

# 3T2: Fourier Transform properties (2 of 2)

***Xavier Serra***

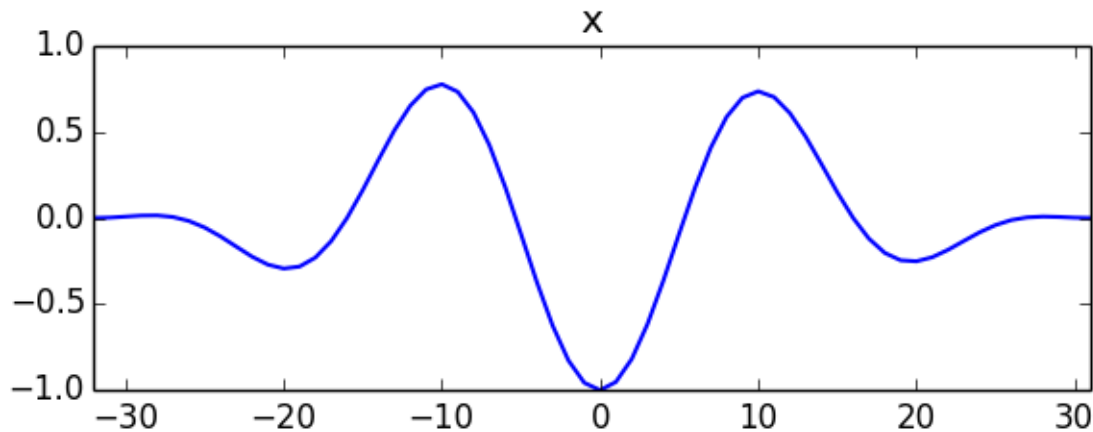
Universitat Pompeu Fabra, Barcelona

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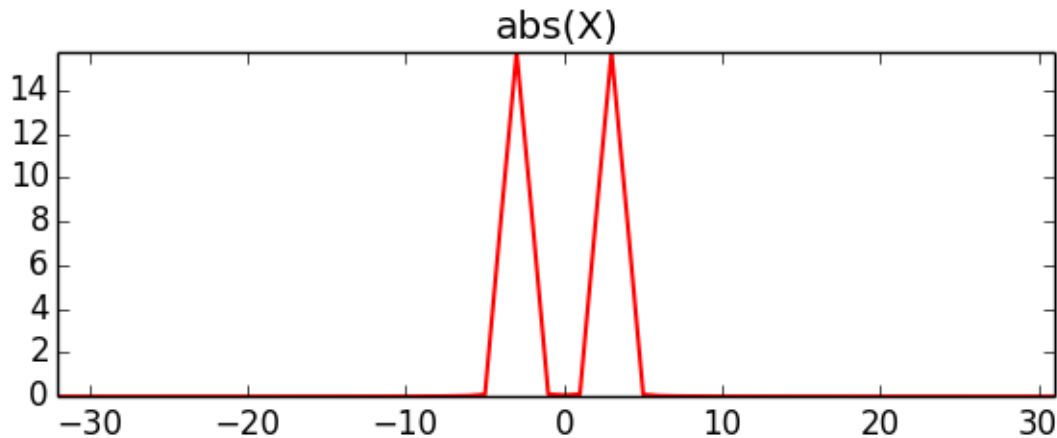
- Energy conservation & decibels
- Phase unwrapping
- Zero padding
- Fast Fourier Transform (FFT)
- FFT and zero-phase windowing
- Analysis/synthesis

