Analiza numeryczna

Wyklad 7. Funkcje sklejane

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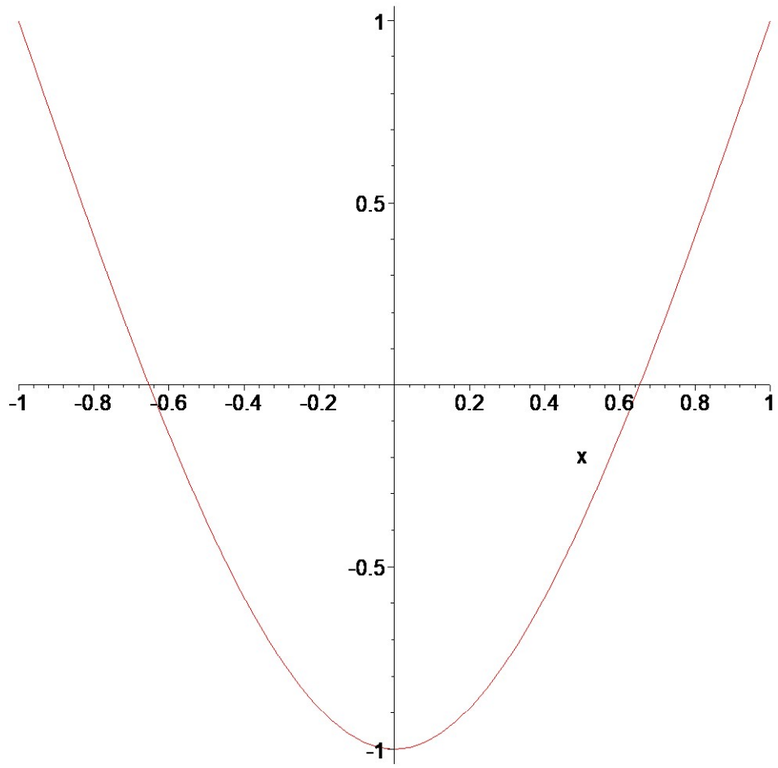
Przyklady

1. Przyklad

```
> 
> 
> restart;
> 
> spline([-1,0,1],[1,-1,1],x);
> 
```

$$\begin{cases} -1+3\,x^2+x^3 & x<0 \\ -1+3\,x^2-x^3 & \textit{otherwise} \end{cases}$$

```
> 
> 
> plot(spline([-1,0,1],[1,-1,1],x),x=-1..1);
> 
```

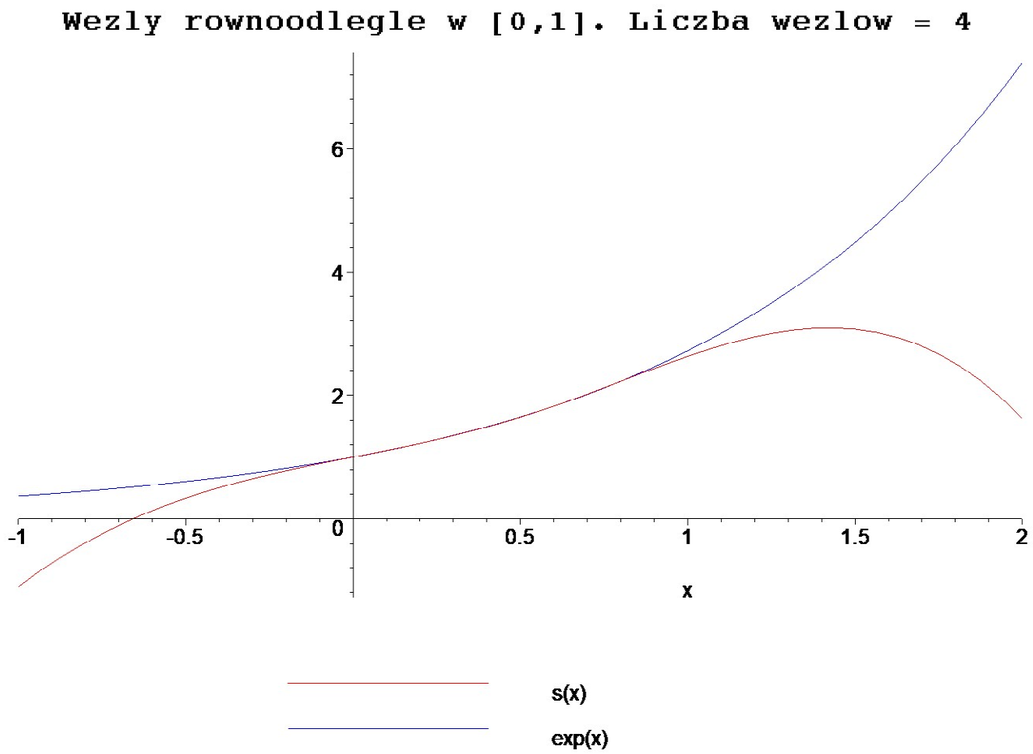


>

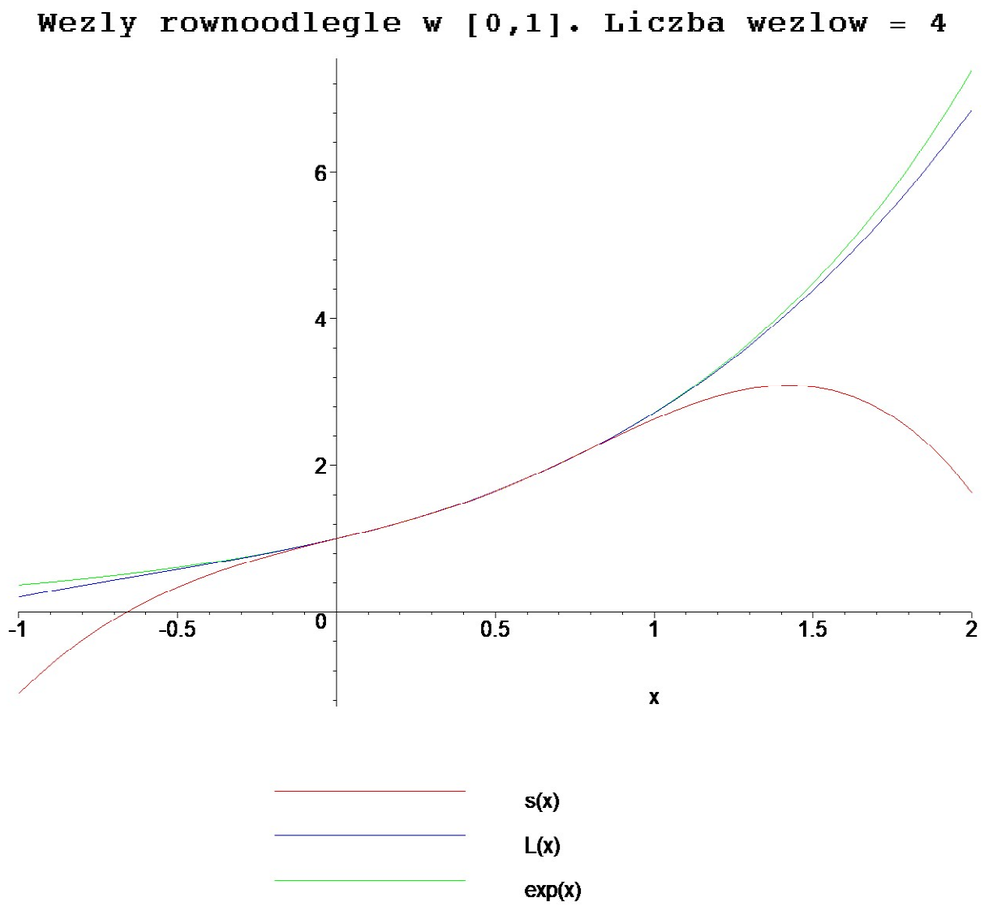
2. Przyklad

```
>
>
> restart:
>
> s:=unapply(spline([0.0,0.2,0.6,0.8],[exp(0.0),exp(0.2),exp(0.6),exp(0.8)],x),x):
>
> s(x);
>
> plot([s(x),exp(x)],x=-1..2,color=[red,blue],legend=["s(x)","exp(x)"],
       title="Wezly rownoodlegle w [0,1]. Liczba wezlow = 4",titlefont=[COURIER,BOLD,15]);
>
>
>
```

