4T1: The Short-Time Fourier Transform (1 of 2)

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Short-time Fourier Transform

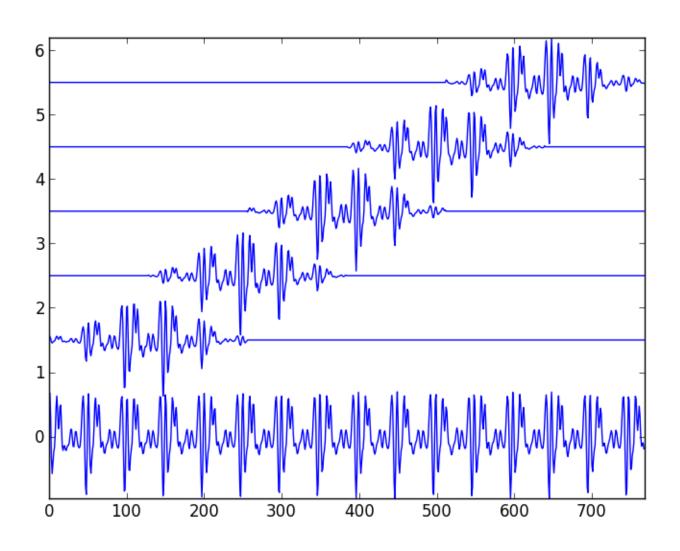
$$X_{l}[k] = \sum_{n=-N/2}^{N/2-1} w[n]x[n+lH]e^{-j2\pi kn/N} \quad l=0,1,...,$$

