MPC Powered Rocket Landing

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1 Motivation

In recent years, commercial aerospace companies such as SpaceX and Blue Origin have achieved to develop new technologies for the reuse of their launch vehicles. More specifically, efforts have been made to recover the first section of multistage rockets via autonomous precision landings, powered by the vehicle's main engines. This modern and sustainable approach allows private entities to perform launches at reduced cost and enables future research in aerospace engineering and astronomy.

2 Position control

The soft rocket landing problem can be separated into translation and attitude control since the rotational dynamics can be controlled at a higher bandwidth [1, 2]. In this case, the attitude controller aligns the rocket's main axis with the thrust vector. We define the following, simple, equations of motion for a vehicle with variable mass under the influence of gravity:

$$\begin{aligned} m(t)\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} &= \mathbf{T}(t) - \mathbf{g}, \\ \frac{\mathrm{d}m(t)}{\mathrm{d}t} &= -\alpha \left\| \mathbf{T}(t) \right\|. \end{aligned} \tag{1}$$

Model predictive control (MPC) [3] can be used to generate a feed-forward input sequence that brings the vehicle into a desired final state. We can define the optimization problem as follows:

$$\begin{aligned} \min_{t_f, \mathbf{T}(\cdot)} \int_0^{t_f} \|\mathbf{T}(t)\|\, dt \quad \text{s.t.} \\ \forall t \in [0, t_f],\ t_f \in \mathbb{R}^+, \\ 0 < T_{\min} \leq \|\mathbf{T}(t)\| \leq T_{\max}, \\ \mathbf{r}(0) = \mathbf{r}_0,\ \dot{\mathbf{r}}(0) = \mathbf{v}_0, \\ \mathbf{e}_3^T \mathbf{r}(t) \geq 0, \\ |\mathbf{e}_i^T \mathbf{r}(t_f)| \leq \epsilon_r,\ |\mathbf{e}_i^T \dot{\mathbf{r}}(t_f)| \leq \epsilon_v, \\ m(0) = m_0, \end{aligned} \tag{2}$$

where the initial position and velocity must correspond to the current state of the vehicle and the final state must reach the origin by some predefined tolerances. The vector $\mathbf{e}_i$ corresponds to the unit vector in the $i-$th direction. The thrust vector magnitude is upper and lower bounded, which is a common constraint for launch vehicles. Nevertheless, this last condition represents a non-convex constraint in $\mathbb{R}^2$ and $\mathbb{R}^3$ [1], making the optimization less robust and sensitive to initial conditions. The problem in (2) can be redefined using lossless convexification. Based on the work of Açıkmeşe *et al.*, a relaxed problem yielding the same optimal solution can be formulated [1, 4].

$$\begin{aligned} \min_{t_f, \Gamma(\cdot)} \int_0^{t_f} \Gamma(t)\, dt \quad \text{s.t.} \\ \forall t \in [0, t_f],\ t_f \in \mathbb{R}^+, \\ 0 < T_{\min} \leq \Gamma(t) \leq T_{\max}, \\ \|\mathbf{T}(t)\| \leq \Gamma(t), \\ \mathbf{r}(0) = \mathbf{r}_0,\ \dot{\mathbf{r}}(0) = \dot{\mathbf{r}_0}, \\ \mathbf{e}_3^T \mathbf{r}(t) \geq 0, \\ |\mathbf{e}_i^T \mathbf{r}(t_f)| \leq \epsilon_r,\ |\mathbf{e}_i^T \dot{\mathbf{r}}(t_f)| \leq \epsilon_v. \\ m(0) = m_0, \end{aligned} \tag{3}$$

For the relaxed version, a slack variable Γ is introduced. This scalar function has upper and lower bounds, which represent a convex constraint. Furthermore, the thrust magnitude is now only upper bounded by Γ, which also yields a convex constraint in higher dimensions. The equations of motion can then be rewritten for the optimization.

$$\begin{aligned} m(t)\frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2} &= \mathbf{T}(t) - \mathbf{g}, \\ \frac{\mathrm{d}m(t)}{\mathrm{d}t} &= -\alpha\, \Gamma(t). \end{aligned} \tag{4}$$

Lossless convexification is applicable to nonlinear systems. Local optima of the relaxed problem are still mapped to the same local optima of the original problem [5].

3 Attitude control

In this section, we intend to discuss the optimality of the generated trajectories by lossless convexification when the attitude of the vehicle is taken into account. For this, we will have a look at the two-dimensional case.

We assume that the trajectory curvature is smooth and that aligning the rocket with the nominal thrust vector requires only small torque inputs. We define T_Δ as the thrust in the perpendicular direction to the rocket's main axis, which should be small compared to the nominal thrust magnitude $T_\Delta \ll T^*$. We neglect its effect on the translational motion of the vehicle.