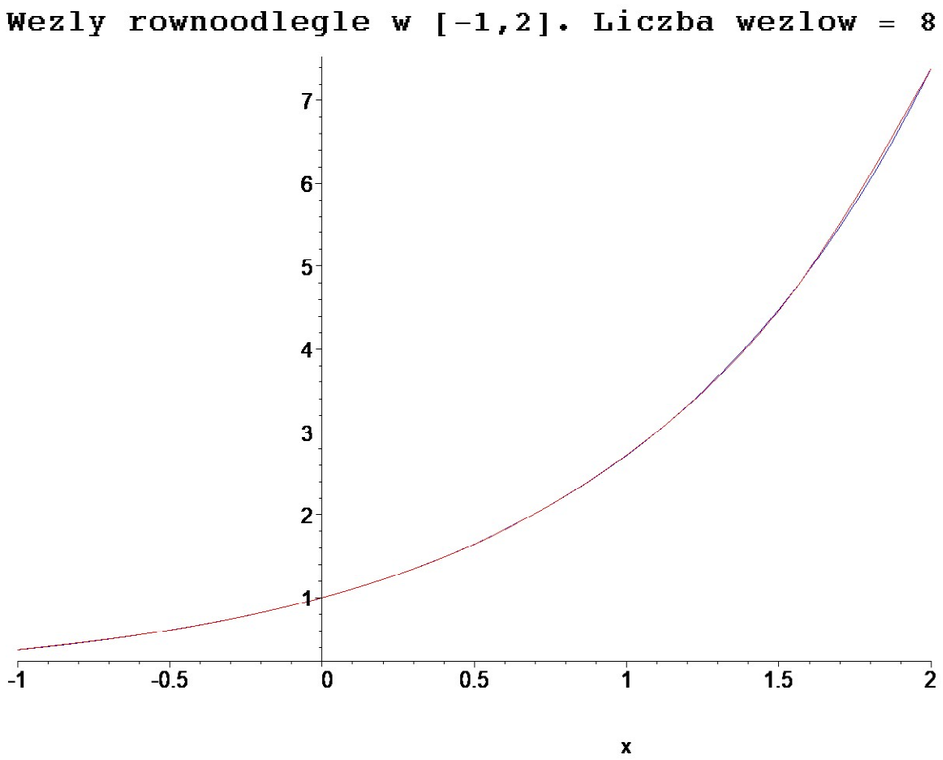
$$\begin{cases} 1. + 1.06520076400000008\,x + 1.04532564100000002\,x^3 & x < 0.2 \\ 0.9832747898 + 1.19063984100000008\,x + 0.627195384375000110\,(x-0.2)^2 + 0.376700687300000004\,(x-0.2)^3 & x < 0.6 \\ 0.698191313 + 1.87321247899999998\,x + 1.07923620937499986\,(x-0.6)^2 - 1.79872701500000010\,(x-0.6)^3 & otherwise \end{cases}$$




```
> restart:
>
> s:=unapply(spline([0.0,0.2,0.6,0.8],[exp(0.0),exp(0.2),exp(0.6),exp(0.8)],x),x):
>
> L:=unapply(interp([0.0,0.2,0.6,0.8],[exp(0.0),exp(0.2),exp(0.6),exp(0.8)],x),x):
>
> plot([s(x),L(x),exp(x)],x=-1..2,color=[red,blue,green],legend=["s(x)","L(x)","exp(x)"],
       title="Wezly rownoodlegle w [0,1]. Liczba wezlow = 4",titlefont=[COURIER,BOLD,15]);
>
>
```




```
>
>
> s:=unapply(spline([seq(evalf(-1+3*i/7),i=0..7)],[seq(evalf(exp(-1+3*i/7)),i=0..7)],x),x):
>
> plot([s(x),exp(x)],x=-1..2,color=[red,blue],legend=["s(x)","exp(x)"],
       title="Wezly rownoodlegle w [-1,2]. Liczba wezlow = 8",titlefont=[COURIER,BOLD,15]);
>
>
```

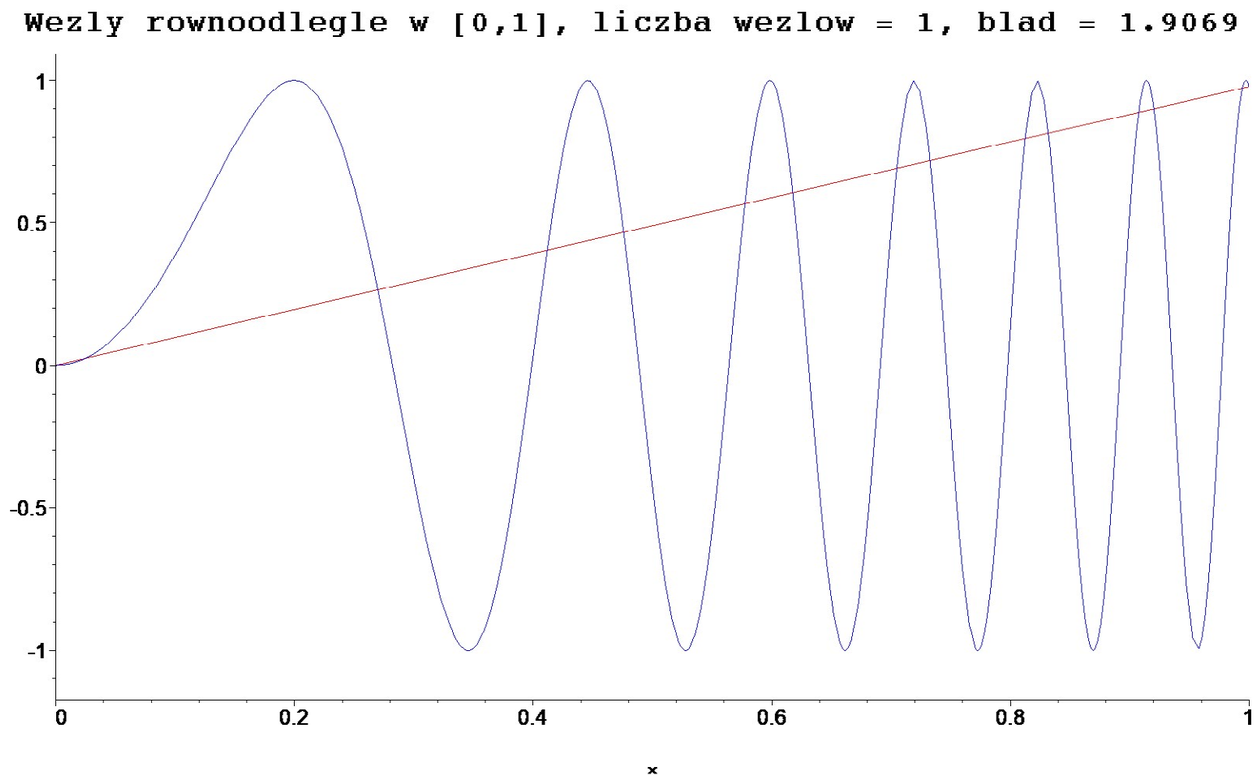



```
> 
```

3. Przyklad

```
> 
> 
> restart:
> 
> N:=45:
> 
> f:=x->sin(4*Pi^2*x^2):
> 
> wykres_f:=plot(f(x),x=0..1,color=blue,axes=frame):
> 
> for i from 1 to N
    do
      s:=unapply(spline([seq(evalf(k/i),k=0..i)],
                        [seq(evalf(f(k/i)),k=0..i)],x),x):

      blad:=max(seq(abs(evalf(s(j/100)-f(j/100))),j=0..100)):
      blads:=convert(evalf(blad,5),string):

      opis:="Wezly rownoodlegle w [0,1], liczba wezlow = "||i||", blad = "||blads:
      wykres_s:=plot(s(x),x=0..1,color=red,axes=frame):
      klatka[i]:=plots[display](wykres_f,wykres_s,title=opis,titlefont=[COURIER,BOLD,15])
```

```
    od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
>
```

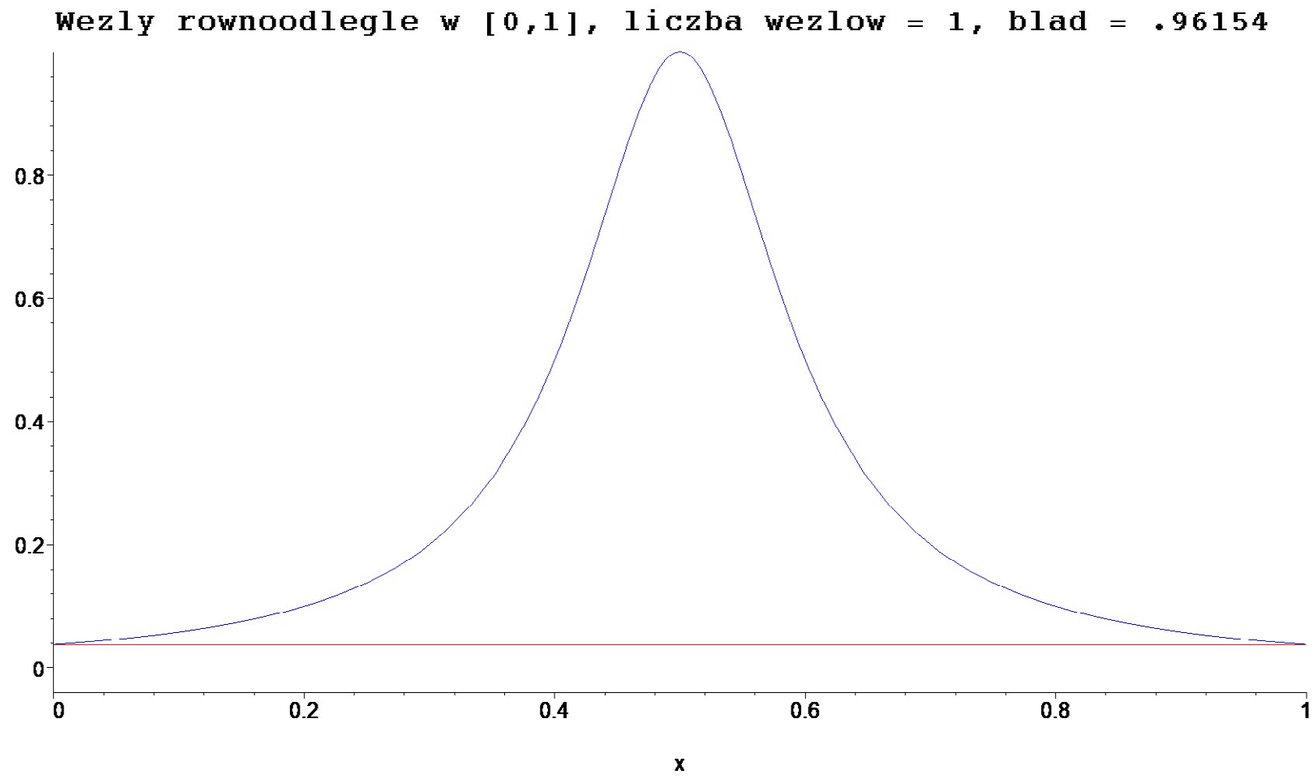



```
[ >
```

4. Przyklad

```
[ >
 >
 > restart:
 >
 > N:=25:
 >
 > f:=x->1/(1+25*(2*x-1)^2):
 >
 > wykres_f:=plot(f(x),x=0..1,color=blue,axes=frame):
 >
 > for i from 1 to N
     do
       s:=unapply(spline([seq(evalf(k/i),k=0..i)],
                         [seq(evalf(f(k/i)),k=0..i)],x),x):

       blad:=max(seq(abs(evalf(s(j/100)-f(j/100))),j=0..100)):
       blads:=convert(evalf(blad,5),string):

       opis:="Wezly rownoodlegle w [0,1], liczba wezlow = "||i||", blad = "||blads:
       wykres_s:=plot(s(x),x=0..1,color=red,axes=frame):
       klatka[i]:=plots[display](wykres_f,wykres_s,title=opis,titlefont=[COURIER,BOLD,15])
```

```
    od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
>
```



