ECE 2001
Fall 2019
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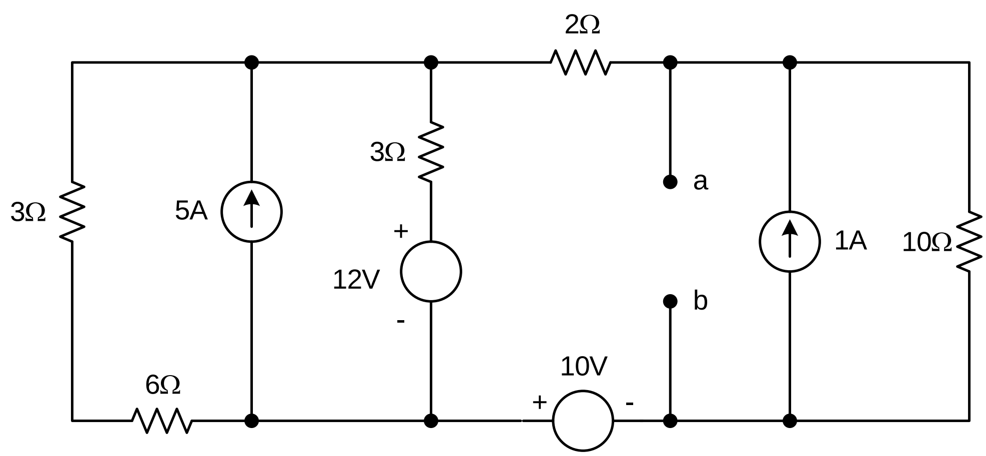
Homework Assignment 5 Solution
Due: Oct. 11, 2019

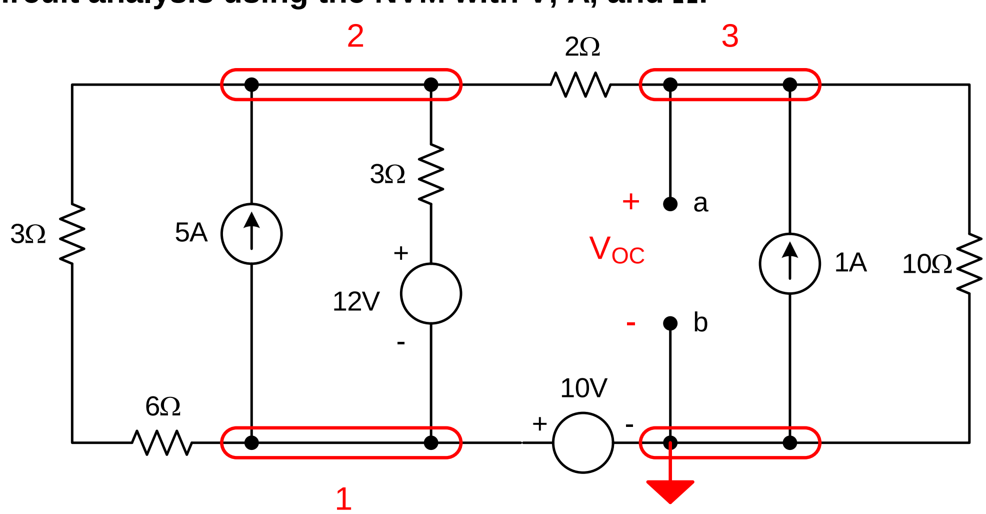
NOTE ON SIGNS: The open-circuit voltage V_{OC} is the open-circuit value of V_{ab}. The short-circuit current is the current which flows from a to b in a wire connected between these two terminals. Please use these conventions throughout this assignment.

1. (20) Find the Thevenin equivalent with respect to the terminals a and b in the circuit shown.



Solution.

Open-circuit analysis using the NVM with V, A, and Ω:



1. $V_1 = 10V$ **(known node voltage)**

2. $\frac{V_2-10}{9} - 5 + \frac{V_2-12-10}{3} + \frac{V_2-V_3}{2} = 0$

3. $\frac{V_3-V_2}{2} - 1 + \frac{V_3}{10} = 0$

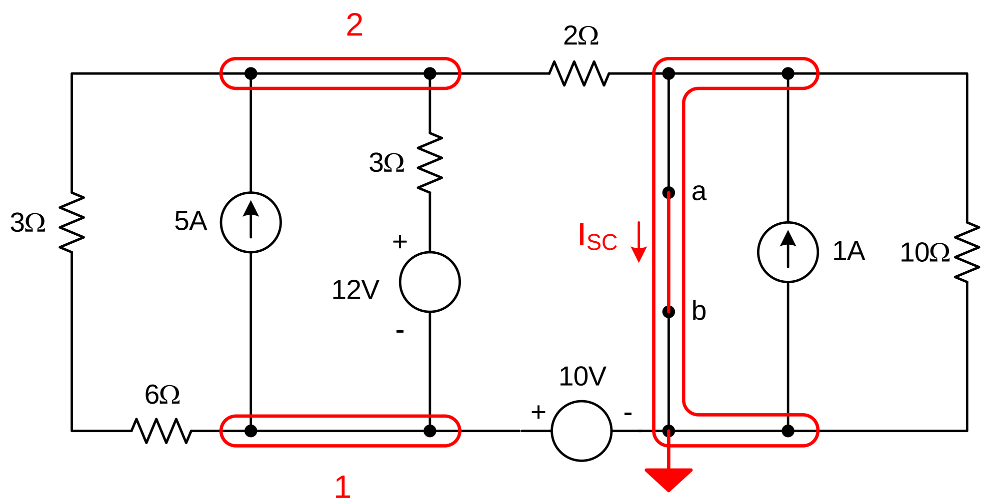
2. $2V_2 - 20 - 90 + 6V_2 - 132 + 9V_2 - 9V_3 = 0$

3. $5V_3 - 5V_2 - 10 + V_3 = 0$

$\begin{bmatrix} 17 & -9 \\ -5 & 6 \end{bmatrix} \begin{bmatrix} V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 242 \\ 10 \end{bmatrix}$ $\quad V_2 = 27.05\ V \quad V_3 = 24.21\ V$

$V_{OC} = 24.21\ V = V_{Th}$

Short-circuit analysis using the NVM with V, A, and Ω:



1. $V_1 = 10V$ **(known node voltage)**

2. $\frac{V_2-10}{9} - 5 + \frac{V_2-12-10}{3} + \frac{V_2}{2} = 0$

2. $2V_2 - 20 - 90 + 6V_2 - 132 + 9V_2 = 0$

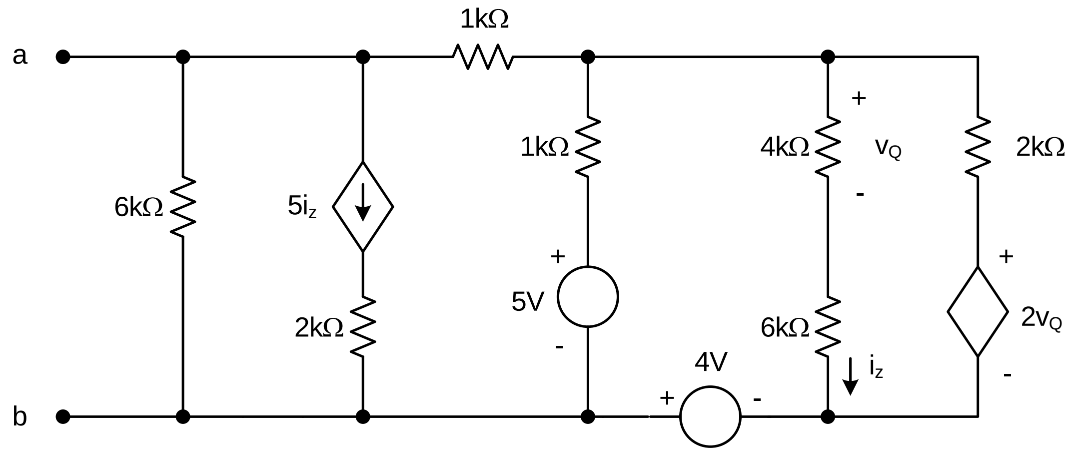
2. $17V_2 - 242 = 0$

$V_2 = 14.24\ V$

$$I_{SC} = \frac{V_2}{2\Omega} + 1A = 8.12\ A$$

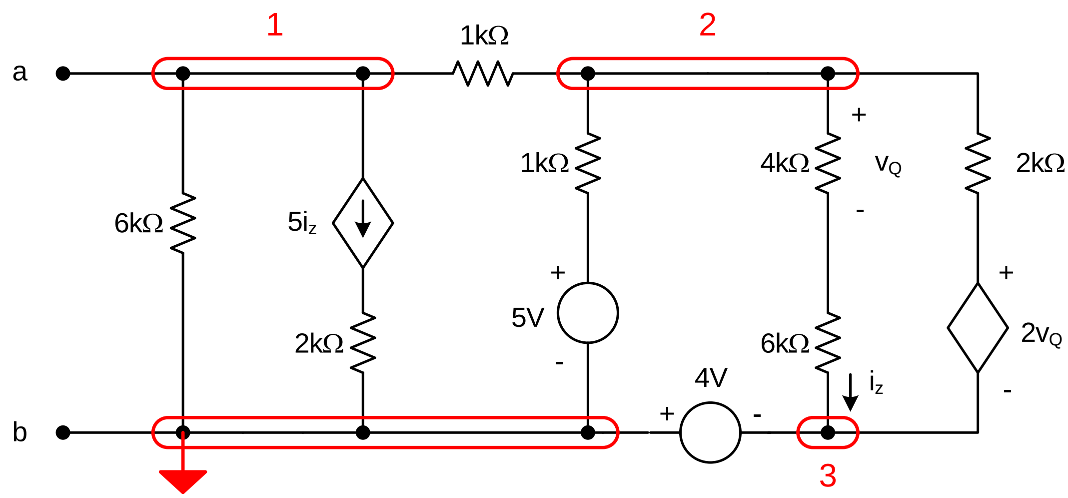
$$R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{24.21V}{8.12A} = 2.98\Omega$$

