ECE 2001
Fall 2019
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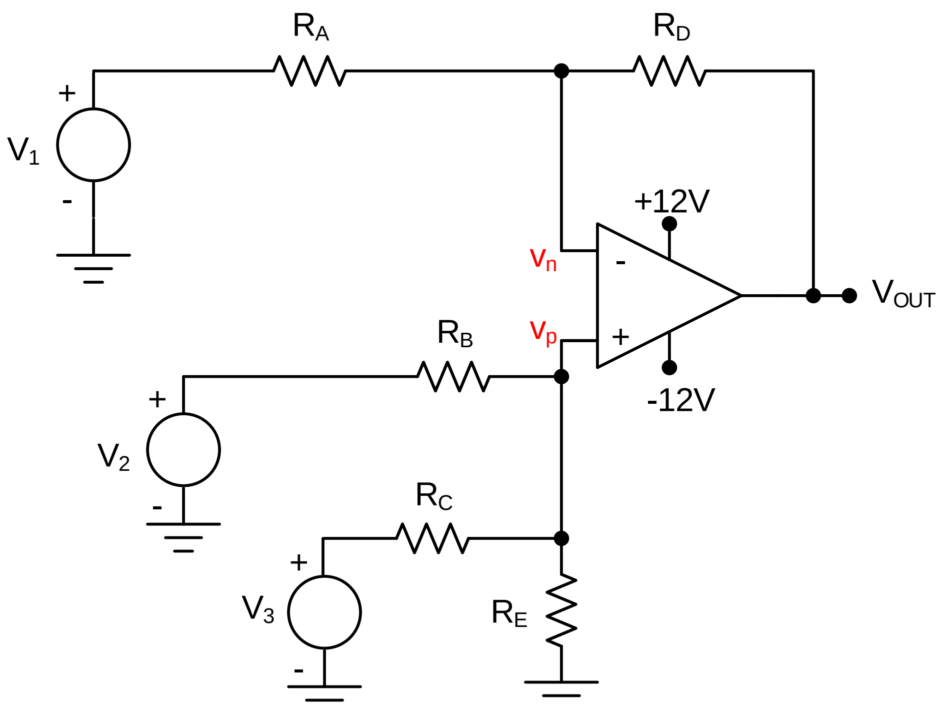
Homework Assignment 4 SOLUTION
Due: Friday, Oct. 4, 2019

1. (20) Consider the op amp circuit shown below. Assume the op amp is ideal.



a. Assuming linear operation, find an expression for V_{OUT} in terms of V_1, V_2, and V_3.

Solution. The voltage v_p at the non-inverting input may be found using the node voltage method.

$$\frac{v_p - V_2}{R_B} + \frac{v_p - V_3}{R_C} + \frac{v_p}{R_E} = 0$$

$$v_p = \frac{\frac{V_2}{R_B} + \frac{V_3}{R_C}}{\frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_E}}$$

Using KCL at the inverting input of the op amp we obtain

$$\frac{v_n - V_1}{R_A} + \frac{v_n - V_{OUT}}{R_D} = 0$$

Using the virtual short concept, $v_p = v_n$ **, we can combine these equations to obtain**

$$V_{OUT} = -\frac{R_D}{R_A}V_1 + \left(1 + \frac{R_D}{R_A}\right)\frac{\frac{V_2}{R_B} + \frac{V_3}{R_C}}{\frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_E}}$$

b. Using this result, choose resistors so that $V_{OUT} = -5V_1 + 2.4V_2 + 1.2V_3$. Use standard resistors with 5% tolerances. The nominal coefficients (before taking into account tolerances) should all be within 10% of the desired coefficients.

Solution. The condition to achieve the desired coefficient for V_1 is

$$\frac{R_D}{R_A} = 5.$$

We can choose $\boldsymbol{R_D = 10k\Omega}$ and $\boldsymbol{R_A = 2k\Omega}$. As a consequence

$$\left(1 + \frac{R_D}{R_A}\right) = 6.$$

Therefore to achieve the desired coefficients for V_2 and V_3

$$\frac{\frac{1}{R_B}}{\frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_E}} = \frac{2.4}{6} = 0.4$$

$$\frac{\frac{1}{R_C}}{\frac{1}{R_B} + \frac{1}{R_C} + \frac{1}{R_E}} = \frac{1.2}{6} = 0.2$$