# Energy conservation

$$\sum_{n=-N/2}^{N/2-1} |x[n]|^2 = \frac{1}{N} \sum_{k=-N/2}^{N/2-1} |X[k]|^2$$

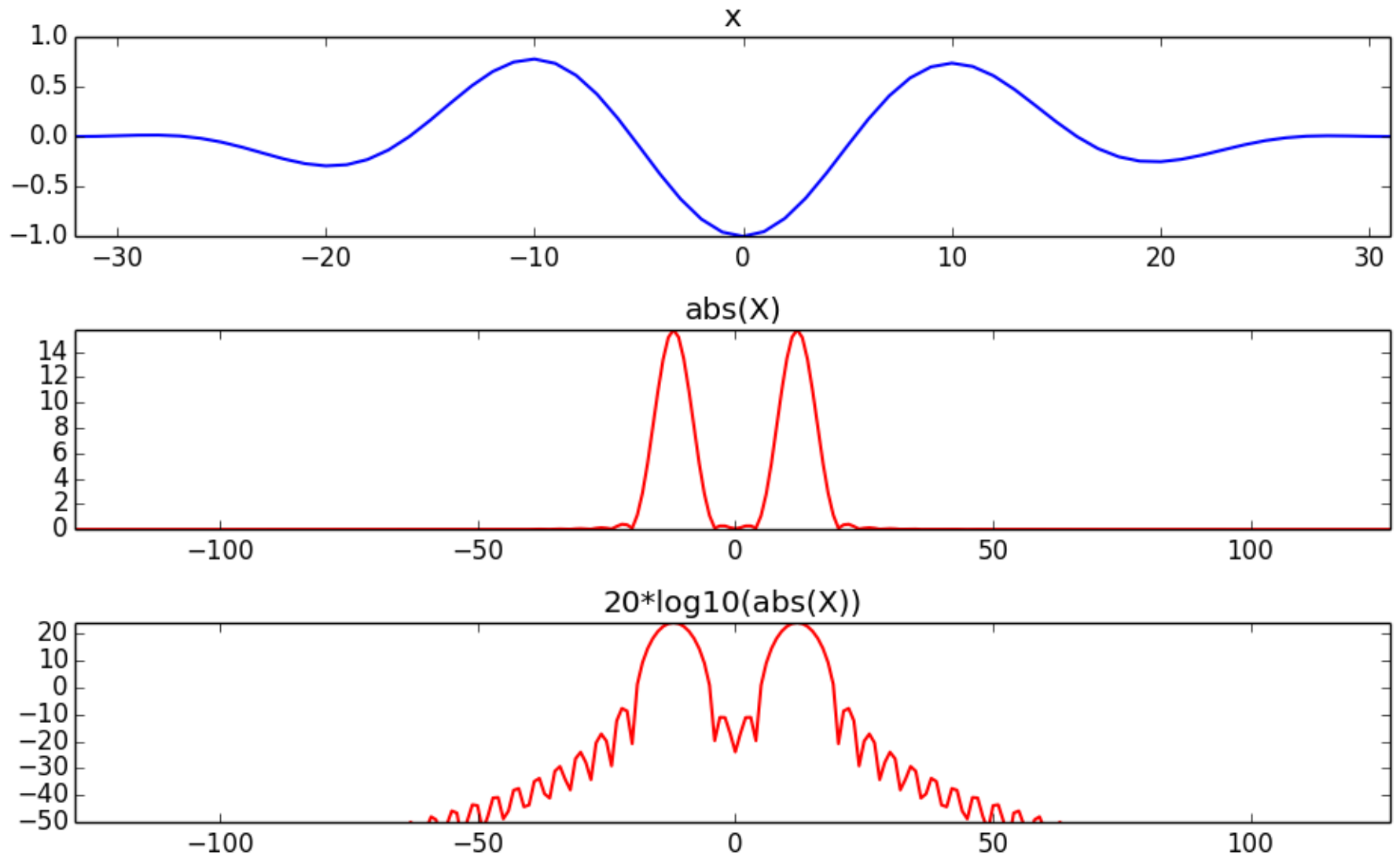


$$\sum_{n=-N/2}^{N/2-1} |x[n]|^2 = 11.81182$$