w: analysis window

l: frame number

H: hop-size

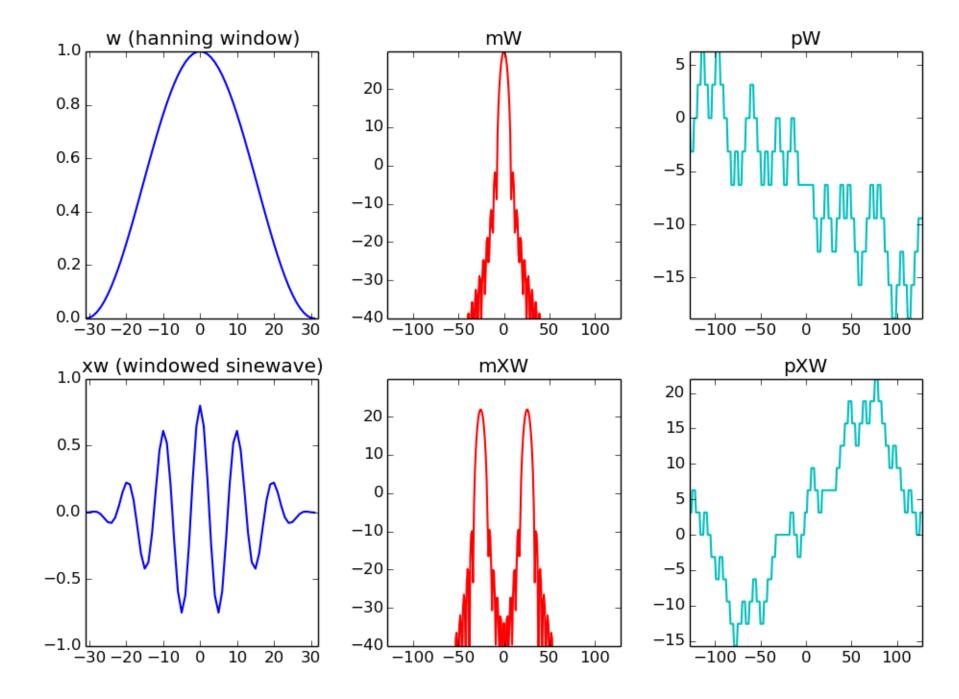
$$xw_{l}[n]=w[n]x[n+lH]$$
 $l=0,1,...,$



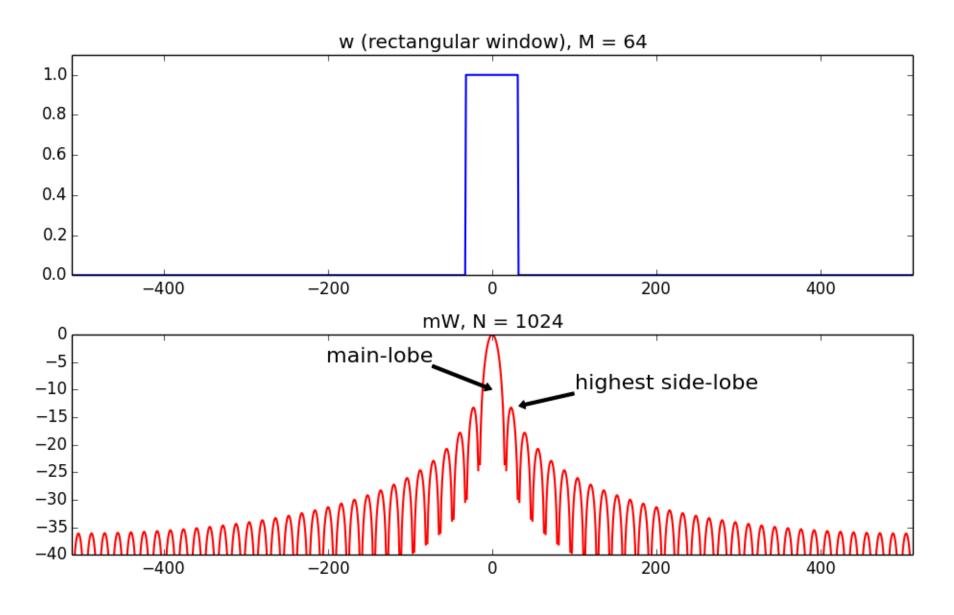
Transform of a windowed sinewave

$$x[n] = A_0 \cos(2\pi k_0 n/N) = \frac{A_0}{2} e^{j2\pi k_0 n/N} + \frac{A_0}{2} e^{-j2\pi k_0 n/N}$$

$$\begin{split} X[k] &= \sum_{n=-N/2}^{N/2-1} w[n]x[n]e^{-j2\pi kn/N} \\ &= \sum_{n=-N/2}^{N/2-1} w[n] (\frac{A_0}{2} e^{j2\pi k_0 n/N} + \frac{A_0}{2} e^{-j2\pi k_0 n/N}) e^{-j2\pi kn/N} \\ &= \sum_{n=-N/2}^{N/2-1} w[n] \frac{A_0}{2} e^{j2\pi k_0 n/N} e^{-j2\pi kn/N} + \sum_{n=-N/2}^{N/2-1} w[n] \frac{A_0}{2} e^{-j2\pi k_0 n/N} e^{-j2\pi kn/N} \\ &= \frac{A_0}{2} \sum_{n=-N/2}^{N/2-1} w[n] e^{-j2\pi (k-k_0)n/N} + \frac{A_0}{2} \sum_{n=-N/2}^{N/2-1} w[n] e^{-j2\pi (k+k_0)n/N} \\ &= \frac{A_0}{2} W[k-k_0] + \frac{A_0}{2} W[k+k_0] \end{split}$$



Analysis window

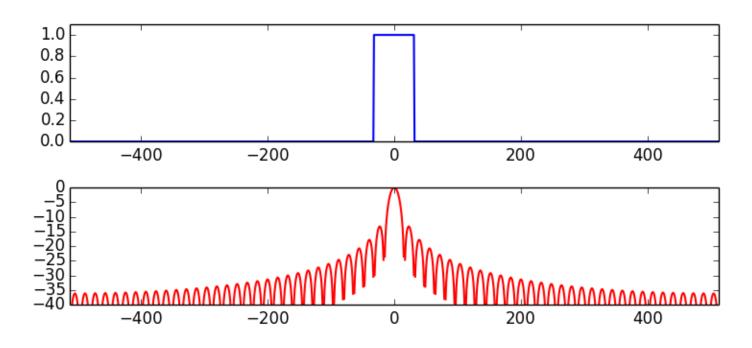


Window functions in Scipy

barthann (M[, sym])	Return a modified Bartlett-Hann window.
bartlett (M[, sym])	Return a Bartlett window.
blackman (M[, sym])	Return a Blackman window.
blackmanharris (M[, sym])	Return a minimum 4-term Blackman-Harris window.
bohman (M[, sym])	Return a Bohman window.
boxcar (M[, sym])	Return a boxcar or rectangular window.
chebwin (M, at[, sym])	Return a Dolph-Chebyshev window.
flattop (M[, sym])	Return a flat top window.
gaussian (M, std[, sym])	Return a Gaussian window.
general-gaussian (M, p, sig[, sym])	Return a window with a generalized Gaussian shape.
hamming (M[, sym])	Return a Hamming window.
hann (M[, sym])	Return a Hann window.
kaiser (M, beta[, sym])	Return a Kaiser window.
nuttall (M[, sym])	Return a minimum 4-term Blackman-Harris window according to Nuttall.
parzen (M[, sym])	Return a Parzen window.
slepian (M, width[, sym])	Return a digital Slepian window.
triang (M[, sym])	Return a triangular window.

