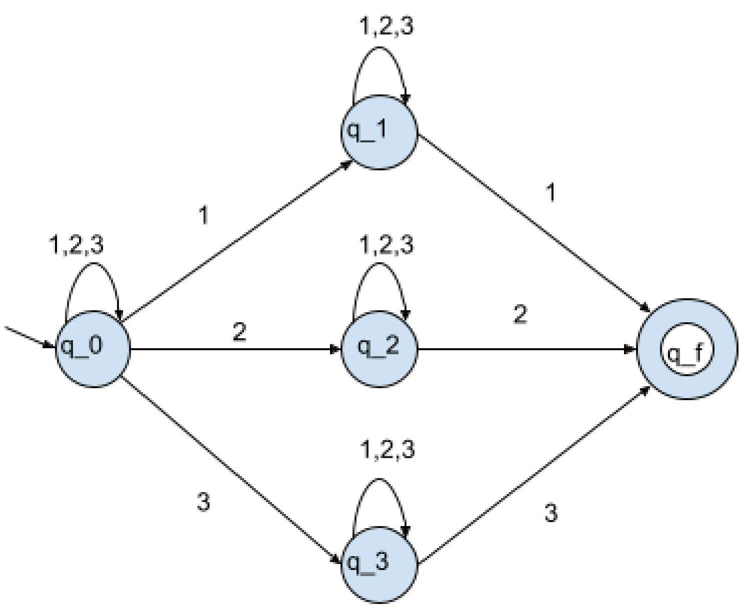
FINITE AUTOMATA (FA)

1. Given the FA: $M = (Q, \Sigma, \delta, q_0, F)$, $Q = \{q_0,\ q_1,\ q_2,\ q_3,\ q_f\}$, $\Sigma = \{1,\ 2,\ 3\}$, $F = \{q_f\}$,

δ	1	2	3
q_0	$\{q_0,\ q_1\}$	$\{q_0,\ q_2\}$	$\{q_0,\ q_3\}$
q_1	$\{q_1,\ q_f\}$	$\{q_1\}$	$\{q_1\}$
q_2	$\{q_2\}$	$\{q_2,\ q_f\}$	$\{q_2\}$
q_3	$\{q_3\}$	$\{q_3\}$	$\{q_3,\ q_f\}$
q_f	$\oslash$	$\oslash$	$\oslash$

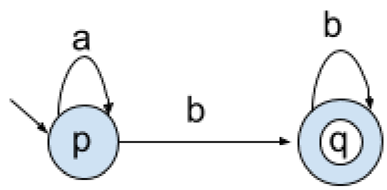
Prove that $w = 12321 \in L(M)$

Sol.: B: [redacted]



$(q_0,\ 12321) \vdash (q_1, 2321) \vdash^{3} (q_1, 1) \vdash (q_f, \varepsilon)$ => $(q_0, w) \vdash^{*} (q_f, \varepsilon)$ =>
$w \in L(M)$

2. Find the language accepted by the FA below.



Sol.: B: [redacted]

$L = \{a^n b^m \mid n \in N,\ m \in N^*\}$
? $L = L(M)$

1. $?\ L \subseteq L(M)$ (all sequences of that shape are accepted by M)
   $\forall n \in N,\ \forall m \in N^*\ \ a^n b^m \in L(M)$
   Let $n \in N,\ m \in N^*$
   $(p,\ a^n b^m) \vdash_{a}^{n} (p,\ b^m) \vdash (q,\ b^{m-1}) \vdash_{b}^{m-1} (q,\ \varepsilon)$ => $a^n b^m \in L(M)$

   a). $(p, a^n) \vdash^{n} (p,\ \varepsilon)$, $\forall n \in N$ oki

   b). $(q,\ b^k) \vdash^{k} (q,\ \varepsilon)$, $\forall k \in N$

   $a).\ P(n):\ (p,\ a^n) \vdash^{n} (p,\ \varepsilon)$

   $P(0):\ (p,\ \varepsilon) \vdash^{0} (p,\ \varepsilon)$, P(0) - true
   ? $P(k) - true \ => P(k+1) - true$

   $P(k) - true$ => $(p,\ a^k) \vdash^{k} (p,\ \varepsilon)$ (induction hypothesis)

   k k+1

$(p,\ a^{k+1}) \mid\!-\ (p,\ a^{k}) \mid\!-\ (p,\ \varepsilon)$ => $(p,\ a^{k+1}) \mid\!-\ (p,\ \varepsilon)$ => P(k+1) -true

Ind. hyp.