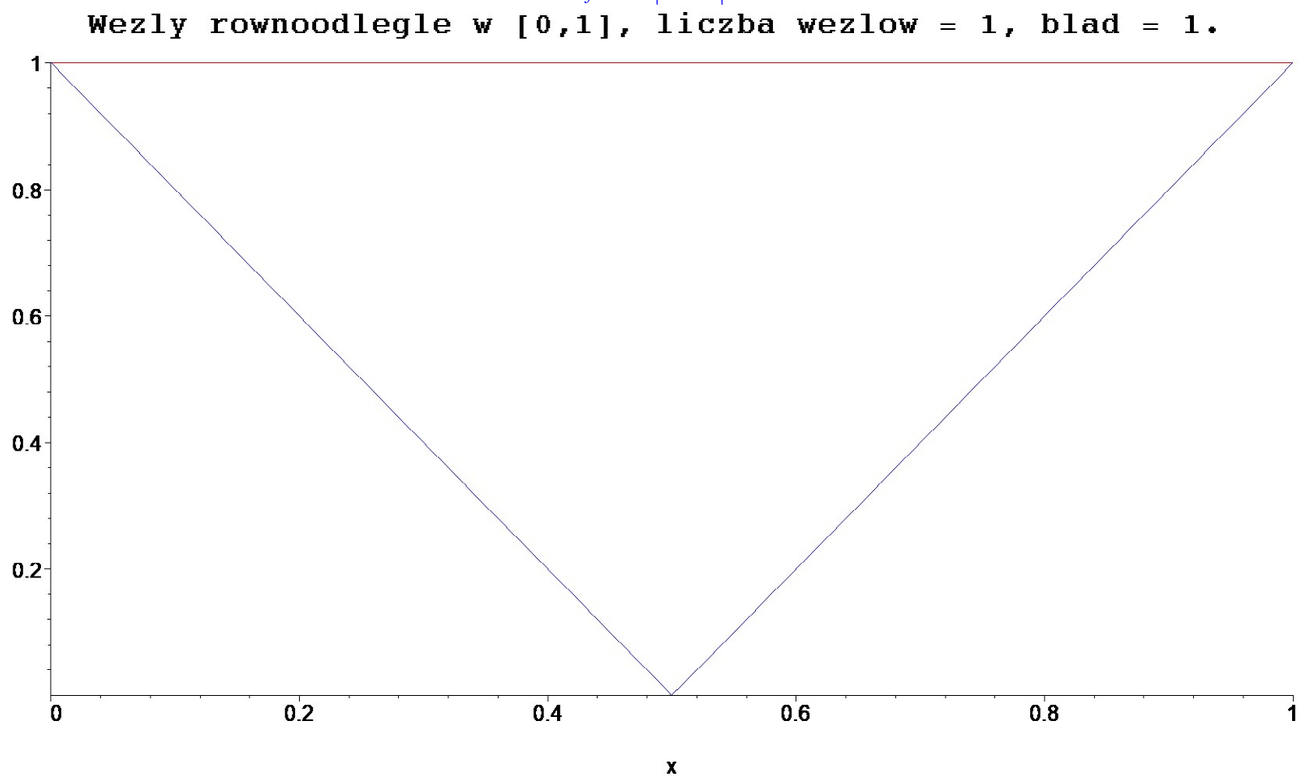

```
>
```

5. Przyklad

```
>
>
> restart:
>
> N:=25:
>
> f:=x->abs(2*x-1);
>
> wykres_f:=plot(f(x),x=0..1,color=blue,axes=frame):
>
> for i from 1 to N
    do
      s:=unapply(spline([seq(evalf(k/i),k=0..i)],
                        [seq(evalf(f(k/i)),k=0..i)],x),x):

      blad:=max(seq(abs(evalf(s(j/100)-f(j/100))),j=0..100)):
      blads:=convert(evalf(blad,5),string):

      opis:="Wezly rownoodlegle w [0,1], liczba wezlow = "||i||", blad = "||blads:
      wykres_s:=plot(s(x),x=0..1,color=red,axes=frame):
      klatka[i]:=plots[display](wykres_f,wykres_s,title=opis,titlefont=[COURIER,BOLD,15])
```

```
    od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
```

$$f := x \to |2x - 1|$$




```
>
```

6. Krzywe parametryczna

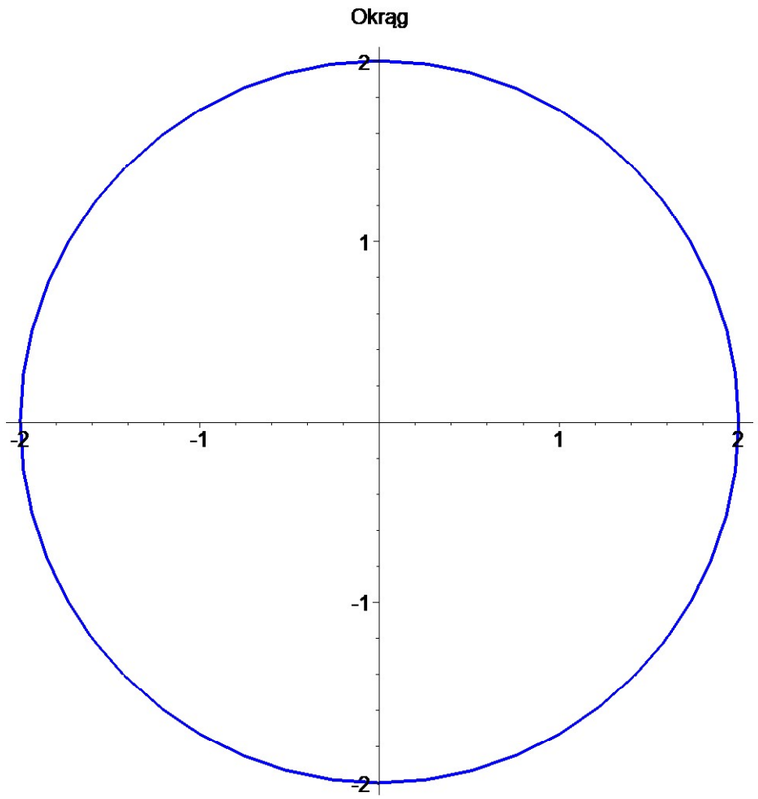
```
>
> r:=2; x:=t->r*cos(t);  y:=t->r*sin(t);
```

$$r := 2$$

$$x := t \to r\cos(t)$$

$$y := t \to r\sin(t)$$

```
> plot([x(t),y(t),t=0..2*Pi],thickness=4,color=blue,title=`Okrąg`);
```



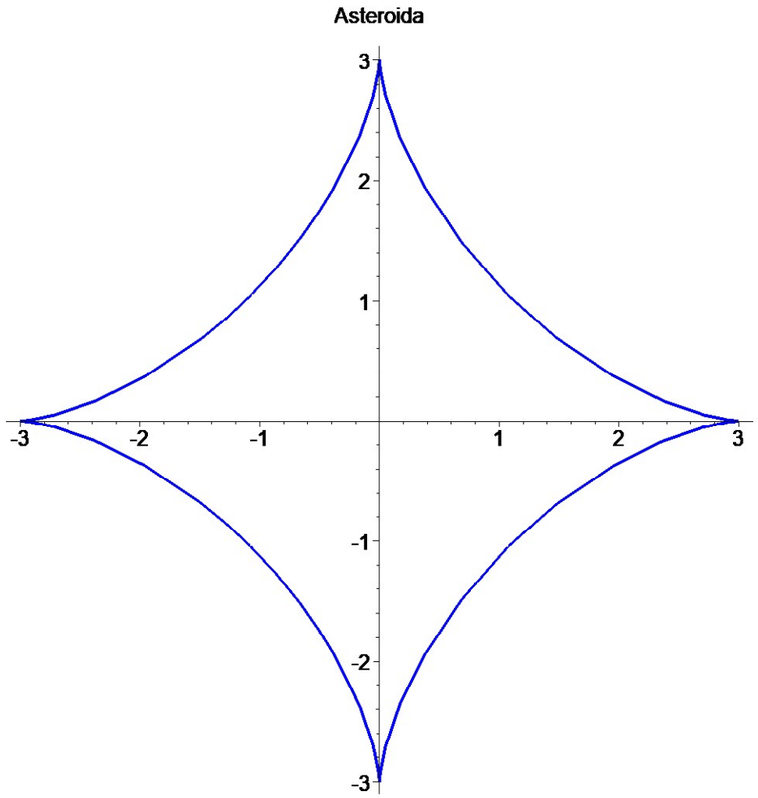

```
> 
> 
> 
> r:=3; x:=t->r*(cos(t))^3;  y:=t->r*(sin(t))^3;
```

$$r := 3$$

$$x := t \to r \cos(t)^3$$

$$y := t \to r \sin(t)^3$$

```
> plot([x(t),y(t),t=0..2*Pi],thickness=4,color=blue,title=`Asteroida`);
```




```
> 
> 
> 
> a:=1; r:=1/3; x:=t->r*t-a*sin(t);  y:=t->r-a*cos(t);
```

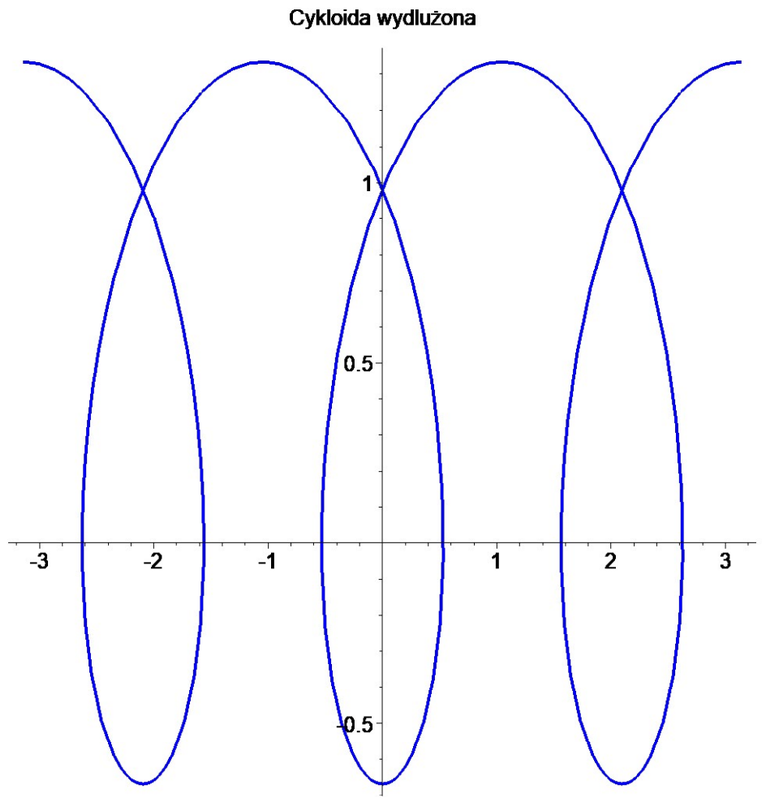
$$a := 1$$

$$r := \frac{1}{3}$$

$$x := t \to r\,t - a\,\sin(t)$$

$$y := t \to r - a\,\cos(t)$$

```
> plot([x(t),y(t),t=-3*Pi..3*Pi],thickness=4,color=blue,title=`Cykloida wydłużona`);
```