For this case, we can define a new upper bound of the thrust magnitude for the position controller. The lower bound will not have to be modified.

$$\sqrt{(T_{\Delta,\max})^2 + (T^*_{\max})^2} \leq T_{\max} \tag{5}$$

We can then write the equation of motion for the rotational dynamics of a body:

$$\begin{aligned} \ddot{\theta} &= \frac{l\,T_\Delta}{I} = \tau, \\ |\tau| &\leq \tau_{\max}, \end{aligned} \tag{6}$$

where $\tau_{\max}$ can easily be computed from (5), l defines the distance between the center of mass and the engine, and I corresponds to the moment of inertia. We now design a high bandwidth attitude controller using MPC. The controller should be able to bring the rocket angle and angular velocity to the expected value after a period of Δt. Since this might now always be feasible, as will be discussed in the results, we introduce a free time parameter T which is lower bounded by Δt. MPC will then minimize this parameter.

$$\begin{aligned} \min_T \int_{k\Delta t}^{k\Delta t+T} dt \; &= \min_T \; T \quad \text{s.t.} \\ \forall t \in [k\Delta t, k\Delta t + T]&, \Delta t \leq T, \\ -\tau_{\max} \leq \|\tau(t)\| &\leq \tau_{\max}, \\ \theta(k\Delta t) = \theta_k,\; \dot{\theta}(k\Delta t) &= \dot{\theta}_k, \\ \theta(k\Delta t + T) = \theta_{k+1},\; \dot{\theta}(k\Delta t + T) &= \dot{\theta}_{k+1}, \end{aligned} \tag{7}$$

where $\Delta t = t_f/N$ and $k \in \{0, 1, ..., N-1\}$ We implement this controller with CasADi and the Forward Euler discretization method to reduce the time of computation. Meanwhile, the generation of the trajectory uses the Runge-Kutta-Fehlberg (RK45) method, which should be more reliable for nonlinearities. The translational motion is also slower, and we can invest more time in the MPC optimization for the trajectory. It is executed offline in our simulations.

4 Results

We use normalized values for the generalization of the soft rocket landing problem.

Param.	**Value**
g	$9.81\ \mathrm{m\,s^{-2}}$
$\mathbf{r}_0$	$[30\ \ -20\ 50]^\top$ m
$\mathbf{v}_0$	$[-1\ \ -2\ 40]^\top$ m s^{-1}
$\mathbf{r}_f$	$[0\ 0\ 0]^\top$ m
$\mathbf{v}_f$	$[0\ 0\ 0]^\top$ m s^{-1}
m_0	100 kg
$T_{\min}/m_0$	$3\ \mathrm{m\,s^{-2}}$
$T_{\max}/m_0$	$15\ \mathrm{m\,s^{-2}}$
α	$0.1\ \mathrm{s\,m^{-1}}$

Furthermore, we compare the performance of lossless convexification to the original MPC formulation for the given initial conditions. We observe that the relaxed problem provides the same optimal solution in less time.

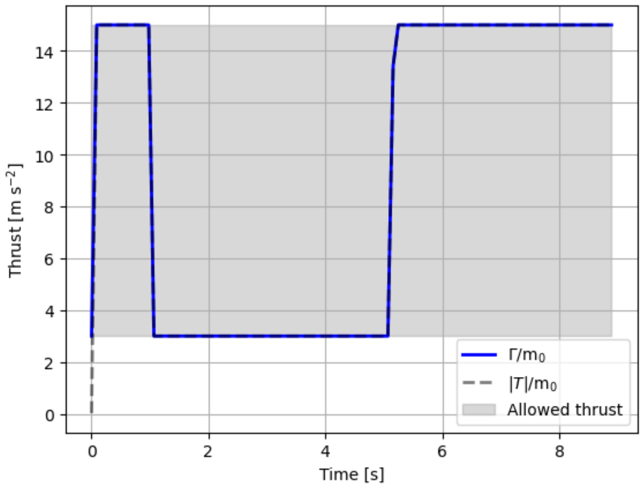

Figure 1: Comparison between the real thrust magnitude and the slack variable as a function of time. Despite that the thrust is only bounded by an upper value in the relaxed problem, it still satisfies the non-convex constraint.