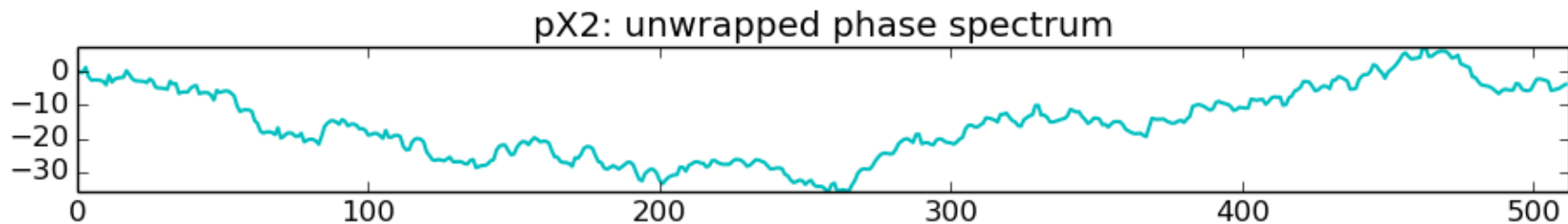
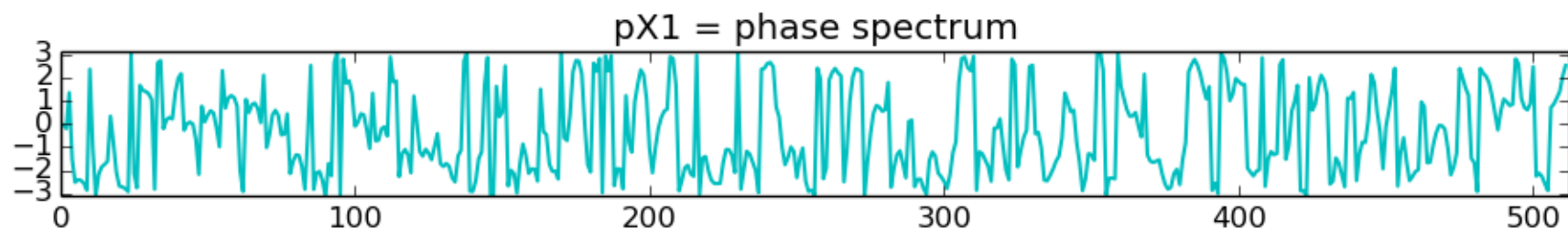
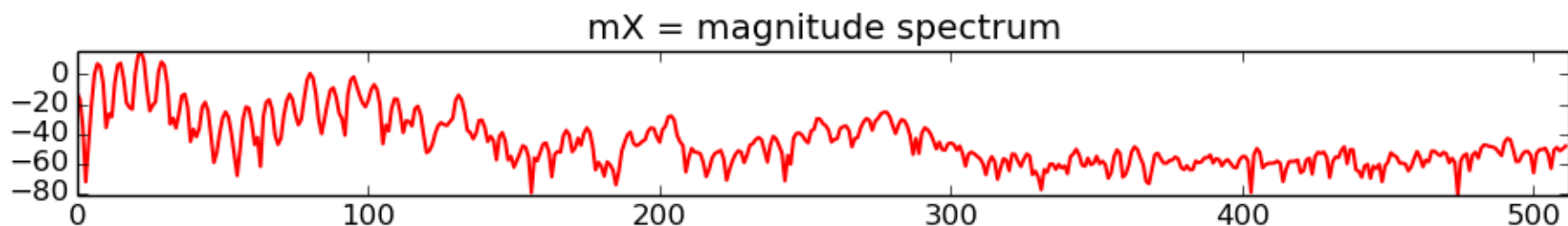
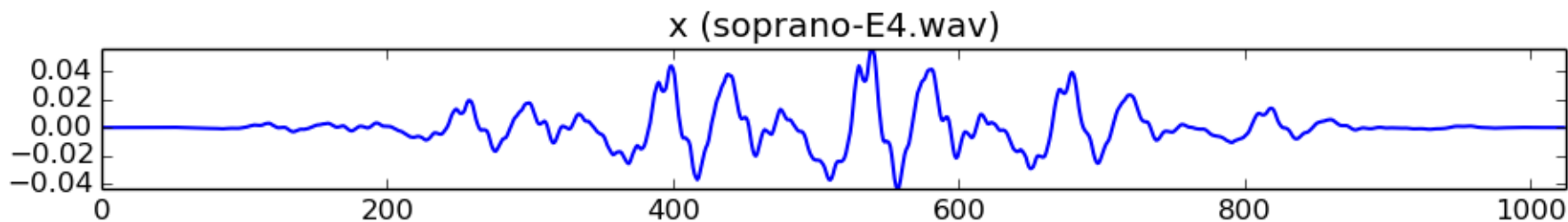