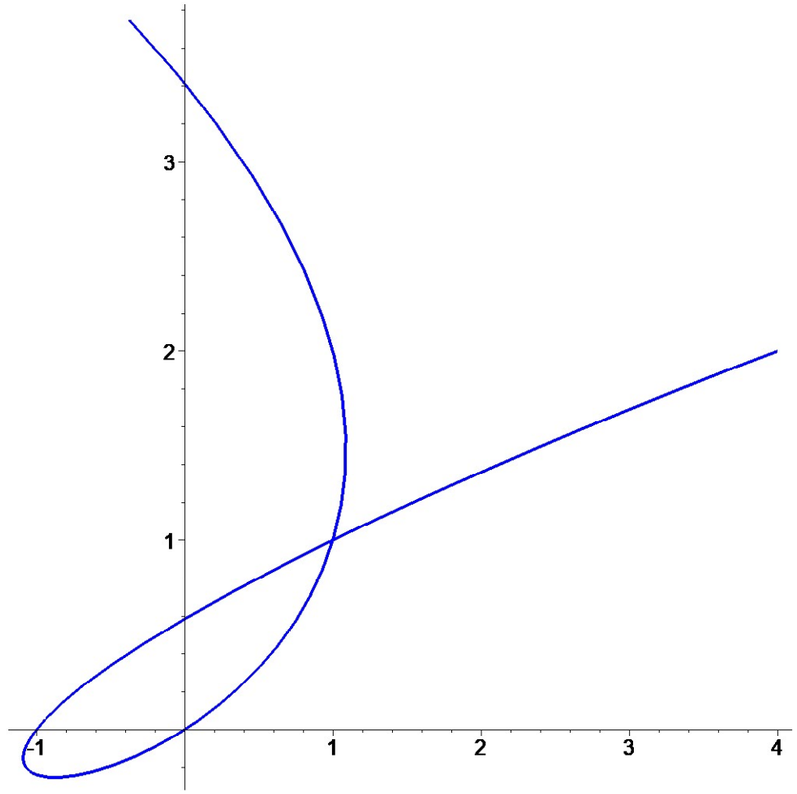



```
> 
> 
> 
> x:=t->t*(t^2-2);  y:=t->t*(t-1);
```

$$x := t \to t\,(t^2 - 2)$$

$$y := t \to t\,(t - 1)$$

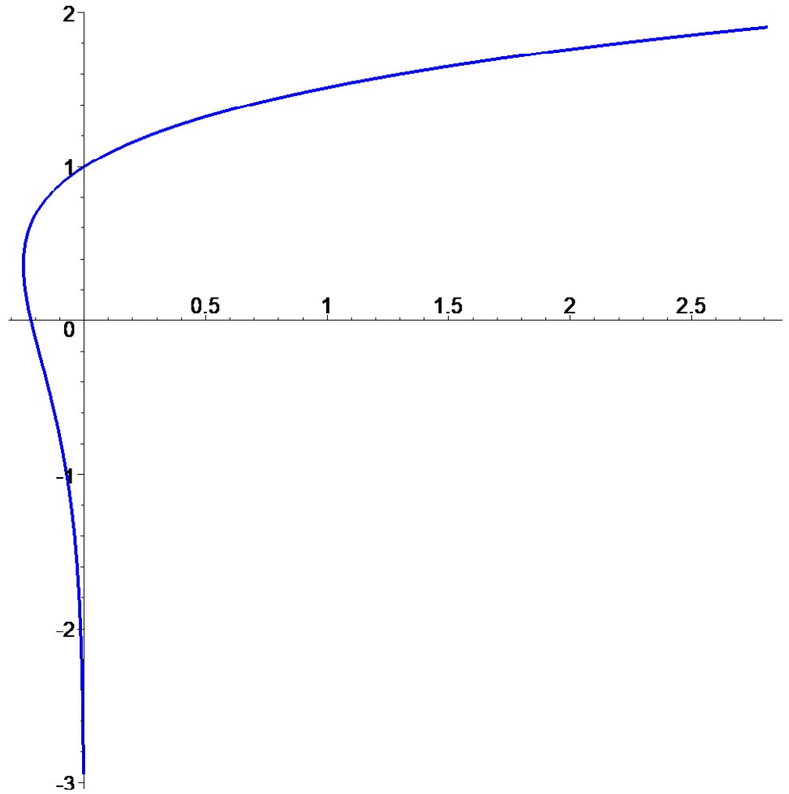
```
> plot([x(t),y(t),t=-1.5..2],thickness=4,color=blue);
```




```
> x:=t->t^2*(t^2-1);  y:=t->t+ln(t);
```

$$x := t \to t^2 (t^2 - 1)$$
$$y := t \to t + \ln(t)$$

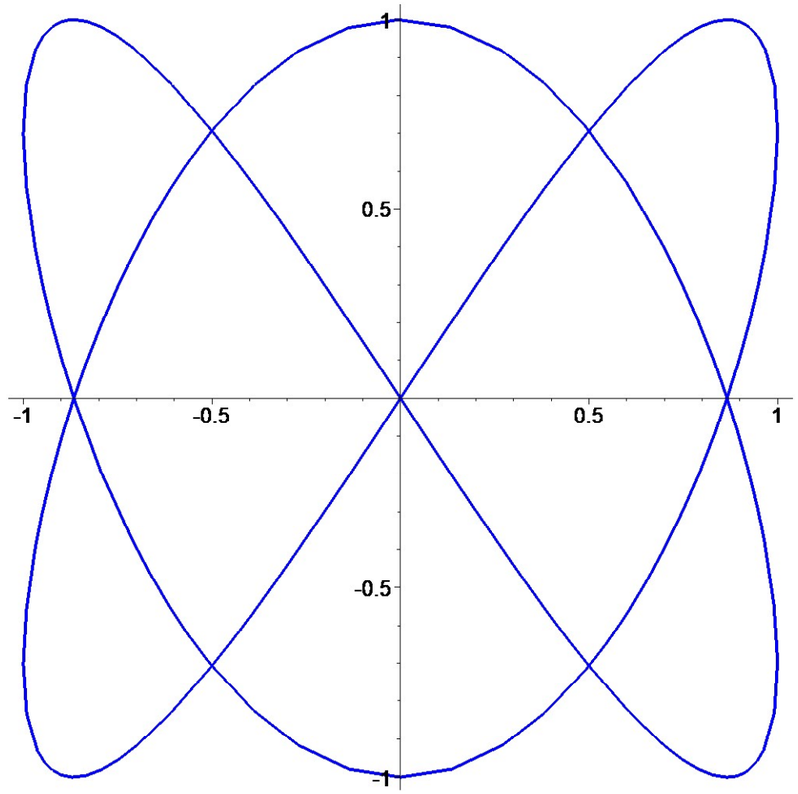
```
> plot([x(t),y(t),t=0.05..1.5],thickness=4,color=blue);
```




```
>
>
>
> x:=t->sin(2*t);  y:=t->cos(3*t);
```

$$x := t \to \sin(2\,t)$$
$$y := t \to \cos(3\,t)$$

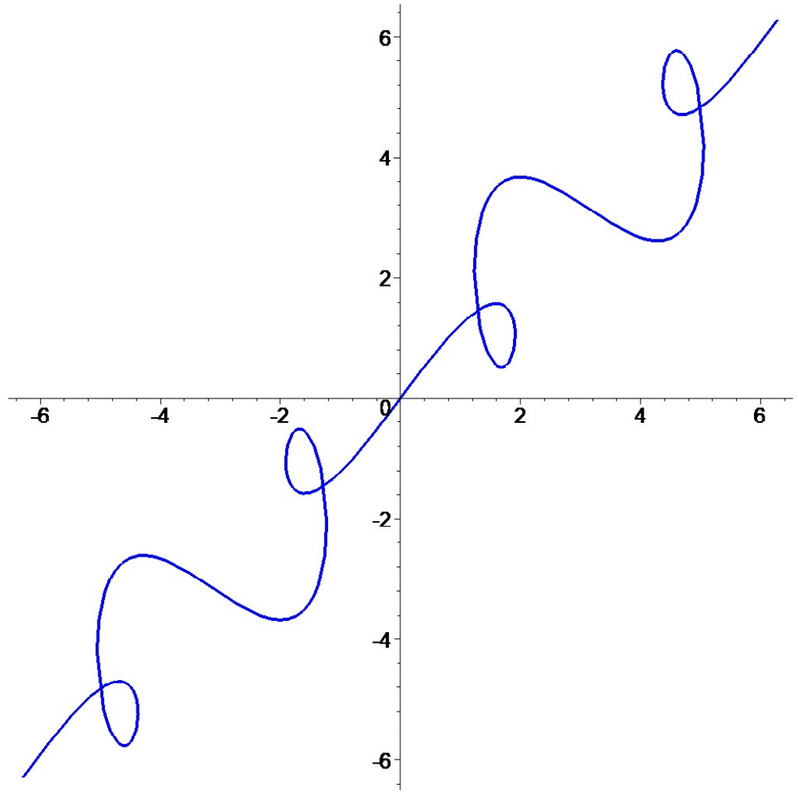
```
> plot([x(t),y(t),t=-Pi..Pi],thickness=4,color=blue);
```




```
>
>
>
> x:=t->t+sin(2*t);  y:=t->t+sin(3*t);
```

