Highlights

The Beggining of Control Revolution: Ofset-free Koopman MPC

Patrik Valábek

- Research highlight 1
- Research highlight 2

The Beggining of Control Revolution: Ofset-free Koopman MPC

Patrik Valábek

[a] *STU, Redlinskeho 9, Bratislava, Slovakia*

Abstract

Keywords:

1. Introduction

2. Preliminaries and Notation

2.1. Parsim-K Identification

2.2. Offset-Free MPC

2.3. Nonlinear MPC

2.4. Koopman Operator

2.5. Koopman MPC

2.6. State Estimation - Observer

3. Koopman MPC with Offset-Free - Easyiest

In this section, we will present a standard offset free optimal framework consisting of Target Optimization, State Estimation - Observer and MPC. As observer is standard Kalman filter or extended Kalman filter, we will discuss mainly remaining two optimization problems. Both remaining components had to be designed in a way that they are able to work with the Koopman operator. The main goal of this section is to show that the standard formulation can be easily transformed for the offset-free MPC use.

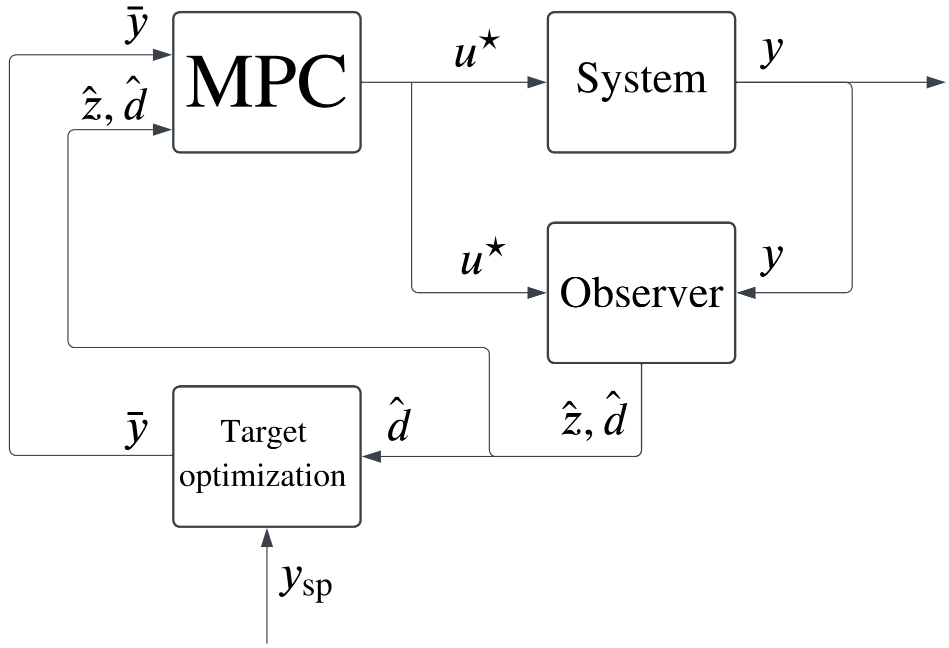

Figure 1: Closed-loop control structure.

3.1. Target Optimization

$$
\begin{aligned}
\min_{\bar{u},\bar{z},\bar{y}} \quad & (\bar{y} - y_{\mathrm{ref}})^{\intercal} Q_{\mathrm{y}} (\bar{y} - y_{\mathrm{ref}}) && \text{(1a)} \\
\text{s.t.} \quad & \bar{z} = A\bar{z} + B\bar{u} && \text{(1b)} \\
& \bar{y} = C\bar{z} + d && \text{(1c)} \\
& u_{\min} \leq \bar{u} \leq u_{\max} && \text{(1d)} \\
& y_{\min} \leq \bar{y} \leq y_{\max} && \text{(1e)} \\
& d = \hat{d}(t), \quad y_{\mathrm{ref}} = y_{\mathrm{sp}} && \text{(1f)}
\end{aligned}
$$

3.2. MPC

$$
\begin{aligned}
\min_{u_0,\ldots,u_{N-1}} \quad & \sum_{k=0}^{N-1} (y_k - \bar{y})^\intercal Q_\mathrm{y} (y_k - \bar{y}) + \sum_{k=0}^{N-1} \Delta u_k^\intercal Q_\mathrm{u} \Delta u_k, && \text{(2a)}\\
\text{s.t.} \quad & z_{k+1} = A z_k + B u_k, \quad k \in \mathbb{N}_0^{N-1}, && \text{(2b)}\\
& y_k = C z_k + d, \quad k \in \mathbb{N}_0^{N-1}, && \text{(2c)}\\
& \Delta u_k = u_k - u_{k-1}, \quad k \in \mathbb{N}_0^{N-1}, && \text{(2d)}\\
& u_\mathrm{min} \le u_k \le u_\mathrm{max}, \quad k \in \mathbb{N}_0^{N-1}, && \text{(2e)}\\
& y_\mathrm{min} \le y_k \le y_\mathrm{max}, \quad k \in \mathbb{N}_0^{N-1}, && \text{(2f)}\\
& z_0 = \hat{z}(t), \quad d = \hat{d}(t), \quad u_{-1} = u(t - T_\mathrm{s}) && \text{(2g)}
\end{aligned}
$$