Similarly, we demonstrate b.

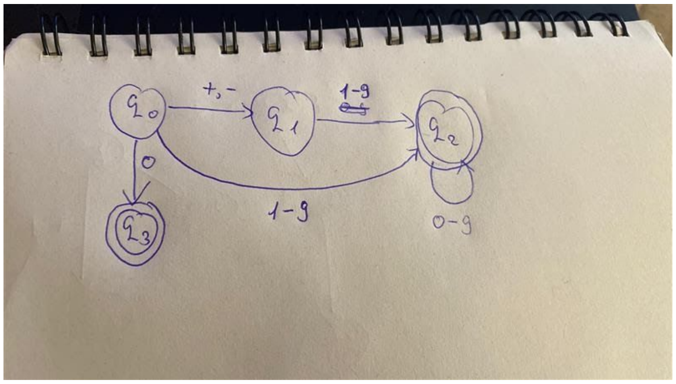
2. $?\quad L(M) \subseteq L$ (M does not accept anything else but sequences of that shape)

   In order to reach the final state q from the initial state p, we should read at least one b. Before the mandatory b, we can read any natural number of a's, while remaining in state p, and after the mandatory b we can read any natural number of b's, while remaining in state q. Therefore, M accepts only sequences of the shape $a^n b b^k,\ n, k\ \in N$

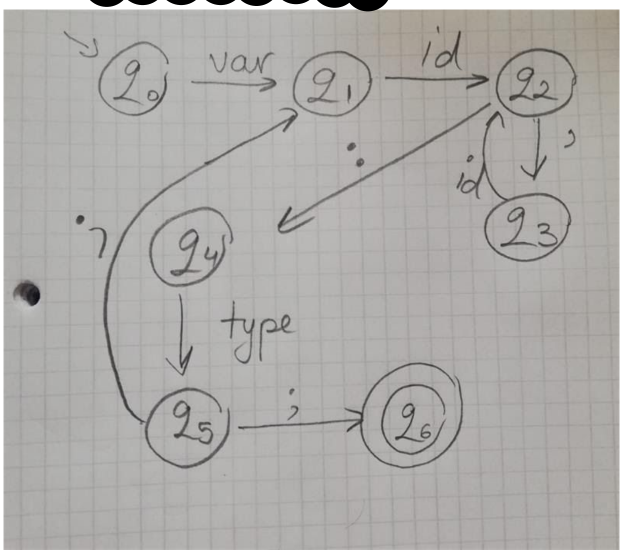
   Obs. In order for such a reasoning to count as proof, you should make sure that you have covered all paths from initial state to final states.

3. Build FAs that accept the following languages
   a. Integer numbers
   b. Variable declarations (Pascal, C, ...)
   c. $L = \{0^n 1^m 0^q \mid n, m\ \in N^*,\ q\ \in N\}$
   d. $L = \{0(01)^n \mid n\ \in N\}$
   e. $L = \{c^{3n} \mid n\ \in N^*\}$
   f. The language over $\Sigma = \{0, 1\}$ having the property that all sequences have at least two consecutive 0's.

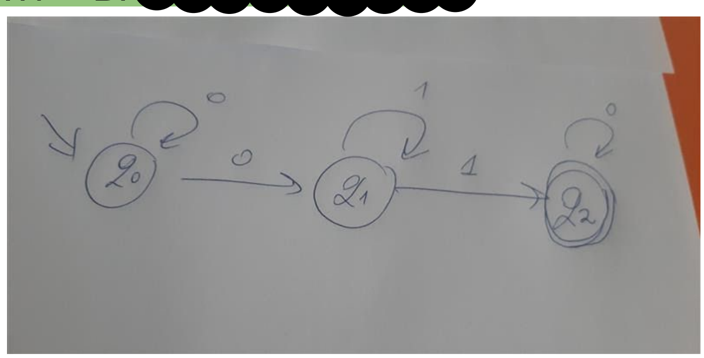
      a. IW -> [redacted]