$$x := t \to t + \sin(2\,t)$$
$$y := t \to t + \sin(3\,t)$$

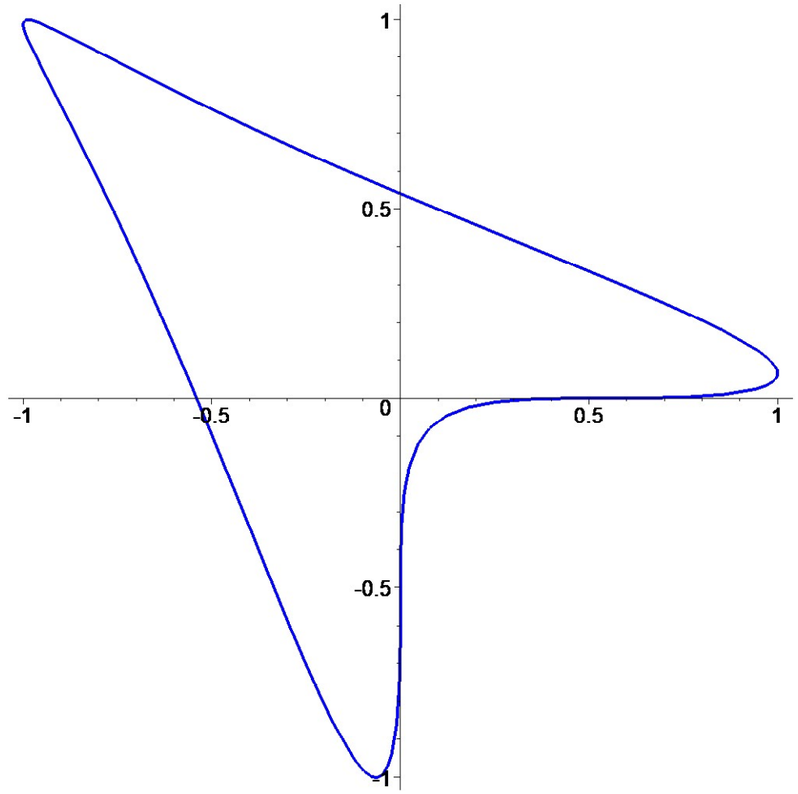
```
> plot([x(t),y(t),t=-2*Pi..2*Pi],thickness=4,color=blue);
```




```
>
>
>
> x:=t->sin(t+sin(t));  y:=t->cos(t+cos(t));
```

$$x := t \to \sin(t + \sin(t))$$
$$y := t \to \cos(t + \cos(t))$$

```
> plot([x(t),y(t),t=-Pi..Pi],thickness=4,color=blue);
```

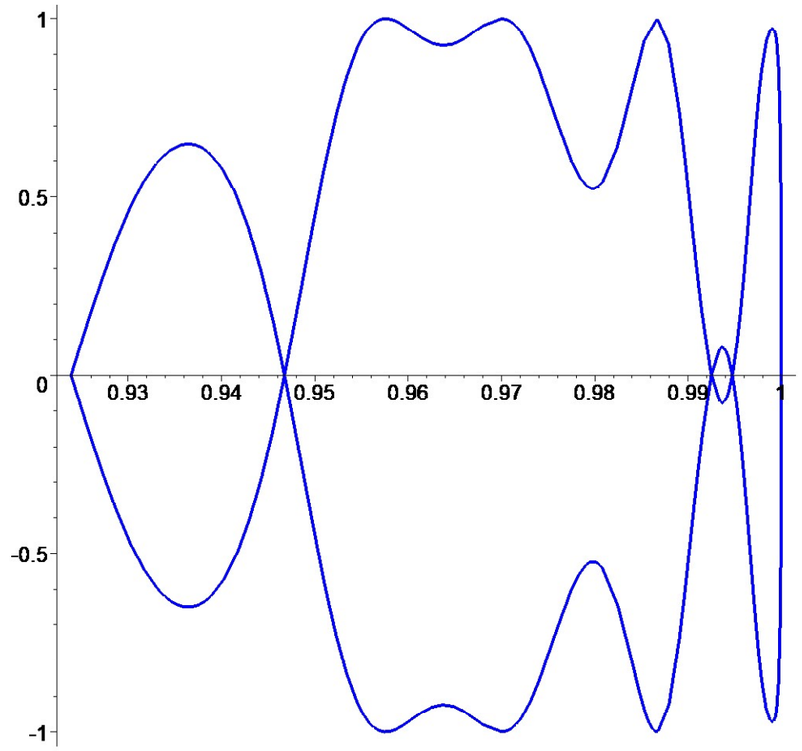



```
> 
> 
> 
> x:=t->cos(t/8);  y:=t->sin(t+sin(5*t));
```

$$x := t \to \cos\left(\frac{1}{8}t\right)$$

$$y := t \to \sin(t + \sin(5\,t))$$

```
> plot([x(t),y(t),t=-Pi..Pi],thickness=4,color=blue);
```

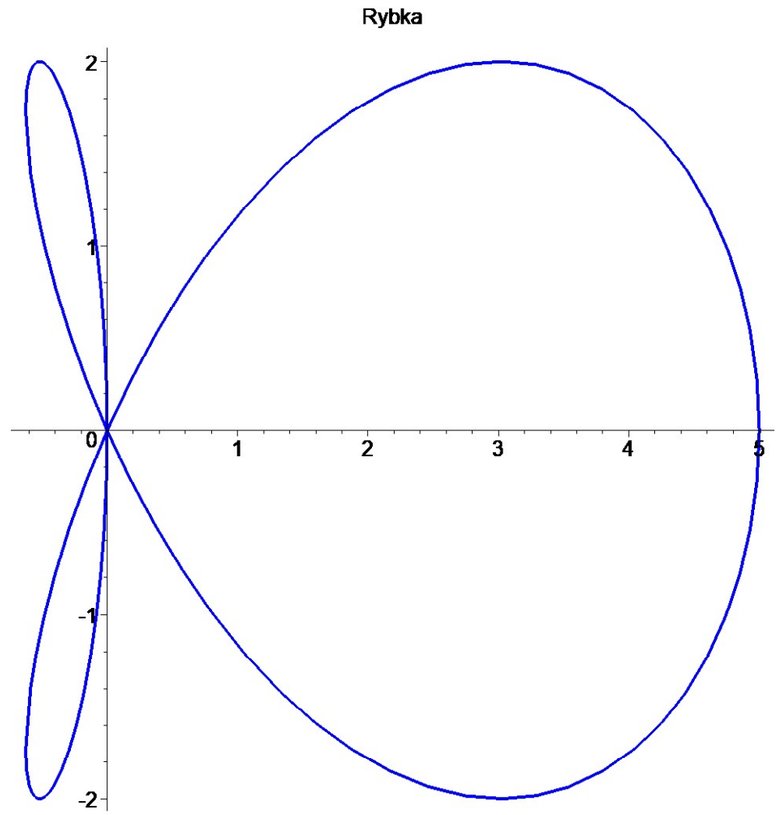



```
>
>
>
>
>
> x:=t->5/2*cos(t)*(cos(t)+1);  y:=t->2*sin(2*t);
```

$$x := t \to \frac{5}{2}\cos(t)\,(\cos(t)+1)$$

$$y := t \to 2\sin(2\,t)$$

```
> plot([x(t),y(t),t=0..2*Pi],thickness=4,color=blue,title='Rybka');
```

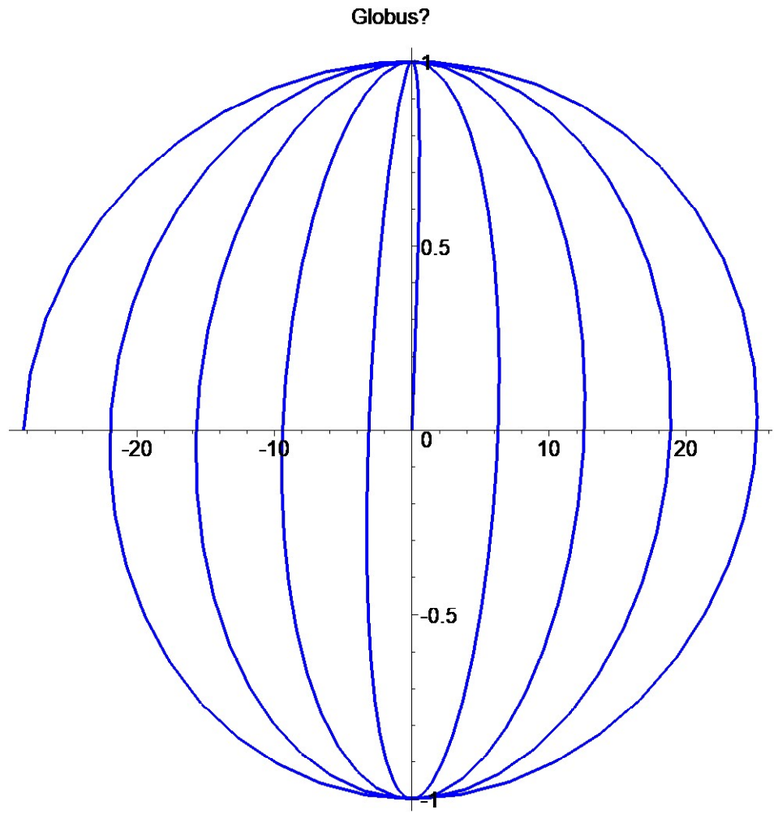



```
> 
> x:=t->t*cos(t);  y:=t->sin(t);
```

$$x := t \to t \cos(t)$$

$$y := \sin$$

```
> plot([x(t),y(t),t=0..9*Pi],color=blue,scaling=UNCONSTRAINED,thickness=4,title='Globus?');
```



