0.01782	0.01125	0.00930189	0.009301895
	-0.02056	0.01297	0.0088061	0.008793
	OIS		0.01656875	0.0147640

Table 5: Calibrated short rate volatilities σ to fit ATM Volatility for the Deterministic basis.

Tenor	α^x	β^x	5Y	10Y	15Y	20Y	25Y
3M	0.0	0.0	3.88%	10.3%	17.66%	25.3%	32.98%
		0.00109	4.27%	11.31%	19.34%	27.68%	36.09%
		0.0166	4.66%	12.23%	20.81%	29.69%	38.41%
		0.02245	5.14%	13.37%	22.61%	32.14%	41.68%
	-0.01313	0.0087	4.1%	10.88%	18.64%	26.7%	34.83%
	-0.02	0.01325	4.35%	11.48%	19.58%	27.99%	36.45%
	-0.02704	0.01792	4.67%	12.23%	20.77%	29.59%	38.46%
	OIS		2.28%	5.41%	8.3%	11.44%	14.91%
6M	0.0	0.0	3.74%	10.06%	17.32%	24.89%	32.53%
		0.01225	4.2%	11.25%	19.31%	27.71%	36.18%
		0.01435	4.33%	11.58%	19.84%	28.44%	37.1%
		0.01655	4.49%	11.96%	20.44%	29.25%	38.12%
	-0.01521	0.0096	3.99%	10.73%	18.45%	26.51%	34.65%
	-0.01782	0.01125	4.08%	10.94%	18.78%	26.97%	35.22%
	-0.02056	0.01297	4.18%	11.17%	19.16%	27.48%	35.86%
	OIS		2.14%	5.34%	8.43%	11.78%	15.44%

Table 6: Price in % of notional for ATM Cap prices for our considered tenor structures and dependence structures.

Tenor	α^x	β^x	5Y	10Y	15Y	20Y	25Y
3M	0.0	0.0	5.45%	13.29%	21.98%	30.9%	39.80%
		0.00109	5.77%	14.13%	23.38%	32.88%	42.38%
		0.0166	6.09%	14.90%	24.62%	34.58%	44.52%
		0.02245	5.50%	15.88%	26.17%	36.69%	47.17%
	-0.01313	0.0087	5.62%	13.75%	22.75%	32.01%	41.26%
	-0.02	0.01325	5.82%	14.23%	23.52%	33.06%	42.58%
	-0.02704	0.01792	6.09%	14.86%	24.51%	34.39%	44.25%
	OIS		3.54%	7.62%	11.30%	15.25%	19.55%
6M	0.0	0.0	5.31%	13.05%	21.64%	30.49%	39.33%
		0.01225	5.69%	14.04%	23.30%	32.84%	42.38%
		0.01435	5.8%	14.32%	23.75%	33.45%	43.15%
		0.01655	5.93%	14.64%	24.26%	34.15%	38.12%
	-0.01521	0.0096	5.51%	13.58%	22.54%	31.78%	41.02%
	-0.01782	0.01125	5.58%	13.75%	22.81%	32.14%	41.48%
	-0.02056	0.01297	5.66%	13.94%	23.12%	32.56%	42.00%
	OIS		3.34%	7.54%	11.46%	15.67%	20.19%

Table 7: Price in % of notional for ITM (with 50 BP) Cap prices for our considered tenor structures and dependence structures.

Tenor	α^x	β^x	5Y	10Y	15Y	20Y	25Y
3M	0.0	0.0	2.61%	7.77%	13.91%	20.35%	26.87%
		0.00109	3.05%	8.92%	15.85%	23.12%	30.49%
		0.0166	3.50%	9.97%	17.53%	25.42%	33.40%
		0.02245	4.03%	11.24%	19.55%	28.18%	36.87%
	-0.01313	0.0087	2.87%	8.45%	15.08%	22.04%	29.11%
	-0.02	0.01325	3.16%	9.15%	16.19%	23.57%	31.03%
	-0.02704	0.01792	3.53%	10.02%	17.56%	25.43%	33.38%
	OIS		1.41%	3.75%	5.98%	8.42%	11.14%
6M	0.0	0.0	2.48%	7.53%	13.57%	19.94%	26.42%
		0.01225	3.0%	8.9%	15.87%	23.21%	30.68%
		0.01435	3.15%	9.27%	16.47%	24.05%	31.73%
		0.01655	3.33%	9.7%	17.15%	24.98%	32.9%
	-0.01521	0.0096	2.77%	8.32 %	14.92%	21.89%	28.99%
	-0.01782	0.01125	2.87%	8.57%	15.31%	22.43%	29.67%
	-0.02056	0.01297	2.99%	8.84%	15.75%	23.03%	30.43%
	OIS		1.32 %	3.7%	6.08%	8.69%	11.56%

Table 8: Price in % of notional for OTM (with 50 BP) Cap prices for our considered tenor structures and dependence structures.

6 Conclusion

Summing everything up this project showcased how a simple single curve model can be extenden through a stochastic basis to price both derivatives with both OIS and xIBOR rates as a reference rate. In addition it showcased that it is almost possible to infer same implied volatilities, however only for Black implied vol., in the OIS and Spread model. Nevertheless a further investigation led an interesting finding that the calibrated short rate volatilies to fit ATM implied volatilities for the deterministic spread model were quite low, which could indicate that the pricing for some parameterization under a identical short rate vol, σ, could be significantly larger. This was confirmed when checking prices for ATM, ITM and OTM prices for caps of different maturities, where the spread model consistently had prices approximately twice as large as the OIS model. This could confirm that the era of OIS swaps leads to lesser change in daily values on derivatives contracts - Nevertheless this should be exmined further.

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