Rectangular window

$$w[n]=1, \quad n=-M/2,...,0,...M/2$$
 $W[k]=\frac{\sin(\pi k)}{\sin(\pi k/M)}$
=0, $n=$ elsewhere

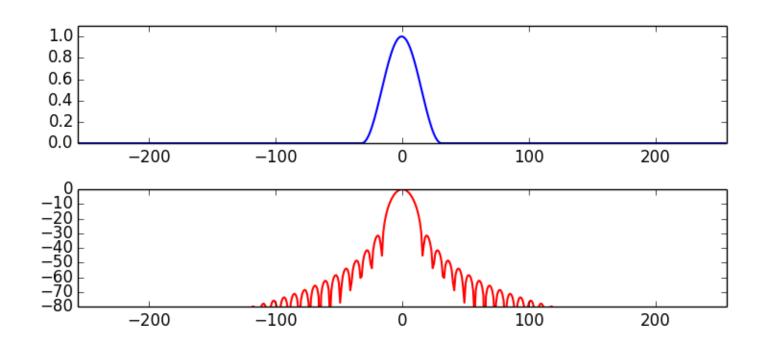


main-lobe width: 2 bins side-lobe level: -13.3 dB

Hanning window

$$w[n]=.5+.5\cos(2\pi n/M), n=-M/2,...,0,...M/2$$

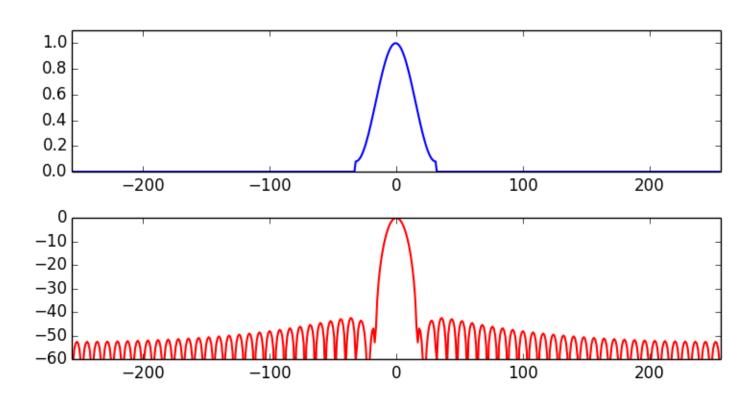
$$W[k] = .5D[k] + .25(D[k-1] + D[k+1])$$
 where $D[k] = \frac{\sin(\pi k)}{\sin(\pi k/M)}$



main-lobe width: 4 bins side-lobe level: -31.5 dB

Hamming window

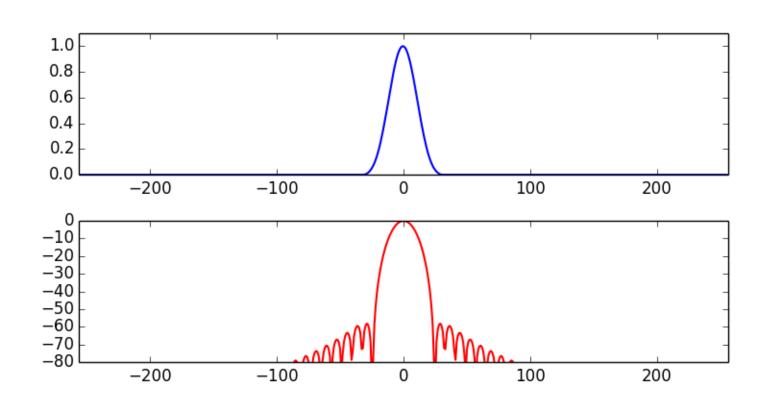
 $w[n]=.54+.46\cos(2\pi n/M), n=-M/2,...,0,...M/2$



