Neural Networks

Recurrent Neural Networks

Rafał Nowak May 13, 2025

8. Deep Learning Classification: Named Entity Recognition (NER)

- The task: find and classify names in text, by labeling word tokens, for example:

  Last night , Paris Hilton wowed in a sequin gown .

  PER PER

  Samuel Quinn was arrested in the Hilton Hotel in Paris in April 1989 .

  PER PER LOC LOC LOC DATE DATE

- Possible uses:
  - Tracking mentions of particular entities in documents
  - For question answering, answers are usually named entities
  - Relating sentiment analysis to the entity under discussion
- Often followed by Entity Linking/Canonicalization into a Knowledge Base such as Wikidata

Simple NER: Window classification using binary logistic classifier

- **Idea:** classify each word in its context window of neighboring words
- Train logistic classifier on hand-labeled data to classify center word {yes/no} for each class based on a concatenation of word vectors in a window
  - Really, we usually use multi-class softmax, but we're trying to keep it simple ☺
- **Example:** Classify "Paris" as +/– location in context of sentence with window length 2:

the museums in Paris are amazing to see .

$X_{window} = [\ x_{museums}\quad x_{in}\quad x_{Paris}\quad x_{are}\quad x_{amazing}\ ]^T$

- Resulting vector $x_{window} = \boxed{x \in R^{5d}}$
- To classify all words: run classifier for each class on the vector centered on each word in the sentence

NER: Binary neural classifier for center word being location

- We do supervised training and want high score if it's a location

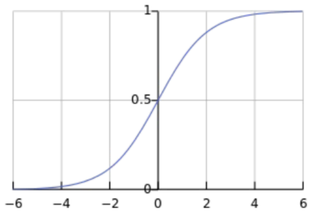
$$J_t(\theta) = \sigma(s) = \frac{1}{1 + e^{-s}}$$

predicted model probability of class

$$s = \boldsymbol{u}^T \boldsymbol{h}$$

$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{x} + \boldsymbol{b})$$

$$\boldsymbol{x} \quad \text{(input)}$$

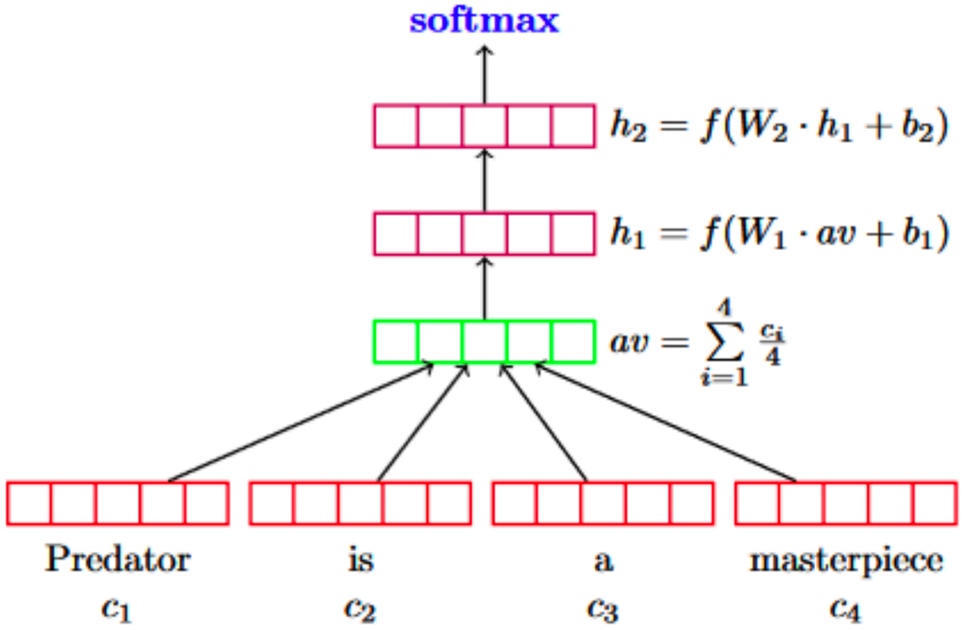


f = Some element-wise non-linear function, e.g., logistic, tanh, ReLU

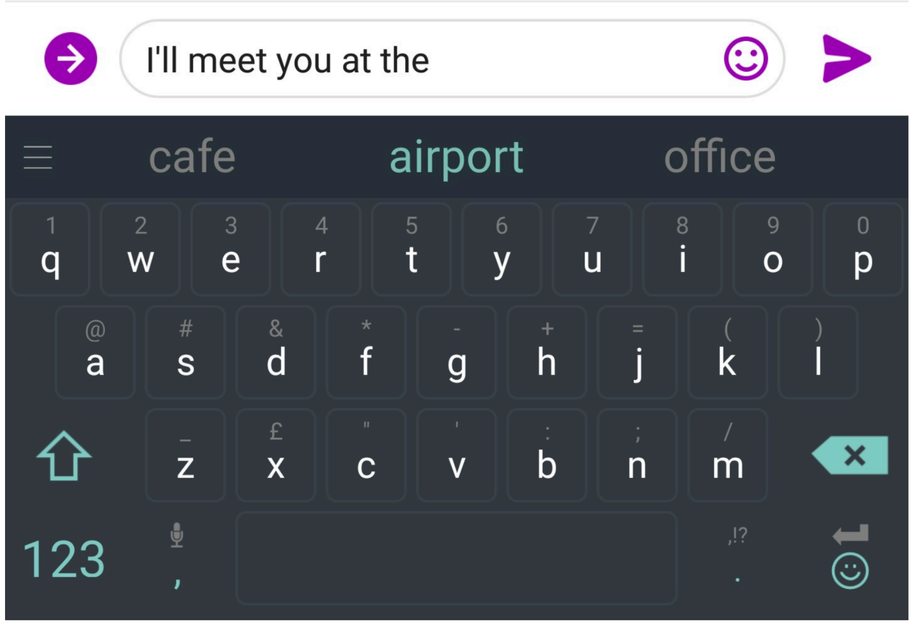
Deep Averaging Network

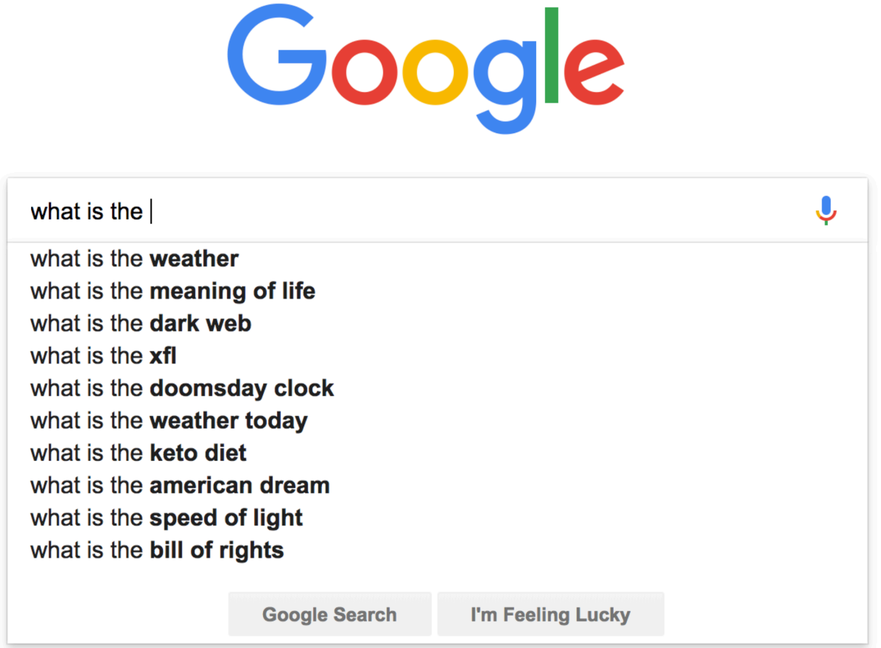
General idea

- For the sentence take the average of embeddings of its words
- Apply 2 layers feed forward NN



Language modeling

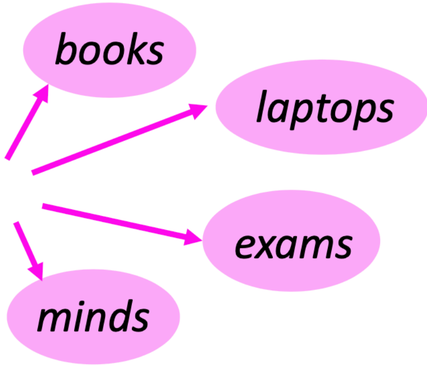




2. Language Modeling

- **Language Modeling** is the task of predicting what word comes next

the students opened their _____



- More formally: given a sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$, compute the probability distribution of the next word $\boldsymbol{x}^{(t+1)}$:

$$P(\boldsymbol{x}^{(t+1)} | \ \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$$

where $\boldsymbol{x}^{(t+1)}$ can be any word in the vocabulary $V = \{\boldsymbol{w}_1, \ldots, \boldsymbol{w}_{|V|}\}$

- A system that does this is called a **Language Model**

Language Modeling

- You can also think of a Language Model as a system that assigns a probability to a piece of text

- For example, if we have some text $\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}$, then the probability of this text (according to the Language Model) is:

$$P(\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(T)}) = P(\boldsymbol{x}^{(1)}) \times P(\boldsymbol{x}^{(2)} | \boldsymbol{x}^{(1)}) \times \cdots \times P(\boldsymbol{x}^{(T)} | \boldsymbol{x}^{(T-1)}, \ldots, \boldsymbol{x}^{(1)})$$

$$= \prod_{t=1}^{T} P(\boldsymbol{x}^{(t)} | \boldsymbol{x}^{(t-1)}, \ldots, \boldsymbol{x}^{(1)})$$

This is what our LM provides

n-gram Language Models

the students opened their _____

- **Question**: How to learn a Language Model?
- **Answer** (pre- Deep Learning): learn an *n*-gram Language Model!

- **Definition:** An *n*-gram is a chunk of *n* consecutive words.
  - unigrams: “the”, “students”, “opened”, ”their”
  - bigrams: “the students”, “students opened”, “opened their”
  - trigrams: “the students opened”, “students opened their”
  - four-grams: “the students opened their”

- **Idea:** Collect statistics about how frequent different n-grams are and use these to predict next word.

n-gram Language Models

- First we make a Markov assumption: $x^{(t+1)}$ depends only on the preceding *n*-1 words

$$P(\boldsymbol{x}^{(t+1)}|\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(1)}) = P(\boldsymbol{x}^{(t+1)}|\overbrace{\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)}}^{n\text{-1 words}}) \quad \text{(assumption)}$$

prob of a n-gram

prob of a (n-1)-gram

$$= \frac{P(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}{P(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})} \quad \text{(definition of conditional prob)}$$

- **Question:** How do we get these *n*-gram and (*n*-1)-gram probabilities?
- **Answer:** By counting them in some large corpus of text!

$$\approx \frac{\text{count}(\boldsymbol{x}^{(t+1)},\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})}{\text{count}(\boldsymbol{x}^{(t)},\ldots,\boldsymbol{x}^{(t-n+2)})} \quad \text{(statistical approximation)}$$

n-gram Language Models: Example

Suppose we are learning a 4-gram Language Model.

~~*as the proctor started the clock, the*~~ *students opened their* _____

discard

condition on this

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\text{count}(\text{students opened their } \boldsymbol{w})}{\text{count}(\text{students opened their})}$$

For example, suppose that in the corpus:

- "students opened their" occurred 1000 times
- "students opened their books" occurred 400 times
  - → P(books | students opened their) = 0.4
- "students opened their exams" occurred 100 times
  - → P(exams | students opened their) = 0.1

Should we have discarded the "proctor" context?

Sparsity Problems with n-gram Language Models

Sparsity Problem 1

Problem: What if *"students opened their w"* never occurred in data? Then *w* has probability 0!

(Partial) Solution: Add small δ to the count for every $w \in V$. This is called *smoothing*.

$$P(\boldsymbol{w}|\text{students opened their}) = \frac{\text{count}(\text{students opened their } \boldsymbol{w})}{\text{count}(\text{students opened their})}$$

Sparsity Problem 2

Problem: What if *"students opened their"* never occurred in data? Then we can't calculate probability for *any w*!

(Partial) Solution: Just condition on *"opened their"* instead. This is called *backoff*.