$$\frac{1}{N} \sum_{k=-N/2}^{N/2-1} |X[k]|^2 = 11.81182$$

# Amplitude in decibels (dB)

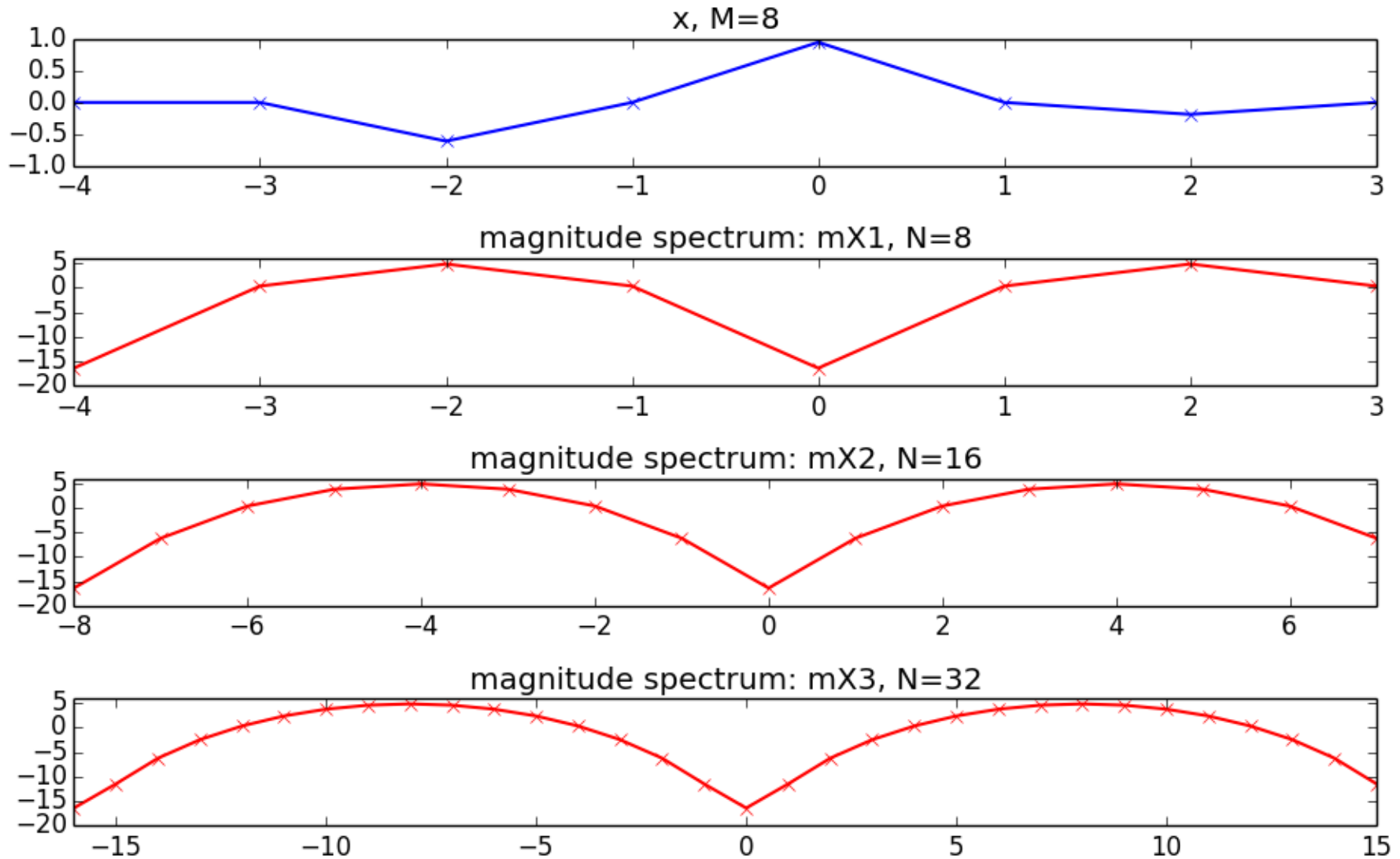


# Phase unwrapping