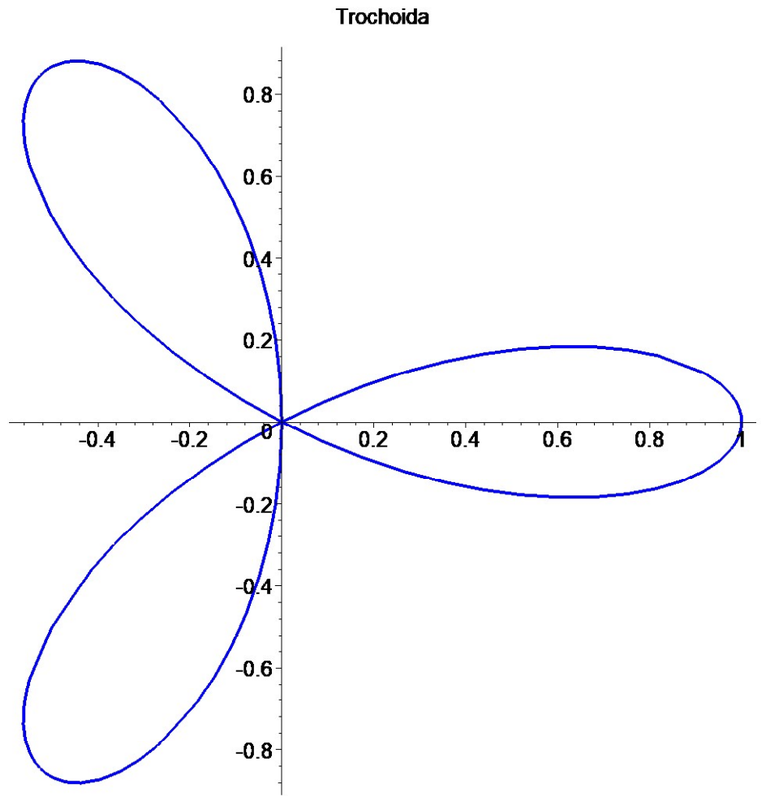

```
>
```

```
> x:=t->cos(3*t)*cos(t); y:=t->cos(3*t)*sin(t);
```

$$x := t \to \cos(3\,t)\cos(t)$$

$$y := t \to \cos(3\,t)\sin(t)$$

```
> plot([x(t),y(t),t=0..Pi],title=`Trochoida`,thickness=4,color=blue);
```




```
>
>
>
>
> x:=t->r*cos(t);  y:=t->r*sin(t); z:=t->h*t; r:=1; h:=1;
```

$$x := t \to r \cos(t)$$
$$y := t \to r \sin(t)$$
$$z := t \to h\, t$$
$$r := 1$$
$$h := 1$$

```
> with(plots):  spacecurve([x(t),y(t),z(t),t=0..6*Pi],numpoints=150,thickness=5,title=`Helisa`);
Warning, the name changecoords has been redefined
```

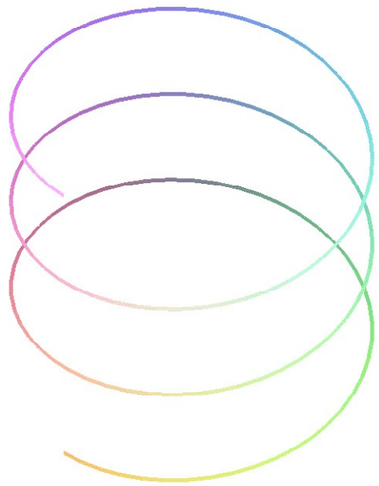
>

7. Przyklad

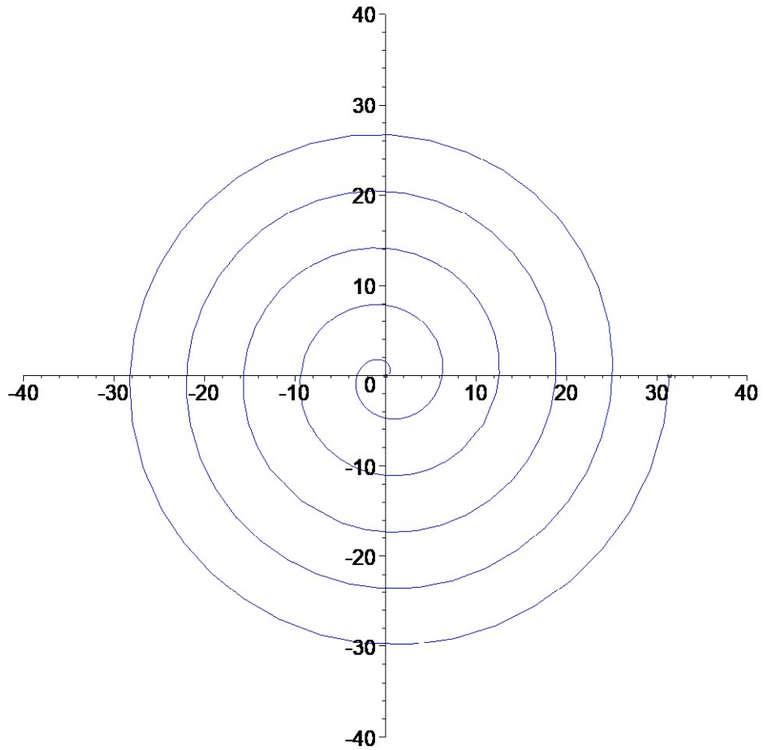
7.1 Przyklad

```
> 
> 
> restart;
> 
> Digits:=18:
> 
> 
> 
> fx:=t->10*Pi*t*cos(10*Pi*t):
> fy:=t->10*Pi*t*sin(10*Pi*t):
> 
> N:=31:
> 
> for n from 1 to N
    do
      w:=[seq(evalf(i/n),i=0..n)]:
      Lx:=unapply(interp(w,[seq(evalf(fx(w[i+1])),i=0..n)],x),x):
      Ly:=unapply(interp(w,[seq(evalf(fy(w[i+1])),i=0..n)],x),x):

      Rys[0]:=plot([Lx(x),Ly(x),x=0..1],color=red,scaling=CONSTRAINED):
      Rys[1]:=PLOT(POINTS(seq([evalf(fx(w[i+1])),evalf(fy(w[i+1]))],i=0..n)),SYMBOL(BOX)):
      Rys[2]:=plot([fx(x),fy(x),x=0..1],color=blue,scaling=CONSTRAINED):

      Klatka[n]:=plots[display]([Rys[0],Rys[1],Rys[2]]):
    od:
```

Helisa



```
>
>
> plots[display]([seq(Klatka[n],n=1..N)],insequence=true,view=[-40..40,-40..40]);
>
```

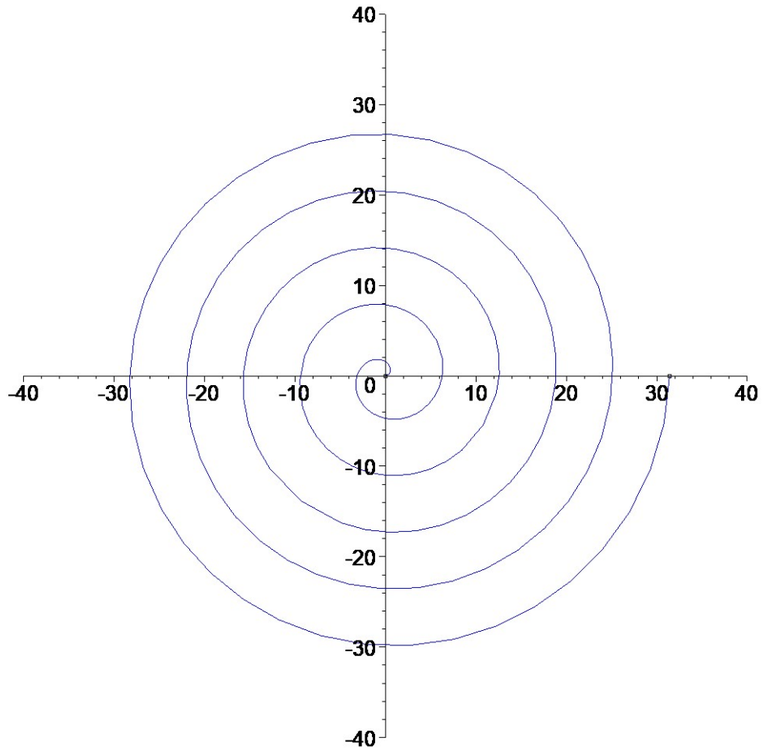



```
>
>
```

7.2 Przyklad

```
>
>
> fx:=t->10*Pi*t*cos(10*Pi*t):
> fy:=t->10*Pi*t*sin(10*Pi*t):
>
> N:=31:
>
> for n from 1 to N
    do
      w:=[seq(evalf(i/n),i=0..n)]:
      Lx:=unapply(spline(w,[seq(evalf(fx(w[i+1])),i=0..n)],x),x):
      Ly:=unapply(spline(w,[seq(evalf(fy(w[i+1])),i=0..n)],x),x):

      Rys[0]:=plot([Lx(x),Ly(x),x=0..1],color=red,scaling=CONSTRAINED):
      Rys[1]:=PLOT(POINTS(seq([evalf(fx(w[i+1])),evalf(fy(w[i+1]))],i=0..n)),SYMBOL(BOX)):
      Rys[2]:=plot([fx(x),fy(x),x=0..1],color=blue,scaling=CONSTRAINED):

      Klatka[n]:=plots[display]([Rys[0],Rys[1],Rys[2]]):
    od:
>
>
> plots[display]([seq(Klatka[n],n=1..N)],insequence=true,view=[-40..40,-40..40]);
>
```