Therefore

$$\frac{R_C}{R_B} = \frac{0.4}{0.2} = 2$$

We can choose $\boldsymbol{R_C = 20k\Omega}$ and $\boldsymbol{R_B = 10k\Omega}$. With this choice

$$\frac{\frac{1}{20k\Omega}}{\frac{1}{10k\Omega} + \frac{1}{20k\Omega} + \frac{1}{R_E}} = 0.2$$

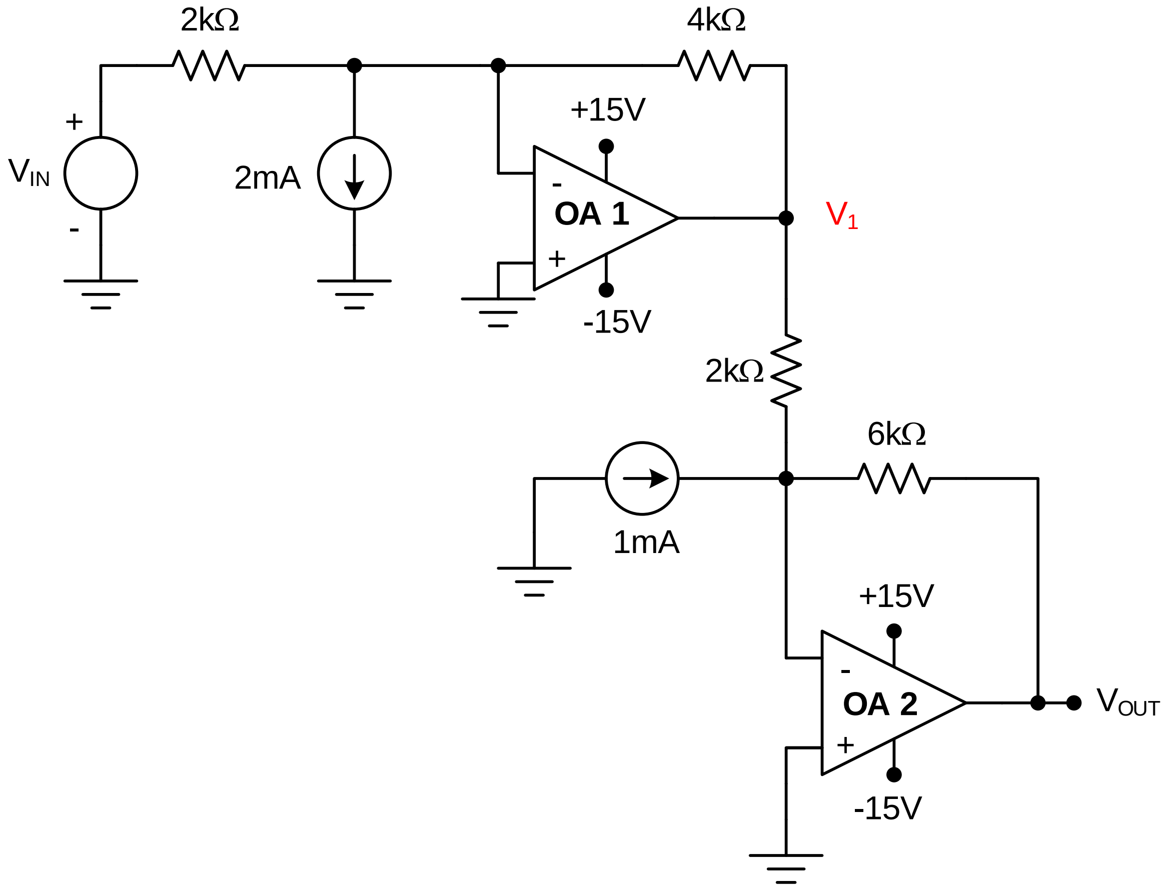
$$R_E = \left(\frac{1}{(0.2)20k\Omega} - \frac{1}{10k\Omega} - \frac{1}{20k\Omega}\right)^{-1} = 10k\Omega$$

These choices are not unique, but the ratios must be maintained as follows:

$$R_D : R_A = 5 : 1$$

$$R_C : R_B : R_E = 2 : 1 : 1$$

2. (20) Consider the system of cascaded op amps shown below. Assume that the op amps are ideal.



a. Find the range of V_{IN} for linear operation of OA1.

