CONTROL ROD WORTH DETERMINATION IN A TRIGA MARK II REACTOR USING THE SUBCRITICAL METHOD

Elsa SEICHAIS - Patrik SHYTAJ

MSc Nuclear engineering
Politecnico di Milano

Figure 1: Triga Mark II reactor

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Abstract

This report, a component of the "Experimental Nuclear Reactor Kinetics" course under the guidance of Professor Stefano Lorenzi at Politecnico di Milano, is dedicated to the calibration of control rods in the experimental TRIGA Mark II reactor. The study specifically delves into the application and significance of the subcritical method for control rod calibration, elucidating its role and advantages. The initial section of the report provides a comprehensive overview of TRIGA reactors, establishing the groundwork for a profound understanding of their operational principles. Subsequently, the focus shifts to control rod calibration, highlighting the intricacies of the subcritical method. The experiment involves a practical component conducted at the Pavia facility, employing the subcritical method for control rod calibration. The obtained results undergo thorough analysis, and calculations lead to the reactivity rod worths of SHIM, REG, and TRANS control rods.

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Introduction

Triga Mark reactors

The TRIGA[1] reactor is an experimental reactor designed and manufactured by General Atomics. From the first TRIGA reactor prototype to the actual TRIGA Mark III, various prototypes have been considered with different reactor designs. This report will focus on the reactor in which the experiments are made which is a TRIGA Mark II located in Pavia, Italy.

A TRIGA reactor operates on the fundamental principle articulated by Edward Teller, a Hungarian-American nuclear physicist, asserting that '*the reactor fuel itself should inherently possess safety characteristics for reactivity insertion events.*' [2] This foundational philosophy guides the design process of the TRIGA Mark II, incorporating numerous inherent safety mechanisms from its conceptualization. Hence, TRIGA reactors can be utilized not only for education and training but also across various domains, including boron neutron capture therapy (BNCT) and the production of radioisotopes for medical and industrial applications.

On the one hand, concerning fuel and moderator characteristics, TRIGA Mark II reactor is operating at a nominal power of 250 kW. This reactor utilizes a Uranium Zirconium Hydride fuel, UZrH which is solid. Hydrogen nuclei serve as a moderator, directly blended with the fuel. Zirconium is employed to create ZrH, known for its chemical stability and excellent thermal conductivity. Additionally, the uranium component is enriched to 20% in ^{235}U.
On the other hand, regarding the pool and inherent safety features, the reactor adopts an open pool configuration, with natural circulation, eliminating the need for pumps. Notably, it is not situated underground like TRIGA Mark I reactors, facilitating easier access for maintenance, ventilation, and cooling, as well as the incorporation of measuring instruments on the beams. The coolant employed is light water, which also functions as a moderator. To monitor core responsiveness, three control rods are employed. Two, crafted from boron carbide (B_4C), are motor-driven – namely, the SHIM and REG (for regulating) control rods. The former enables coarse reactivity adjustment, while the latter facilitates fine reactivity tuning. The third control rod, TRANS (for transient), is composed of borated graphite and serves a safety role, activated by a pneumatic system.

Some of the characteristics of the TRIGA Mark II in Pavia are presented in the following table[1] :

Maximum Power (Steady State)	250 kW
Maximum Flux (Central Channel)	$1,8.10^{13}$ $cm^{-2}s^{-1}$
Mass[a] of Fissile (235-U),	$2,2$ kg [b]
Negative temperature coefficient of the fuel-moderator	$-1,2.10^{-4}$ $\Delta k/k$ C° at 50°C
Moderator	HZr,H_2O
Reflector	Graphite
Thermal coolant	H_2O
n° of control rods	3
Fuel temperature at 250 kW	230 °C
Coolant temperature at 250 kW	35-40 °C

Table 1: Characteristics of the reactor Triga in LENA

[a] mass of U^{235} in the fuel to achieve an effective multiplication coefficient k equal to 1 (critical reactor)
[b] equivalent to 62 new elements (initial fuel load)

[1] Training Research Isotope production General Atomic

Control rod worth measurement

In nuclear reactors, the effective regulation of neutron population is crucial for maintaining control and ensuring safe operation. Control rods, composed of absorbent materials, play a pivotal role in this process by modulating reactivity within the core. Understanding the precise impact of control rods on reactivity is essential for reactor start-up, shutdown, and overall safety protocols. Calibrating control rods involves experimentally determining the relationship between the rod's position in the core and the introduced reactivity. This calibration enables operators to assess core excess and shutdown margin, which are critical parameters for reactor stability.

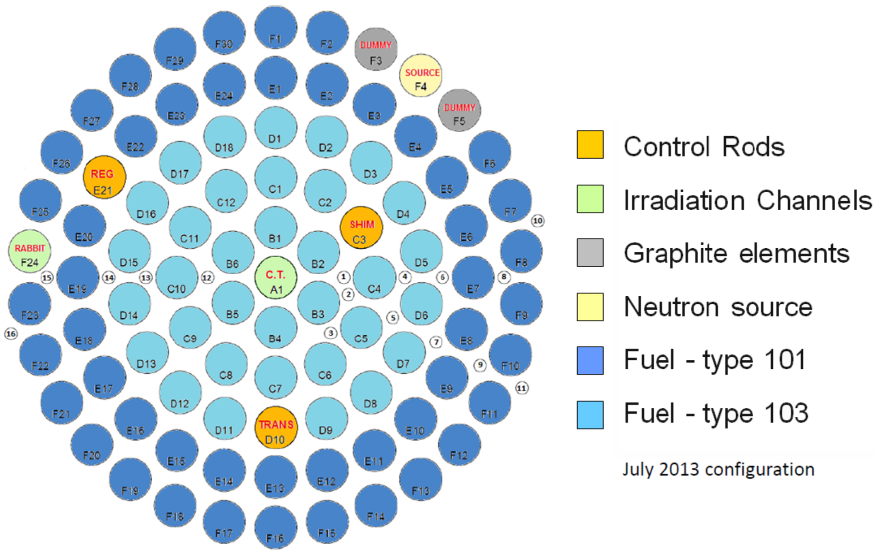

Figure 2: Triga Mark II reactor core

In the context of TRIGA reactors, various control rods exhibit distinct reactivity worths, contributing to the complexity of neutron population control. This report delves into the measurement of control rod worth (for REG, SHIM and TRANS), focusing on the *Subcritical Reactor Method*. This method, while simple and efficient with a short measuring time, poses challenges such as low accuracy, dependency on neutron source location and instrumentation, and the need for calibration adjustments due to fission chamber burnup. The exploration of control rod worth measurement methods is paramount for both operational considerations and reactor design optimization.