Note: Increasing *n* makes sparsity problems *worse*.
Typically, we can't have *n* bigger than 5.

n-gram Language Models in practice

- You can build a simple trigram Language Model over a 1.7 million word corpus (Reuters) in a few seconds on your laptop*

Business and financial news

today the _____

get probability distribution

company	0.153
bank	0.153
price	0.077
italian	0.039
emirate	0.039
…	

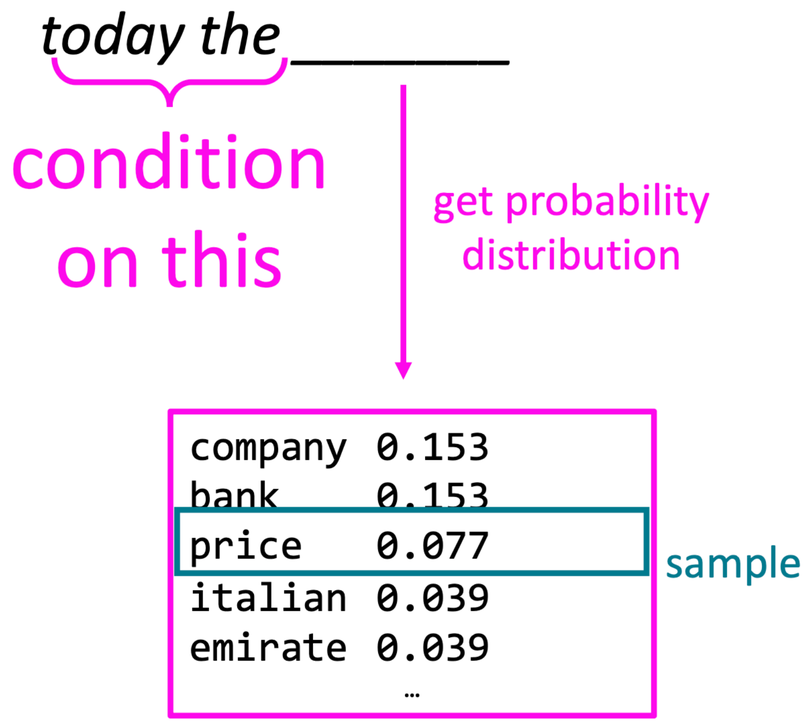
Sparsity problem: not much granularity in the probability distribution

Otherwise, seems reasonable!

*** Try for yourself:** https://nlpforhackers.io/language-models/

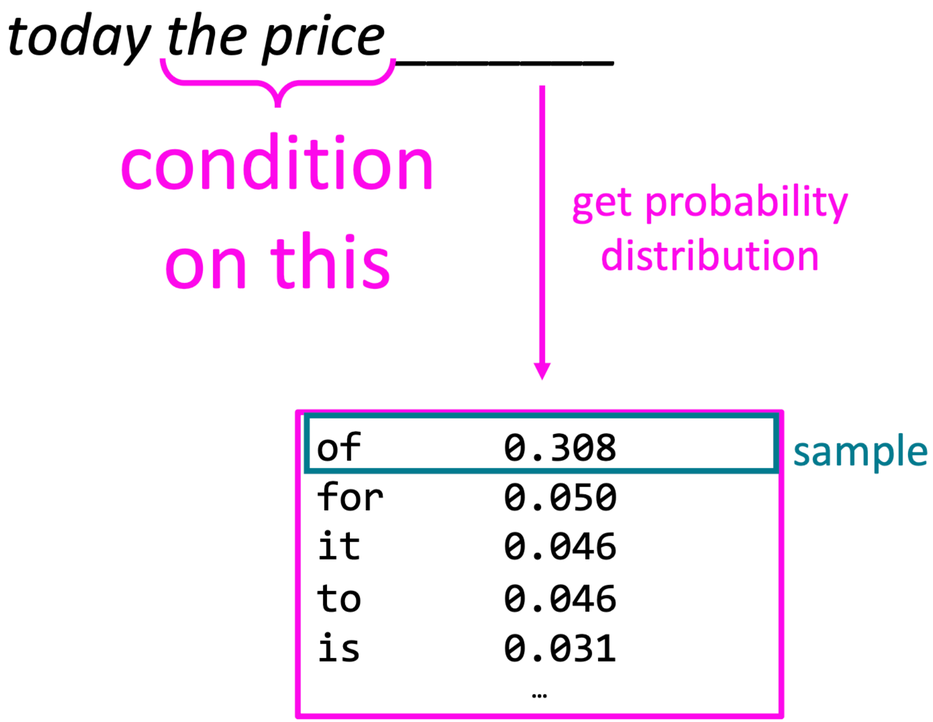
Generating text with a n-gram Language Model

You can also use a Language Model to generate text



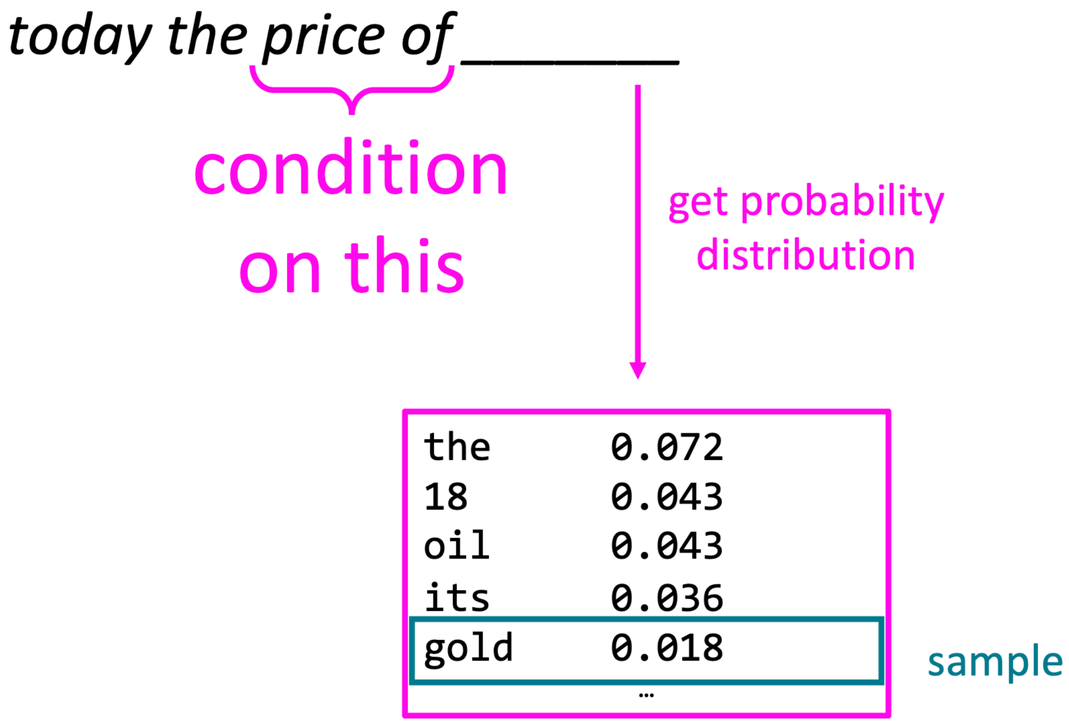
Generating text with a n-gram Language Model

You can also use a Language Model to generate text



Generating text with a n-gram Language Model

You can also use a Language Model to generate text



Generating text with a n-gram Language Model

You can also use a Language Model to generate text

today the price of gold per ton , while production of shoe lasts and shoe industry , the bank intervened just after it considered and rejected an imf demand to rebuild depleted european stocks , sept 30 end primary 76 cts a share .

Surprisingly grammatical!

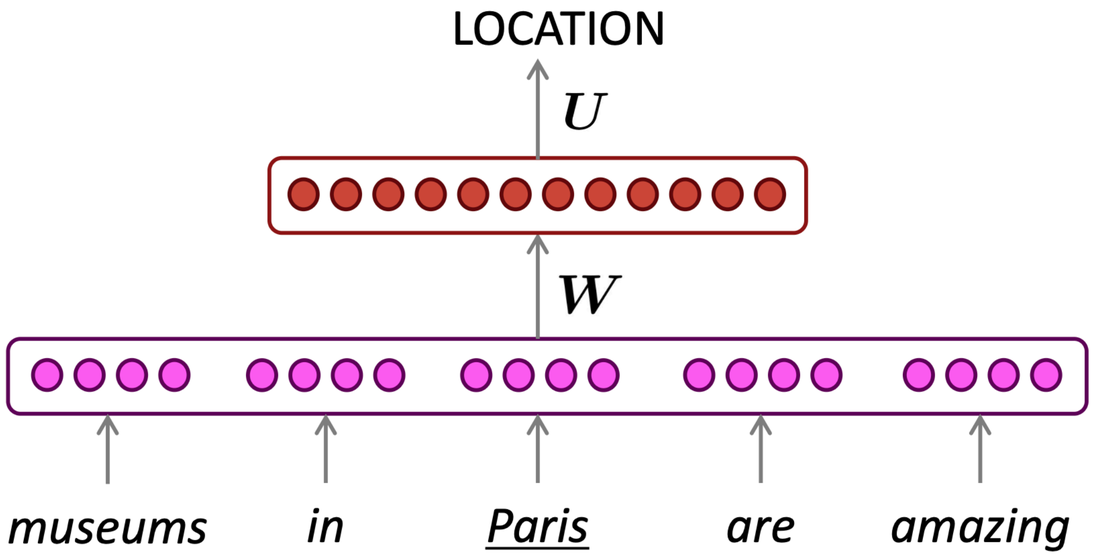
…but **incoherent.** We need to consider more than three words at a time if we want to model language well.

But increasing *n* worsens sparsity problem, and increases model size…

How to build a *neural* language model?

- Recall the Language Modeling task:
  - Input: sequence of words $\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \ldots, \boldsymbol{x}^{(t)}$
  - Output: prob. dist. of the next word $P(\boldsymbol{x}^{(t+1)} | \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})$

- How about a window-based neural model?
  - We saw this applied to Named Entity Recognition in Lecture 2:



A fixed-window neural Language Model

~~as the proctor started the clock~~ the students opened their _____

discard

fixed window

A fixed-window neural Language Model

output distribution

$$\hat{\boldsymbol{y}} = \text{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$$

hidden layer

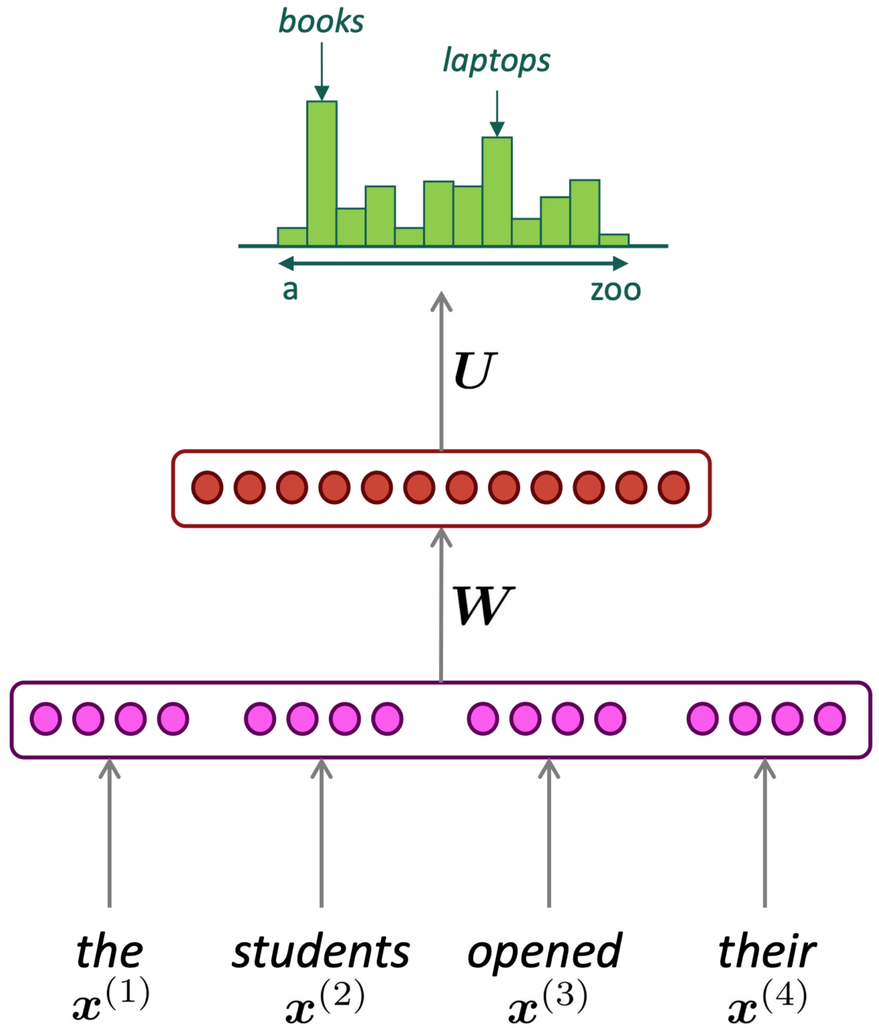
$$\boldsymbol{h} = f(\boldsymbol{W}\boldsymbol{e} + \boldsymbol{b}_1)$$

concatenated word embeddings

$$\boldsymbol{e} = [\boldsymbol{e}^{(1)}; \boldsymbol{e}^{(2)}; \boldsymbol{e}^{(3)}; \boldsymbol{e}^{(4)}]$$

words / one-hot vectors

$$\boldsymbol{x}^{(1)}, \boldsymbol{x}^{(2)}, \boldsymbol{x}^{(3)}, \boldsymbol{x}^{(4)}$$



A fixed-window neural Language Model

Approximately: Y. Bengio, et al. (2000/2003): A Neural Probabilistic Language Model

