

2T1: The Discrete Fourier Transform (1 of 2)

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Discrete Fourier Transform

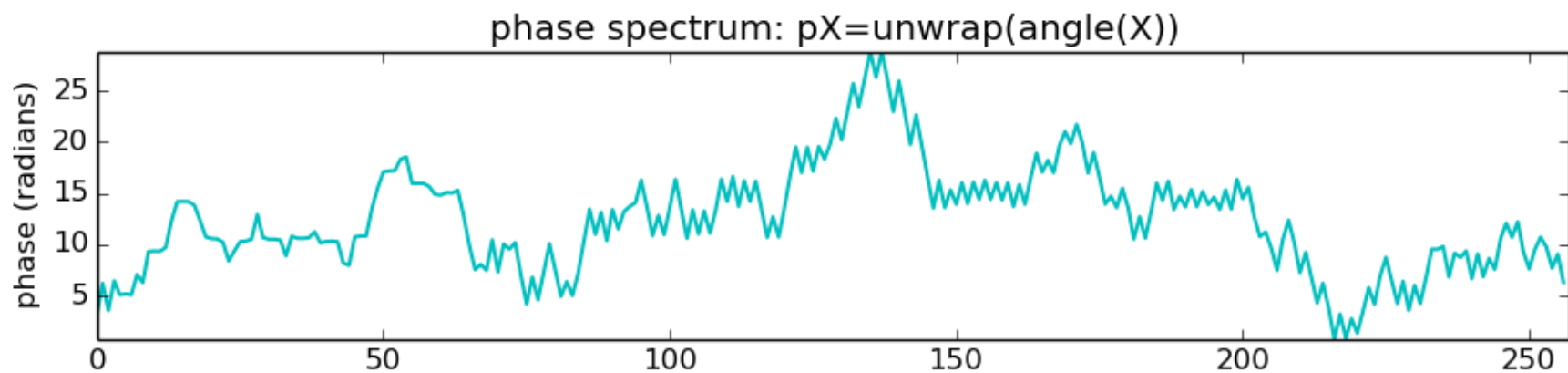
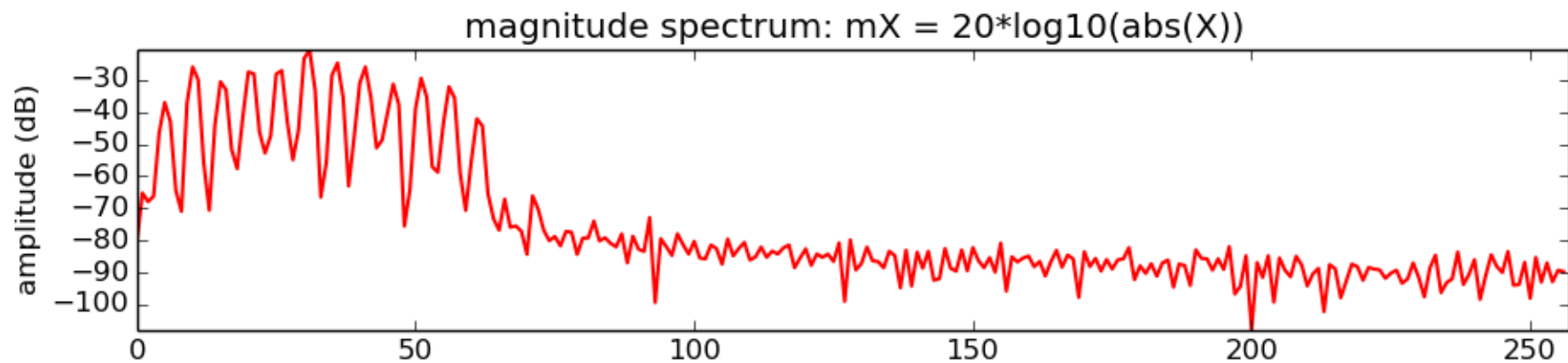
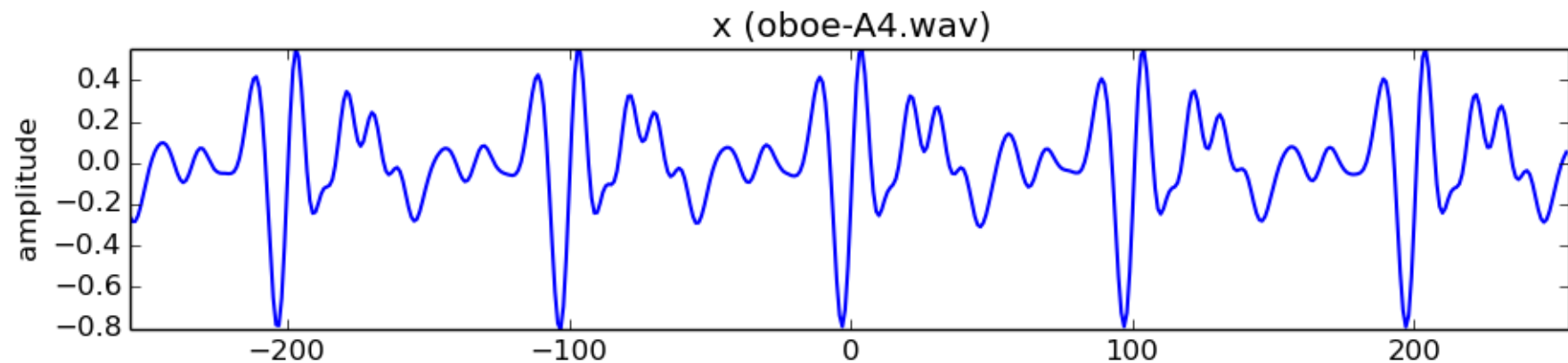
$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j2\pi kn/N} \quad k=0, \dots, N-1$$