2. (20) Determine the open-circuit voltage, the short-circuit current, the Thevenin equivalent, and the Norton equivalent with respect to the terminals a and b in the circuit shown.



Solution.

Open-circuit analysis using the NVM with V, mA, and kΩ:



3. $V_3 = -4V$ (known node voltage)

1. $\frac{V_1}{6} + 5i_z + \frac{V_1 - V_2}{1} = 0$

2. $\frac{V_2 - V_1}{1} + \frac{V_2 - 5}{1} + \frac{V_2 + 4}{10} + \frac{V_2 - 2v_Q + 4}{2} = 0$

CS. $i_z = \frac{V_2 + 4}{10}$

VS. $v_Q = \left(\frac{V_2 + 4}{10}\right) 4 = \frac{4V_2 + 16}{10}$

1CS. $\frac{V_1}{6} + \frac{V_2 + 4}{2} + \frac{V_1 - V_2}{1} = 0$

2VS. $\frac{V_2 - V_1}{1} + \frac{V_2 - 5}{1} + \frac{V_2 + 4}{10} + \frac{V_2 + 4}{2} - \frac{4V_2 + 16}{10} = 0$

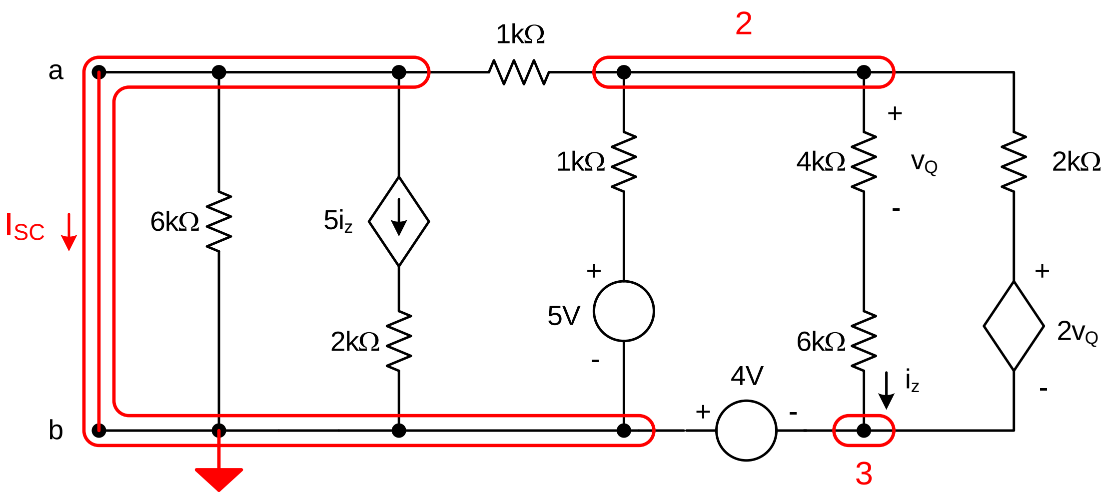
1CS. $V_1 + 3V_2 + 12 + 6V_1 - 6V_2 = 0$

2VS. $10V_2 - 10V_1 + 10V_2 - 50 + V_2 + 4 + 5V_2 + 20 - 4V_2 - 16 = 0$

$\begin{bmatrix} 7 & -3 \\ -10 & 22 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -12 \\ 42 \end{bmatrix}$ $\qquad$ $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} -1.113V \\ 1.403V \end{bmatrix}$

$V_{OC} = V_1 = -1.113V = V_{Th}$

Short-circuit analysis using the NVM with V, mA, and kΩ:



3. $V_3 = -4V$ (known node voltage)

2. $\frac{V_2}{1} + \frac{V_2-5}{1} + \frac{V_2+4}{10} + \frac{V_2-2v_Q+4}{2} = 0$

CS. $i_z = \frac{V_2+4}{10}$

VS. $v_Q = \left(\frac{V_2+4}{10}\right)4 = \frac{4V_2+16}{10}$

2VS. $\frac{V_2}{1} + \frac{V_2-5}{1} + \frac{V_2+4}{10} + \frac{V_2+4}{2} - \frac{4V_2+16}{10} = 0$