For the Triga Mark II reactor at LENA, the characteristics of the control rods can be summarised in Table 2 below.

Control rod	Material	Reactivity worth
SHIM	B_4C	$3 - 3.5$ \$
REG	B_4C	$1 - 1.5$ \$
TRANS	Borated graphite	$2 - 2.5$ \$

Table 2: Characteristics of the control rods in LENA's Triga reactor.

In this report, the first section will elucidate the fundamentals of the subcritical method, succeeded by a subsequent section dedicated to detailing the conducted experiment and its application. Subsequently, the acquired measurements will be presented and analyzed in the concluding section.

1 Theorical background

1.1 The subcritical method

The subcritical method is an experimental method that permits to determine the reactivity of a subcritical core by measuring the flux in steady state.

The subcritical method, as its name suggests, is performed on a reactor core which is subcritical. The core is in a subcritical state if the multiplication factor respects : $k_{eff} < 1$. This implies that the core cannot maintain the chain reaction on its own. To prevent the neutron density n from dying out, an external neutron source must be introduced into the core. In the case of the TRIGA reactor, the neutron source will be Ra-Be.

The key point of the subcritical method lies in the fact that when a constant source of neutrons $q_0(n/cm^3 s)$ is inserted into the core, it is possible to observe a *geometric progression* type of evolution among the neutron generations which leads the neutron density towards a constant value lMq_0, where l is the lifetime of neutrons in the core and M another way to express the multiplication factor, called the *subcritical multiplication factor* defined as following :

$$M = \frac{1}{1-k} \tag{1}$$

Let's explain the previous assertion. In fact, overall, there are two offsetting effects. On one hand, each generation will always have a neutron contribution due to the presence of the source that in one generation produces $lq_0(n/cm3)$. On the other hand, the multiplication factor k in a subcritical state will act on the generation i and decrease the number of neutrons in generation $i+1$. These phenomena are repeated in each generation so the population n_i of the the generation i[2] is given by a *finite geometrical series*, follows that the steady state value of the neutron population is given by a *geometrical series*[3]:

$$n_i = \sum_{j=0}^{i} lq_0 k^j \underset{i \to +\infty}{\longrightarrow} n_\infty = lq_0 \frac{1}{1-k} \tag{2}$$

This asymptotic result obtained after a large amount of generations will be the base of the experiment, as it is shown in section 2.2. Is possible to express the steady state neutron density in other forms, taking into account the subcritical multiplication factor M or the reactivity ρ:

$$\boxed{n_\infty = lq_0 \frac{1}{1-k} = lq_0 M = lq_0 \frac{\rho - 1}{\rho}} \tag{3}$$

The measure of n_∞ can be used to determine the negative reactivity in a subcritical core. As this is an asymptotic value, it is necessary to determine a time after which the measurement is feasible. To do so the *Point kinetics* equations are used.

$$\begin{cases} \dfrac{dn}{dt} = \dfrac{k(\rho - \beta)}{l} n + \sum\limits_{i=1}^{6} \lambda_i c_i + q_0 \\ \\ \dfrac{dc_i}{dt} = \dfrac{k\beta_i}{l} n - \lambda_i c_i, \quad i = 1, ..., 6 \end{cases} \tag{4}$$

[2] The 0-th generation is considered with the source inserted only.
[3] The convergence is assured because $k < 1$.

In the case of one-group of precursors approximation[4], by posing :

$$\begin{cases} \beta = \sum\limits_{i=1}^{6} \beta_i \\ \\ \lambda = \left(\dfrac{1}{\beta} \sum\limits_{i=1}^{6} \dfrac{\beta_i}{\lambda_i} \right)^{-1} \end{cases} \tag{5}$$

the system of equations (4) can be simplified as follow :

$$\begin{cases} \dfrac{dn}{dt} = \dfrac{k(\rho - \beta)}{l} n + \lambda c + q_0 \\ \\ \dfrac{dc}{dt} = \dfrac{k\beta}{l} n - \lambda c \end{cases} \tag{6}$$

The solution of the system is:

$$n(t) = n_\infty [1 + A_p \exp(-\frac{t}{\tau_p}) + A_d \exp(-\frac{t}{\tau_d})] \tag{7}$$

where :

- τ_p and τ_d are the characteristic time constants of prompt and delayed neutrons respectively, where $\tau_d \ggg \tau_p$
- A_p and A_d are constants

The steady state can be considered reached after $5\tau_d$[5]. This gives a first idea of how long it is needed to wait before doing a measurement. Obviously, this time is an approximation since is obtained from the one-group approximation of the point kinetics equation.

1.2 Assumptions

As it will appear in section *Experimental application of the method* (2), during the experiment some sources of uncertainties will be taken into account, whereas others can be easily neglected. Here are the main points that can be discussed :

- Decoupled control rods: the control rod worth of one control rod is considered independent on the position of the other control rods
- When the core changes configuration, the minimum time before doing a measurement is $150s$: this is assumed as the time needed to the reactor to reach a stationary state as will be shown in section 1.1
- The uncertainty due to the electronic system of acquisition is neglected
- $\alpha\phi_s$, the calibration factor in the fission chamber, is considered constant to determine the control rods worth, even if depends on burnup of the detector. In our case, the time of the experiment is so short that this approximation can hold
- The control rods worth given as data are considered without uncertainty
- The counts measured by the fission chamber are considered Poisson variables
- Some counts are given by the background: cosmic radiations and radioactive decays

[4] Valid approximation because this method is not used for the high accuracy, but more for the fast operation.
[5] $\tau_d \simeq \frac{\beta - \rho}{\lambda\rho}$ and in the case of the experiment object of this paper $\tau_d \simeq 30s$

2 Experimental application of the method

2.1 Instrumentation

To perform the experimental procedure described later, the used instruments are:

- Neutron source[6];
- Fission chamber;
- Electronic system to support the fission chamber.