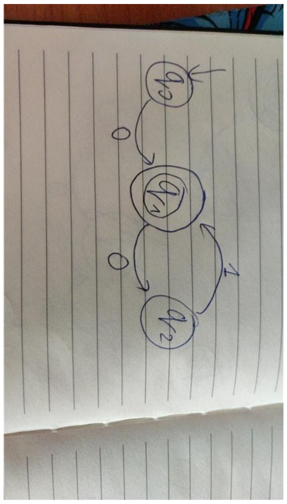
b. IW->[redacted]



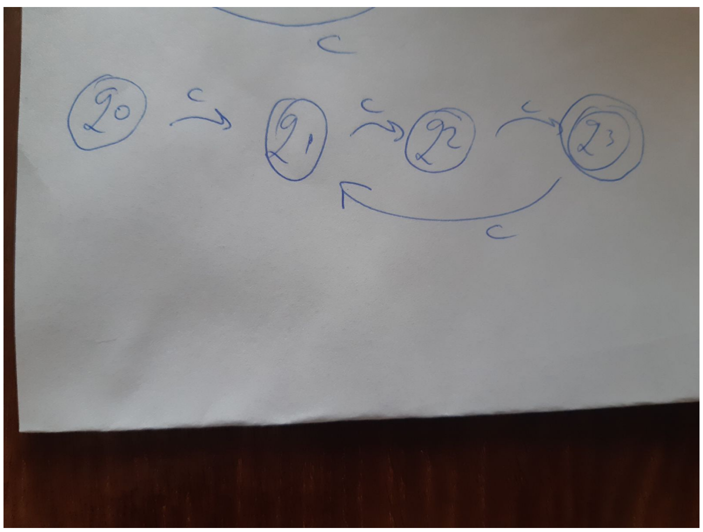
c. IW ->B:[redacted]



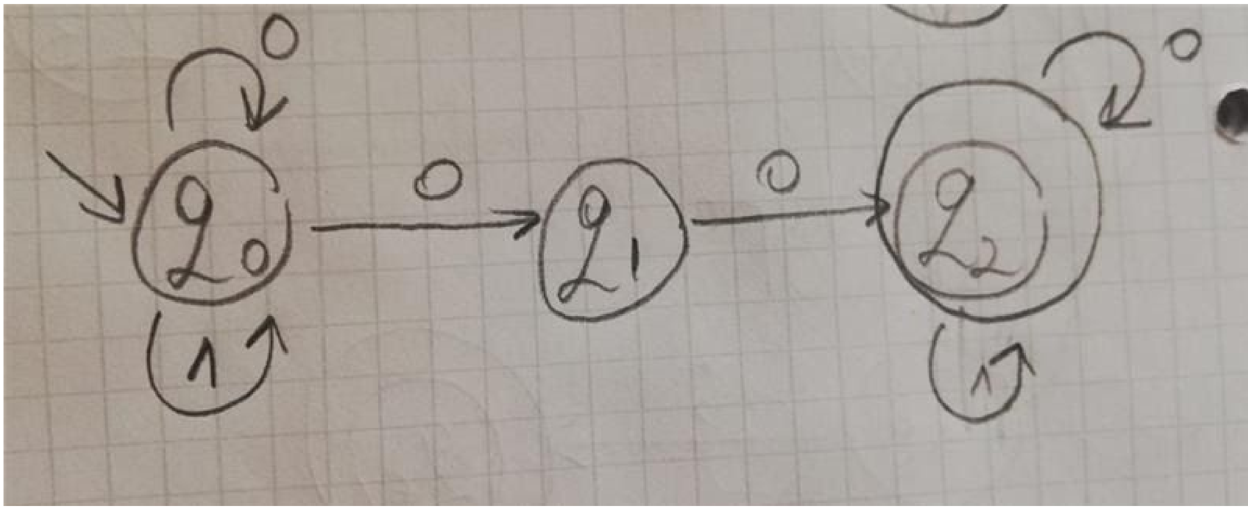
d. IW ->B:[redacted]



e. IW ->B: [redacted] (+mark q as initial state)



f. IW->B: [redacted]



0
q0
1
0
q1
0
q2
0
1