main-lobe width: 4 bins side-lobe level: -42.7 dB

Blackman window

 $w[n] = 0.42 - 0.5\cos(2\pi n/M) + 0.08\cos(4\pi n/M)$

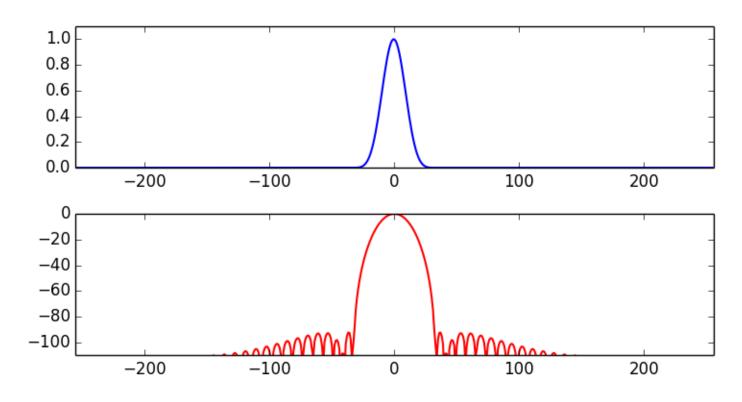


main-lobe width: 6 bins side-lobe level: -58 dB

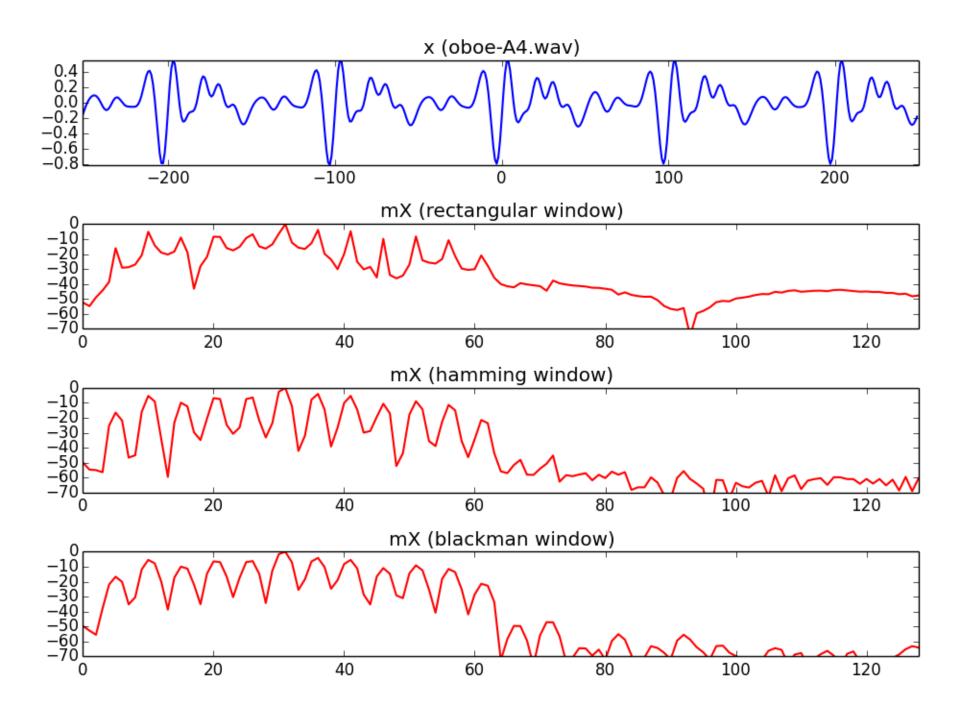
Blackman-Harris window

$$w(n) = \frac{1}{M} \sum_{l=0}^{3} \alpha_l \cos(2nl\pi/M), \quad n = -M/2, ...0, ...M/2$$

where $\alpha_0 = 0.35875, \alpha_1 = 0.48829, \alpha_2 = 0.14128, \alpha_3 = 0.01168$



main lobe width: 8 bins side-lobe level: -92dB



References and credits

- More information in: https://en.wikipedia.org/wiki/STFT https://en.wikipedia.org/wiki/Window_function
- Reference on the STFT by Julius O. Smith: https://ccrma.stanford.edu/~jos/sasp/
- Sounds from: http://www.freesound.org/people/xserra/packs/13038/
- Slides and code released using the CC Attribution-Noncommercial-Share Alike license or the Affero GPL license and available from https://github.com/MTG/sms-tools

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