2VS. $10V_2 + 10V_2 - 50 + V_2 + 4 + 5V_2 + 20 - 4V_2 - 16 = 0$

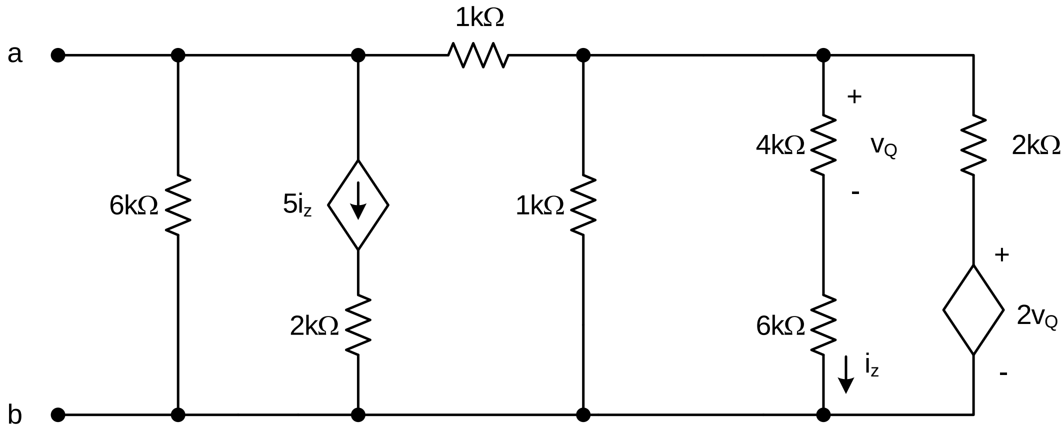
2VS. $22V_2 = 42$

$V_2 = 1.909V$

$I_{SC} = \frac{V_2}{1k\Omega} - 5i_z = \frac{V_2}{1k\Omega} - \frac{V_2+4V}{2k\Omega} = -1.046mA$

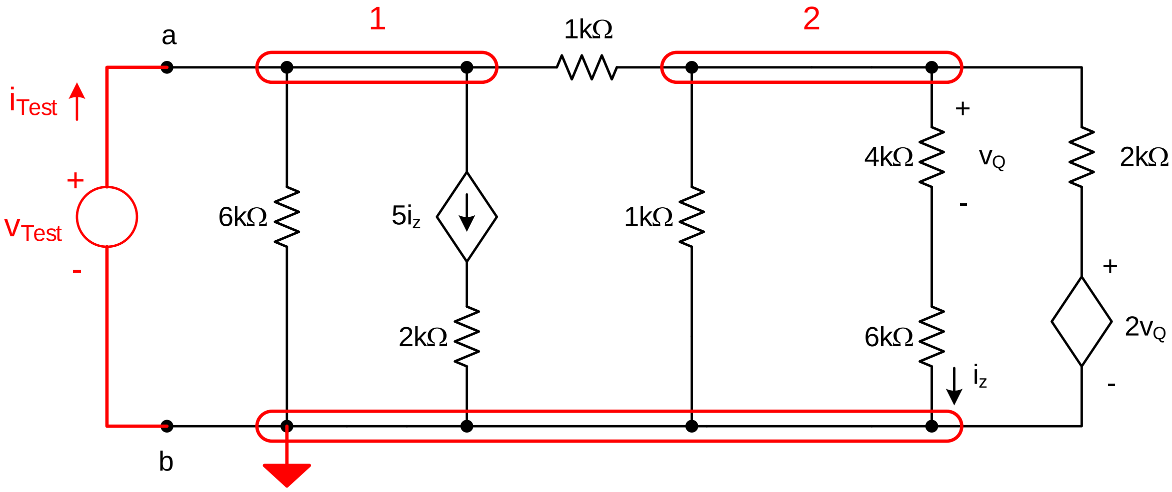
$R_{Th} = \frac{V_{OC}}{I_{SC}} = \frac{-1.113V}{-1.046mA} = 1.064k\Omega$

3. (20) Determine the Thevenin resistance for the network shown below.



Solution.