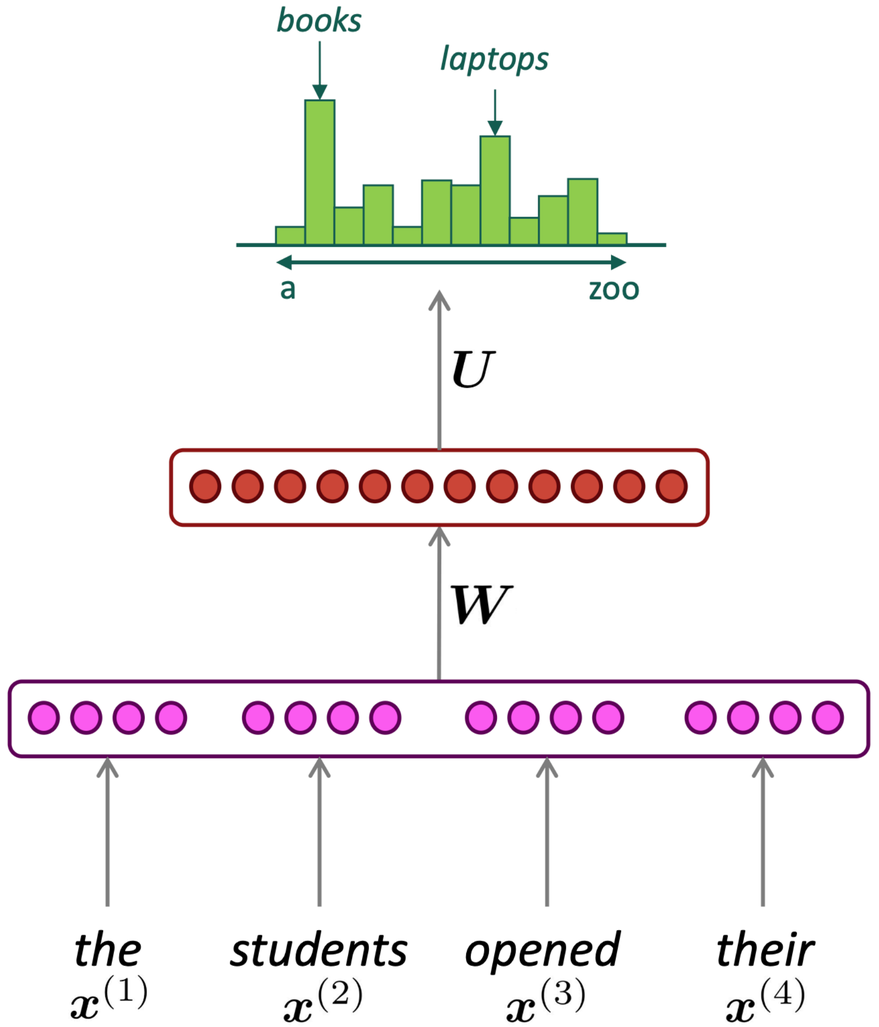
Improvements over *n*-gram LM:

- No sparsity problem
- Don't need to store all observed *n*-grams

Remaining **problems**:

- Fixed window is too small
- Enlarging window enlarges W
- Window can never be large enough!
- $x^{(1)}$ and $x^{(2)}$ are multiplied by completely different weights in W. No symmetry in how the inputs are processed.

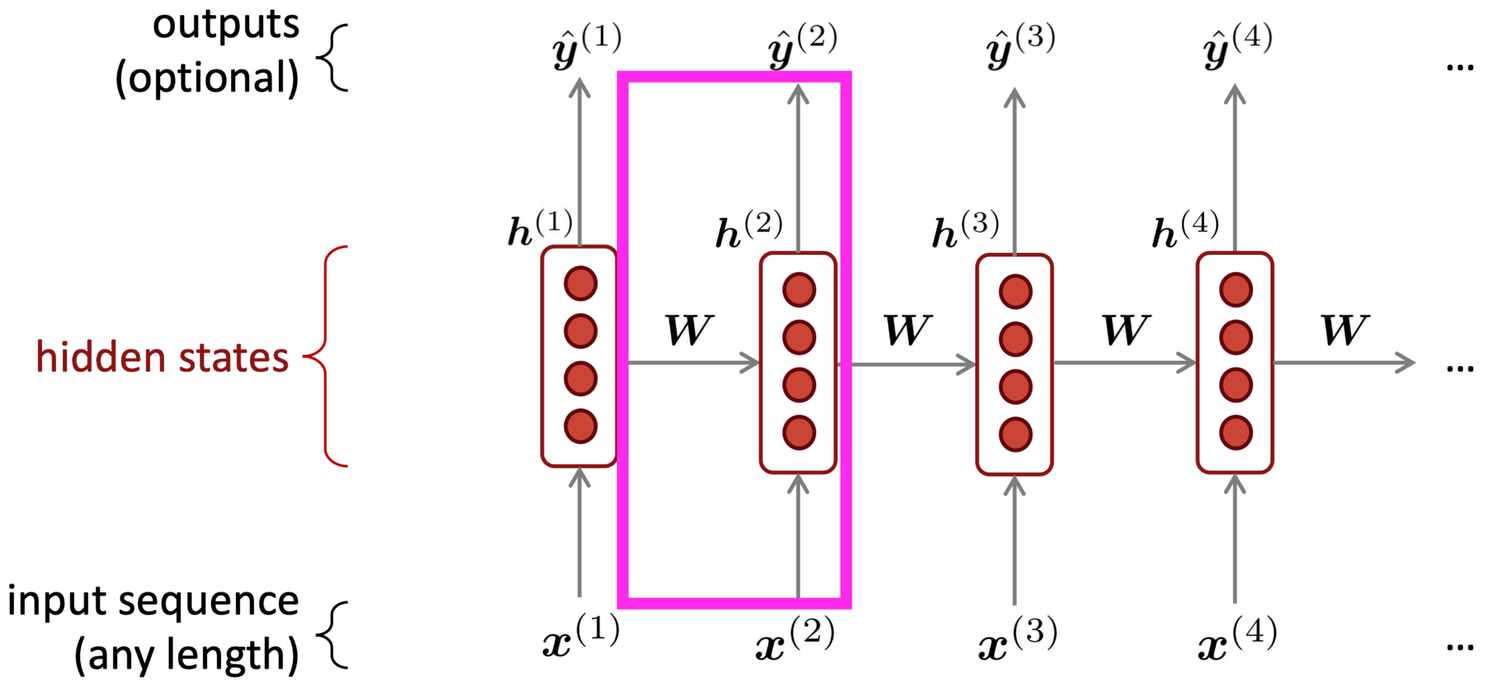
We need a neural architecture that can process *any length input*



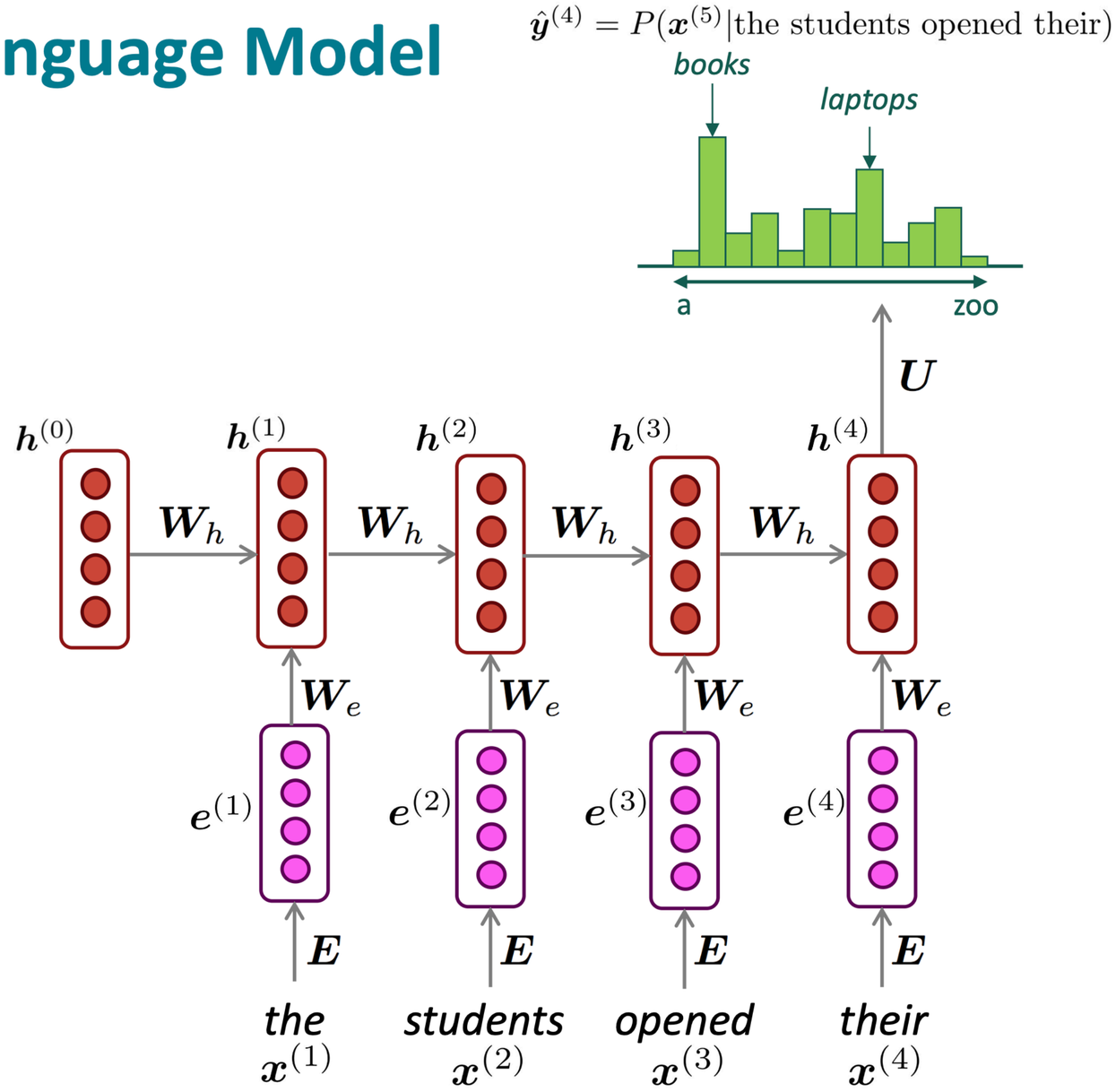
3. Recurrent Neural Networks (RNN)

A family of neural architectures

Core idea: Apply the same weights *W repeatedly*



A Simple RNN Language Model



output distribution

$$\hat{y}^{(t)} = \text{softmax}\left(Uh^{(t)} + b_2\right) \in \mathbb{R}^{|V|}$$

hidden states

$$h^{(t)} = \sigma\left(W_h h^{(t-1)} + W_e e^{(t)} + b_1\right)$$

$h^{(0)}$ is the initial hidden state

word embeddings

$$e^{(t)} = Ex^{(t)}$$

words / one-hot vectors

$$x^{(t)} \in \mathbb{R}^{|V|}$$

***Note**: this input sequence could be much longer now!*

RNN Language Models

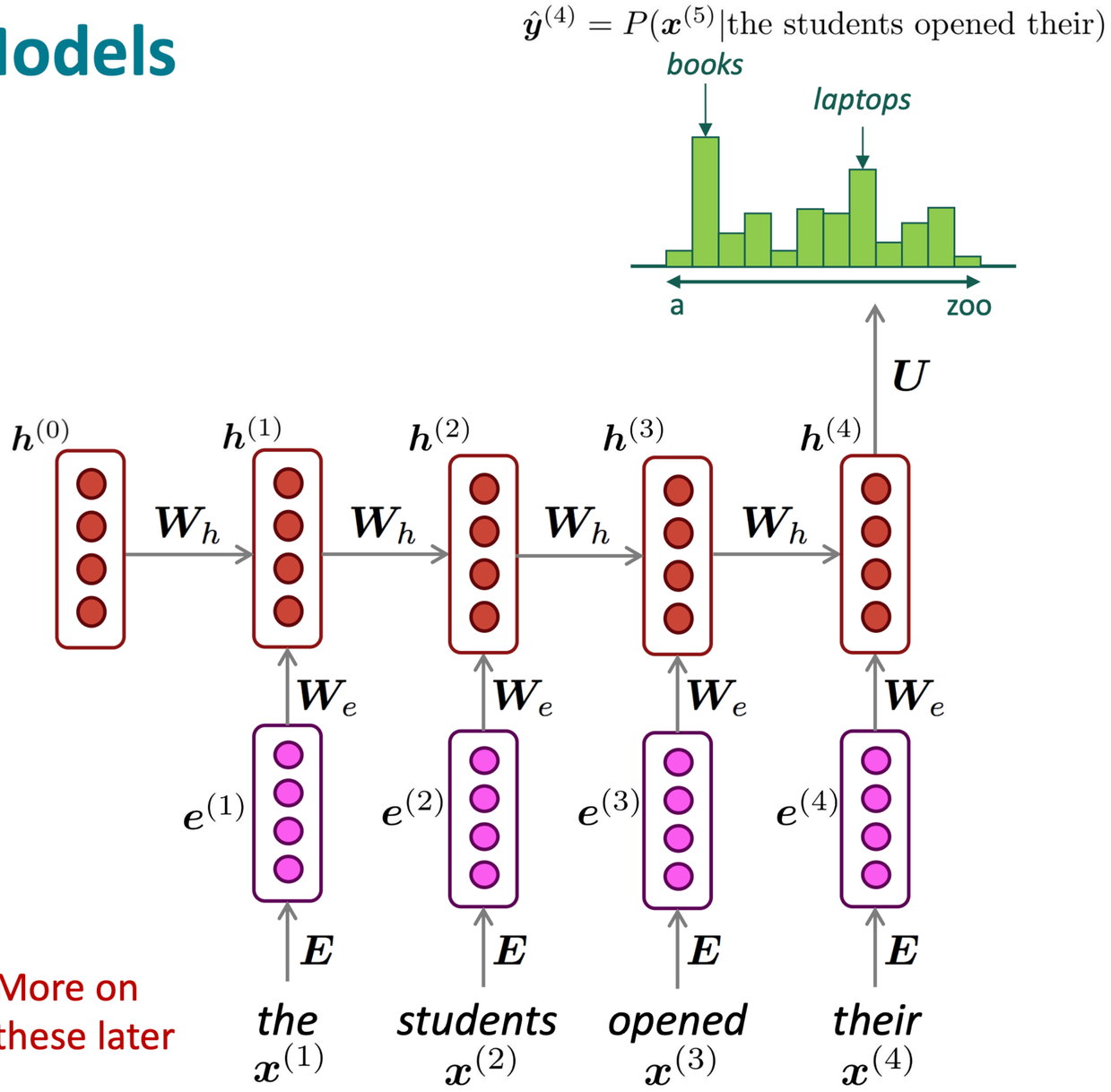
RNN **Advantages**:

- Can process any length input
- Computation for step *t* can (in theory) use information from many steps back
- Model size doesn't increase for longer input context
- Same weights applied on every timestep, so there is symmetry in how inputs are processed.