Solution. For linear operation of OA1 the virtual short applies.

$$\frac{0 - V_{IN}}{2k\Omega} + \frac{0 - V_1}{4k\Omega} + 2mA = 0$$

$$V_1 = 8V - 2V_{IN}$$

For positive saturation,

$$+15V = 8V - 2V_{IN}$$

and

$$V_{IN} = -3.5V.$$

For negative saturation,

$-15V = 8V - 2V_{IN}$

and

$V_{IN} = +11.5V$.

For linear operation of OA1,

$-3.5V < V_{IN} < 11.5V$.

b. Find the range of V_{IN} for linear operation of OA2.

Solution. For linear operation of OA2 the virtual short applies.

$$\frac{0 - V_1}{2k\Omega} + \frac{0 - V_{OUT}}{6k\Omega} - 1mA = 0$$

$V_{OUT} = -3V_1 - 6V = -30V + 6V_{IN}$

For positive saturation,

$+15V = -30V + 6V_{IN}$

and

$V_{IN} = 7.5V$.

For negative saturation,

$-15V = -30V + 6V_{IN}$

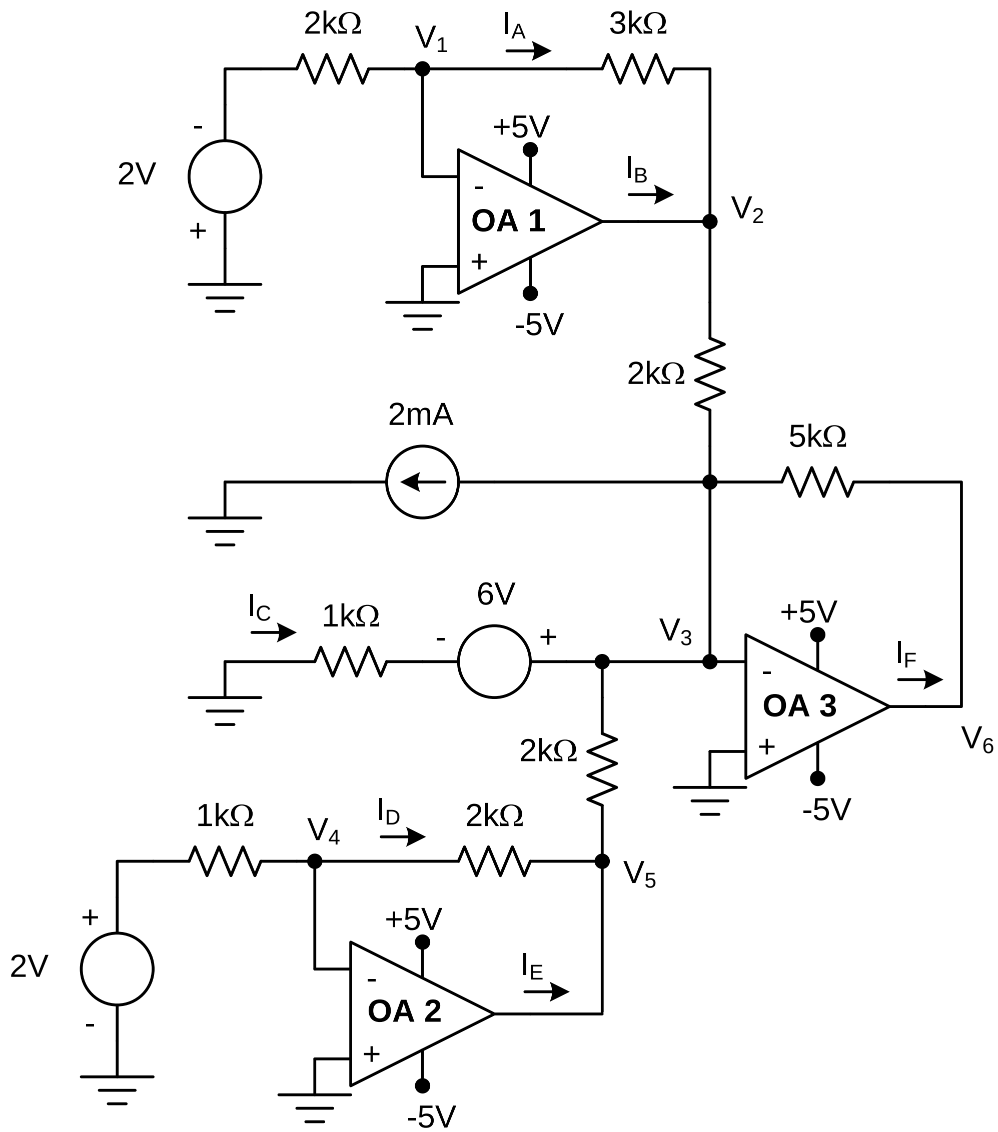
and

$V_{IN} = 2.5V$.

