

Solve the following system using partial Gaussian elimination,  
with maximal pivoting: 
$$\begin{cases} 4x + z &= 5 \\ x + 4y + z &= 3 \\ x + 4z &= -10 \end{cases}$$

a) Establish the error bound for the approximation of  $f(x) = e^x$  by Taylor polynomial of 8-th degree  $(T_8 f)(x)$  on the interval  $|x| \leq 1$ , with  $x_0$  being the center of the interval.

b) Write the expression of Taylor polynomial  $(T_8 f)(x)$  from a).

Find the coefficients  $A$ ,  $B$  and  $C$  of the following quadrature formula:

$$\int_0^1 f(x)dx = Af'(0) + Bf''(0) + Cf(1) + R(f)$$

Solve the following using partial Gaussian elimination, with maximal pivoting:

$$\begin{cases} 2x_1 + 4x_2 - 2x_3 &= 2 \\ 4x_1 + 9x_2 - 3x_3 &= 8 \\ -2x_1 - 3x_2 + 7x_3 &= 10 \end{cases}$$

Find the polynomial interpolating the function  $f(x) = e^{-x^2}$  at the nodes  $x_0 = -1$  double,  $x_1 = 0$  double and  $x_2 = 1$  double.