Since for this experiment the reactor is subcritical the power level depends only on the external source, so is very low: a direct measure on the flux is performed, so on the counts of the fission chamber.

2.2 Procedure

Conceptual path: given the control rod worth (CRW) of one control rod, the aim is to find the worth of the other two control rods.

The core is operated in subcritical state so it is characterised by a *subcritical multiplication factor* M: the flux of the source ϕ_s is amplified by the core and becomes ϕ following the relation

$$\phi \propto M\phi_s \tag{8}$$

This flux ϕ is the responsible of the counting rate CR measured in the fission chamber, which goes as

$$CR \propto \phi \tag{9}$$

So considering a proportionality constant α which takes in account the positions of source and fission chamber and the sensitivity of the fission chamber, is possible to determine experimentally M

$$CR = \alpha\phi_s M \tag{10}$$

Or determine the reactivity ρ

$$CR = \alpha\phi_s \frac{\rho - 1}{\rho} \tag{11}$$

It would be easy to evaluate ρ from this relation, but since $\alpha\phi_s$ is not known it is necessary to adopt another approach: is possible to consider two different configurations of the reactor characterised by ρ_1 and ρ_2 in which the measured counting rates are respectively CR_1 and CR_2 and they are related in the following way

$$\gamma = \frac{\rho_2 - 1}{\rho_2}\frac{\rho_1}{\rho_1 - 1} \qquad\qquad \gamma = \frac{CR_2}{CR_1} \tag{12}$$

[6] In our case a $2Ci$ Ra-Be neutron source

In the experiment the starting point is the known CRW, which is the one of the REG[7], whose CRW has been measured using the *reactor period method* in another experimental activity.

In order to collect all the needed data, the CR are obtained in 5 different configurations[8] of the core:

	Source	REG	SHIM	TRANS
Configuration 0	out	in	in	in
Configuration ALL	in	in	in	in
Configuration REG	in	out	in	in
Configuration SHIM	in	in	out	in
Configuration TRANS	in	in	in	out

The following passages explain how to obtain the CRW of the $SHIM$ rod and $TRANS$ rod:

- The first step is cleaning the counting rates from the background contribution. The measurement on $configuration$ 0 is performed with this aim, this way the CR of the other configurations can be corrected.

- $Configuration\ ALL$ and $configuration\ REG$ are the two configurations that will be compared since :

$$CRW_{REG,ex} = \rho_{REG} - \rho_{ALL} \tag{13}$$

  Substituting this in equation (12) is found a relation from where to obtain ρ_{ALL} :

$$\gamma = \frac{(\rho_{ALL} + CRW_{REG,ex}) - 1}{(\rho_{ALL} + CRW_{REG,ex})} \frac{\rho_{ALL}}{\rho_{ALL} - 1} \qquad \gamma = \frac{CR_{REG,ex}}{CR_{ALL}} \tag{14}$$

  It can be found in an analytic way as one root of the parabola :

$$\rho_{ALL} = -\frac{(CRW_{REG,ex}) - 1}{2} - \frac{1}{2}\sqrt{(CRW_{REG,ex} - 1)^2 - \frac{4\gamma CRW_{REG,ex}}{\gamma - 1}} \tag{15}$$

- The following step, now that ρ_{ALL} is known, is to estimate $\alpha\phi_s$ using equation (11) in the case of $configuration\ ALL$.

- In the final step are used $Configuration\ SHIM$ and $configuration\ TRANS$, which are needed to find CRW_{SHIM} and CRW_{TRANS} using equation (11) to obtain ρ_{SHIM} and ρ_{TRANS} and finally

$$CRW_{SHIM} = \rho_{SHIM} - \rho_{ALL} \tag{16}$$

$$CRW_{TRANS} = \rho_{TRANS} - \rho_{ALL} \tag{17}$$

[7] In the following sections have been considered also other cases with other rods as reference

[8] All the configurations are subcritical

2.3 Measurements

The experiment is performed with the core in clean condition. For each configuration the procedure is the same: measure the integral value of counts of the fission chamber during an interval Δt=100s and perform this measure several times[9] to improve the statistics.

When passing from one configuration to the other is needed to wait at least 150s as specified in the assumptions (1.2).

2.4 Uncertainty analysis

To obtain the control rods worth, it is explained in the previous part that the basis of the experiment is the measure of the CR in the five configurations of the core.

The concrete result observed from the experiment is the number of counts from the fission chamber (x_i), that is performed in some samples (N_s the number of samples) as said before to reduce the uncertainty on the measure.

So from the experimentally measured x_i, is obtained $\overline{X}$:

$$\overline{X} = \frac{1}{N_s}\sum_{i=1}^{N_s} x_i \tag{18}$$

Here appears the first source of error, the Poisson law followed by the data and so the standard deviation which is generated. Indeed, in a Poisson law, the variance is equal to the mean value. Thus implies :

$$\sigma_{\overline{X}} = \sqrt{\frac{\sigma_p^2}{N_s}} = \sqrt{\frac{\overline{X}}{N_s}} \tag{19}$$

Since the measurement is performed on the integral value of counts, with the aim of reducing the relative error on them, as said in the previous part, to obtain the CR of each configuration, the mean number of counts measured before is divided by Δt:

$$\overline{CR} = \frac{\overline{X}}{\Delta t} \qquad and \qquad \sigma_{\overline{CR}} = \frac{\sigma_{\overline{X}}}{\Delta t} \tag{20}$$

These uncertainties due to the measure of the counts propagate through the final result, which is the control rod worth. To take these uncertainties into account, the procedure mentioned in [3], by R.J. Moffat is used.

In a long story short, the principle of the method is the determination of the uncertainty in the final result, after calculations, that comes from errors in the measure of physical quantities as input of the calculations. If a physical quantity X_i is affected by an uncertainty δX_i, then the result of an operation to finally obtain R will be affected by an error δR given by the following relation, with N the number of variables in R :

$$\delta R = \sqrt{\sum_{i=1}^{N} (\frac{\partial R}{\partial X_i}\delta X_i)^2} \tag{21}$$

[9]Have been done 3 times for $configuration$ 0 and 5 times for the other configurations

However, for simplicity and faster calculations, R.J. Moffat proposed an approximation of the sensitive coefficients ($\frac{\partial R}{\partial X_i}$) that leads to :

$$\delta R = \sqrt{\sum_{i=1}^{N} (\frac{|C_{i^+}| + |C_{i^-}|}{2})^2} \tag{22}$$

where :

- R_{i^+} and R_{i^-} are respectively used by increasing (respectively reducing) the value of the *i-th* variable, X_i, by its uncertainty interval δX_i
- $C_{i^+} = (R_{i^+} - R)$
- $C_{i^-} = (R - R_{i^-})$

All the procedure applied to the experiment is written in the code in the in the section *Appendix* (4.3)

3 Data collection

For each configuration are showed the number of counts in each interval Δt.