RNN **Disadvantages**:

- Recurrent computation is slow
- In practice, difficult to access information from many steps back

More on these later



Training an RNN Language Model

- Get a big corpus of text which is a sequence of words $x^{(1)}, \ldots, x^{(T)}$
- Feed into RNN-LM; compute output distribution $\hat{\boldsymbol{y}}^{(t)}$ for *every step t*.
  - i.e., predict probability dist of *every word*, given words so far

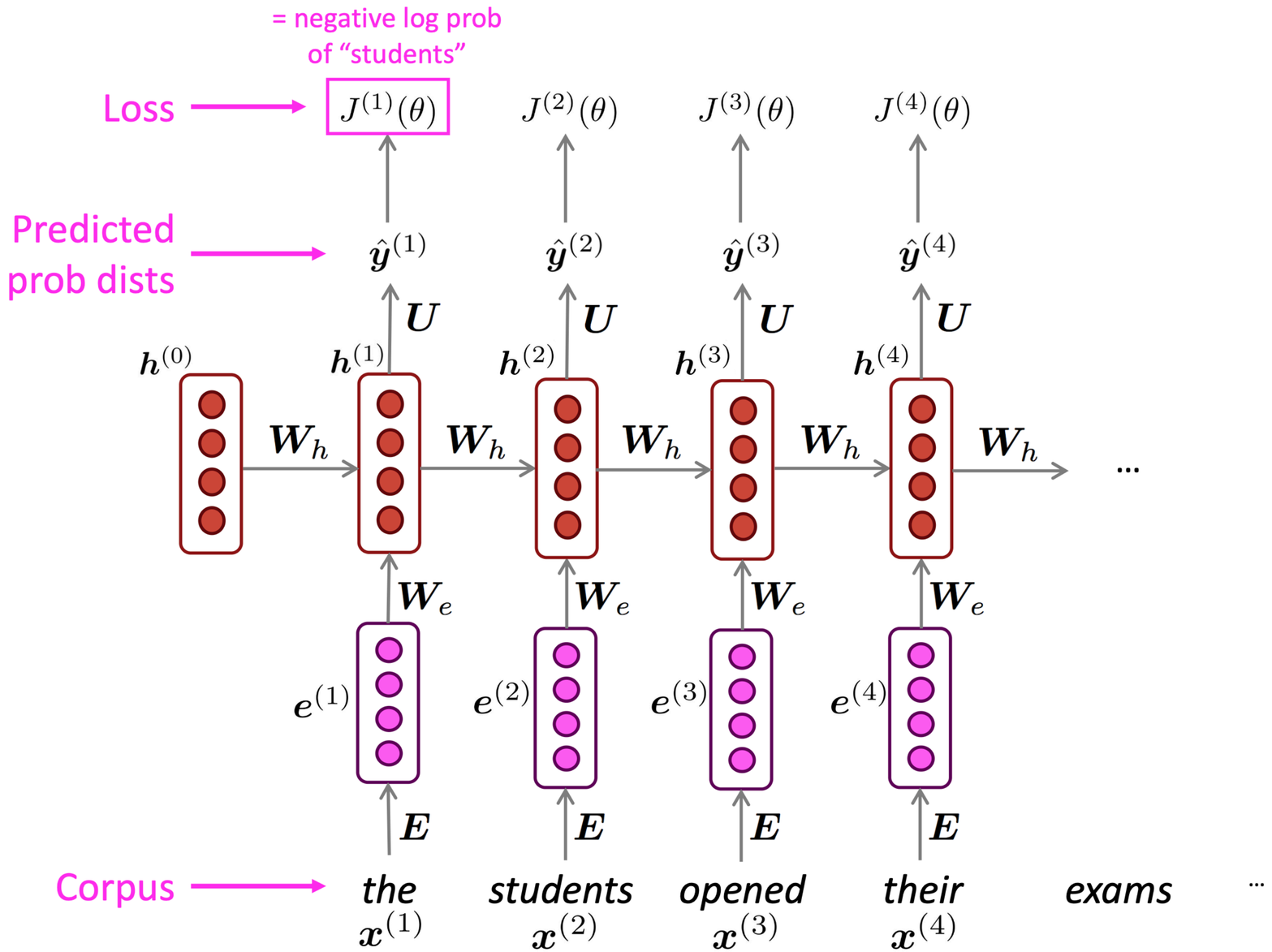
- Loss function on step *t* is cross-entropy between predicted probability distribution $\hat{\boldsymbol{y}}^{(t)}$, and the true next word $\boldsymbol{y}^{(t)}$ (one-hot for $\boldsymbol{x}^{(t+1)}$):

$$J^{(t)}(\theta) = CE(\boldsymbol{y}^{(t)}, \hat{\boldsymbol{y}}^{(t)}) = -\sum_{w \in V} \boldsymbol{y}_w^{(t)} \log \hat{\boldsymbol{y}}_w^{(t)} = -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

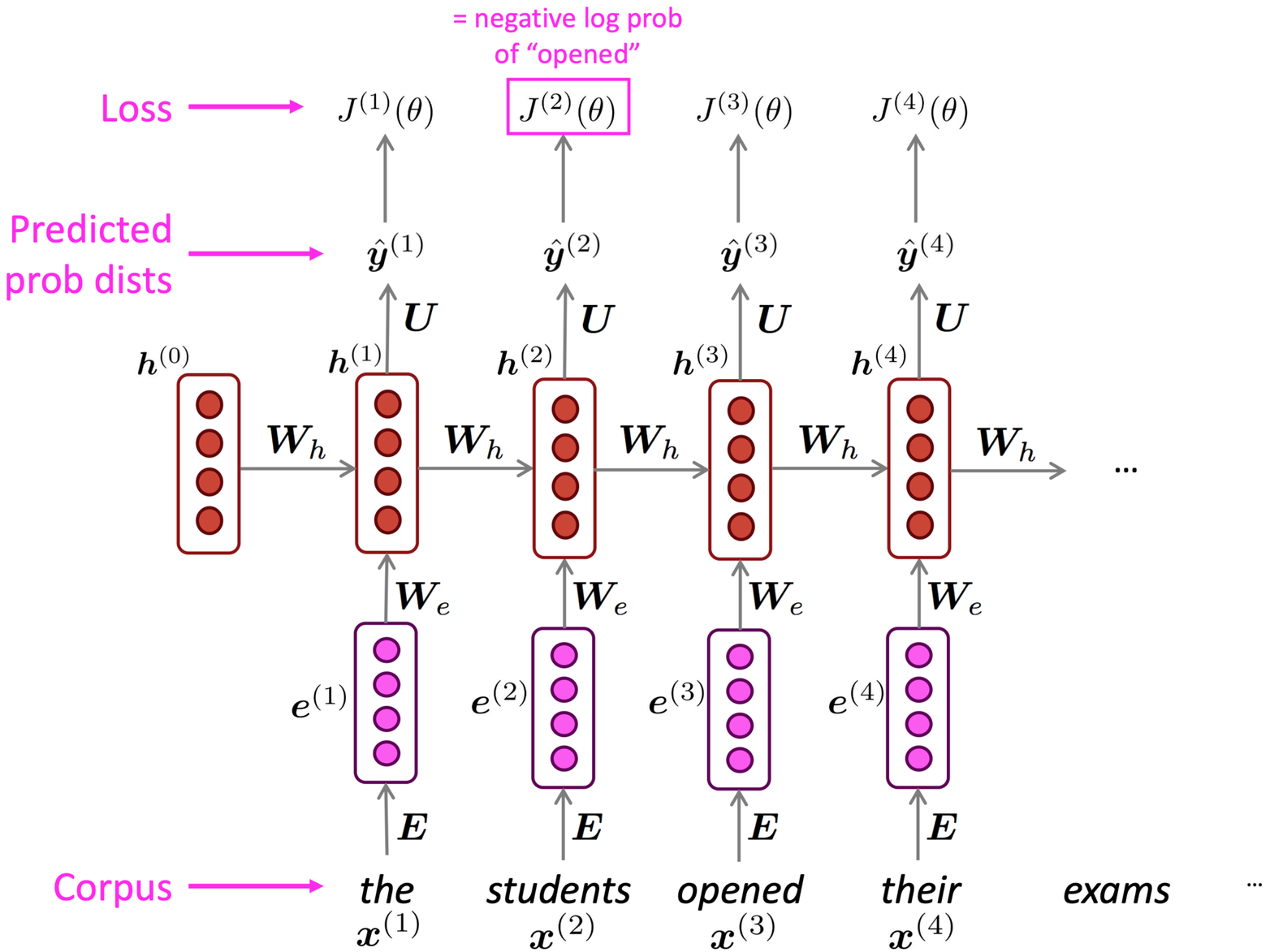
- Average this to get overall loss for entire training set:

$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta) = \frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}_{\boldsymbol{x}_{t+1}}^{(t)}$$