Test-source analysis using the NVM with V, mA, and kΩ:



1. $V_1 = V_{Test}$ (known node voltage)

2. $\frac{V_2 - V_{Test}}{1} + \frac{V_2}{1} + \frac{V_2}{10} + \frac{V_2 - 2v_Q}{2} = 0$

CS. $i_z = \frac{V_2}{10}$

VS. $v_Q = \frac{4V_2}{10}$

2VS. $\frac{V_2 - V_{Test}}{1} + \frac{V_2}{1} + \frac{V_2}{10} + \frac{V_2}{2} - \frac{4V_2}{10} = 0$

2VS. $10V_2 - 10V_{Test} + 10V_2 + V_2 + 5V_2 - 4V_2 = 0$

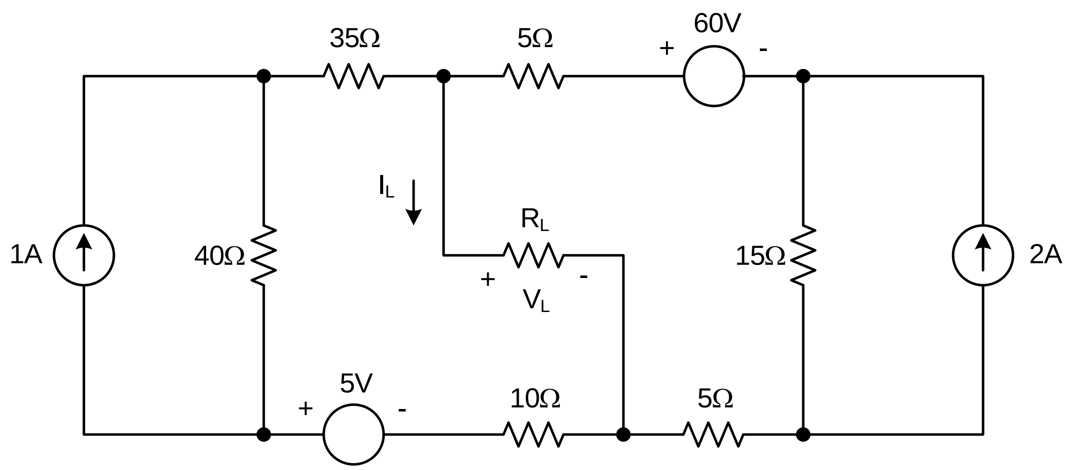
2VS. $22V_2 = 10V_{Test}$

$V_2 = \left(\frac{10}{22}\right) V_{Test}$

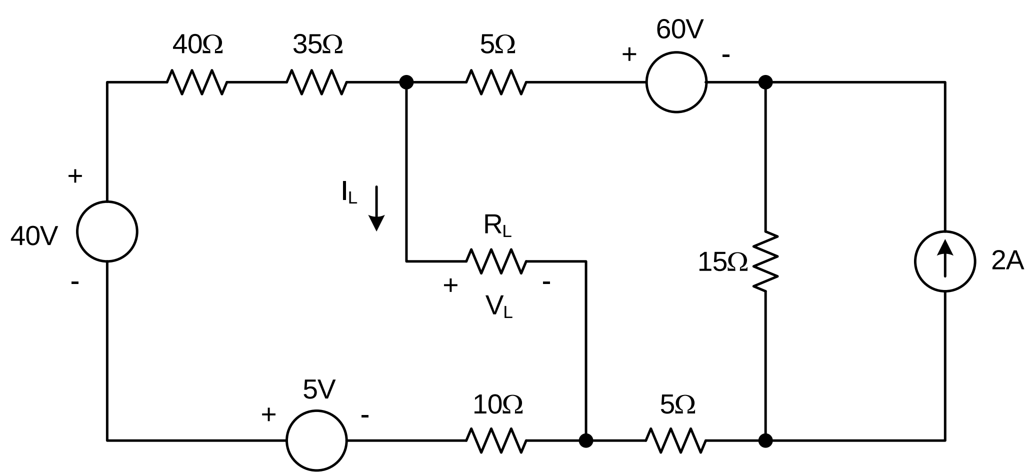
$$I_{Test} = \frac{V_{Test}}{6k\Omega} + 5i_z + \frac{V_{Test} - V_2}{1k\Omega} = \frac{V_{Test}}{6k\Omega} + 5\left(\frac{10V_{Test}/22}{10k\Omega}\right) + \frac{V_{Test} - 10V_{Test}/22}{1k\Omega}$$