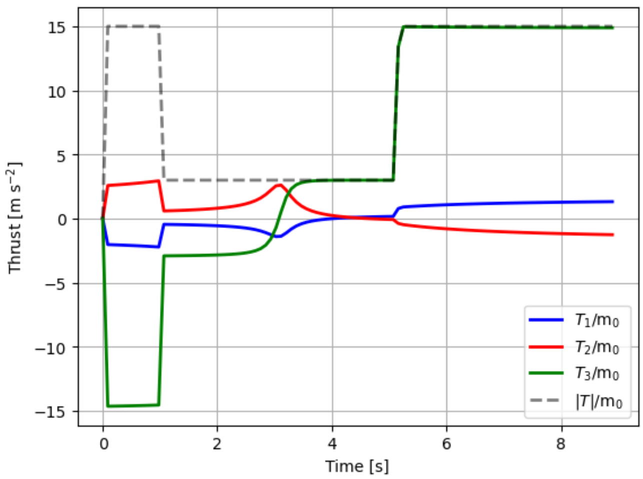

Figure 2: Magnitude of the thrust vector components as a function of time.

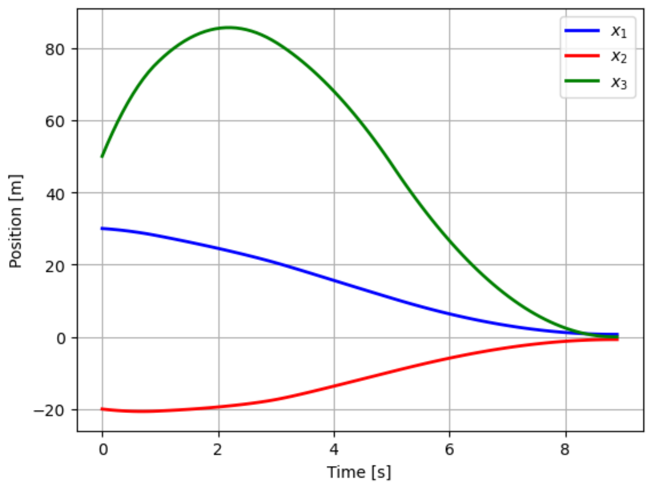

Figure 3: Position as a function of time. The vehicle reaches the target successfully at the final time.

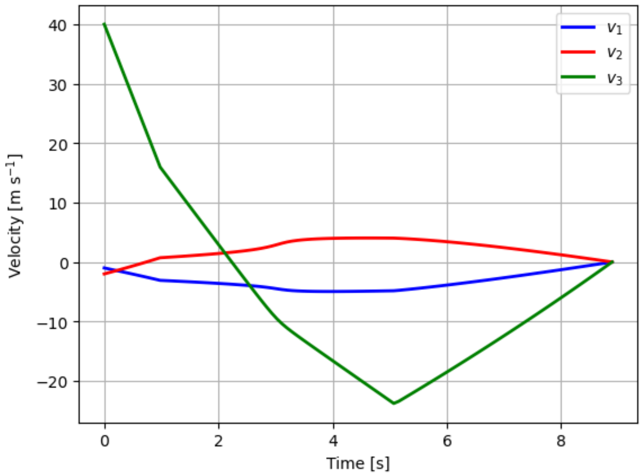

Figure 4: Velocity of the vehicle as a function of time. The vehicle reaches the origin with nearly zero velocities.

Method.	**Value**
Convex	1.134 s
Non-convex	1.433 s

Param.	**Value**
g	$9.81\ \mathrm{m\,s^{-2}}$
$\mathbf{r}_0$	$[50\ 120]^\top$ m
$\mathbf{v}_0$	$[0\ 0]^\top$ m s^{-1}
$\mathbf{r}_f$	$[0\ 0]^\top$ m
$\mathbf{v}_f$	$[0\ 0]^\top$ m s^{-1}
m_0	100 kg
$T_{\min}/m_0$	$3\ \mathrm{m\,s^{-2}}$
$T_{\max}/m_0$	$15\ \mathrm{m\,s^{-2}}$
α	$0\ \mathrm{s\,m^{-1}}$
$\tau_{\max}$	$5\ \mathrm{rad\,s^{-2}}$
N	50

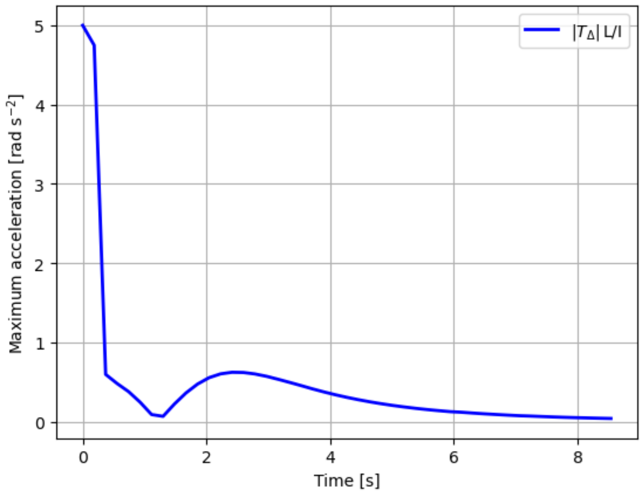

Figure 5: The required angular acceleration is initially very high. This would not be feasible for a system with a large moment of inertia. Thus, the vehicle should already start with initial conditions that facilitate the landing procedure. A decision algorithm should decide when to ignite the engine.