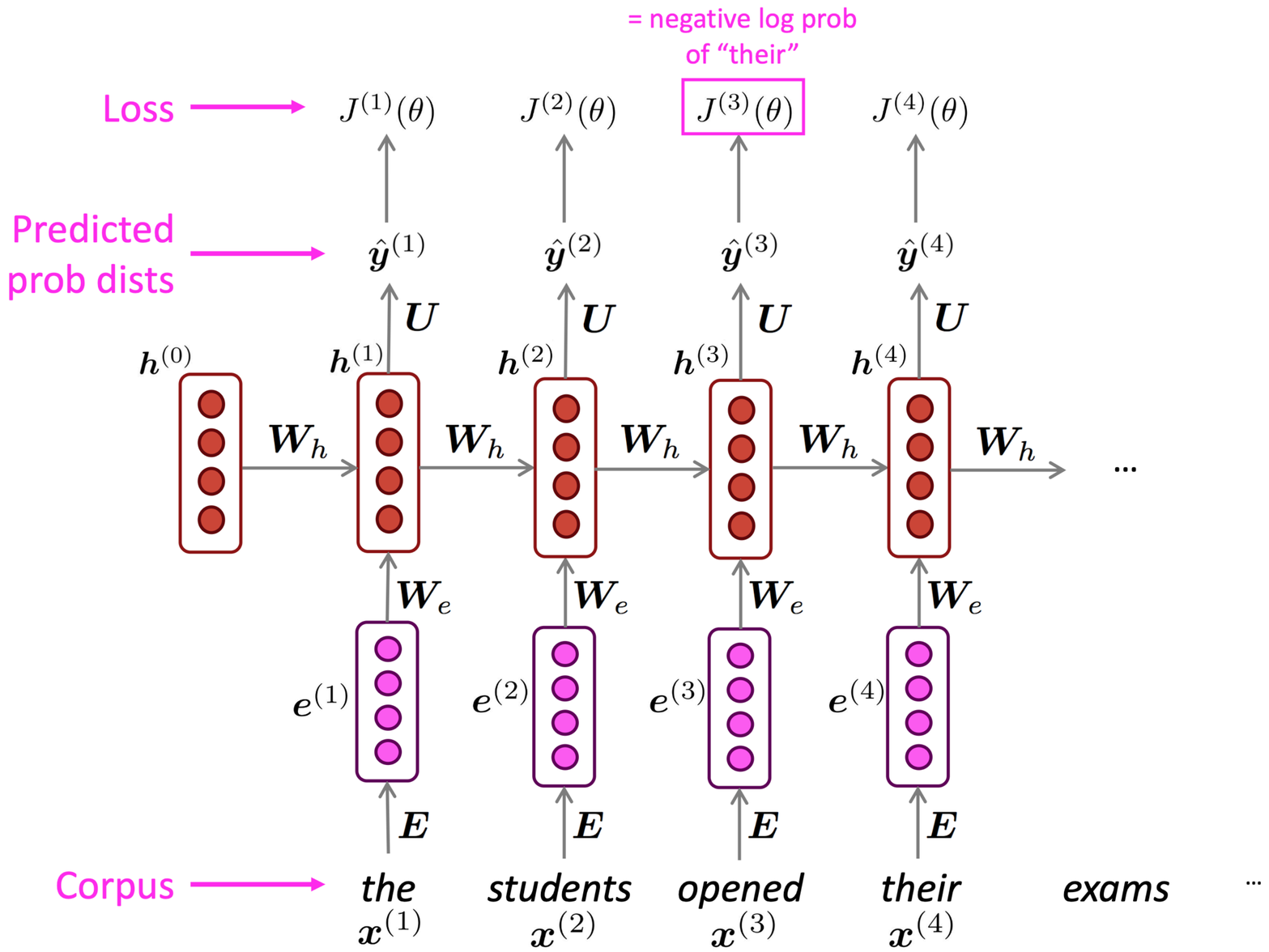
Training an RNN Language Model



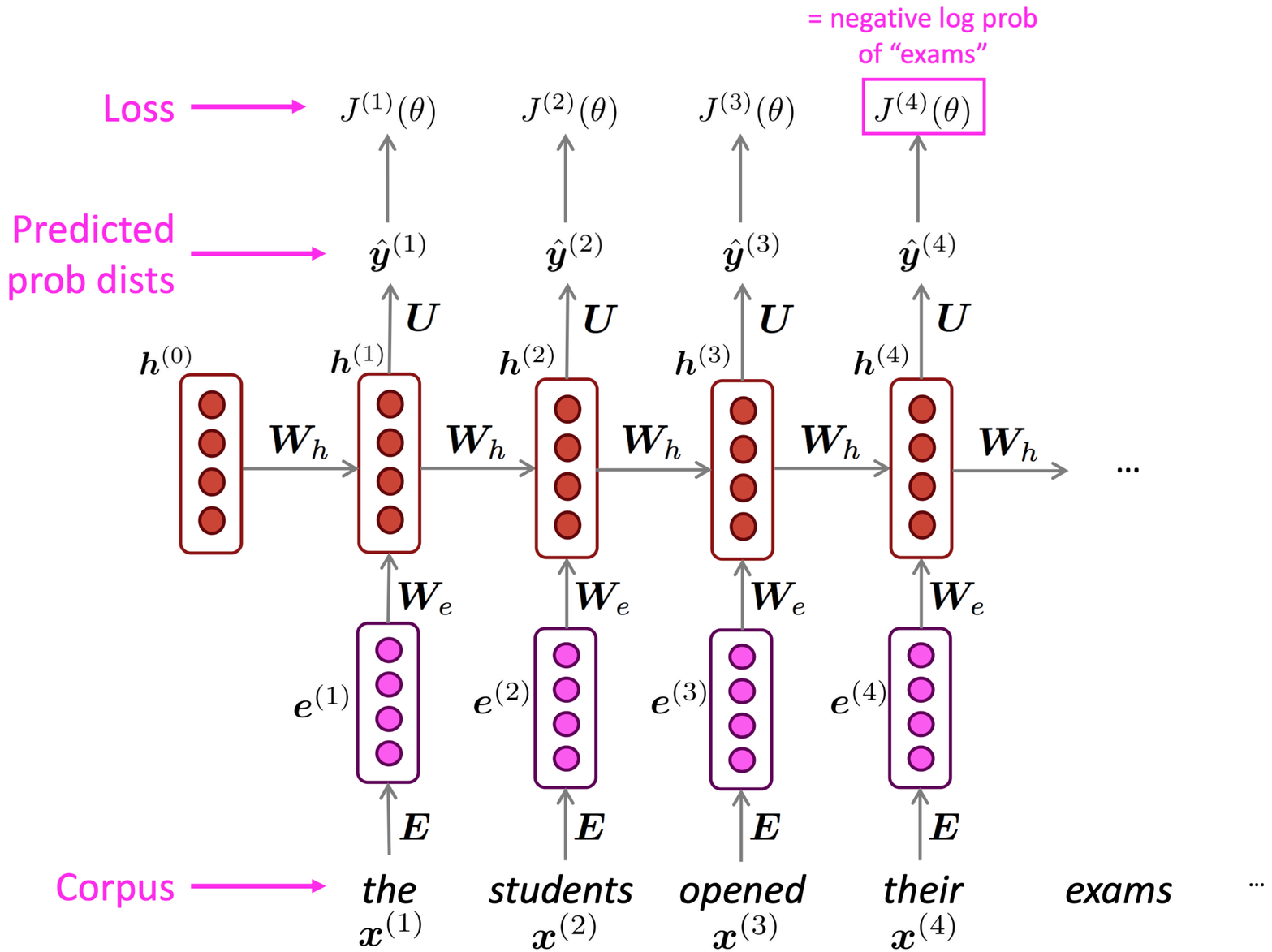
Training an RNN Language Model



Training an RNN Language Model

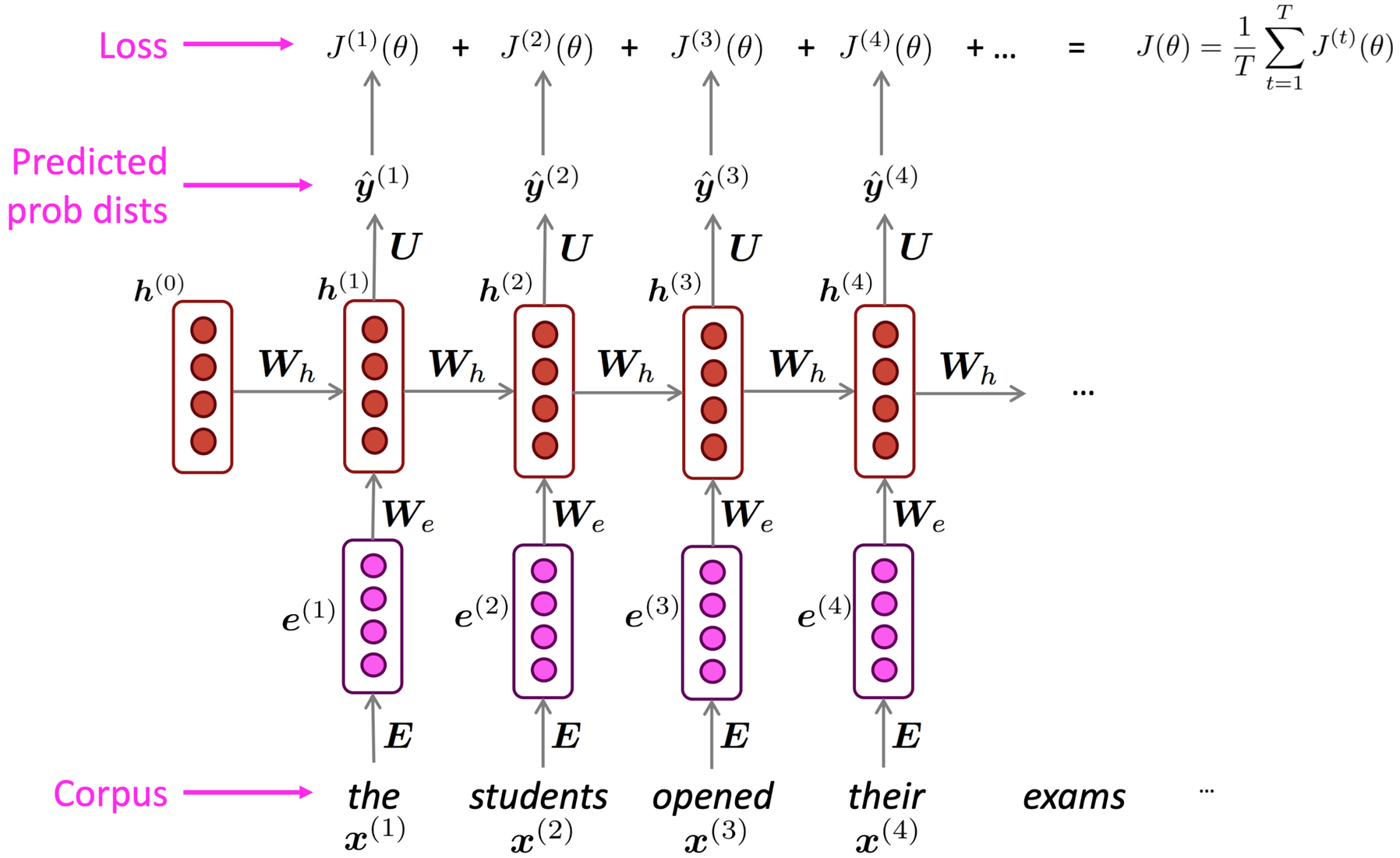


Training an RNN Language Model



Training an RNN Language Model

"Teacher forcing"



Training a RNN Language Model

- However: Computing loss and gradients across entire corpus $x^{(1)}, \ldots, x^{(T)}$ at once is too expensive (memory-wise)!

$$J(\theta) = \frac{1}{T} \sum_{t=1}^{T} J^{(t)}(\theta)$$

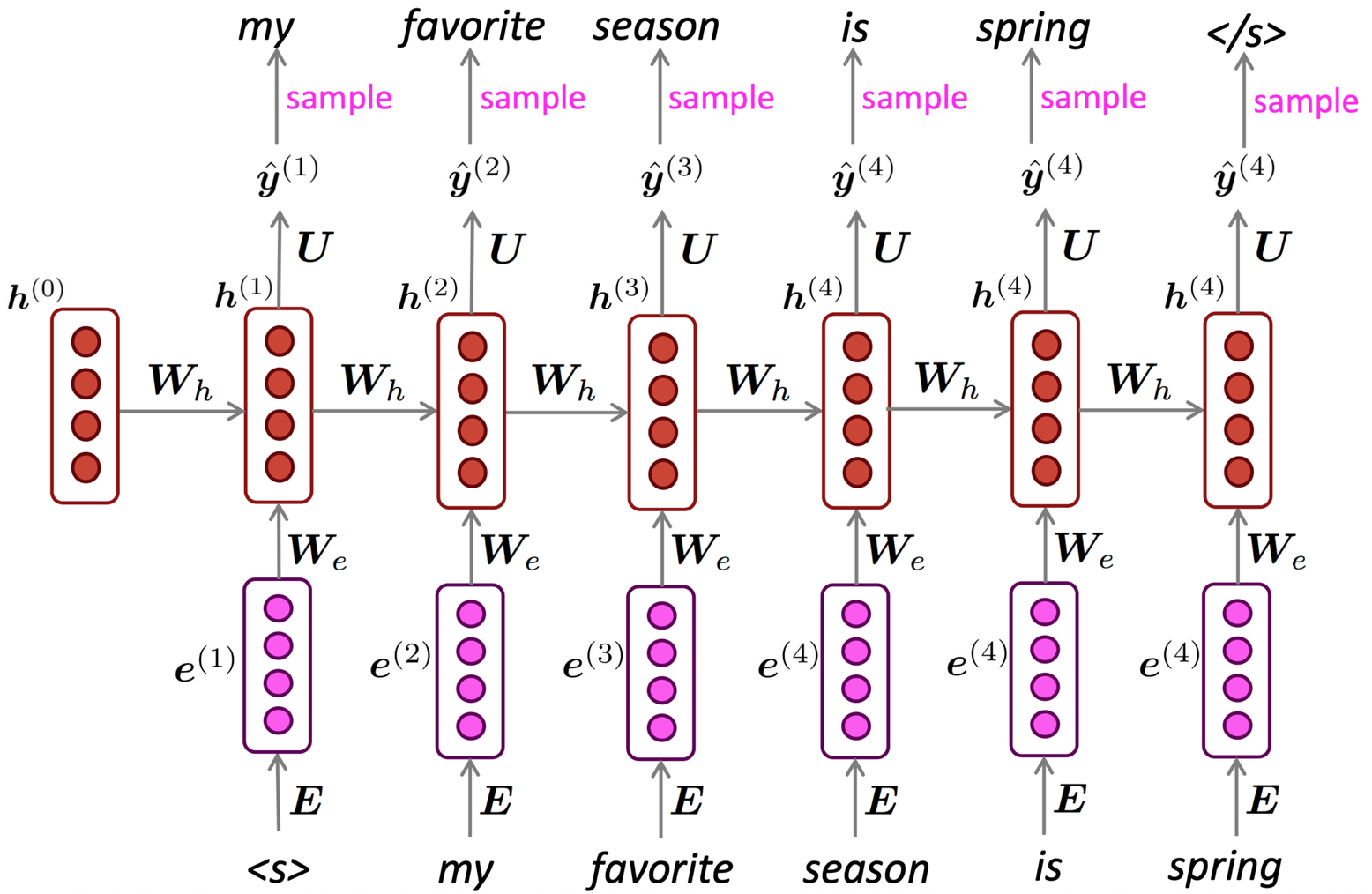
- In practice, consider $x^{(1)}, \ldots, x^{(T)}$ as a sentence (or a document)

- Recall: Stochastic Gradient Descent allows us to compute loss and gradients for small chunk of data, and update.

- Compute loss $J(\theta)$ for a sentence (actually, a batch of sentences), compute gradients and update weights. Repeat on a new batch of sentences.

Generating with an RNN Language Model ("Generating roll outs")

Just like an n-gram Language Model, you can use a RNN Language Model to generate text by repeated sampling. Sampled output becomes next step's input.



Evaluating Language Models

- The standard evaluation metric for Language Models is perplexity.

$$\text{perplexity} = \prod_{t=1}^{T} \left(\frac{1}{P_{\text{LM}}(\boldsymbol{x}^{(t+1)} | \ \boldsymbol{x}^{(t)}, \ldots, \boldsymbol{x}^{(1)})} \right)^{1/T}$$

Normalized by number of words

Inverse probability of corpus, according to Language Model

- This is equal to the exponential of the cross-entropy loss $J(\theta)$:

$$= \prod_{t=1}^{T} \left(\frac{1}{\hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}}} \right)^{1/T} = \exp\left(\frac{1}{T} \sum_{t=1}^{T} -\log \hat{\boldsymbol{y}}^{(t)}_{\boldsymbol{x}_{t+1}} \right) = \exp(J(\theta))$$

Lower perplexity is better!

