FA ⇔ RG ⇔ RE

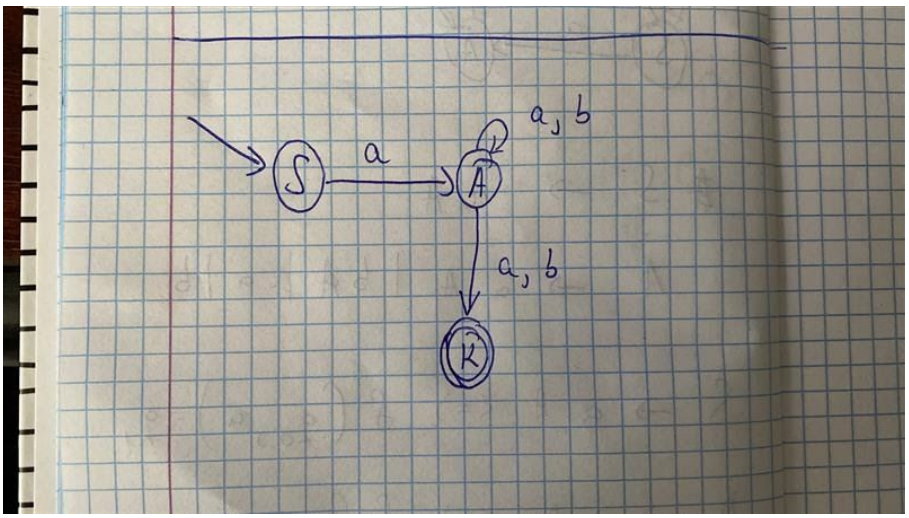
I) FA ⇔ RG (team work)

T1. Given the regular grammar $G = (\{S, A\},\ \{a, b\},\ P,\ S)$

$$P:\ S \to aA$$
$$A \to aA \mid bA \mid a \mid b\,,$$

build the equivalent FA.

Sol.:



T2. Given the regular grammar $G = (\{S, A\},\ \{a, b\},\ P,\ S)$

$$P:\ S \to \varepsilon \mid aA$$
$$A \to aA \mid bA \mid a \mid b\,,$$

build the equivalent FA.

Sol.:

$M = (Q,\ \Sigma,\ \delta,\ q_0,\ F)$
$Q = \{S,\ A,\ K\},\ q_0 = S,\ F = \{K,\ S\},\ \Sigma = \{a,\ b\}$

δ	a	b
S	$\{A\}$	$\oslash$
A	$\{A,\ K\}$	$\{A,\ K\}$
K	$\oslash$	$\oslash$

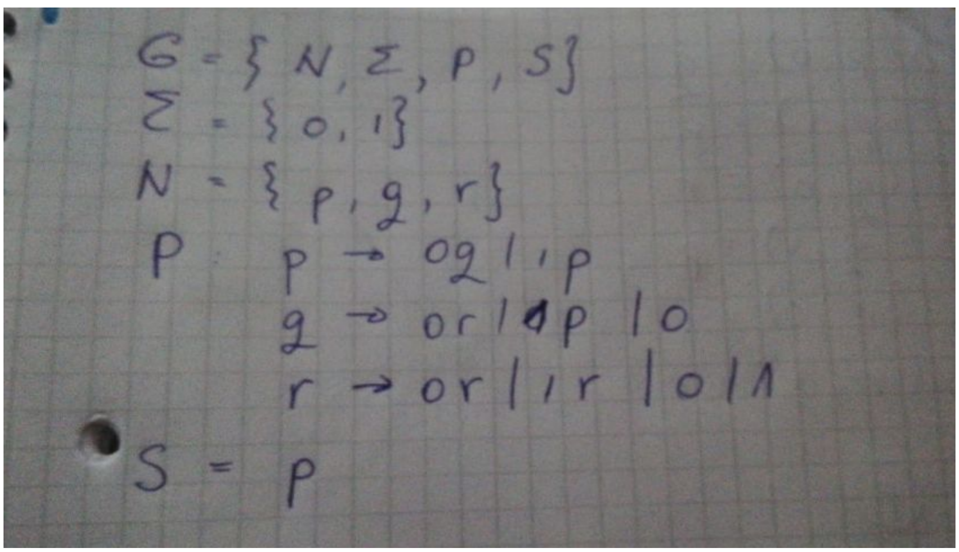
T3. Given the following FA $M = (Q, \Sigma, \delta, q_0, F)$

$$Q = \{p, q, r\},\ q_0 = p,\ F = \{r\},\ \Sigma = \{0, 1\}$$

δ	0	1
p	q	p
q	r	p
r	r	r

build the equivalent right linear grammar.