Configuration 0

Counts
4
2
0

Configuration ALL

Counts
1002
1118
1079
1135
1006

Configuration REG

Counts
1381
1580
1592
1497
1577

Configuration SHIM

Counts
4440
4360
4340
4294
4447

Configuration TRANS

Counts
2578
2590
2431
2392
2482

4 Data analysis

In the following passages will be considered only the mean value of the integral counts for each configuration and, since in the formula appear the CR, these values are divided by Δt=100s. Of course all the data will be accompanied by their respective uncertainty, as explained in section 2.4

	$CR(cps)$	$\sigma(cps)$
Configuration 0	0.02	0.01
Configuration ALL	10.68	0.15
Configuration REG	15.25	0.17
Configuration SHIM	43.76	0.30
Configuration TRANS	24.95	0.22

Comparing the CR of *configuration 0* with the CR of *configuration ALL* results that the background activity is $\sim$ 500 times lower than the Ra-Be source activity. Now, the need for an external source to perform the experiment is evident. While conducting the experiment solely with the natural source is possible, the CR would be too low, resulting in significant uncertainty.

It is also possible to notice that in *configuration SHIM*, the CR is the highest because the SHIM rod is the one with the highest CRW, so when it is extracted the subcritical multiplication factor increases permitting the presence of a higher neutron flux in the core and consequently more signal in the fission chamber.

The analysis will be performed with four different references: the first one is the $CRW_{REG,ex}$ measured in the other experimental activity, while the other three[10] are the official values of the CRW of $TRIGA$[11]

	$CRW(\$)$
$CRW_{REG,ex}$	1.19
$CRW_{REG,official}$	1.26
$CRW_{SHIM,official}$	3.22
$CRW_{TRANS,official}$	2.23

4.1 Background removal

The background contribution is removed from *configuration ALL*, *configuration REG*, *configuration SHIM* and *configuration TRANS* by subtracting to all of them CR_0, while the uncertainties are increased summing up the variances.

	$CR(cps)$	$\sigma(cps)$
Configuration ALL	10.66	0.15
Configuration REG	15.23	0.17
Configuration SHIM	43.74	0.30
Configuration TRANS	24.93	0.22

The changing in the CR is lower than 0.2% in all the configurations, while the uncertainty is around 1% or more, this means that the Ra-Be source that is used has an activity capable to make the background a not relevant factor in the experiment: even without removing the background to the final values, it would not change in a relevant way the result.

[10] Assignment
[11] Calibration of the TRIGA control rods of 27-07-2015

4.2 Estimation of the calibration factor $\alpha\Phi_s$

As explained in the section *Procedure* (2.2), to obtain $\alpha\Phi_s$ are used the data of *configuration ALL* and the data of the configuration in which the reference control rod is extracted, while for the uncertainties is used the relation (22).

	$\alpha\phi_s(cps)$	$\sigma(cps)$
Reference REG_{ex}	0.294	0.015
Reference $REG_{official}$	0.311	0.016
Reference $SHIM_{official}$	0.319	0.006
Reference $TRANS_{official}$	0.291	0.007

The difference between these values can not be attributed only to the uncertainty, but there is also an influence of the position and worth of the control rod that is used as reference. In particular it is possible to notice that the highest value of $\alpha\phi_s$ is obtained using as reference the $SHIM$, which is the rod with the highest CRW in the core and the closest to the source.

It can also be noticed that, since the uncertainty considered is only due to the stochastic nature of the counts in the fission chamber, in the case of reference $SHIM$ and reference $TRANS$ the uncertainties are smaller: the higher is the CRW of the reference and the higher is the CR, so the lower the relative error. This trend will be reflected also in the uncertainties of the CRW.

4.3 Determination of the control rods worth

Following what said in the section *Procedure* (2.2) comes the determination of the control rod worth in all the cases, while for the uncertainties the method is the usual one (22).

	$CRW_{REG}(\$)$	$\sigma_{REG}(\$)$	$CRW_{SHIM}(\$)$	$\sigma_{SHIM}(\$)$	$CRW_{TRANS}(\$)$	$\sigma_{TRANS}(\$)$
Reference REG_{ex}	(1.19)	-	2.96	0.11	2.25	0.08
Reference $REG_{official}$	(1.26)	-	3.13	0.12	2.38	0.08
Reference $SHIM_{official}$	1.29	0.05	(3.22)	-	2.45	0.02
Reference $TRANS_{official}$	1.18	0.04	2.93	0.03	(2.23)	-

For the control rod which is considered as reference it is useless to calculate the CRW passing from the procedure of the $\alpha\phi_s$ calculation, the result of doing so would only end up in adding uncertainty to a quantity which is already known.

The values of CRW that are obtained show a coherent trend with the physics of the reactor, but they are far from the official CRW values. Even by considering a confidence interval $\pm 2\sigma$, there is no superposition with the official values.

This means that the reason of discrepancy between official values and experimentally measured values is not attributed to the stochastic nature of the CR measured by the fission chamber: there is another explanation. Some relevant reasons to explain the discrepancy are the following:

- The main reason, as seen in the $\alpha\phi_s$ calculation, is the strong relevance of the relative position between reference control rod and source
- Another relevant issue, that is neglected in the assumptions (1.2) is the coupling between control rods: the CRW actually depends on the relative position of the control rods
- The last main contribution to this discrepancy is the assumption of clean core[12]: the concentration of poisonous fission products in the core increases the neutron absorption leading to a lower CRW.