Building an efficient neural language model over a billion words

Achieving a perplexity of 45 on the 1-billion word on a single GPU in a couple of days

In terms of technical details, beside the adatpive softmax, we use a relatively standard setting: For our small model we use a LSTM with 1 layer and 2048 units each and for the larger one 2 layers with 2048 neurons each. We train the model using Adagrad (citation needed) and a L2 regularization for the weights. We use a batch size of 128 and we set the back-propagation window size to 20.

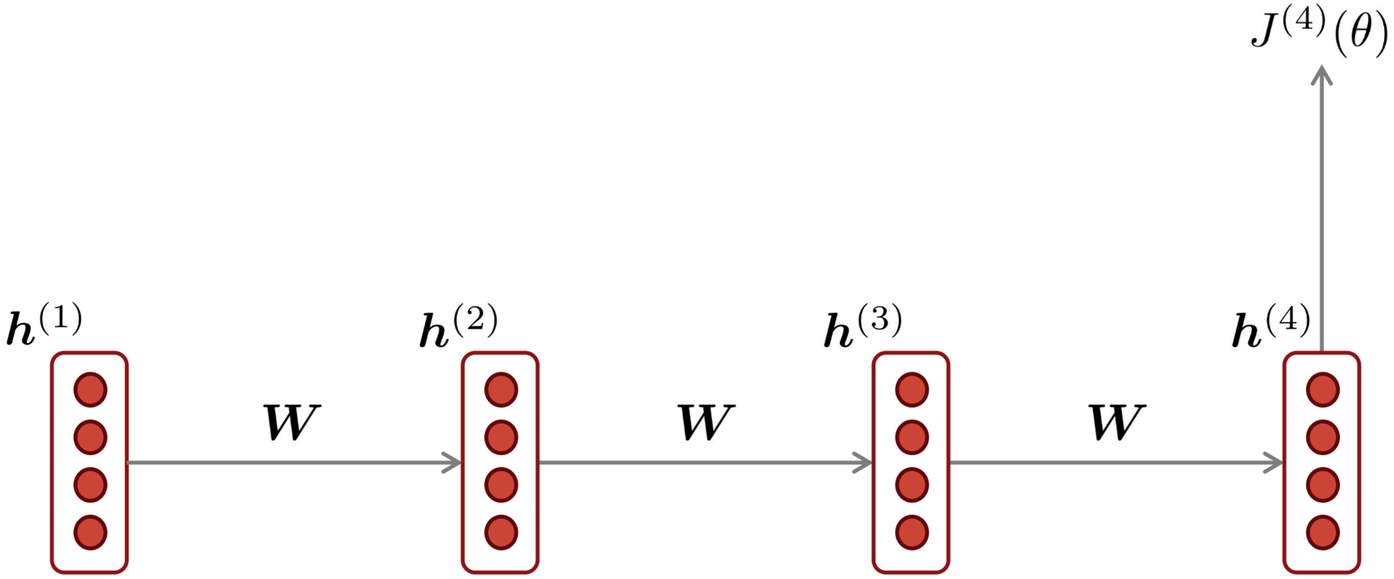
Concerning the Adaptative softmax, we use the optimal setting for the distribution of the train set of 1B words, that is 4 clusters with a dimension reduction of x4 over each cluster (see the paper for details).

Model	Perplexity
Interpolated Kneser-Ney 5-gram (Chelba et al., 2013)	67.6
RNN-1024 + MaxEnt 9-gram (Chelba et al., 2013)	51.3
RNN-2048 + BlackOut sampling (Ji et al., 2015)	68.3
Sparse Non-negative Matrix factorization (Shazeer et al., 2015)	52.9
LSTM-2048 (Jozefowicz et al., 2016)	43.7
2-layer LSTM-8192 (Jozefowicz et al., 2016)	30
Ours small (LSTM-2048)	43.9
Ours large (2-layer LSTM-2048)	39.8

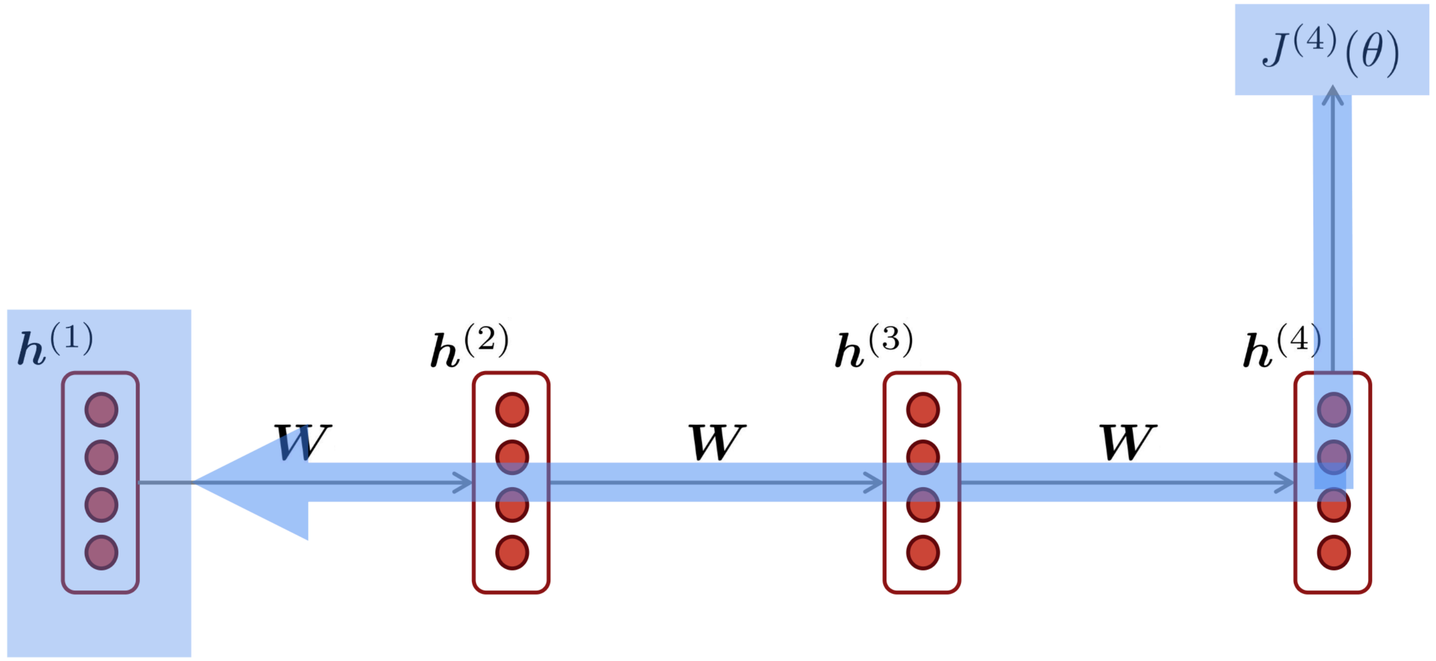
Table 2. Comparison on 1B word in perplexity (lower the better). Note that Jozefowicz et al., uses 32 GPUs for training. We only use 1 GPU.

https://engineering.fb.com/2016/10/25/ml-applications/building-an-efficient-neural-language-model-over-a-billion-words/

4. Problems with RNNs: Vanishing and Exploding Gradients

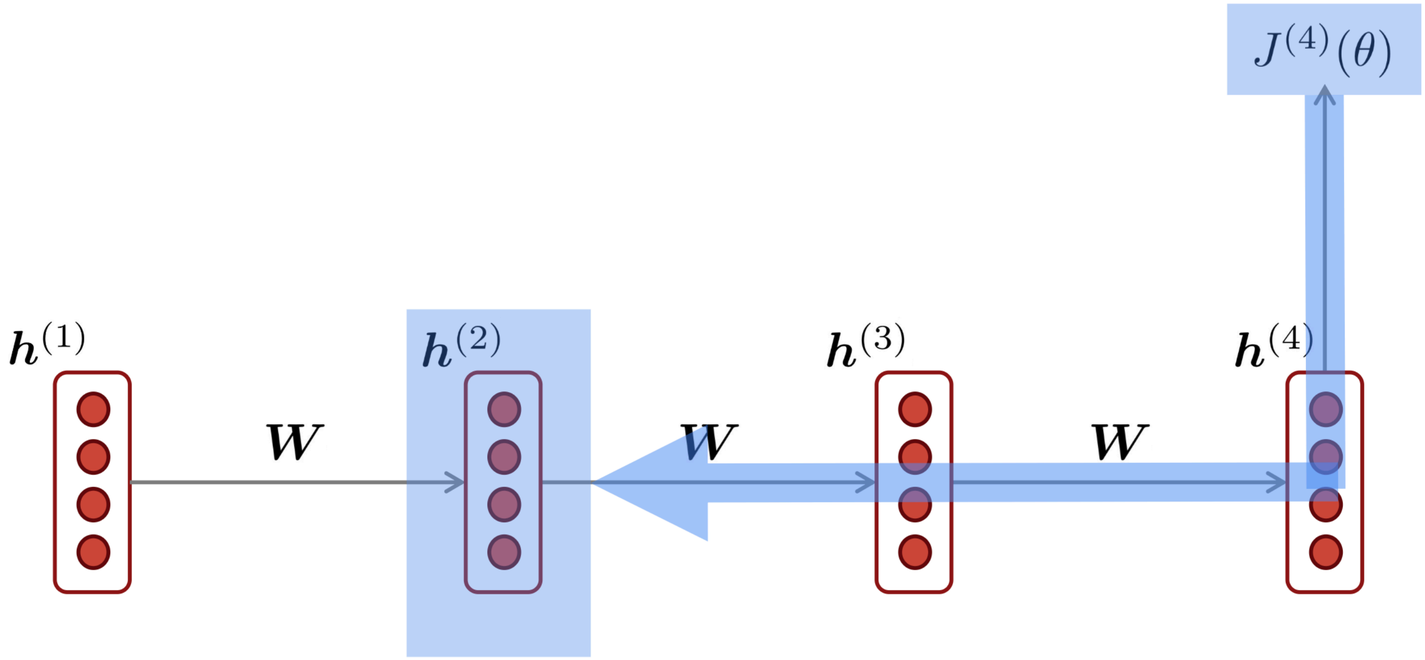


Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial \boldsymbol{h}^{(1)}} = \ ?$$

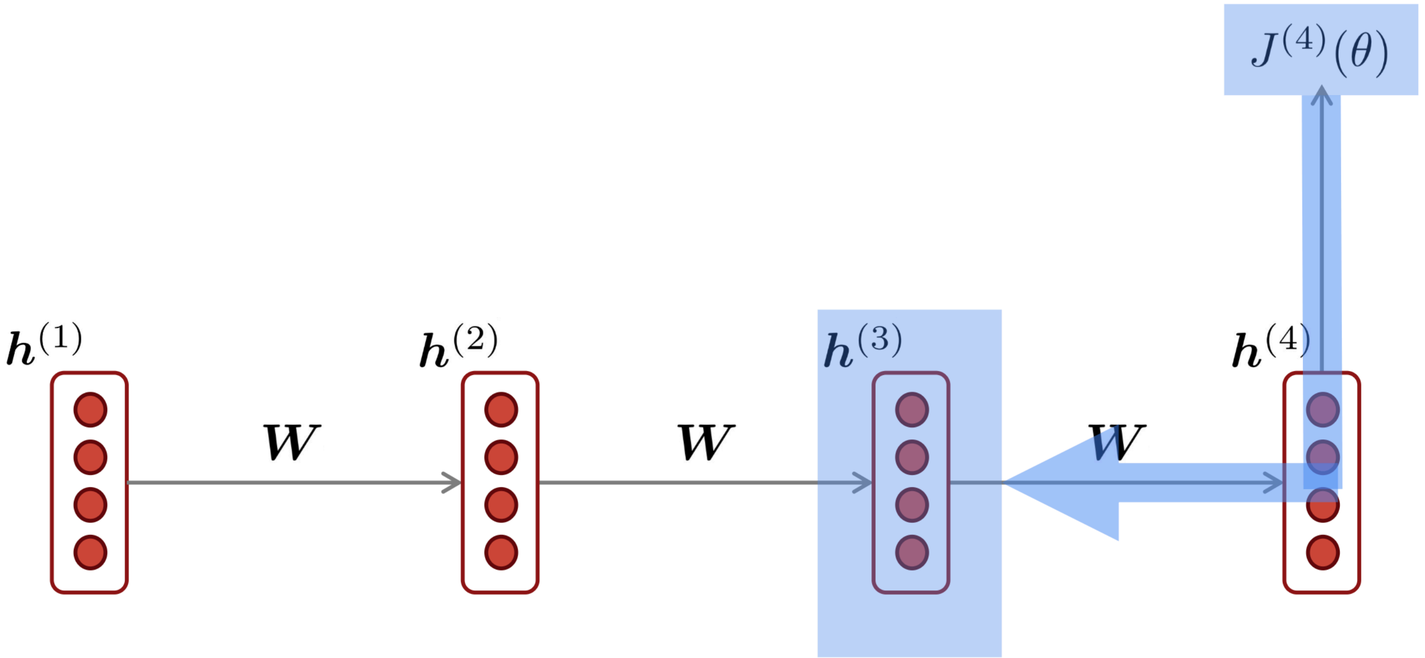
Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial h^{(1)}} = \frac{\partial h^{(2)}}{\partial h^{(1)}} \times \frac{\partial J^{(4)}}{\partial h^{(2)}}$$

chain rule!