n : discrete time index (normalized time, $T=1$)

k : discrete frequency index

$\omega_k = 2\pi k/N$: frequency in radians

$f_k = f_s k/N$: frequency in Hz (f_s : sampling rate)



DFT: complex exponentials

$$s_k^* = e^{-j2\pi kn/N} = \cos(2\pi kn/N) - j \sin(2\pi kn/N)$$

for $N=4$, thus for $n=0,1,2,3$; $k=0,1,2,3$

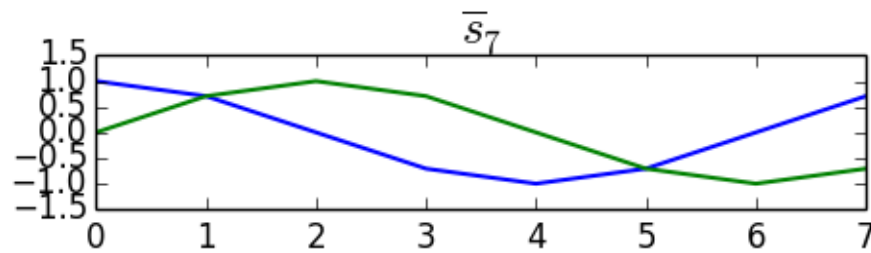
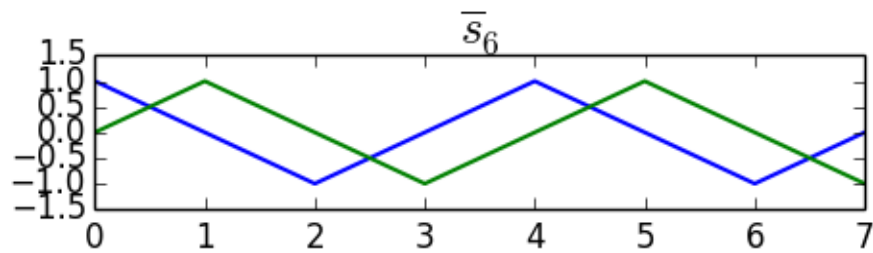
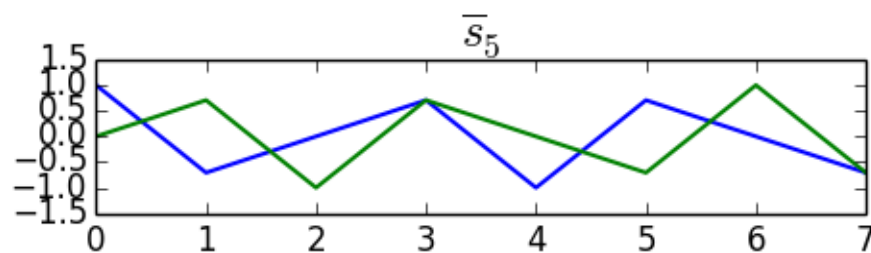
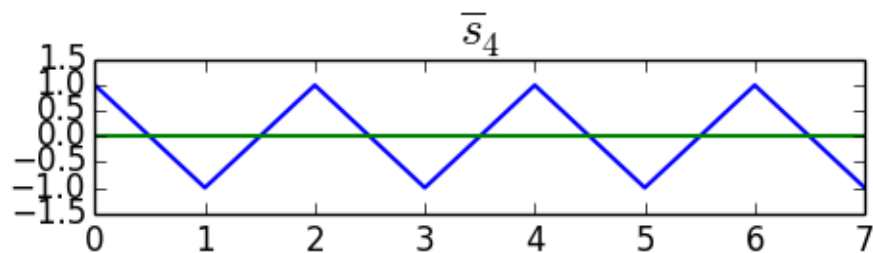
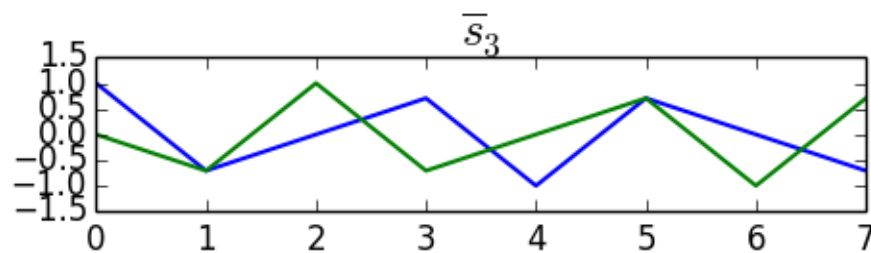
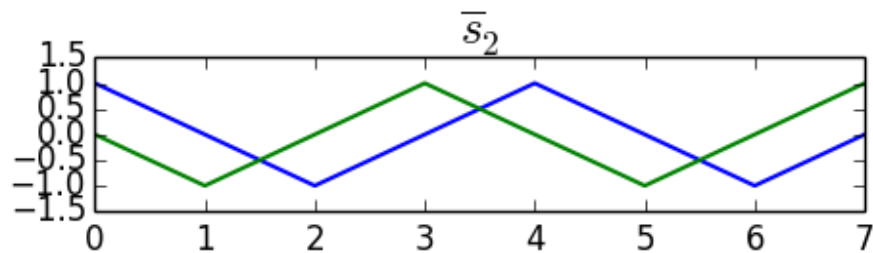
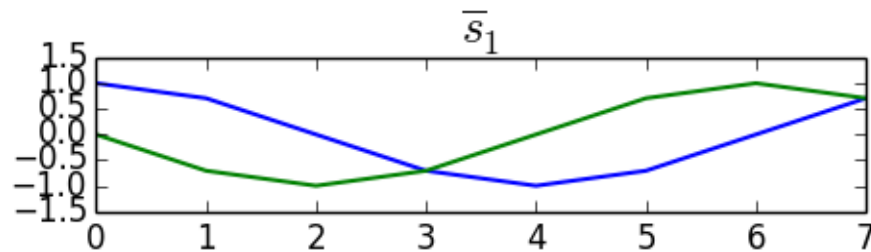
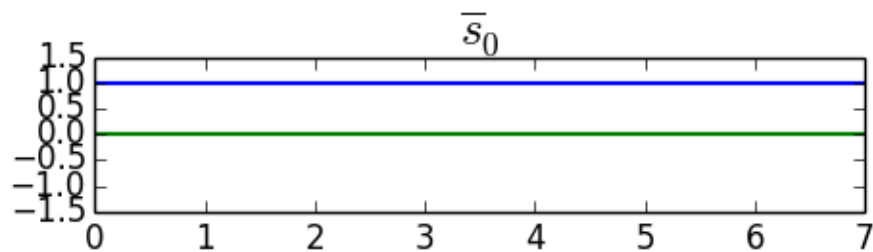
$$s_0^* = \cos(2\pi \times 0 \times n/4) - j \sin(2\pi \times 0 \times n/4) = [1, 1, 1, 1]$$