4. Koopman MPC with Offset-Free - Proposed

Main drawback that we observe in a current Koopman MPC (offset-free, robust, or any other), is that the optimization is done without leveraging knowledge of the lifted space. Of course we are predicting in the lifted space, but in objective function we are using original space, that is created derived as linear transformation of the lifted space. This transformation is not linear by nature, but to use linear MPC, we think we had no other choice. Some papers are not using this transformation directly, but after closer look, they are usualy using equivalent transformations. We will show that we can use the lifted space directly in the objective function, show how to handle the lifted space and transformation of an objective function. We can fully eliminate the neccessity of the back transformation used in objective function in MPC, but we still need them to compute the target for the lifted space. In this case and case of constraints, we use a first order Taylor expansion, to better approximate the transformation.

4.1. Target Optimization

$$\min_{\bar{u},\bar{z}} \quad (\bar{y} - y_{\mathrm{ref}})^{\mathsf{T}} Q_{\mathrm{y}} (\bar{y} - y_{\mathrm{ref}}) \tag{3a}$$

$$\text{s.t.} \quad \bar{z} = A\bar{z} + B\bar{u} \tag{3b}$$

$$\bar{y} = H(z_k)\bar{z} + h(z_k) - H(z_k)z_k + d \tag{3c}$$

$$u_{\min} \leq \bar{u} \leq u_{\max} \tag{3d}$$

$$y_{\min} \leq \bar{y} \leq y_{\max} \tag{3e}$$

$$d = \hat{d}(t), \quad y_{\mathrm{ref}} = y_{\mathrm{sp}}, \quad z_k = \hat{z}(t) \tag{3f}$$

4.2. MPC

$$\min_{u_0,\dots,u_{N-1}} \quad \sum_{k=0}^{N-1} (z_k - \bar{z})^{\mathsf{T}} Q_{\mathrm{z}} (z_k - \bar{z}) + \sum_{k=0}^{N-1} \Delta u_k^{\mathsf{T}} Q_{\mathrm{u}} \Delta u_k, \tag{4a}$$

$$\text{s.t.} \quad z_{k+1} = Az_k + Bu_k, \quad k \in \mathbb{N}_0^{N-1}, \tag{4b}$$

$$y_k = H(z_0)z_k + h(z_0) - H(z_0)z_0 + d, \quad k \in \mathbb{N}_0^{N-1}, \tag{4c}$$

$$\Delta u_k = u_k - u_{k-1}, \quad k \in \mathbb{N}_0^{N-1}, \tag{4d}$$

$$u_{\min} \leq u_k \leq u_{\max}, \quad k \in \mathbb{N}_0^{N-1}, \tag{4e}$$

$$y_{\min} \leq y_k \leq y_{\max}, \quad k \in \mathbb{N}_0^{N-1}, \tag{4f}$$

$$z_0 = \hat{z}(t), \quad d = \hat{d}(t), \quad u_{-1} = u(t - T_{\mathrm{s}}) \tag{4g}$$

5. Block Diagonal Deep Koopman Transformation

Table 1: Comparison of structure of A

Structure	Time TE/MPC	Optimal? [%]	OBJ	ST h_1	ST h_2
Full	0.2392 / 2.3884	26.7	100.0	43	48
Block Diagonal	0.2191 / 1.6204	100.0	94.1	25	31

6. Simulation Setup

7. Simulation Results

Table 2: Comparison of several Tunings

LP MPC	LP TE	Tuning	OBJ
-	-	$J(z_s)$	171.22
C	C	$J(z_s)$	147.26
$J(z_s)$ w/o y-con	$J(z_{s,k-1})$	$J(z_s)$	149.18
$J(z_k)$	$J(z_k)$	$J(z_s)$	163.69
$J(z_k)$	$J(z_s)$	$J(z_s)$	149.35
-	-	C	156.21
C	C	C	151.73
$J(z_s)$ w/o y-con	$J(z_{s,k-1})$	C	158.80
$J(z_k)$	$J(z_k)$	C	177.33
$J(z_k)$	$J(z_s)$	C	158.68

Table 3: Comparison of several Tunings

ALG	MODEL	OBJ
NMPC	Tru	88.09
MPC	Pa.-K (3)	167.51
MPC	DK	156.21
TMPC	DK (C)	147.22
TMPC	DK (J(z))	149.35