[12] Assignment

Conclusion

The experimental measure of the control rod worth using the *subcritical method* permitted a **fast**, **simple** and **safe** measure of the control rods worth, but the drawback, as expected, is the low accuracy of the results. It is on one hand due to the coupling between control rods and on the other hand due to the limit of the lumped reactor model: the analysis of the subcritical condition of the reactor with the *point kinetics equations* hides the importance of the spatial distribution of flux for this experiment, in which the relative position between source, control rods and fission chamber is of primary importance. Following this observation it is possible to say that a more detailed analysis in this direction would give much better results, but also increase the complexity of the calculations by far.

More accurate results can be obtained using the *reactor period method*, but sacrificing the simplicity of the method and also the safety. In particular focusing on this last point, with the *subcritical method* it was possible to perform also the SHIM control rod worth measure, which would not be possible in other way for safety reasons.

An interesting point to notice about this method is the fact that it can also be used for another kind of experiment: the control rod calibration. The procedure is the same as before with the only difference that the measurements are performed at different steps of extraction of the control rod of interest. In case of future experiment it is important to keep in mind that all the procedure has to be repeated, in particular the factor $\alpha\phi_s$ has to be evaluated again, since it suffers of the fission chamber burnup.

References

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Appendix

4.4 Python code for calculations

```
import numpy as np


# Data
beta = 730e-5
CRW_reg_lab1 = 1.19*beta
CRW_reg = 1.26*beta
CRW_trans = 2.23*beta
CRW_shim = 3.22*beta

CR_background = 0.02 #cps
sig_CR_background = 0.01 #cps
CR_allin_mes = 10.68 #cps
sig_CR_allin_mes = 0.15 #cps
CR_reg_mes = 15.25 #cps
sig_CR_reg_mes = 0.17 #cps
CR_trans_mes = 24.95 #cps
sig_CR_trans_mes = 0.22 #cps
CR_shim_mes = 43.76 #cps
sig_CR_shim_mes = 0.30 #cps

# correction using background

CR_allin = CR_allin_mes - CR_background
CR_reg = CR_reg_mes - CR_background
CR_trans = CR_trans_mes - CR_background
CR_shim = CR_shim_mes - CR_background

# sigma corrected by background

sig_CR_allin = np.sqrt(sig_CR_allin_mes**2+sig_CR_background**2)
sig_CR_reg = np.sqrt(sig_CR_reg_mes**2+sig_CR_background**2)
sig_CR_trans = np.sqrt(sig_CR_trans_mes**2+sig_CR_background**2)
sig_CR_shim = np.sqrt(sig_CR_shim_mes**2+sig_CR_background**2)

# solution parabola

def rho_parabola(CRW,gamma):
  return -(CRW-1)/2-np.sqrt((CRW-1)**2+4*gamma*CRW/(gamma-1))/2

# alpha phi_s

def alphaphis(CR_ref,CR_all, CRW):

  gamma = CR_ref/CR_all
  rho_all = rho_parabola(CRW, gamma)
  k_all = 1/(1-rho_all)
  M_all = 1/(1-k_all)
  return CR_all/M_all

# CRW function

def FCRW(CR_specific, CR_ref, CR_all, CRW_ref):

  gamma = CR_ref/CR_all
  rho_all = rho_parabola(CRW_ref, gamma)
  k_all = 1/(1-rho_all)
  M_all = 1/(1-k_all)
  alphaphis = CR_all/M_all
```

```
    M_specific = CR_specific/alphaphis
    k_specific = (M_specific-1)/M_specific
    rho_specific = (k_specific-1)/k_specific
    return (rho_specific-rho_all)/beta





# REFERENCE REG LAB1 (_RR1)

print('REG AS REFERENCE(LAB 1)')

# alpha phi_s value
alphaphis_RR1 = alphaphis(CR_reg, CR_allin, CRW_reg_lab1)

# alpha phi_s uncertainty
R_plus_alphaphis_RR1 = np.array([ alphaphis(CR_reg+sig_CR_reg, CR_allin,
    CRW_reg_lab1), alphaphis(CR_reg, CR_allin+sig_CR_allin, CRW_reg_lab1)])
R_minus_alphaphis_RR1 = np.array([ alphaphis(CR_reg-sig_CR_reg, CR_allin,
    CRW_reg_lab1), alphaphis(CR_reg, CR_allin-sig_CR_allin, CRW_reg_lab1)])

c_plus_alphaphis_RR1 = R_plus_alphaphis_RR1-alphaphis_RR1
c_minus_alphaphis_RR1 = R_minus_alphaphis_RR1-alphaphis_RR1


sig_alphaphis_RR1 = np.sqrt(np.sum(((np.abs(c_plus_alphaphis_RR1)+
                    np.abs(c_minus_alphaphis_RR1))/2)**2))
print(f'\nThe calibration factor with the REG as reference(lab 1) is
      {alphaphis_RR1} +/- {sig_alphaphis_RR1}')



# Control rod worth values
CRW_reg_RR1 = FCRW(CR_reg, CR_reg, CR_allin, CRW_reg_lab1)