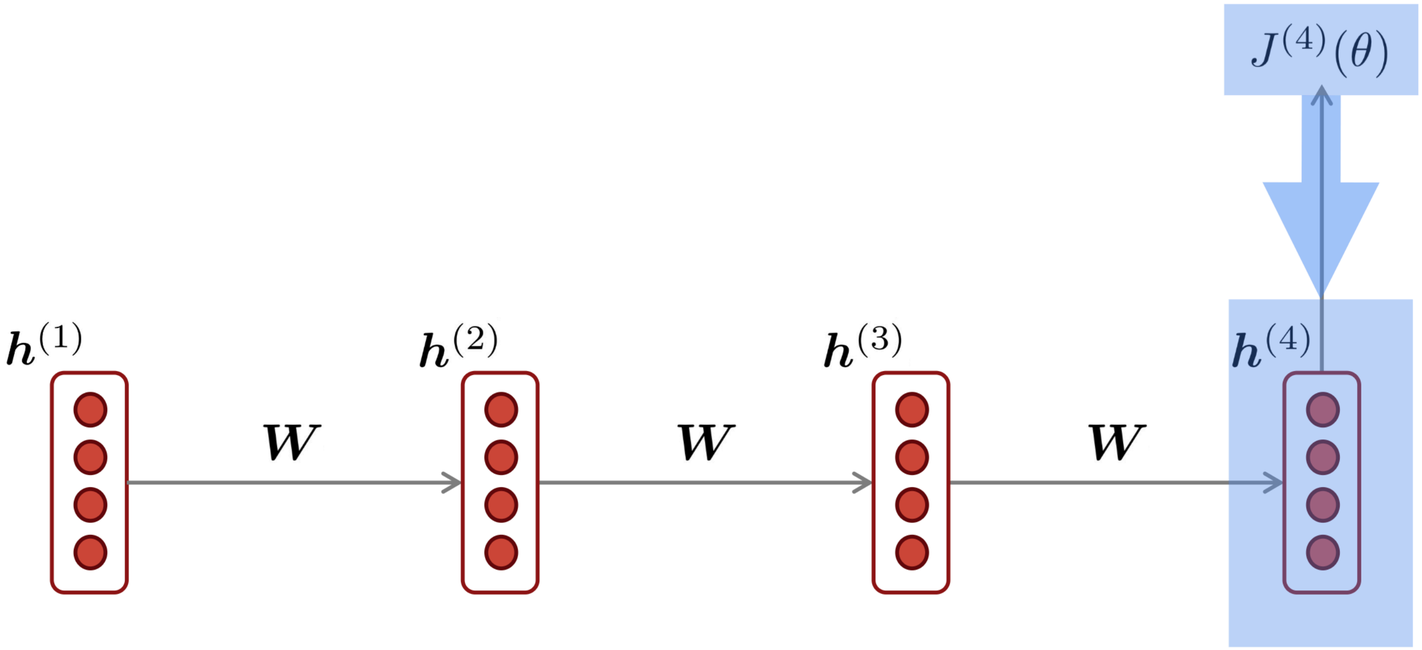
Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial h^{(1)}} = \frac{\partial h^{(2)}}{\partial h^{(1)}} \times \frac{\partial h^{(3)}}{\partial h^{(2)}} \times \frac{\partial J^{(4)}}{\partial h^{(3)}}$$

chain rule!

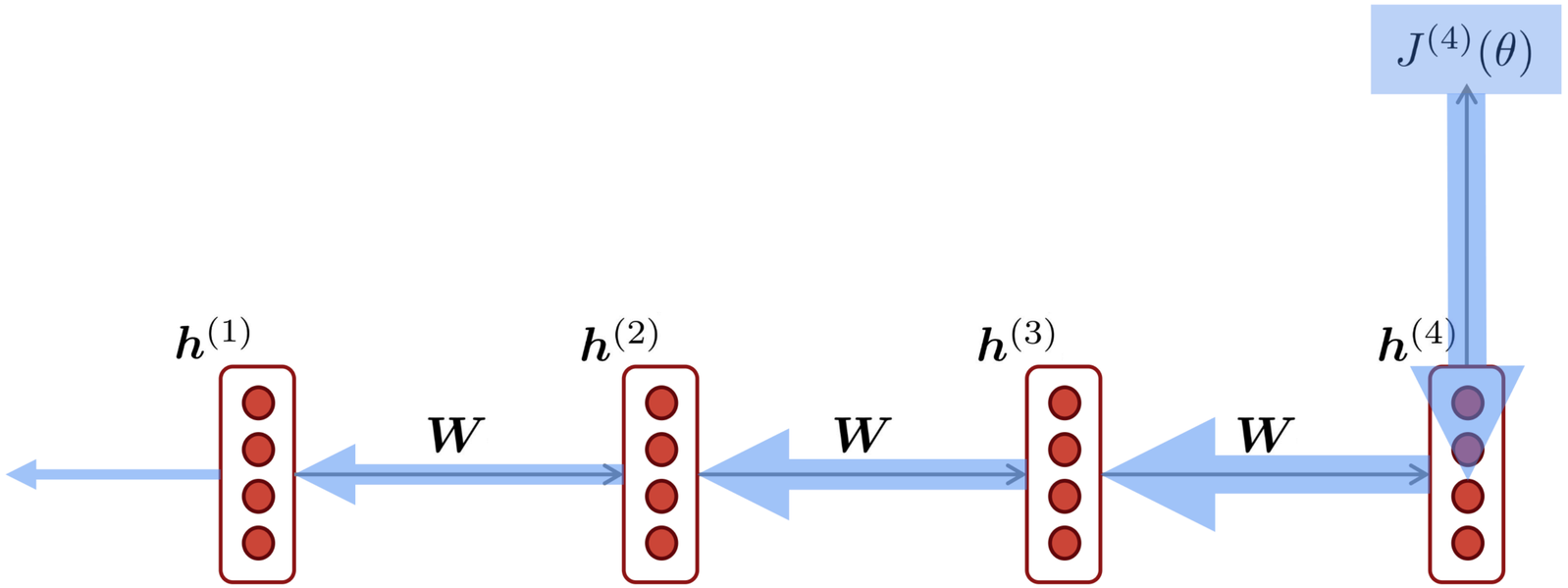
Vanishing gradient intuition



$$\frac{\partial J^{(4)}}{\partial h^{(1)}} = \frac{\partial h^{(2)}}{\partial h^{(1)}} \times \frac{\partial h^{(3)}}{\partial h^{(2)}} \times \frac{\partial h^{(4)}}{\partial h^{(3)}} \times \frac{\partial J^{(4)}}{\partial h^{(4)}}$$

chain rule!

Vanishing gradient intuition

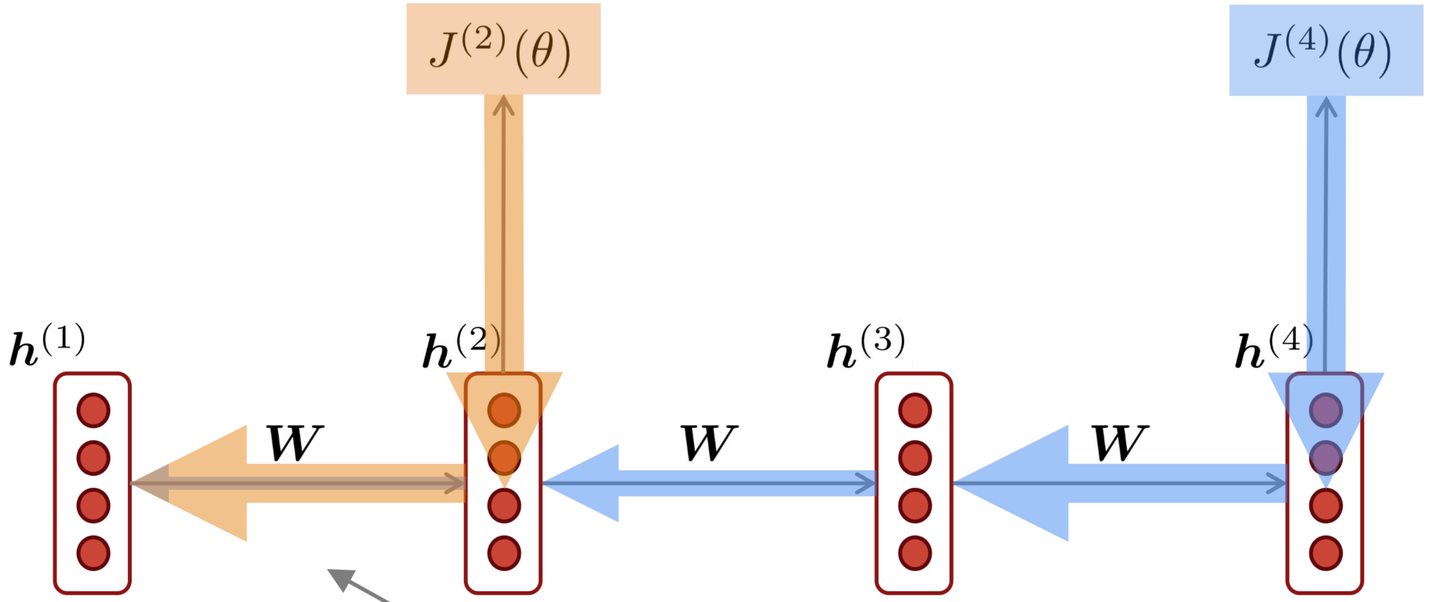


$$\frac{\partial J^{(4)}}{\partial h^{(1)}} = \frac{\partial h^{(2)}}{\partial h^{(1)}} \times \frac{\partial h^{(3)}}{\partial h^{(2)}} \times \frac{\partial h^{(4)}}{\partial h^{(3)}} \times \frac{\partial J^{(4)}}{\partial h^{(4)}}$$

What happens if these are small?

Vanishing gradient problem:
When these are small, the gradient signal gets smaller and smaller as it backpropagates further

Why is vanishing gradient a problem?



Gradient signal from far away is lost because it's much smaller than gradient signal from close-by.

So, model weights are updated only with respect to near effects, not long-term effects.

Effect of vanishing gradient on RNN-LM

- **LM task:** *When she tried to print her tickets, she found that the printer was out of toner. She went to the stationery store to buy more toner. It was very overpriced. After installing the toner into the printer, she finally printed her _____*

- To learn from this training example, the RNN-LM needs to model the dependency between *"tickets"* on the 7th step and the target word *"tickets"* at the end.

- But if the gradient is small, the model can't learn this dependency
  - So, the model is unable to predict similar long-distance dependencies at test time

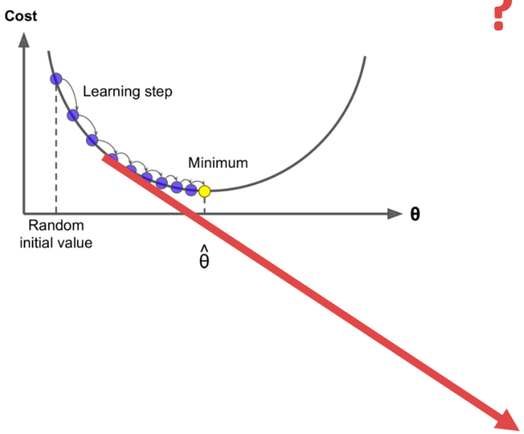
- In practice, a simple RNN will only condition ~7 tokens back [vague rule-of-thumb]

Why is exploding gradient a problem?

- If the gradient becomes too big, then the SGD update step becomes too big:

$$\theta^{new} = \theta^{old} - \overbrace{\alpha}^{\text{learning rate}} \underbrace{\nabla_\theta J(\theta)}_{\text{gradient}}$$

- This can cause bad updates: we take too large a step and reach a weird and bad parameter configuration (with large loss)
  - You think you've found a hill to climb, but suddenly you're in Iowa



?

- In the worst case, this will result in Inf or NaN in your network (then you have to restart training from an earlier checkpoint)

Gradient clipping: solution for exploding gradient

- **Gradient clipping**: if the norm of the gradient is greater than some threshold, scale it down before applying SGD update

Algorithm 1 Pseudo-code for norm clipping

$\hat{\mathbf{g}} \leftarrow \frac{\partial \mathcal{E}}{\partial \theta}$
if $\|\hat{\mathbf{g}}\| \geq threshold$ **then**
  $\hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$
end if

- **Intuition**: take a step in the same direction, but a smaller step

- In practice, **remembering to clip gradients is important**, but exploding gradients are an easy problem to solve

Source: "On the difficulty of training recurrent neural networks", Pascanu et al, 2013. http://proceedings.mlr.press/v28/pascanu13.pdf

How to fix the vanishing gradient problem?

- The main problem is that *it's too difficult for the RNN to learn to preserve information over many timesteps*.

- In a vanilla RNN, the hidden state is constantly being rewritten

$$\boldsymbol{h}^{(t)} = \sigma\left(\boldsymbol{W}_h \boldsymbol{h}^{(t-1)} + \boldsymbol{W}_x \boldsymbol{x}^{(t)} + \boldsymbol{b}\right)$$

- First off next time: How about an RNN with separate memory which is added to?
  - LSTMs

- And then: Creating more direct and linear pass-through connections in model
  - Attention, residual connections, etc.