>



> >

8. Ukochany kraj...

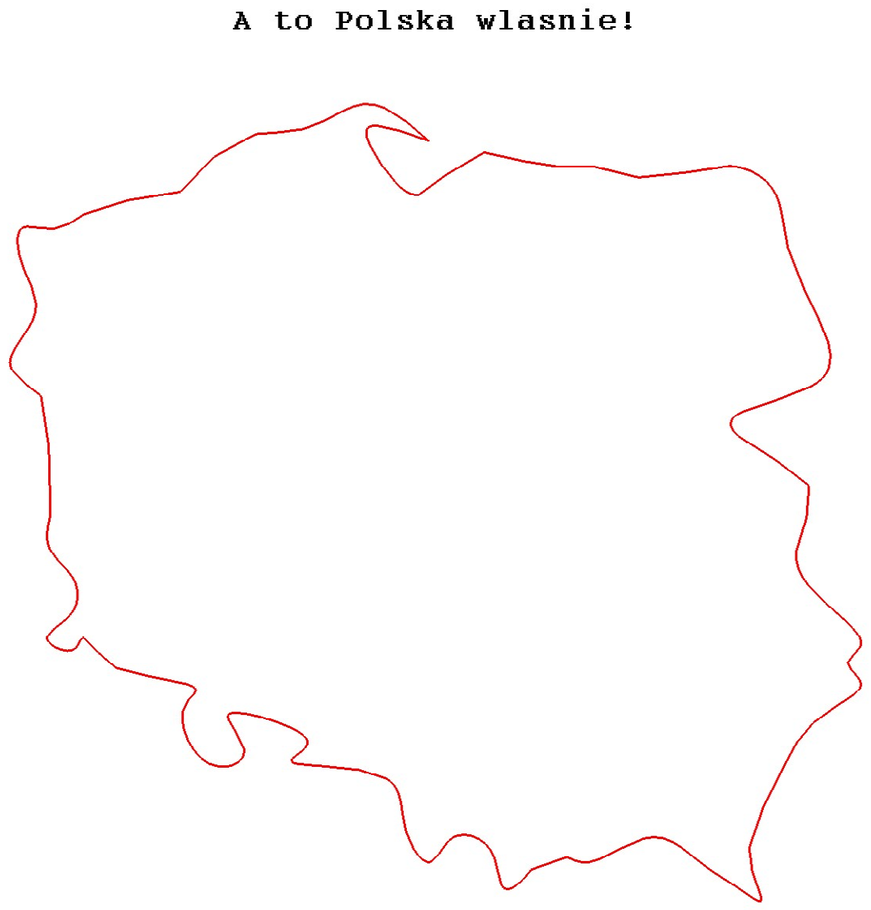
8.1. Przyklad

>

>

```
> restart:
>
> n:=65:
>
> w:=[seq(evalf(i/n),i=0..n)]:
>
> p_x:=[3.7,3.2,2.7,2.1,1.7,1.1,0.7,0.4,
        0.4,0.5,0.3,0.6,0.6,0.7,0.6,0.8,
        0.8,0.6,0.8,0.9,1.1,1.4,1.8,1.7,
        1.9,2.2,2.1,2.7,2.6,3.3,3.5,3.7,
        3.9,4.2,4.3,4.5,4.7,5.0,5.5,5.9,
        6.2,6.4,6.3,6.5,6.8,7.2,7.1,7.2,
        6.8,6.7,6.8,6.4,6.2,6.9,6.8,6.6,
        6.5,6.1,5.5,5.0,4.6,4.1,3.7,3.4,
        3.2,3.7]:
>
> p_y:=[6.4,6.7,6.5,6.4,6.0,5.9,5.7,5.7,
        5.4,5.0,4.6,4.3,4.0,3.7,3.2,2.9,
        2.6,2.4,2.3,2.4,2.2,2.1,2.0,1.8,
        1.4,1.5,1.8,1.6,1.4,1.3,0.9,0.6,
        0.8,0.7,0.4,0.5,0.7,0.6,0.8,0.6,
```

```
        0.4,0.3,0.7,1.2,1.7,2.0,2.2,2.4,
        2.8,3.2,3.6,3.9,4.2,4.5,5.1,5.6,
        6.0,6.2,6.1,6.2,6.2,6.3,6.0,6.1,
        6.5,6.4]:
>
> s_x:=unapply(spline(w,p_x,t),t,periodic)-3.5:
>
> s_y:=unapply(spline(w,p_y,t),t,periodic)-3.25:
>
>
>
> plot([s_x(t),s_y(t),t=0..1],axes=none,scaling=constrained,thickness=3,
               title="A to Polska wlasnie!",titlefont=[COURIER,BOLD,15]);
>
>
```




```
>
```

8.2. Przyklad

```
>
>
> restart:
>
> n:=65:
>
```

```
> w:=[seq(evalf(i/n),i=0..n)]:
>
> p_x:=[3.7,3.2,2.7,2.1,1.7,1.1,0.7,0.4,
        0.4,0.5,0.3,0.6,0.6,0.7,0.6,0.8,
        0.8,0.6,0.8,0.9,1.1,1.4,1.8,1.7,
        1.9,2.2,2.1,2.7,2.6,3.3,3.5,3.7,
        3.9,4.2,4.3,4.5,4.7,5.0,5.5,5.9,
        6.2,6.4,6.3,6.5,6.8,7.2,7.1,7.2,
        6.8,6.7,6.8,6.4,6.2,6.9,6.8,6.6,
        6.5,6.1,5.5,5.0,4.6,4.1,3.7,3.4,
        3.2,3.7]:
>
> p_y:=[6.4,6.7,6.5,6.4,6.0,5.9,5.7,5.7,
        5.4,5.0,4.6,4.3,4.0,3.7,3.2,2.9,
        2.6,2.4,2.3,2.4,2.2,2.1,2.0,1.8,
        1.4,1.5,1.8,1.6,1.4,1.3,0.9,0.6,
        0.8,0.7,0.4,0.5,0.7,0.6,0.8,0.6,
        0.4,0.3,0.7,1.2,1.7,2.0,2.2,2.4,
        2.8,3.2,3.6,3.9,4.2,4.5,5.1,5.6,
        6.0,6.2,6.1,6.2,6.2,6.3,6.0,6.1,
        6.5,6.4]:
>
> L_x:=unapply(interp(w,p_x,t)-3.5,t):
>
> L_y:=unapply(interp(w,p_y,t)-3.25,t):
>
>
>
>
> plot([L_x(t),L_y(t),t=0..1],axes=none,scaling=constrained,thickness=1,
        title="A to Polska (interpolowana)",titlefont=[COURIER,BOLD,15]);
>
>
>
>
```

Floating Point Overflow. Please shorten axes.

>

8.3. Przyklad

>

>

> with(plots):

>

Warning, the name changecoords has been redefined

>

```
>
> animate([cos(u)*s_x(t)-sin(u)*s_y(t),sin(u)*s_x(t)+cos(u)*s_y(t),t=0..1],u=0..8*Pi,
          axes=none,scaling=constrained,thickness=3,numpoints=200,frames=50,
          title="Zakrecony kraj w srodku Europy",titlefont=[COURIER,BOLD,15]);
>
>
>
```


```
>
```

8.4. Przyklad

```
>
>
> animate([(1-u)*s_x(t)+u*s_x(t),(1-u)*s_y(t)-u*s_y(t),t=0..1],u=0..1,
          axes=none,scaling=constrained,thickness=3,numpoints=200,frames=50);
>
>
```

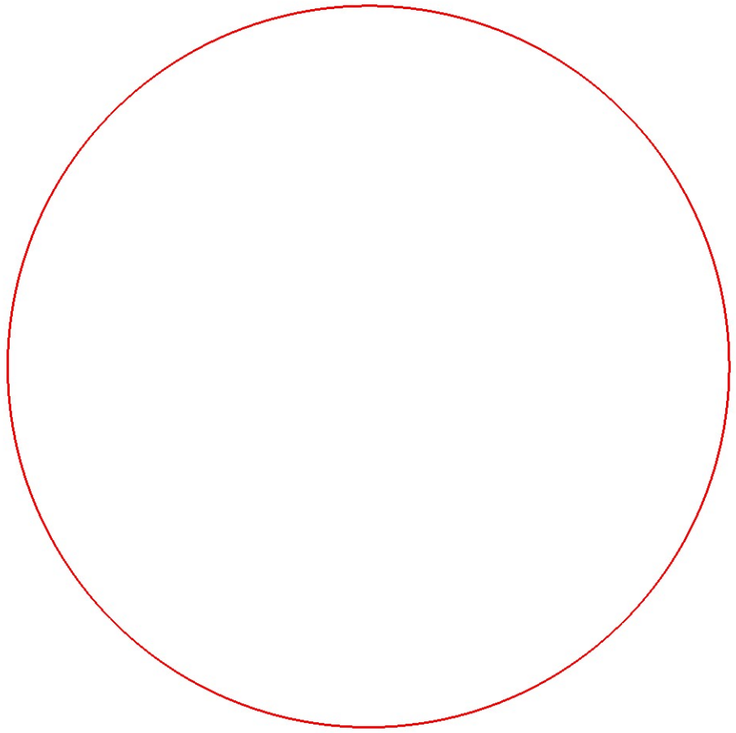
```
>
```

8.5. Przyklad

```
>
>
> animate([u*s_x(t)+(1-u)*3.5*cos(2*Pi*t),u*s_y(t)+(1-u)*3.5*sin(2*Pi*t),t=0..1],u=0..1,
          axes=none,scaling=constrained,thickness=3,numpoints=200,frames=50);
>
>
```

8.5. Przyklad

```
>
>
> animate([u*s_x(t)+(1-u)*3.5*cos(2*Pi*t),u*s_y(t)+(1-u)*3.5*sin(2*Pi*t),t=0..1],u=0..1,
          axes=none,scaling=constrained,thickness=3,numpoints=200,frames=50);
>
>
```



>

8.6. Przyklad

8.6. Przyklad

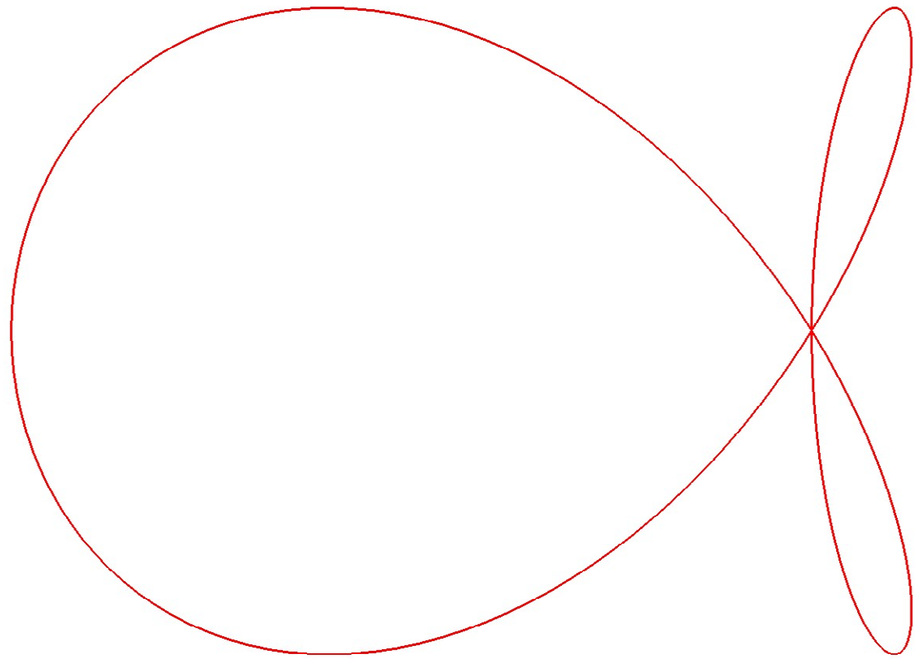
>

>

```
> animate([u*s_x(t)+(1-u)*(-5/2*cos(2*Pi*t)*(cos(2*Pi*t)+1)),
           u*s_y(t)+(1-u)*2*sin(4*Pi*t),t=0..1],u=0..1,
           axes=none,scaling=constrained,thickness=3,numpoints=200,frames=50);
```