Sol.:



$G = \{N, \Sigma, P, S\}$

$\Sigma = \{0, 1\}$

$N = \{p, q, r\}$

P: $p \to 0q \mid 1p$

$q \to 0r \mid 1p \mid 0$

$r \to 0r \mid 1r \mid 0 \mid \Lambda$

$S = p$

T4. Given the following FA $M = (Q, \Sigma, \delta, q_0, F)$

$$Q = \{p, q, r\},\ q_0 = p,\ F = \{p, r\},\ \Sigma = \{0, 1\}$$

δ	0	1
p	q	p
q	r	p
r	r	r

build the equivalent right linear grammar.

Sol.:

$G = \{N, \Sigma, P, S\}$

$\Sigma = \{0, 1\}$

$N = \{p, q, r\}$

$S = p$

$P: \quad p \to 0q \mid 1p \mid 1 \mid \varepsilon$

$\quad\quad q \to 0r \mid 1p \mid 0 \mid 1$

$\quad\quad r \to 0r \mid 1r \mid 0 \mid 1.$

II) RG ⇔ RE

1. Give the RG corresponding to the following RE $0(0+1)^*1$. //[redacted]

$0: \quad G_1 = (\{S_1\}, \{0,1\}, \{S_1 \to 0\}, S_1)$

$1: \quad G_2 = (\{S_2\}, \{0,1\}, \{S_2 \to 1\}, S_2)$

$0+1: \quad G_3 = (\{S_1, S_2, S_3\}, \{0,1\}, \{S_1 \to 0, S_2 \to 1, S_3 \to 0 \mid 1\}, S_3)$

$\quad\quad G'_3 = (\{S_3\}, \{0,1\}, \{S_3 \to 0 \mid 1\}, S_3)$

$(0+1)^*: \quad G_4 = (\{S_3\}, \{0,1\}, \{S_3 \to 0 \mid 1, S_3 \to 0S_3 \mid 1S_3, S_3 \to \varepsilon\})$

$\quad\quad G'_4 = (\{S_3\}, \{0,1\}, \{S_3 \to 0S_3 \mid 1S_3 \mid \varepsilon\}, S_3)$! not regular

$0(0+1)^*: \quad G_5 = (\{S_1, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 \mid 1S_3 \mid \varepsilon\}, S_1)$

! not regular

$0(0+1)^*1$:

$G_6 = (\{S_1, S_2, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 \mid 1S_3 \mid S_2, S_2 \to 1\}, S_1)$! not regular

$G'_6 = (\{S_1, S_3\}, \{0,1\}, \{S_1 \to 0S_3, S_3 \to 0S_3 \mid 1S_3 \mid 1\}, S_1)$

(TW)

2. Give the RE corresponding to the following grammar

$G = (\{S, A, B\},\ \{a, b\},\ P,\ S)$

$P:\ S \to aA$

$A \to aA \mid bB \mid b$

$B \to bB \mid b$

Sol.: T4

$S = aA$

$A = aA + bB + b$

$B = bB + b \Rightarrow B = b^* b = b^+$

$\Rightarrow A = aA + B$

$A = a^* B = a^* b^+$

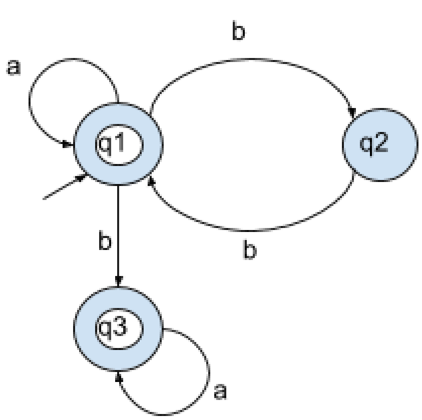
$S = aA \Rightarrow S = a a^* b = a^+ b^+$

III) FA ⇔ RE

1. Give the FA corresponding to the following RE $01(1+0)^*1^*$.

#board, pdf attached to Seminar 7 meet in MSTeams

2. Give the regular expression corresponding to the FA below.



//[redacted]

$q_1 = \varepsilon + q_1 a + q_2 b$

$q_2 = q_1 b$

$q_3 = q_1 b + q_3 a$

$X = Xa + b$ => $X = ba^*$ *solution* [redacted]

$q_3 = q_1 ba^*$

$q_1 = \varepsilon + q_1 a + q_1 bb = q_1(a + bb) + \varepsilon$ => $q_1 = (a + bb)^*$ => $q_3 = (a + bb)^* ba^*$

$RE = q_1 + q_3 = (a + bb)^* + (a + bb)^* ba^* = (a + bb)^*(\varepsilon + ba^*)$