CRW_shim_RR1 = FCRW(CR_shim, CR_reg, CR_allin, CRW_reg_lab1)
CRW_trans_RR1 = FCRW(CR_trans, CR_reg, CR_allin, CRW_reg_lab1)

# Uncertainty on REG control rod worth
R_plus_CRW_reg_RR1 = np.array([ FCRW(CR_reg+sig_CR_reg, CR_reg+sig_CR_reg,
    CR_allin, CRW_reg_lab1), FCRW(CR_reg, CR_reg, CR_allin+sig_CR_allin,
    CRW_reg_lab1)])
R_minus_CRW_reg_RR1 = np.array([ FCRW(CR_reg-sig_CR_reg, CR_reg-sig_CR_reg,
    CR_allin, CRW_reg_lab1), FCRW(CR_reg, CR_reg, CR_allin-sig_CR_allin,
    CRW_reg_lab1)])

c_plus_CRW_reg_RR1 = R_plus_CRW_reg_RR1-CRW_reg_RR1
c_minus_CRW_reg_RR1 = R_minus_CRW_reg_RR1-CRW_reg_RR1

sig_CRW_reg_RR1 = np.sqrt(np.sum(((np.abs(c_plus_CRW_reg_RR1)+
                  np.abs(c_minus_CRW_reg_RR1))/2)**2))
print(f'\nThe control rod worth of the REG with the REG as reference(lab 1)
      is {CRW_reg_RR1} +/- {sig_CRW_reg_RR1} $')

# Uncertainty on SHIM control rod worth
R_plus_CRW_shim_RR1 = np.array([ FCRW(CR_shim+sig_CR_shim, CR_reg, CR_allin,
    CRW_reg_lab1), FCRW(CR_shim, CR_reg+sig_CR_reg, CR_allin, CRW_reg_lab1),
    FCRW(CR_shim, CR_reg, CR_allin+sig_CR_allin, CRW_reg_lab1)])
R_minus_CRW_shim_RR1 = np.array([ FCRW(CR_shim-sig_CR_shim, CR_reg, CR_allin,
    CRW_reg_lab1), FCRW(CR_shim, CR_reg-sig_CR_reg, CR_allin, CRW_reg_lab1),
    FCRW(CR_shim, CR_reg, CR_allin-sig_CR_allin, CRW_reg_lab1)])

c_plus_CRW_shim_RR1 = R_plus_CRW_shim_RR1-CRW_shim_RR1
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c_minus_CRW_shim_RR1 = R_minus_CRW_shim_RR1-CRW_shim_RR1

sig_CRW_shim_RR1 = np.sqrt(np.sum(((np.abs(c_plus_CRW_shim_RR1)+
                    np.abs(c_minus_CRW_shim_RR1))/2)**2))
print(f'\nThe control rod worth of the SHIM with the REG as reference(lab 1)
      is {CRW_shim_RR1} +/- {sig_CRW_shim_RR1} $')

# Uncertainty on TRANS control rod worth
R_plus_CRW_trans_RR1 = np.array([ FCRW(CR_trans+sig_CR_trans, CR_reg, CR_allin,
    CRW_reg_lab1),  FCRW(CR_trans, CR_reg+sig_CR_reg, CR_allin, CRW_reg_lab1),
    FCRW(CR_trans, CR_reg, CR_allin+sig_CR_allin, CRW_reg_lab1)])
R_minus_CRW_trans_RR1 = np.array([ FCRW(CR_trans-sig_CR_trans, CR_reg,
    CR_allin, CRW_reg_lab1),  FCRW(CR_trans, CR_reg-sig_CR_reg, CR_allin,
    CRW_reg_lab1),  FCRW(CR_trans, CR_reg, CR_allin-sig_CR_allin,
    CRW_reg_lab1)])

c_plus_CRW_trans_RR1 = R_plus_CRW_trans_RR1-CRW_trans_RR1
c_minus_CRW_trans_RR1 = R_minus_CRW_trans_RR1-CRW_trans_RR1

sig_CRW_trans_RR1 = np.sqrt(np.sum(((np.abs(c_plus_CRW_trans_RR1)+
                     np.abs(c_minus_CRW_trans_RR1))/2)**2))
print(f'\nThe control rod worth of the TRANS with the REG as reference(lab 1)
      is {CRW_trans_RR1} +/- {sig_CRW_trans_RR1} $\n\n\n\n')



# REFERENCE REG (_RR)

print('REG AS REFERENCE')

# alpha phi_s value
alphaphis_RR = alphaphis(CR_reg, CR_allin, CRW_reg)

# alpha phi_s uncertainty
R_plus_alphaphis_RR = np.array([ alphaphis(CR_reg+sig_CR_reg, CR_allin,
    CRW_reg),  alphaphis(CR_reg, CR_allin+sig_CR_allin, CRW_reg)])
R_minus_alphaphis_RR = np.array([ alphaphis(CR_reg-sig_CR_reg, CR_allin,
    CRW_reg),  alphaphis(CR_reg, CR_allin-sig_CR_allin, CRW_reg)])

c_plus_alphaphis_RR = R_plus_alphaphis_RR-alphaphis_RR
c_minus_alphaphis_RR = R_minus_alphaphis_RR-alphaphis_RR


sig_alphaphis_RR = np.sqrt(np.sum(((np.abs(c_plus_alphaphis_RR)+
                    np.abs(c_minus_alphaphis_RR))/2)**2))
print(f'\nThe calibration factor with the REG as reference is
      {alphaphis_RR} +/- {sig_alphaphis_RR}')

# Control rod worth values
CRW_reg_RR = FCRW(CR_reg, CR_reg, CR_allin, CRW_reg)
CRW_shim_RR = FCRW(CR_shim, CR_reg, CR_allin, CRW_reg)
CRW_trans_RR = FCRW(CR_trans, CR_reg, CR_allin, CRW_reg)

# Uncertainty on REG control rod worth
R_plus_CRW_reg_RR = np.array([ FCRW(CR_reg+sig_CR_reg, CR_reg+sig_CR_reg,
    CR_allin, CRW_reg),  FCRW(CR_reg, CR_reg, CR_allin+sig_CR_allin, CRW_reg)])
R_minus_CRW_reg_RR = np.array([ FCRW(CR_reg-sig_CR_reg, CR_reg-sig_CR_reg,
    CR_allin, CRW_reg),  FCRW(CR_reg, CR_reg, CR_allin-sig_CR_allin, CRW_reg)])

c_plus_CRW_reg_RR = R_plus_CRW_reg_RR-CRW_reg_RR
c_minus_CRW_reg_RR = R_minus_CRW_reg_RR-CRW_reg_RR

sig_CRW_reg_RR = np.sqrt(np.sum(((np.abs(c_plus_CRW_reg_RR)+
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                np.abs(c_minus_CRW_reg_RR))/2)**2))