For linear operation of OA2,

$2.5V < V_{IN} < 7.5V$.

3. (20) Find I_A, I_B, I_C, I_D, I_E, I_F, V_1, V_2, V_3, V_4, V_5, and V_6. Assume that the op amps are ideal.



Solution. We will start by assuming linear operation of OA1.

$$\frac{0+2V}{2k\Omega}+\frac{0-V_2}{3k\Omega}=0$$

$$V_2 = 3V$$

Therefore OA1 is linear and

$V_1 = 0 \qquad I_A = \frac{-2V}{2k\Omega} = -1mA$

We will next assume that OA2 is linear.

$$\frac{0-2V}{1k\Omega} + \frac{0-V_5}{2k\Omega} = 0$$

$V_2 = -4V$

Therefore OA2 is linear and

$V_4 = 0 \qquad I_D = \frac{2V}{1k\Omega} = 2mA$

We will next assume that OA3 is linear.

$$\frac{0-3V}{2k\Omega} + 2mA + \frac{0-6V}{1k\Omega} + \frac{0-(-4V)}{2k\Omega} + \frac{0-V_6}{5k\Omega} = 0$$

$V_6 = -17.5V$

Therefore OA3 is saturated.

$V_6 = -5V$

$$\frac{V_3-3V}{2k\Omega} + 2mA + \frac{V_3-6V}{1k\Omega} + \frac{V_3-(-4V)}{2k\Omega} + \frac{V_3-(-5V)}{5k\Omega} = 0$$

$$5V_3 - 15V + 20V + 10V_3 - 60V + 5V_3 + 20V + 2V_3 + 10V = 0$$

$V_3 = 1.136V$

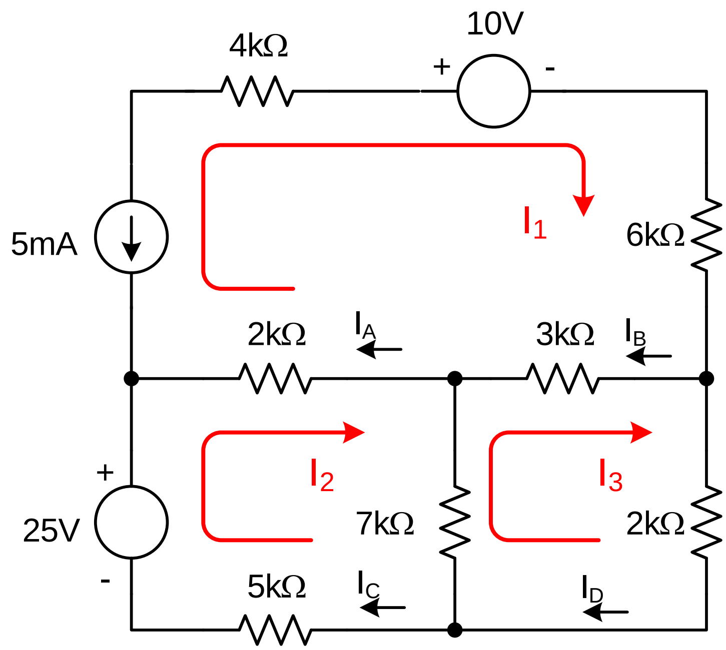
$I_C = \frac{6V-1.136V}{1k\Omega} = 4.864mA$

$$I_F = \frac{-5V-1.136V}{5k\Omega} = -1.227mA$$

$$I_B = \frac{3V-0}{3k\Omega} + \frac{3V-1.136V}{2k\Omega} = 1.932mA$$

$$I_E = \frac{-4V-1.136V}{2k\Omega} + \frac{-4V-0}{2k\Omega} = -4.568mA$$

4. (20) Determine I_A, I_B, I_C, I_D, and the power for the 25V source.