>

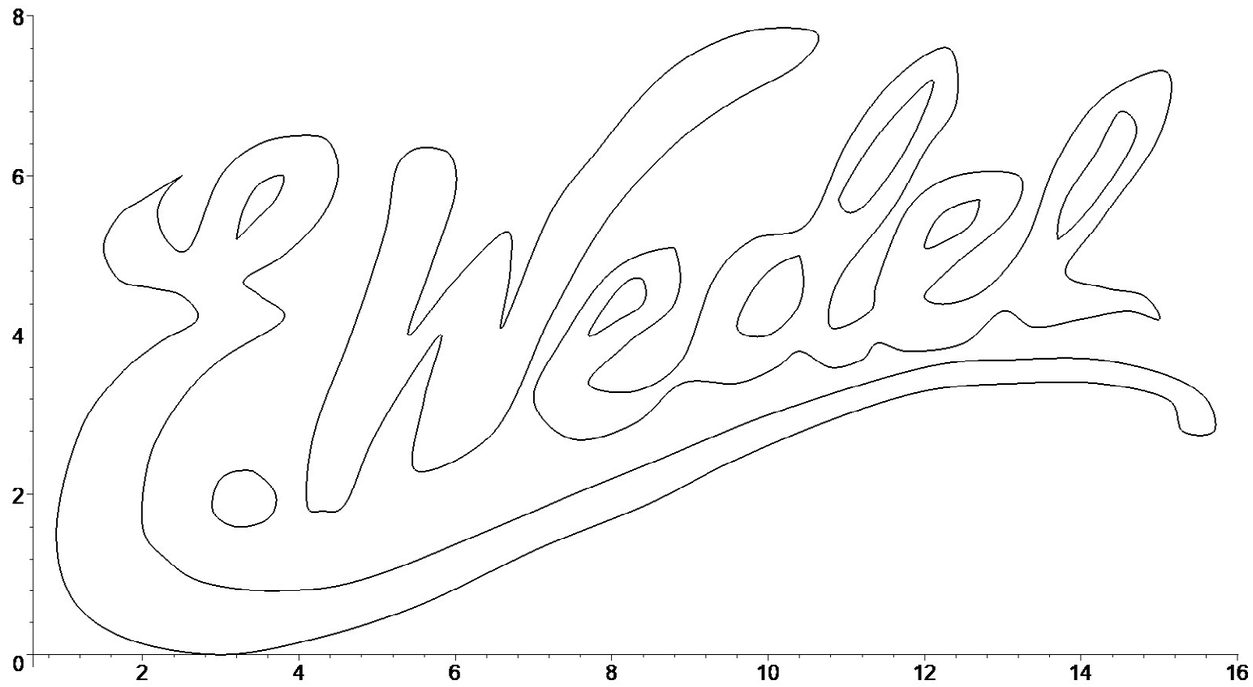
>



- >

9. Cos slodkiego

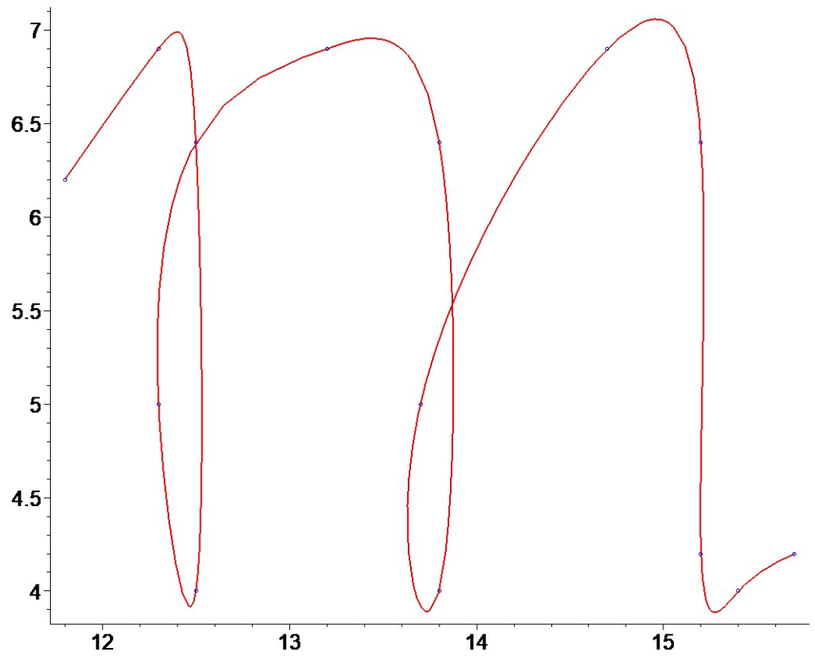
- >
- >




```
>
>
```

10. Literki i napisy

```
> restart;
> with(plots): with(plottools):
Warning, the name changecoords has been redefined

Warning, the name arrow has been redefined

> read `flamenco.cor`:
literka "f"literka "l"literka "a"literka "m"literka "e"literka "n"literka "c"literka "o"oko1oko2usmiech
> readlib(spline):
> n:=4:
> Sx[n]:=spline([seq(i,i=1..nops(L[n]))],[seq(L[n][i][1],i=1..nops(L[n]))],t,cubic,periodic):
> Sy[n]:=spline([seq(i,i=1..nops(L[n]))],[seq(L[n][i][2],i=1..nops(L[n]))],t,cubic,periodic):
> pl[n]:=plot([Sx[n],Sy[n],t=1..nops(L[n])],thickness=2):
> pp[n]:=pointplot(L[n],symbol=circle,color=blue):
> display(pl[n],pp[n],scaling=constrained);
```




```
>
>
> napis:=proc(f,L,k) local D,n,Sx,Sy,pl,pp,i,j:
> for n from 1 to k do D[n]:=L[n] od:
> for n from 1 to k do
> D[n]:=map(f,D[n]):
> od:
> for n from 1 to k do
> Sx[n]:= spline([seq(i,i=1..nops(D[n]))],[seq(D[n][i][1],i=1..nops(D[n]))],t,cubic,periodic):
> Sy[n]:=
  spline([seq(i,i=1..nops(D[n]))],[seq(D[n][i][2],i=1..nops(D[n]))],t,cubic,periodic):
> pl[n]:=plot([Sx[n],Sy[n],t=1..nops(D[n])],thickness=2):
> pp[n]:=pointplot(D[n],symbol=circle,color=blue):
> od:
> display(seq(pl[j],j=1..k),scaling=constrained,axes=none);
> end:
>
> II:=proc(L);
>   L;
> end:
> napis(II,L,11);
```

flamenco :)

```
> yr3:=proc(punkt);
>   [punkt[1],3.0*punkt[2]]
> end:
> napis(yr3,L,11);
```

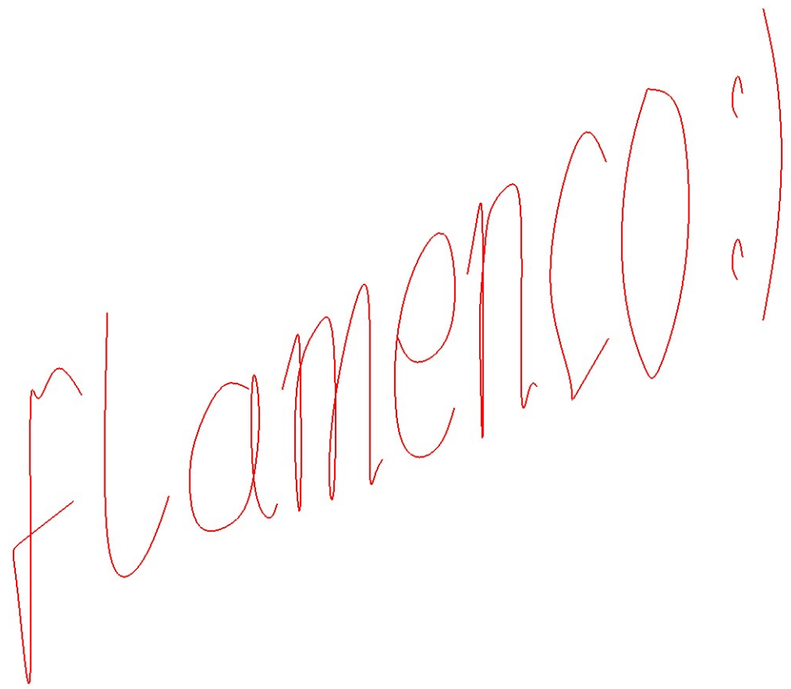

```
> yd3:=proc(punkt);
>    [punkt[1],punkt[2]/3.0]
> end:
> napis(yd3,L,11);
```

flamenco :)

```
> poch:=proc(punkt);
>   [punkt[1]+0.5*punkt[2],punkt[2]]
> end:
> napis(poch,L,11);
```

flamenco :)

```
> up:=proc(punkt);
>   [punkt[1],punkt[2]*(1+punkt[1]/10)]
> end:
> napis(up,L,11);
```




```
> upsidedown:=proc(punkt);
>   [punkt[1],5.5-(punkt[2]-5.5)]
> end:
> napis(upsidedown,L,11);
```

```
> coto:=proc(punkt);
>     [-cos(punkt[1]/5)*(3+punkt[2]),sin(punkt[1]/5)*(3+punkt[2])]
> end:
> napis(coto,L,11);
```

```
> coto:=proc(punkt);
```

flamenco:)

11. „Podrabianie" podpisow