print(f'\nThe control rod worth of the REG with the REG as reference is
      {CRW_reg_RR} +/- {sig_CRW_reg_RR} $')

# Uncertainty on SHIM control rod worth
R_plus_CRW_shim_RR = np.array([ FCRW(CR_shim+sig_CR_shim, CR_reg, CR_allin,
    CRW_reg), FCRW(CR_shim, CR_reg+sig_CR_reg, CR_allin, CRW_reg),
    FCRW(CR_shim, CR_reg, CR_allin+sig_CR_allin, CRW_reg)])
R_minus_CRW_shim_RR = np.array([ FCRW(CR_shim-sig_CR_shim, CR_reg, CR_allin,
    CRW_reg), FCRW(CR_shim, CR_reg-sig_CR_reg, CR_allin, CRW_reg),
    FCRW(CR_shim, CR_reg, CR_allin-sig_CR_allin, CRW_reg)])

c_plus_CRW_shim_RR = R_plus_CRW_shim_RR-CRW_shim_RR
c_minus_CRW_shim_RR = R_minus_CRW_shim_RR-CRW_shim_RR

sig_CRW_shim_RR = np.sqrt(np.sum(((np.abs(c_plus_CRW_shim_RR)+
                  np.abs(c_minus_CRW_shim_RR))/2)**2))
print(f'\nThe control rod worth of the SHIM with the REG as reference is
      {CRW_shim_RR} +/- {sig_CRW_shim_RR} $')

# Uncertainty on TRANS control rod worth
R_plus_CRW_trans_RR = np.array([ FCRW(CR_trans+sig_CR_trans, CR_reg, CR_allin,
    CRW_reg), FCRW(CR_trans, CR_reg+sig_CR_reg, CR_allin, CRW_reg),
    FCRW(CR_trans, CR_reg, CR_allin+sig_CR_allin, CRW_reg)])
R_minus_CRW_trans_RR = np.array([ FCRW(CR_trans-sig_CR_trans, CR_reg, CR_allin,
    CRW_reg), FCRW(CR_trans, CR_reg-sig_CR_reg, CR_allin, CRW_reg),
    FCRW(CR_trans, CR_reg, CR_allin-sig_CR_allin, CRW_reg)])

c_plus_CRW_trans_RR = R_plus_CRW_trans_RR-CRW_trans_RR
c_minus_CRW_trans_RR = R_minus_CRW_trans_RR-CRW_trans_RR

sig_CRW_trans_RR = np.sqrt(np.sum(((np.abs(c_plus_CRW_trans_RR)+
                   np.abs(c_minus_CRW_trans_RR))/2)**2))
print(f'\nThe control rod worth of the TRANS with the REG as reference is
      {CRW_trans_RR} +/- {sig_CRW_trans_RR} $\n\n\n\n')




# REFERENCE SHIM (_RS)

print('SHIM AS REFERENCE')

# alpha phi_s value
alphaphis_RS = alphaphis(CR_shim, CR_allin, CRW_shim)

# alpha phi_s uncertainty
R_plus_alphaphis_RS = np.array([ alphaphis(CR_shim+sig_CR_shim, CR_allin,
    CRW_shim), alphaphis(CR_shim, CR_allin+sig_CR_allin, CRW_shim)])
R_minus_alphaphis_RS = np.array([ alphaphis(CR_shim-sig_CR_shim, CR_allin,
    CRW_shim), alphaphis(CR_shim, CR_allin-sig_CR_allin, CRW_shim)])

c_plus_alphaphis_RS = R_plus_alphaphis_RS-alphaphis_RS
c_minus_alphaphis_RS = R_minus_alphaphis_RS-alphaphis_RS


sig_alphaphis_RS = np.sqrt(np.sum(((np.abs(c_plus_alphaphis_RS)+
                   np.abs(c_minus_alphaphis_RS))/2)**2))
print(f'\nThe calibration factor with the SHIM as reference is
      {alphaphis_RS} +/- {sig_alphaphis_RS}')

# Control rod worth values
CRW_reg_RS = FCRW(CR_reg, CR_shim, CR_allin, CRW_shim)
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CRW_shim_RS = FCRW(CR_shim, CR_shim, CR_allin, CRW_shim)
CRW_trans_RS = FCRW(CR_trans, CR_shim, CR_allin, CRW_shim)

# Uncertainty on REG control rod worth
R_plus_CRW_reg_RS = np.array([ FCRW(CR_reg+sig_CR_reg, CR_shim, CR_allin,
    CRW_shim),  FCRW(CR_reg, CR_shim+sig_CR_shim, CR_allin, CRW_shim),
    FCRW(CR_reg, CR_shim, CR_allin+sig_CR_allin, CRW_shim)])
R_minus_CRW_reg_RS = np.array([ FCRW(CR_reg-sig_CR_reg, CR_shim, CR_allin,
    CRW_shim),  FCRW(CR_reg, CR_shim-sig_CR_shim, CR_allin, CRW_shim),
    FCRW(CR_reg, CR_shim, CR_allin-sig_CR_allin, CRW_shim)])

c_plus_CRW_reg_RS = R_plus_CRW_reg_RS-CRW_reg_RS
c_minus_CRW_reg_RS = R_minus_CRW_reg_RS-CRW_reg_RS

sig_CRW_reg_RS = np.sqrt(np.sum(((np.abs(c_plus_CRW_reg_RS)+
                 np.abs(c_minus_CRW_reg_RS))/2)**2))
print(f'\nThe control rod worth of the REG with the SHIM as reference is
      {CRW_reg_RS} +/- {sig_CRW_reg_RS} $')

# Uncertainty on SHIM control rod worth
R_plus_CRW_shim_RS = np.array([ FCRW(CR_shim+sig_CR_shim, CR_shim+sig_CR_shim,
    CR_allin, CRW_shim),  FCRW(CR_shim, CR_shim, CR_allin+sig_CR_allin,
    CRW_shim)])
R_minus_CRW_shim_RS = np.array([ FCRW(CR_shim-sig_CR_shim, CR_shim-sig_CR_shim,
    CR_allin, CRW_shim),  FCRW(CR_shim, CR_shim, CR_allin-sig_CR_allin,
    CRW_shim)])

c_plus_CRW_shim_RS = R_plus_CRW_shim_RS-CRW_shim_RS
c_minus_CRW_shim_RS = R_minus_CRW_shim_RS-CRW_shim_RS

sig_CRW_shim_RS = np.sqrt(np.sum(((np.abs(c_plus_CRW_shim_RS)+
                  np.abs(c_minus_CRW_shim_RS))/2)**2))
print(f'\nThe control rod worth of the SHIM with the SHIM as reference is
      {CRW_shim_RS} +/- {sig_CRW_shim_RS} $')