Solution. MCM using V, kΩ, and mA:

1. $I_1 = -5mA$ **(known mesh current)**

2. $-25 + 2(I_2 + 5) + 7(I_2 - I_3) + 5I_2 = 0$

3. $7(I_3 - I_2) + 3(I_3 + 5) + 2I_3 = 0$

$\begin{bmatrix} 14 & -7 \\ -7 & 12 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 15 \\ -15 \end{bmatrix}$ $\quad I_2 = 0.630\ mA \quad I_3 = -0.882\ mA$

$I_A = I_1 - I_2 = -5.63\ mA$

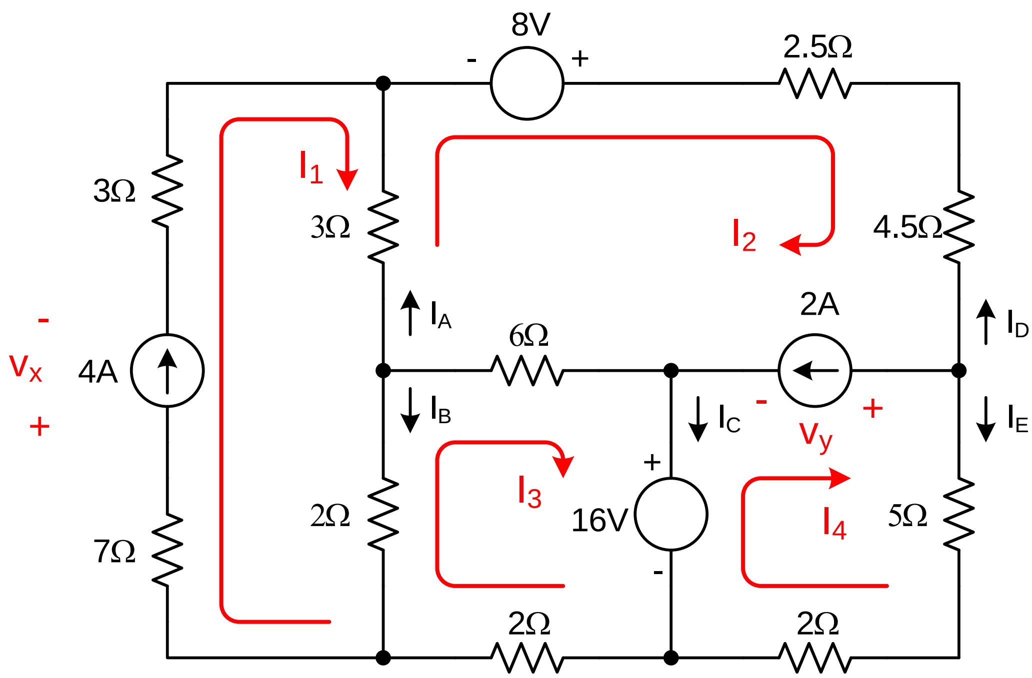
$I_B = I_1 - I_3 = -4.12\ mA$

$I_C = I_2 = 0.630\ mA$

$I_D = I_3 = -0.88\ mA$

$P_{25V} = -(25V)I_2 = -15.75mW$

5. (20) For the circuit shown, find I_A, I_B, I_C, I_D, I_E, and the power for each of the sources.



Solution. MCM using V, Ω, and A:

1. $I_1 = 4A$ **(known mesh current)**

3. $2(I_3 - 4) + 6(I_3 - I_2) + 16 + 2I_3 = 0$

24. $-8 + 7I_2 + 7I_4 - 16 + 6(I_2 - I_3) + 3(I_2 - 4) = 0$ **(supermesh)**

CS. $I_2 - I_4 = 2$

24CS. $-8 + 7I_2 + 7(I_2 - 2) - 16 + 6(I_2 - I_3) + 3(I_2 - 4) = 0$

$$\begin{bmatrix} 23 & -6 \\ -6 & 10 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 50 \\ -8 \end{bmatrix}$$

$I_2 = 2.33A$ $I_3 = 0.60A$ $I_4 = 0.33A$

$I_A = I_2 - I_1 = -1.67\ A$

$I_B = I_1 - I_3 = 3.4\ A$

$I_C = I_3 - I_4 = 0.27\ A$

$I_D = -I_2 = -2.33\ A$

$I_E = I_4 = 0.33A$

$v_x + 15I_1 - 3I_2 - 2I_3 = 0$

$v_x = -51.8\,V$

$P_{4A} = (-51.8)(4A) = -207\,W$

$-v_y + 7I_4 - 16 = 0$

$v_y = -13.69V$

$P_{2A} = (2A)v_y = -27.4\,W$

$P_{8V} = -(8V)(I_2) = -18.64\,W$

$P_{16V} = (16V)(I_3 - I_4) = 4.32W$