GRAMMARS

1. Given the grammar $G = (N,\ \Sigma,\ P,\ S)$

   $N = \{S, C\}$

   $\Sigma = \{a, b\}$

   $P:\ S\ \to ab \mid aCSb$

   $\quad\quad C\ \to S \mid bSb$

   $\quad\quad CS\ \to b$

   ? Prove that $w\ =\ ab(ab^2)^2\ \ \in L(G)$

   Obs.: $(ab)^2\ =\ abab\ \neq a^2b^2\ = aabb$

   Sol.: **B:** [redacted]

   $S \Rightarrow_2 aCSb \Rightarrow_4 abSbSb \Rightarrow^2{}_1\ ababbabb\ = w$ => $S \Rightarrow^* w$ => $w\ \in L(G)$

2. Given the grammar $G = (N,\ \Sigma,\ P,\ S)$

   $N = \{S\}$

   $\Sigma = \{a, b, c\}$

   $P:\ S\ \to a^2S \mid bc$

   find $L(G)$.

   Sol.: **B:** [redacted]

   $L = \{a^{2n}bc \mid n\ \in N\}$

   ? L= L(G)

   1. $L\ \subseteq L(G)$

      $\forall n \in N,\ a^{2n}bc\ \in L(G)$

      $P(n)\ :\ a^{2n}bc\ \in L(G)$

      Prove that P(n) is true for any natural n, by math. Induction

      i). verification step n=0

      $S \Rightarrow bc\ \ => \ bc\ \in L(G)$ => P(0) - true

      ii). proof step P(k) => P(k+1), $k \in N$

$P(k) - true \; => \; S \Rightarrow^* \; a^{2k}bc$ (induction hypothesis)

$S \Rightarrow a^2S \Rightarrow^* \; a^2a^{2k}bc \; = a^{2k+2}bc$ => P(k+1) - true

From i) and ii) we have that $L \subseteq L(G)$

2. ? $L(G) \subseteq L$

$S \Rightarrow bc \;\; = \; a^0bc$

$\quad \Rightarrow a^2S \; \Rightarrow a^2bc$

$\qquad\qquad \Rightarrow a^4S \; \Rightarrow a^4bc$

$\qquad\qquad\qquad\qquad \Rightarrow a^6S \Rightarrow$ ...

3. Find a grammar that generates $L = \{0^n1^n2^m \mid n, m \; \in \; N^*\}$

Sol.: B: [redacted]

$N = \{S, \; C, \; D\}$

$\Sigma = \{0, \; 1, \; 2\}$

$P : \; S \; \rightarrow CD$

$\qquad C \rightarrow 0C1 \mid 01$

$\qquad D \rightarrow 2D \mid 2$

$? \; L(G) \; = \; L$

1.? $L \; \subseteq L(G)$

$\qquad ? \; \forall n \, , \; m \; \in N^*, \; 0^n1^n2^m \;\; \in \; L(G)$

Let $n, \; m \; \in N^*$ 2

$$S \Rightarrow CD \overset{n}{\underset{(a)}{\Rightarrow}} \; 0^n1^nD \overset{m}{\underset{(b)}{\Rightarrow}} 0^n1^n2^m$$

(a): $\forall n \; \in N^*, \; C \overset{n}{\Rightarrow} 0^n1^n$

(b): $\forall m \; \in N^*, \; D \overset{m}{\Rightarrow} 2^m$

2. ? $L(G) \subseteq L$

-tree for C
-tree for D
S can only generate concatenations of C and D ...