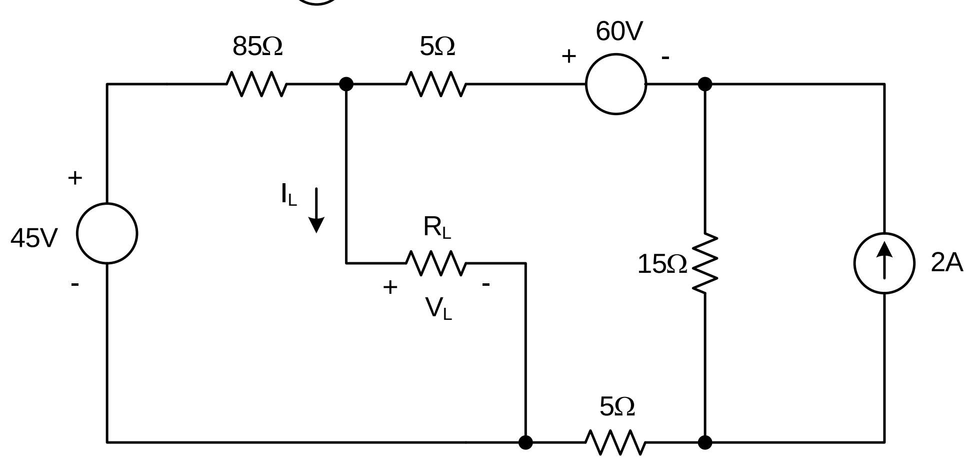
$$R_{Th} = \frac{V_{Test}}{I_{Test}} = \left[\frac{1}{6k\Omega} + \frac{50/22}{10k\Omega} + \frac{1}{1k\Omega} - \frac{10/22}{1k\Omega}\right]^{-1} = 1.0645k\Omega$$

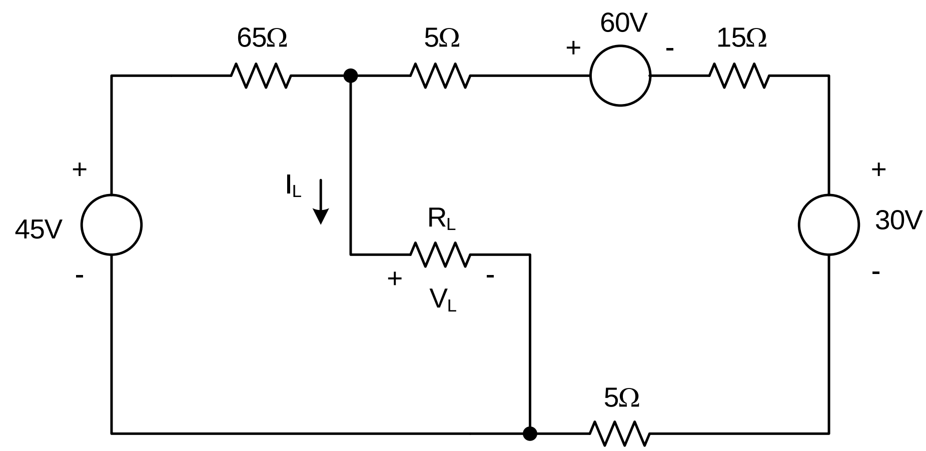
4. (20) For the network shown below, plot I_L as a function of R_L, plot V_L as a function of R_L, and plot P_L as a function of R_L. Use a range of RL from $R_{Th}/4$ to $4R_{Th}$.

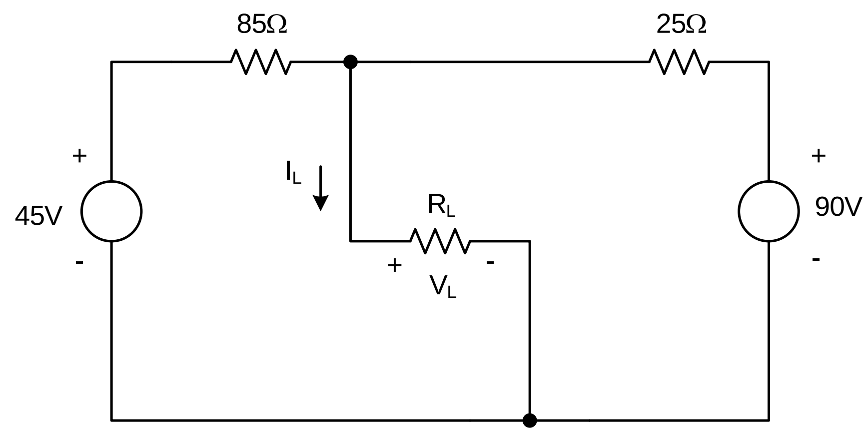


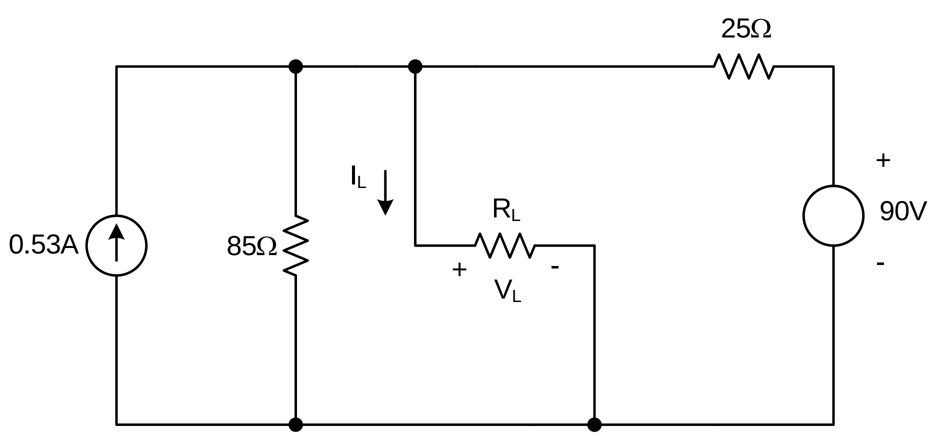
Solution. Source transformations may be used in conjunction with simplifications.