$$s_1^* = \cos(2\pi \times 1 \times n/4) - j \sin(2\pi \times 1 \times n/4) = [1, -j, -1, j]$$

$$s_2^* = \cos(2\pi \times 2 \times n/4) - j \sin(2\pi \times 2 \times n/4) = [1, -1, 1, -1]$$

$$s_3^* = \cos(2\pi \times 3 \times n/4) - j \sin(2\pi \times 3 \times n/4) = [1, j, -1, -j]$$

DFT: complex exponentials



DFT: scalar product

$$\langle x, s_k \rangle = \sum_{n=0}^{N-1} x[n] s_k^*[n] = \sum_{n=0}^{N-1} x[n] e^{-j 2\pi kn/N}$$

Example:

$$x[n] = [1, -1, 1, -1]; N=4$$

$$\langle x, s_0 \rangle = 1 \times 1 + (-1) \times 1 + 1 \times 1 + (-1) \times 1 = 0$$

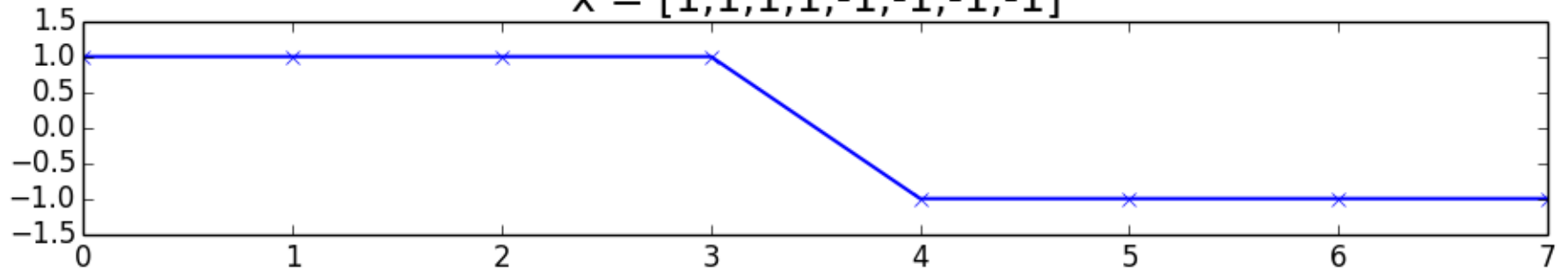
$$\langle x, s_1 \rangle = 1 \times 1 + (-1) \times (-j) + 1 \times (-1) + (-1) \times j = 0$$

$$\langle x, s_2 \rangle = 1 \times 1 + (-1) \times (-1) + 1 \times 1 + (-1) \times (-1) = 4$$

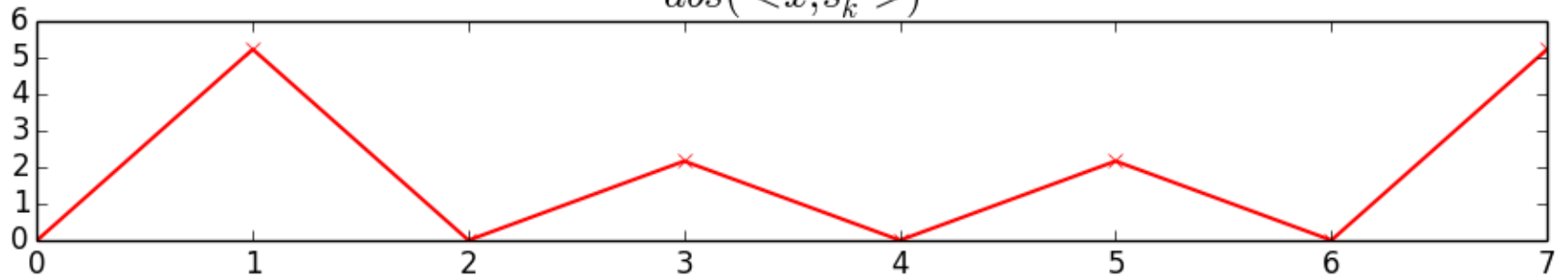
$$\langle x, s_3 \rangle = 1 \times 1 + (-1) \times j + 1 \times (-1) + (-1) \times (-j) = 0$$

DFT: scalar product

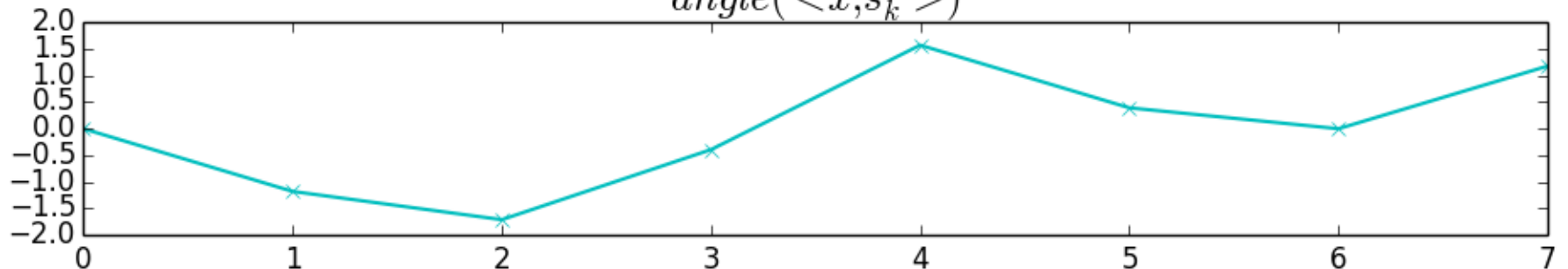
$$x = [1, 1, 1, 1, -1, -1, -1, -1]$$



$$abs(\langle x, s_k \rangle)$$



$$angle(\langle x, s_k \rangle)$$



References and credits

- More information in:
https://en.wikipedia.org/wiki/Discrete_Fourier_transform
- Reference on the mathematics of the DFT from Julius O. Smith: <https://ccrma.stanford.edu/~jos/mdft/>
- Sounds from:
<http://www.freesound.org/people/xserra/packs/13038>
- Slides released under CC Attribution-Noncommercial-Share Alike license and code under Affero GPL license; available from <https://github.com/MTG/sms-tools>

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