

# Comparative Analysis of FitzHugh-Nagumo and Hindmarsh-Rose Neural Models

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## 1 Introduction

This project undertakes a comprehensive analysis and comparison of two neural models: the FitzHugh-Nagumo model, better from a biological plausibility perspective, and the Hindmarsh-Rose model, but in a simplified version, removing the equation for the adaptation current, to perform an appropriate comparison with the FHN model, reducing also the computational cost. The analysis consisted in an exploration of their dynamics, stability characteristics, and responses to external stimuli, highlighting the distinct attributes and overlapping features that characterize the behaviors of these models in neuronal contexts.

## 2 Methods

In our study, we conducted a thorough examination and comparison of the FitzHugh-Nagumo (FHN) and Hindmarsh-Rose (HR) models, delving into their ability to accurately simulate essential aspects of neuronal behavior. Our analytical journey commenced with the FHN model, where we meticulously explored the nullclines—curves in the phase space where the rate of change for each variable is zero. This examination was crucial for visualizing the system’s behavior over time, enabling us to identify equilibrium points and assess stability. Furthermore, we investigated the trajectories in the phase space to understand how the system evolves from various initial conditions, influenced by changes in membrane potential, recovery variable, and external current. Transitioning to the HR model, we applied a similar approach to analyze its nullclines and trajectories, assessing the model’s dynamics under diverse conditions. This analysis was augmented by a stability study, using phase planes to observe how variations in external current affect the system’s dynamics. Through iterative examinations, we pinpointed critical Iext values that indicate significant shifts in the model’s behavior, thus identifying regions of stability and instability. Additionally, we explored the HR model’s response to time-dependent external currents, focusing on its ability to generate action potentials and adapt to fluctuating input signals. This comprehensive methodological framework allowed us to draw meaningful comparisons between the FHN and HR models, highlighting their respective strengths and limitations in replicating complex neuronal dynamics.

## 3 Results

In our study of the FitzHugh-Nagumo (FHN) and Hindmarsh-Rose (HR) models, we delved into how these frameworks simulate neuronal dynamics, particularly in response to external currents. The FHN model, with its straightforward approach, provides a clear visualization of neuronal stability through a single stable equilibrium. This model effectively demonstrates basic neuronal responses, with its recovery and potential nullclines predicting a predictable, stable behavior across a range of parameters. Transitioning to the more complex HR model, we encountered a richer tapestry of neuronal behaviors. This model’s

capability to oscillate between values in response to a highly negative current (-11A) or to exhibit damped oscillations under a positive current (14A) showcases its versatility. The HR model adeptly mirrors the nuanced responses of neurons to varying external stimuli, including the transition from spiking activity to near-static equilibrium with sudden current changes. The comparative analysis highlights the HR model's superior ability to capture a wider array of neuronal dynamics, from oscillatory patterns indicative of rhythmic firing to the subtleties of adaptation and response to stimuli. While the FHN model serves well for illustrating basic neuronal behaviors, the HR model offers a deeper dive into the complex mechanisms underlying neuronal activity, making it a more encompassing tool for neuroscientific exploration. This study reinforces the importance of selecting the appropriate model based on the specific dynamics and phenomena under investigation in neuroscience research.

## 4 Conclusion

This comparative study of the Fitzhugh-Nagumo and Hindmarsh-Rose models underscores the importance of model selection in neuroscience research. Depending on the specific aspect of neuronal behavior under investigation, researchers must choose the model that best aligns with their objectives, whether it be for its simplicity and clarity or for its detailed and comprehensive representation of neuronal dynamics.