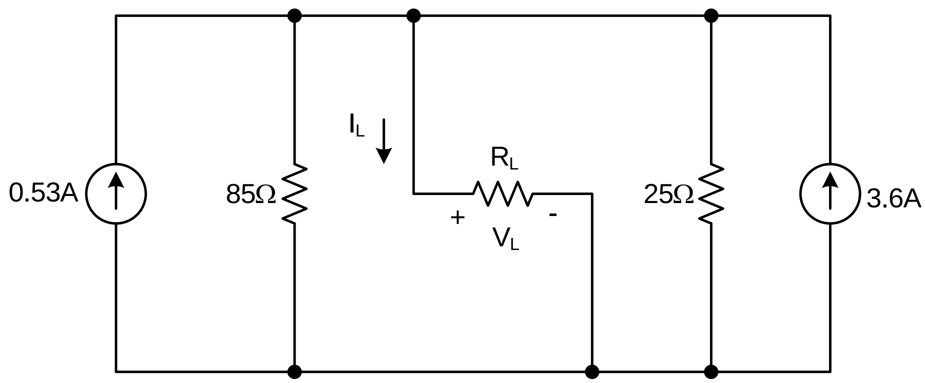


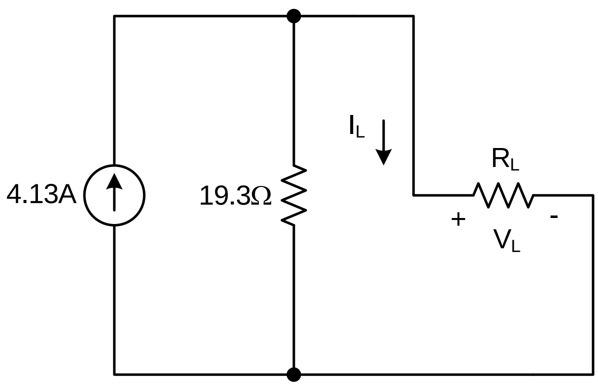


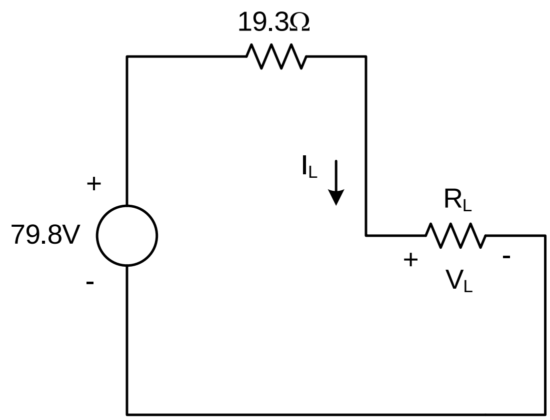

65Ω
5Ω
60V
+
-
15Ω
+
45V
-
I_L
R_L
+
V_L
-
+
30V
-
5Ω


85Ω
25Ω
+
45V
-
I_L
R_L
+
V_L
-
+
90V
-


25Ω
0.53A
85Ω
I_L
R_L
+
V_L
-
+
90V
-



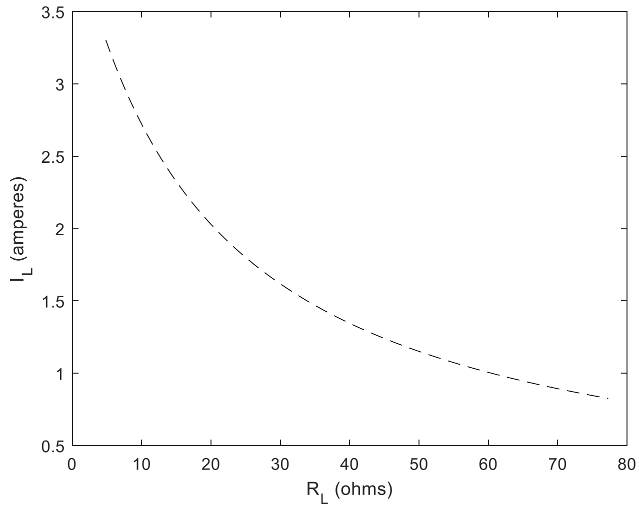




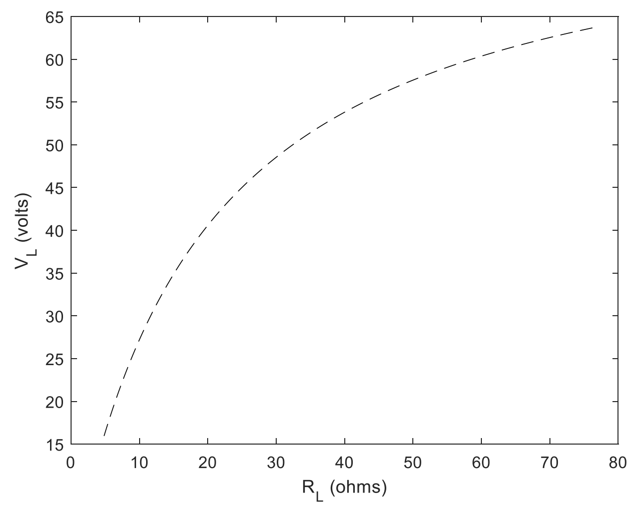
$$I_L = \frac{79.8V}{19.3\Omega + R_L}$$

$$V_L = \left(\frac{79.8V}{19.3\Omega + R_L}\right) R_L$$

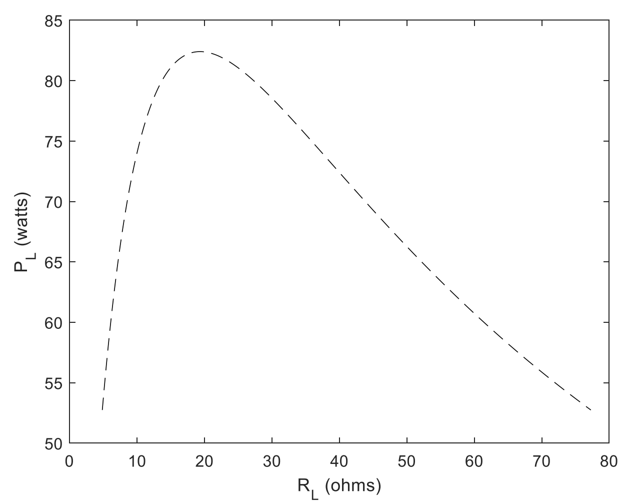
$$P_L = I_L V_L = \left(\frac{79.8V}{19.3\Omega + R_L}\right)^2 R_L$$



I_L (amperes)
R_L (ohms)

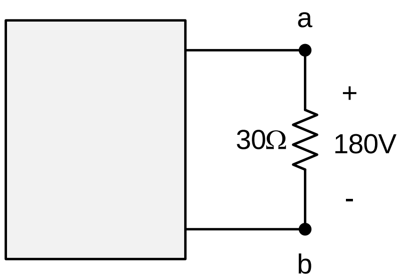


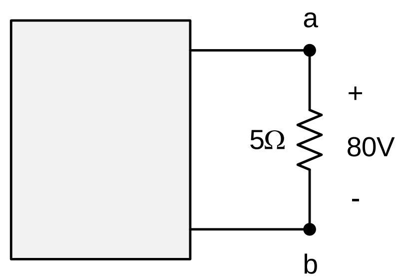
V_L (volts)
R_L (ohms)



P_L (watts)
R_L (ohms)

5. (20) The figures below represent two measurements made using two different load resistances connected to <u>the same two-terminal network</u> contained within an enclosure. The network in the enclosure is a linear network involving resistances, independent sources, and dependent sources.
(a) Find the open-circuit voltage for the network contained in the box.
(b) Find the value of the resistance R_L which will dissipate maximum power.





Solution.

Test 1. $180V = V_{Th}\left(\frac{30\Omega}{R_{Th}+30\Omega}\right)$

Test 2. $80V = V_{Th}\left(\frac{5\Omega}{R_{Th}+5\Omega}\right)$

Test 1. $(180V)R_{Th} + (180V)(30\Omega) = V_{Th}(30\Omega)$

Test 2. $(80V)R_{Th} + (80V)(5\Omega) = V_{Th}(5\Omega)$

Test 2. $(180V)R_{Th} + (900V\Omega) = V_{Th}(11.25\Omega)$

Subtracting Test 2 from Test 1: $(4500V\Omega) = V_{Th}(18.75\Omega)$

$V_{Th} = 240V$

Test 1. $(180V)R_{Th} + (180V)(30\Omega) = 240V(30\Omega)$

$R_{Th} = 10V$

a. **The open-circuit voltage is** $V_{OC} = 240V$.
b. **The load resistance which will dissipate maximum power is** $R_L = 10\Omega$.