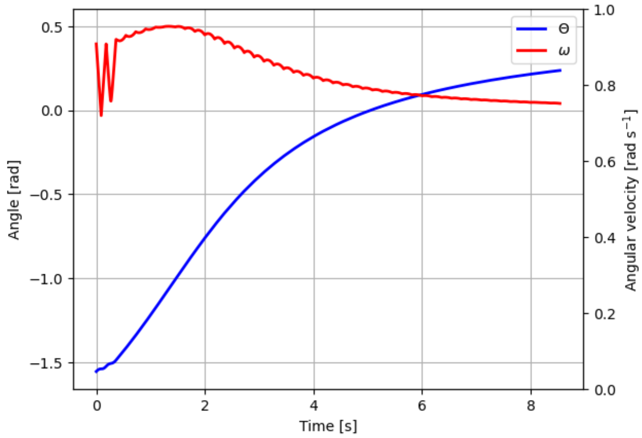

Figure 6: Angle and angular velocity of the vehicle as a function of time. Initially, tracking the nominal attitude requires a large torque. It also requires more time than Δt. This is the reason why we introduced flexibility in the final time of the attitude MPC. The reference function is not natural for a second-order system, which leads to chattering in the initial period of the flight. This demonstrates that the reference trajectory is suboptimal for the attitude. However, in this case, the initial conditions do not represent reality. The vehicle usually ignites its engines when there is a high negative velocity in the vertical direction, which would lead to only small angles of the thrust vector. In the final part of the graph, chattering disappears and the trajectory is smoother and more suitable for the attitude control. One could add an additional constraint to the thrust angle. This is suggested by Açıkmeşe *et al.* However, lossless convexification can fail, which we encounter in (7). This is also observed by the author [4].

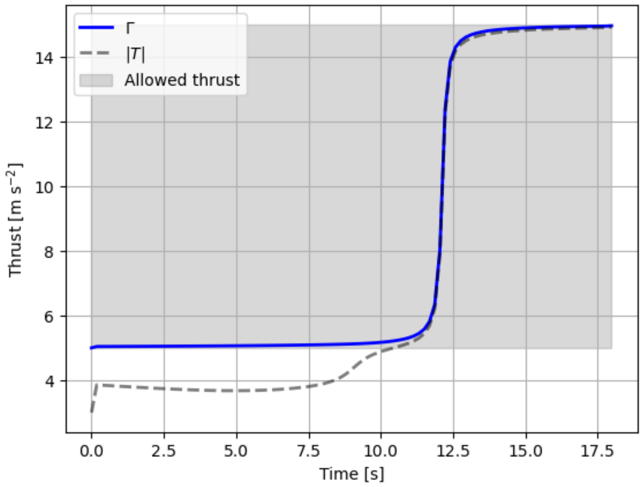

Figure 7: Failure of lossless convexification. By setting the vertical component of the thrust vector to be non-negative, the thrust magnitude does not satisfy the non-convex constraint anymore. We verified this problem on both Pyomo and CasADi. We also tested this case under linear dynamics, which still generated this solution.

5 Source code

The code can be found on the following GitHub repository:
`https://github.com/PatrissTV/MPC_Rocket_Landing`

References

[1] B. Açıkmeşe, J. M. Carson, and L. Blackmore, "Lossless convexification of nonconvex control bound and pointing constraints of the soft landing optimal control problem," *IEEE Transactions on Control Systems Technology*, vol. 21, no. 6, pp. 2104–2113, 2013.

[2] M. Sagliano, M. Dumke, and S. Theil, "Simulations and flight tests of a new nonlinear controller for the eagle lander," *Journal of Spacecraft and Rockets*, vol. 56, no. 1, pp. 259–272, 2019.

[3] J. Rawlings, D. Mayne, and M. Diehl, *Model Predictive Control: Theory, Computation, and Design*, 01 2017.

[4] B. Acikmese and S. R. Ploen, "Convex programming approach to powered descent guidance for mars landing," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 5, pp. 1353–1366, 2007.

[5] D. Malyuta, T. P. Reynolds, M. Szmuk, T. Lew, R. Bonalli, M. Pavone, and B. Açıkmeşe, "Convex optimization for trajectory generation: A tutorial on generating dynamically feasible trajectories reliably and efficiently," *IEEE Control Systems Magazine*, vol. 42, no. 5, pp. 40–113, 2022.