

Started on	Monday, 14 November 2022, 3:35 PM
State	Finished
Completed on	Monday, 14 November 2022, 4:10 PM
Time taken	34 mins 57 secs
Grade	1.50 out of 1.50 (100%)

Question 1

Correct

Mark 0.75 out of 0.75

Use Fermat's method to determine the decomposition of the number  $n = 6811$  into two factors.

Important note: All answer boxes should be filled in using the convention that those not applicable must be filled in with x.

Solution.

Initialization:

$t_0 = \lfloor \sqrt{n} \rfloor =$   ✓

Iterations:

$t = t_0 + 1: t^2 - n =$	<input type="text" value="78"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 2: t^2 - n =$	<input type="text" value="245"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 3: t^2 - n =$	<input type="text" value="414"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 4: t^2 - n =$	<input type="text" value="585"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 5: t^2 - n =$	<input type="text" value="758"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 6: t^2 - n =$	<input type="text" value="933"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 7: t^2 - n =$	<input type="text" value="1110"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 8: t^2 - n =$	<input type="text" value="1289"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 9: t^2 - n =$	<input type="text" value="1470"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 10: t^2 - n =$	<input type="text" value="1653"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 11: t^2 - n =$	<input type="text" value="1838"/> ✓	perfect square (yes/no)	<input type="text" value="no"/> ✓
$t = t_0 + 12: t^2 - n =$	<input type="text" value="2025"/> ✓	perfect square (yes/no)	<input type="text" value="yes"/> ✓
$t = t_0 + 13: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓
$t = t_0 + 14: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓
$t = t_0 + 15: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓
$t = t_0 + 16: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓
$t = t_0 + 17: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓
$t = t_0 + 18: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓
$t = t_0 + 19: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓
$t = t_0 + 20: t^2 - n =$	<input type="text" value="x"/> ✓	perfect square (yes/no)	<input type="text" value="x"/> ✓

Values:

$s =$   ✓  $t =$   ✓

Conclusion:

The obtained two factors of  $n$  are (in increasing order!)  ✓ and  ✓ .

Question **2**

Correct

Mark 0.75 out of 0.75

Use Pollard's  $\rho$  method with  $x_0 = 2$  and  $f(x) = x^2 + 1$  to determine the decomposition of the number  $n = 5539$  into two factors.

Important note: All answer boxes should be filled in using the convention that those not applicable must be filled in with  $x$ .  
All numbers must be filled in as positive numbers mod  $n$ .

Solution.

Iterations (results mod  $n$ ):

$x_1 =$	<input type="text" value="5"/>	✓	$x_2 =$	<input type="text" value="26"/>	✓	$( x_2 - x_1 , n) =$	<input type="text" value="1"/>	✓
$x_3 =$	<input type="text" value="677"/>	✓	$x_4 =$	<input type="text" value="4132"/>	✓	$( x_4 - x_2 , n) =$	<input type="text" value="1"/>	✓
$x_5 =$	<input type="text" value="2227"/>	✓	$x_6 =$	<input type="text" value="2125"/>	✓	$( x_6 - x_3 , n) =$	<input type="text" value="1"/>	✓
$x_7 =$	<input type="text" value="1341"/>	✓	$x_8 =$	<input type="text" value="3646"/>	✓	$( x_8 - x_4 , n) =$	<input type="text" value="1"/>	✓
$x_9 =$	<input type="text" value="5256"/>	✓	$x_{10} =$	<input type="text" value="2544"/>	✓	$( x_{10} - x_5 , n) =$	<input type="text" value="1"/>	✓
$x_{11} =$	<input type="text" value="2385"/>	✓	$x_{12} =$	<input type="text" value="5212"/>	✓	$( x_{12} - x_6 , n) =$	<input type="text" value="1"/>	✓
$x_{13} =$	<input type="text" value="1689"/>	✓	$x_{14} =$	<input type="text" value="137"/>	✓	$( x_{14} - x_7 , n) =$	<input type="text" value="1"/>	✓
$x_{15} =$	<input type="text" value="2153"/>	✓	$x_{16} =$	<input type="text" value="4806"/>	✓	$( x_{16} - x_8 , n) =$	<input type="text" value="29"/>	✓
$x_{17} =$	<input type="text" value="x"/>	✓	$x_{18} =$	<input type="text" value="x"/>	✓	$( x_{18} - x_9 , n) =$	<input type="text" value="x"/>	✓
$x_{19} =$	<input type="text" value="x"/>	✓	$x_{20} =$	<input type="text" value="x"/>	✓	$( x_{20} - x_{10} , n) =$	<input type="text" value="x"/>	✓

Conclusion:

The obtained two factors of  $n$  are (in increasing order!)  ✓ and  ✓ .