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| Started on   | Tuesday, 8 November 2022, 7:04 PM    |
| State        | Finished                             |
| Completed on | Wednesday, 9 November 2022, 12:24 AM |
| Time taken   | 5 hours 20 mins                      |
| Grade        | 1.29 out of 1.50 (86%)               |

Question 1

Partially correct

Mark 0.69 out of 0.75

Use the Miller-Rabin test to decide whether the number  $n = 2193$  is prime or not. Check for 3 different bases only if necessary.

Important note: All answer boxes should be filled in using the convention that those not applicable must be filled in with  $x$ . All numbers must be filled in as positive numbers mod  $n$ .

Solution.

Decomposition:

$s = 4$  ✓  $t = 137$  ✓  $t$  in binary = 10001001 ✓

Iteration  $k = 1$  for  $a = 2$  (results mod  $n$ ):

$2^{(2^0)} = 2$  ✓  $2^{(2^1)} = 4$  ✓  $2^{(2^2)} = 16$  ✓  $2^{(2^3)} = 256$  ✓  $2^{(2^4)} = 1939$  ✓  
 $2^{(2^5)} = 919$  ✓  $2^{(2^6)} = 256$  ✓  $2^{(2^7)} = 1939$  ✓  $2^{(2^8)} = x$  ✓  $2^{(2^9)} = x$  ✓

$2^t = 1532$  ✓  $2^{2t} = 514$  ✓  $2^{2^2t} = 1036$  ✓  $2^{2^3t} = 919$  ✓  $2^{2^4t} = x$  ✗

Iteration  $k = 2$  for  $a = 3$  (results mod  $n$ ):

$3^t = x$  ✓  $3^{2t} = x$  ✓  $3^{2^2t} = x$  ✓  $3^{2^3t} = x$  ✓  $3^{2^4t} = x$  ✓

Iteration  $k = 3$  for  $a = 5$  (results mod  $n$ ):

$5^t = x$  ✓  $5^{2t} = x$  ✓  $5^{2^2t} = x$  ✓  $5^{2^3t} = x$  ✓  $5^{2^4t} = x$  ✓

Conclusion:

$n$  is prime (yes/no) = no ✓

Question 2

Partially correct

Mark 0.60 out of 0.75

Use the Miller-Rabin test to decide whether the number  $n = 1657$  is prime or not. Check for 3 different bases only if necessary.

Important note: All answer boxes should be filled in using the convention that those not applicable must be filled in with  $x$ . All numbers must be filled in as positive numbers mod  $n$ .

Solution.

Decomposition:

$s = 3$  ✓  $t = 207$  ✓  $t$  in binary = 11001111 ✓

Iteration  $k = 1$  for  $a = 2$  (results mod  $n$ ):

$2^{(2^0)} = 2$  ✓  $2^{(2^1)} = 4$  ✓  $2^{(2^2)} = 16$  ✓  $2^{(2^3)} = 256$  ✓  $2^{(2^4)} = 913$  ✓  
 $2^{(2^5)} = 98$  ✓  $2^{(2^6)} = 1319$  ✓  $2^{(2^7)} = 1568$  ✓  $2^{(2^8)} = x$  ✓  $2^{(2^9)} = x$  ✓

$2^t = 874$  ✓  $2^{2t} = -1$  ✗  $2^{2^2t} = 1$  ✓  $2^{2^3t} = 1$  ✓  $2^{2^4t} = 1$  ✗

Iteration  $k = 2$  for  $a = 3$  (results mod  $n$ ):

$3^t = 1$  ✓  $3^{2t} = 1$  ✓  $3^{2^2t} = 1$  ✓  $3^{2^3t} = 1$  ✓  $3^{2^4t} = 1$  ✗

Iteration  $k = 3$  for  $a = 5$  (results mod  $n$ ):

$5^t = 239$  ✓  $5^{2t} = 783$  ✓  $5^{2^2t} = -1$  ✗  $5^{2^3t} = 1$  ✓  $5^{2^4t} = 1$  ✗

Conclusion:

$n$  is prime (yes/no) = yes ✓