# Uncertainty on TRANS control rod worth
R_plus_CRW_trans_RS = np.array([ FCRW(CR_trans+sig_CR_trans, CR_shim, CR_allin,
    CRW_shim),  FCRW(CR_trans, CR_shim+sig_CR_shim, CR_allin, CRW_shim),
    FCRW(CR_trans, CR_shim, CR_allin+sig_CR_allin, CRW_shim)])
R_minus_CRW_trans_RS = np.array([ FCRW(CR_trans-sig_CR_trans, CR_shim,
    CR_allin, CRW_shim),  FCRW(CR_trans, CR_shim-sig_CR_shim, CR_allin,
    CRW_shim),  FCRW(CR_trans, CR_shim, CR_allin-sig_CR_allin, CRW_shim)])

c_plus_CRW_trans_RS = R_plus_CRW_trans_RS-CRW_trans_RS
c_minus_CRW_trans_RS = R_minus_CRW_trans_RS-CRW_trans_RS

sig_CRW_trans_RS = np.sqrt(np.sum(((np.abs(c_plus_CRW_trans_RS)+
                   np.abs(c_minus_CRW_trans_RS))/2)**2))
print(f'\nThe control rod worth of the TRANS with the SHIM as reference is
      {CRW_trans_RS} +/- {sig_CRW_trans_RS} $\n\n\n\n')




# REFERENCE SHIM (_RT)

print('TRANS AS REFERENCE')

# alpha phi_s value
alphaphis_RT = alphaphis(CR_trans, CR_allin, CRW_trans)

# alpha phi_s uncertainty
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R_plus_alphaphis_RT = np.array([ alphaphis(CR_trans+sig_CR_trans, CR_allin, CRW_trans), alphaphis(CR_trans, CR_allin+sig_CR_allin, CRW_trans)])
R_minus_alphaphis_RT = np.array([ alphaphis(CR_trans-sig_CR_trans, CR_allin, CRW_trans), alphaphis(CR_trans, CR_allin-sig_CR_allin, CRW_trans)])

c_plus_alphaphis_RT = R_plus_alphaphis_RT-alphaphis_RT
c_minus_alphaphis_RT = R_minus_alphaphis_RT-alphaphis_RT


sig_alphaphis_RT = np.sqrt(np.sum(((np.abs(c_plus_alphaphis_RT)+
                   np.abs(c_minus_alphaphis_RT))/2)**2))
print(f'\nThe calibration factor with the TRANS as reference is
      {alphaphis_RT} +/- {sig_alphaphis_RT}')

# Control rod worth values
CRW_reg_RT = FCRW(CR_reg, CR_trans, CR_allin, CRW_trans)
CRW_shim_RT = FCRW(CR_shim, CR_trans, CR_allin, CRW_trans)
CRW_trans_RT = FCRW(CR_trans, CR_trans, CR_allin, CRW_trans)

# Uncertainty on REG control rod worth
R_plus_CRW_reg_RT = np.array([ FCRW(CR_reg+sig_CR_reg, CR_trans, CR_allin, CRW_trans), FCRW(CR_reg, CR_trans+sig_CR_trans, CR_allin, CRW_trans), FCRW(CR_reg, CR_trans, CR_allin+sig_CR_allin, CRW_trans)])
R_minus_CRW_reg_RT = np.array([ FCRW(CR_reg-sig_CR_reg, CR_trans, CR_allin, CRW_trans), FCRW(CR_reg, CR_trans-sig_CR_trans, CR_allin, CRW_trans), FCRW(CR_reg, CR_trans, CR_allin-sig_CR_allin, CRW_trans)])

c_plus_CRW_reg_RT = R_plus_CRW_reg_RT-CRW_reg_RT
c_minus_CRW_reg_RT = R_minus_CRW_reg_RT-CRW_reg_RT

sig_CRW_reg_RT = np.sqrt(np.sum(((np.abs(c_plus_CRW_reg_RT)+
                 np.abs(c_minus_CRW_reg_RT))/2)**2))
print(f'\nThe control rod worth of the REG with the TRANS as reference is
      {CRW_reg_RT} +/- {sig_CRW_reg_RT} $')

# Uncertainty on SHIM control rod worth
R_plus_CRW_shim_RT = np.array([ FCRW(CR_shim+sig_CR_shim, CR_trans, CR_allin, CRW_trans), FCRW(CR_shim, CR_trans+sig_CR_trans, CR_allin, CRW_trans), FCRW(CR_shim, CR_trans, CR_allin+sig_CR_allin, CRW_trans)])
R_minus_CRW_shim_RT = np.array([ FCRW(CR_shim-sig_CR_shim, CR_trans, CR_allin, CRW_trans), FCRW(CR_shim, CR_trans-sig_CR_trans, CR_allin, CRW_trans), FCRW(CR_shim, CR_trans, CR_allin-sig_CR_allin, CRW_trans)])

c_plus_CRW_shim_RT = R_plus_CRW_shim_RT-CRW_shim_RT
c_minus_CRW_shim_RT = R_minus_CRW_shim_RT-CRW_shim_RT

sig_CRW_shim_RT = np.sqrt(np.sum(((np.abs(c_plus_CRW_shim_RT)+
                  np.abs(c_minus_CRW_shim_RT))/2)**2))
print(f'\nThe control rod worth of the SHIM with the TRANS as reference is
      {CRW_shim_RT} +/- {sig_CRW_shim_RT} $')

# Uncertainty on TRANS control rod worth
R_plus_CRW_trans_RT = np.array([ FCRW(CR_trans+sig_CR_trans, CR_trans+sig_CR_trans, CR_allin, CRW_trans), FCRW(CR_trans, CR_trans, CR_allin+sig_CR_allin, CRW_trans)])
R_minus_CRW_trans_RT = np.array([ FCRW(CR_trans-sig_CR_trans, CR_trans-sig_CR_trans, CR_allin, CRW_trans), FCRW(CR_trans, CR_trans, CR_allin-sig_CR_allin, CRW_trans)])
c_plus_CRW_trans_RT = R_plus_CRW_trans_RT-CRW_trans_RT
c_minus_CRW_trans_RT = R_minus_CRW_trans_RT-CRW_trans_RT

sig_CRW_trans_RT = np.sqrt(np.sum(((np.abs(c_plus_CRW_trans_RT)+
                   np.abs(c_minus_CRW_trans_RT))/2)**2))
```

```
print(f'\nThe control rod worth of the TRANS with the TRANS as reference is
      {CRW_trans_RT} +/- {sig_CRW_trans_RT} $\n\n\n\n')
```

Listing 1: Script Python