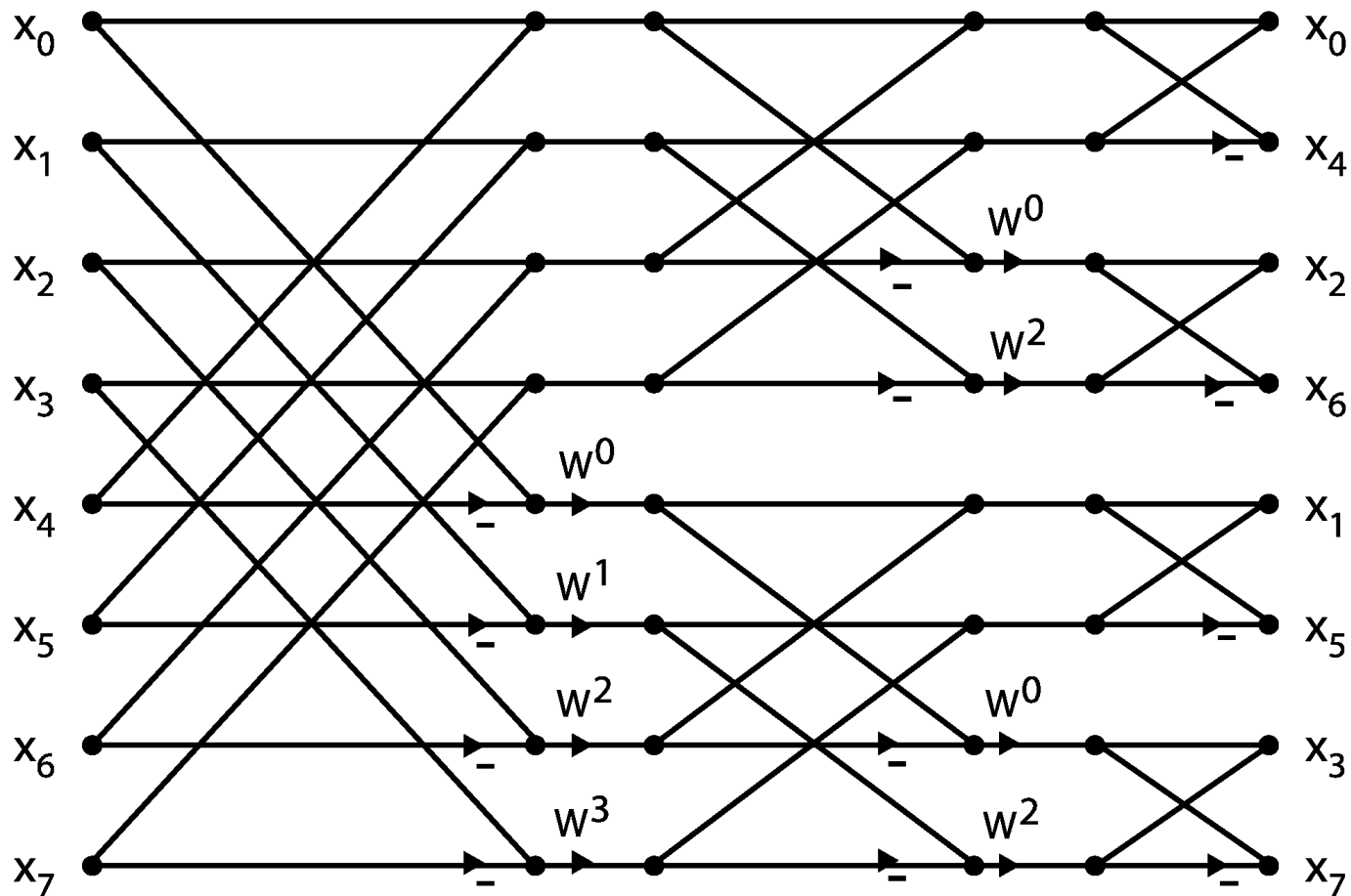
# Zero-padding

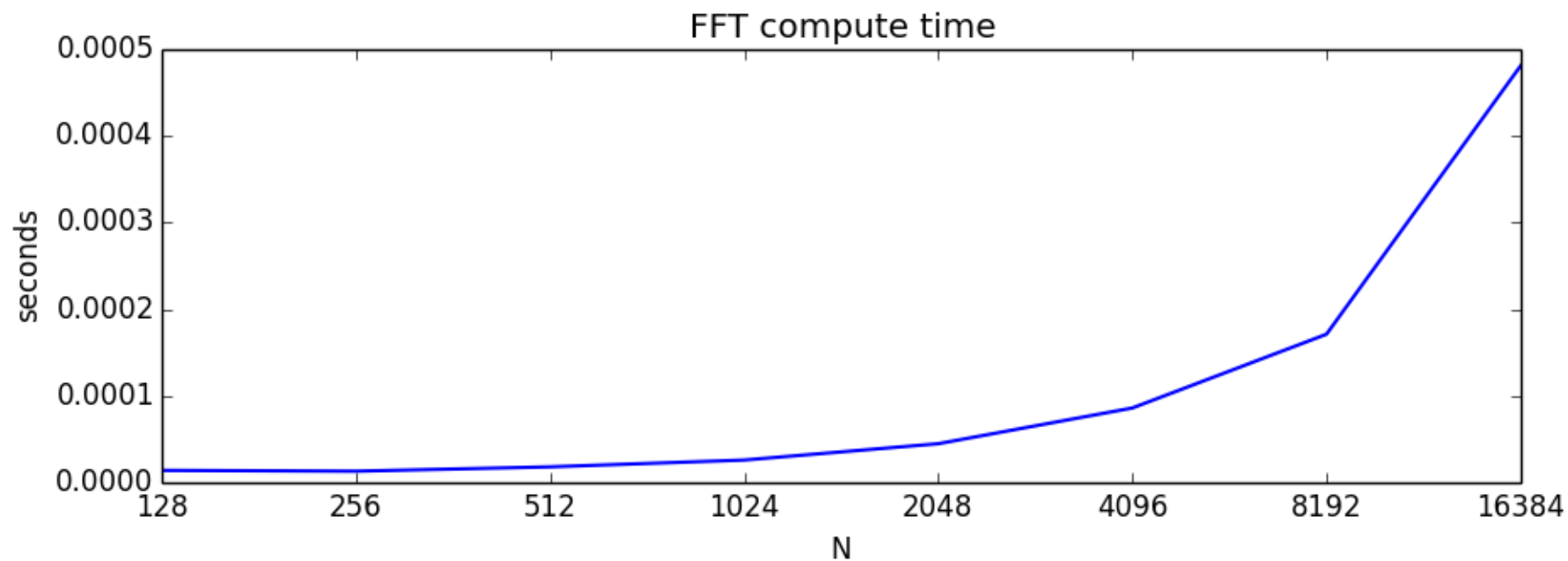
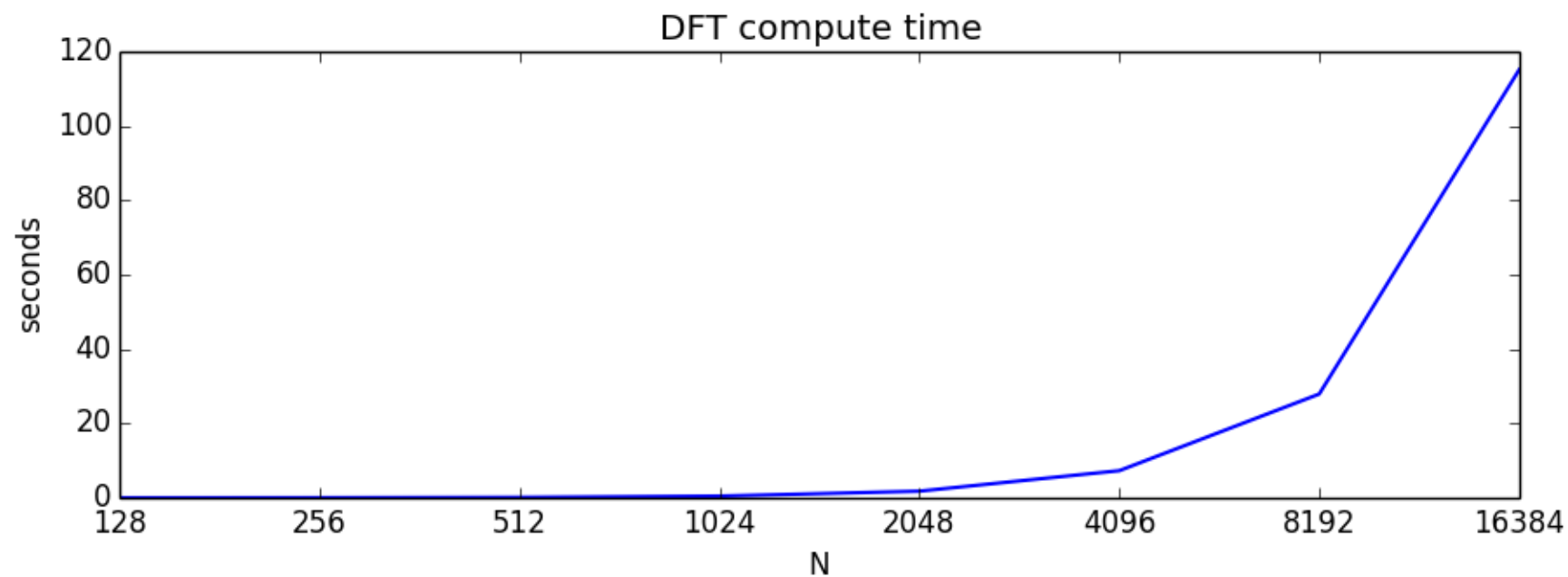
zero padding  $\leftrightarrow$  interpolation



# Fast Fourier Transform

*Cooley-Tukey algorithm:* breaks down recursively the DFT of a power of 2 size into two pieces of size  $N/2$ .

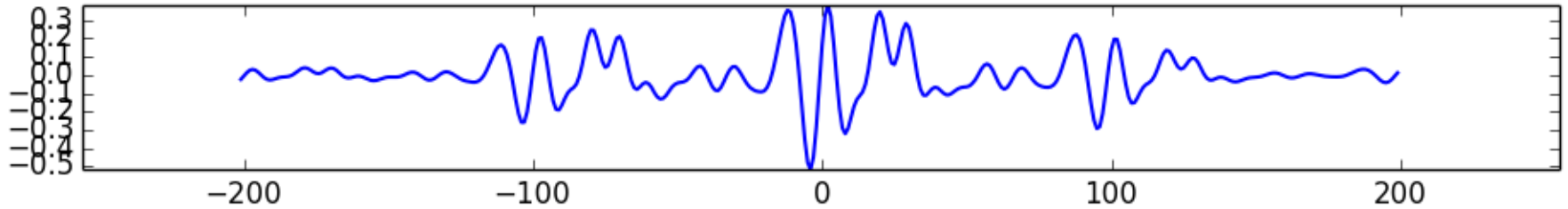




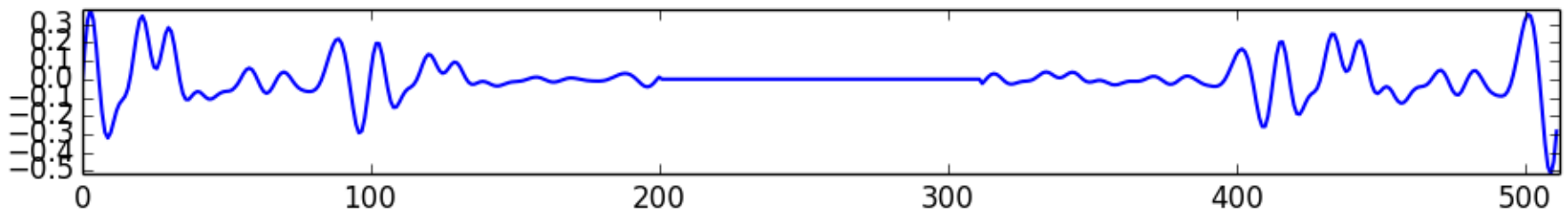


# FFT and zero-phase windowing

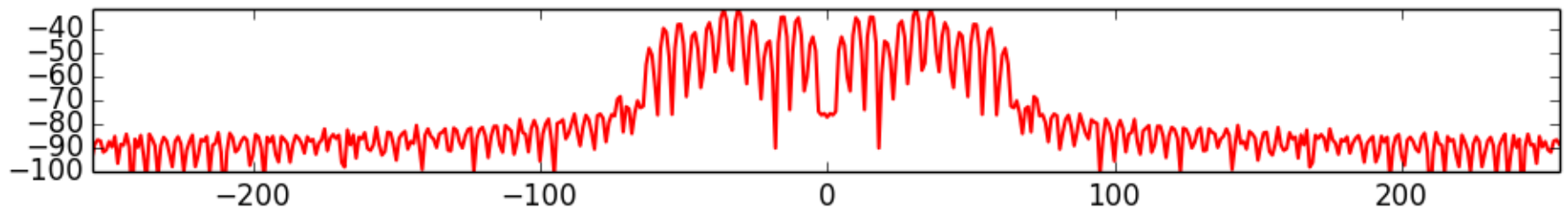
x (oboe-A4.wav), M = 401



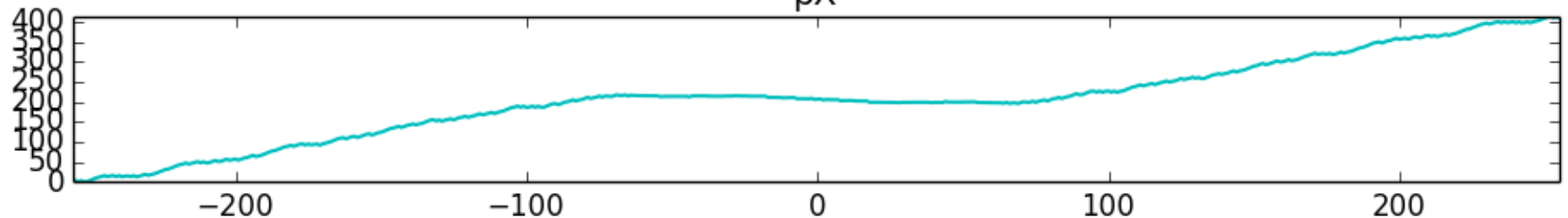
fftbuffer: N = 512



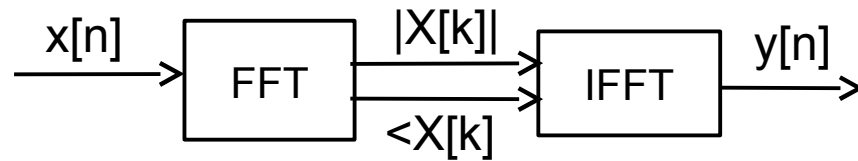
mX



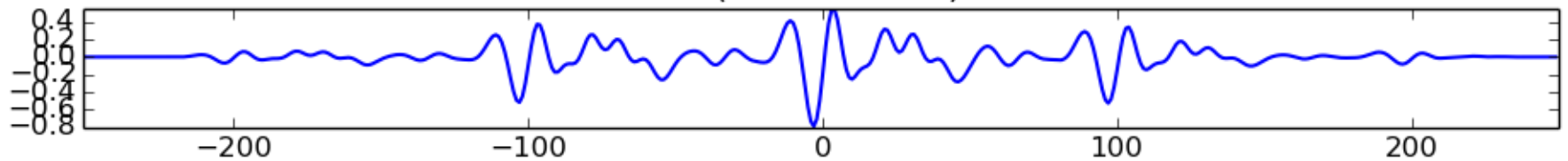
pX



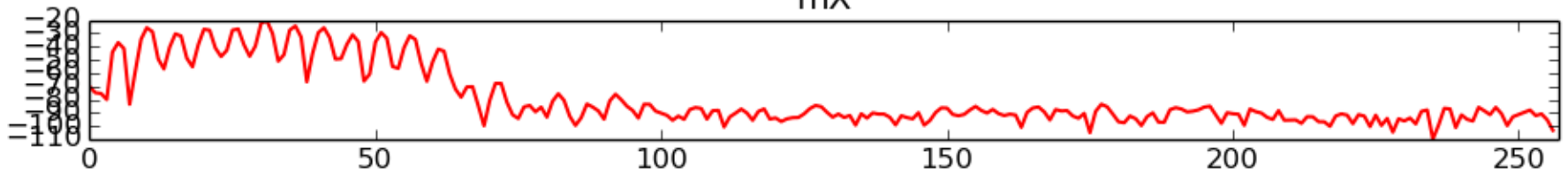
# Analysis/synthesis



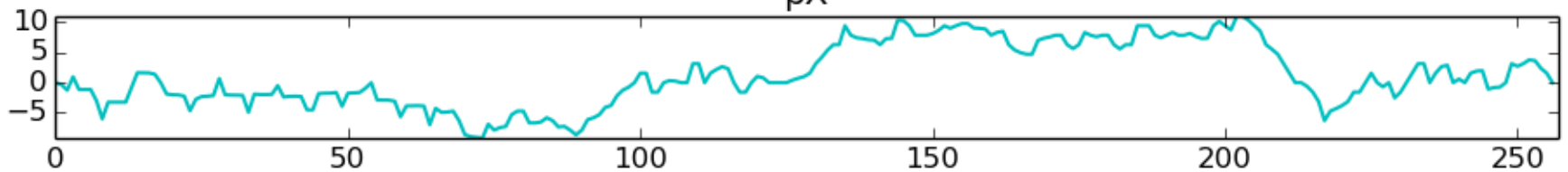
x (oboe-A4.wav)



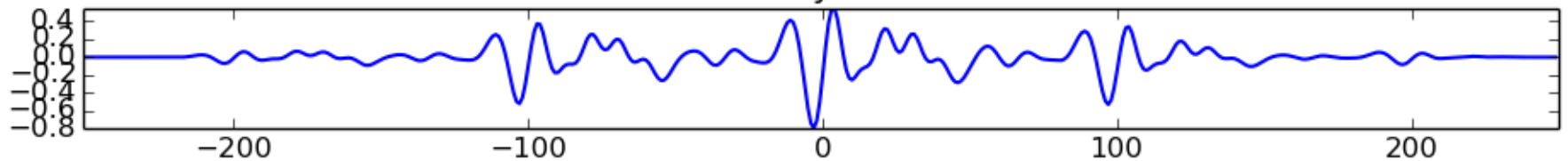
mX



pX



y



# References and credits

- More information on:  
[https://en.wikipedia.org/wiki/Discrete\\_Fourier\\_transform](https://en.wikipedia.org/wiki/Discrete_Fourier_transform)  
[https://en.wikipedia.org/wiki/Fast\\_Fourier\\_transform](https://en.wikipedia.org/wiki/Fast_Fourier_transform)
- Sounds from:  
<http://www.freesound.org/people/xserra/packs/13038/>
- Reference for the DFT by Julius O. Smith:  
<https://ccrma.stanford.edu/~jos/mdft/>
- Slides released under CC Attribution-Non Commercial-Share Alike license and code under Affero GPL license; available from <https://github.com/MTG/sms-tools>

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