Long Short-Term Memory RNNs (LSTMs)

- On step *t*, there is a hidden state $\boldsymbol{h}^{(t)}$ and a cell state $\boldsymbol{c}^{(t)}$
  - Both are vectors length *n*
  - The cell stores long-term information
  - The LSTM can read, erase, and write information from the cell
    - The cell becomes conceptually rather like RAM in a computer

- The selection of which information is erased/written/read is controlled by three corresponding gates (gates are calculated things whose values are probabilities)
  - The gates are also vectors of length *n*
  - On each timestep, each element of the gates can be open (1), closed (0), or somewhere in-between
  - The gates are dynamic: their value is computed based on the current context

Long Short-Term Memory (LSTM)

We have a sequence of inputs $x^{(t)}$, and we will compute a sequence of hidden states $h^{(t)}$ and cell states $c^{(t)}$. On timestep t:

Sigmoid function: all gate values are between 0 and 1

Forget gate: controls what is kept vs forgotten, from previous cell state

$$\boldsymbol{f}^{(t)} = \sigma\left(\boldsymbol{W}_f \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_f \boldsymbol{x}^{(t)} + \boldsymbol{b}_f\right)$$

Input gate: controls what parts of the new cell content are written to cell

$$\boldsymbol{i}^{(t)} = \sigma\left(\boldsymbol{W}_i \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_i \boldsymbol{x}^{(t)} + \boldsymbol{b}_i\right)$$

Output gate: controls what parts of cell are output to hidden state

$$\boldsymbol{o}^{(t)} = \sigma\left(\boldsymbol{W}_o \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_o \boldsymbol{x}^{(t)} + \boldsymbol{b}_o\right)$$

New cell content: this is the new content to be written to the cell

$$\tilde{\boldsymbol{c}}^{(t)} = \tanh\left(\boldsymbol{W}_c \boldsymbol{h}^{(t-1)} + \boldsymbol{U}_c \boldsymbol{x}^{(t)} + \boldsymbol{b}_c\right)$$

Cell state: erase ("forget") some content from last cell state, and write ("input") some new cell content

$$\boldsymbol{c}^{(t)} = \boldsymbol{f}^{(t)} \odot \boldsymbol{c}^{(t-1)} + \boldsymbol{i}^{(t)} \odot \tilde{\boldsymbol{c}}^{(t)}$$

Hidden state: read ("output") some content from the cell

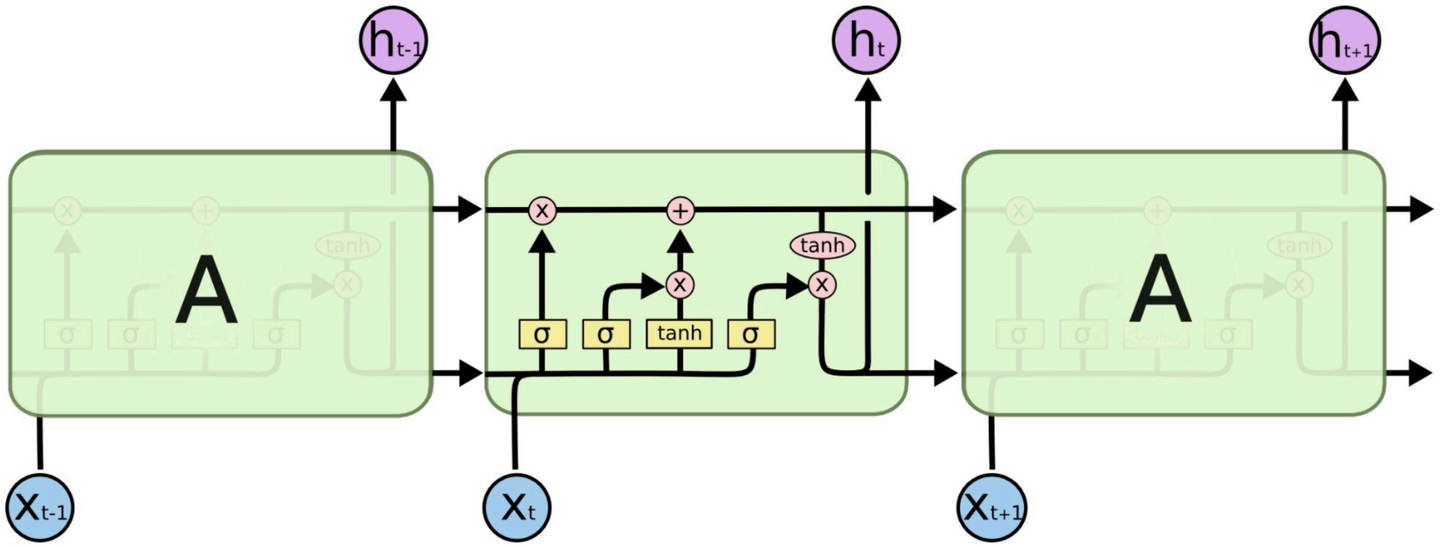
$$\boldsymbol{h}^{(t)} = \boldsymbol{o}^{(t)} \odot \tanh \boldsymbol{c}^{(t)}$$

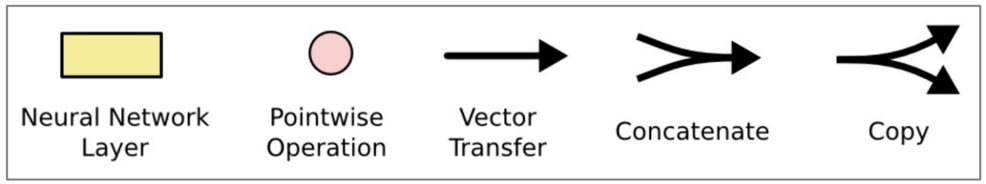
All these are vectors of same length n

Gates are applied using element-wise (or Hadamard) product: $\odot$

Long Short-Term Memory (LSTM)

You can think of the LSTM equations visually like this:



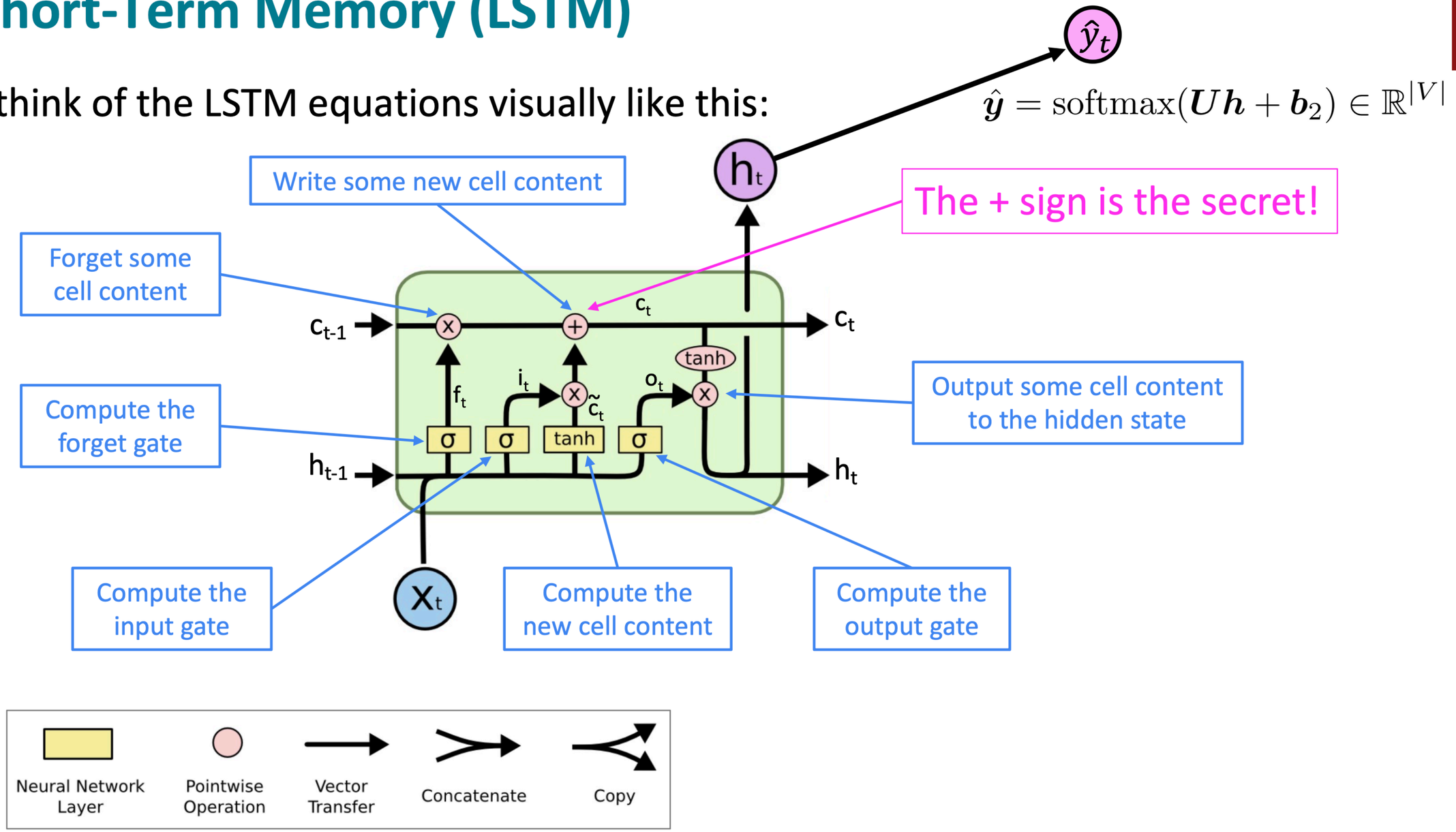


Source: http://colah.github.io/posts/2015-08-Understanding-LSTMs/

Long Short-Term Memory (LSTM)

You can think of the LSTM equations visually like this:

$$\hat{\boldsymbol{y}} = \text{softmax}(\boldsymbol{U}\boldsymbol{h} + \boldsymbol{b}_2) \in \mathbb{R}^{|V|}$$



Source: http://colah.github.io/posts/2015-08-Understanding-LSTMs/

ARTIFICIAL INTELLIGENCE

LSTM Networks | A Detailed Explanation

A Comprehensive Introduction to LSTMs

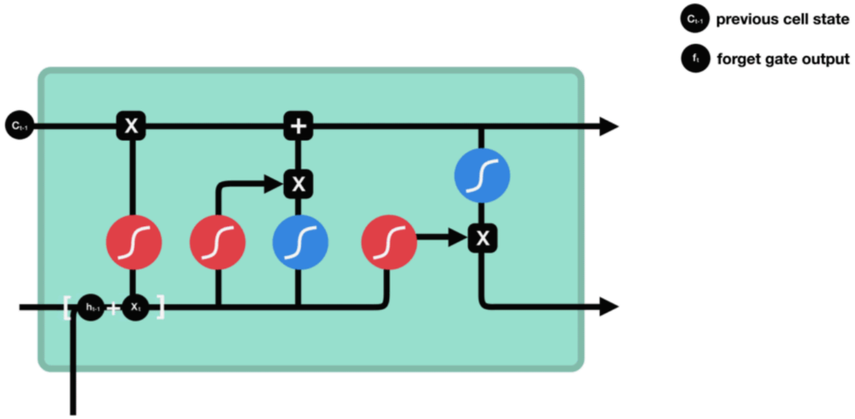
https://medium.com/data-science/illustrated-guide-to-lstms-and-gru-s-a-step-by-step-explanation-44e9eb85bf21

https://colah.github.io/posts/2015-08-Understanding-LSTMs/

https://towardsdatascience.com/lstm-networks-a-detailed-explanation-8fae6aefc7f9/

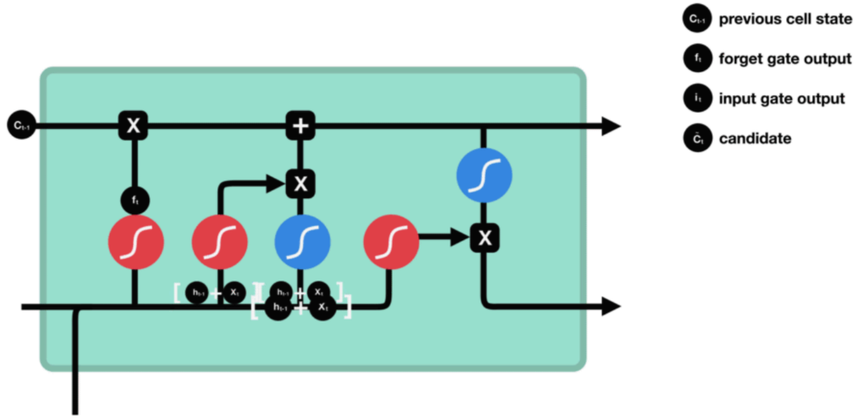
Forget gate

First, we have the forget gate. This gate decides what information should be thrown away or kept. Information from the previous hidden state and information from the current input is passed through the sigmoid function. Values come out between 0 and 1. The closer to 0 means to forget, and the closer to 1 means to keep.



Input Gate

To update the cell state, we have the input gate. First, we pass the previous hidden state and current input into a sigmoid function. That decides which values will be updated by transforming the values to be between 0 and 1. 0 means not important, and 1 means important. You also pass the hidden state and current input into the tanh function to squish values between -1 and 1 to help regulate the network. Then you multiply the tanh output with the sigmoid output. The sigmoid output will decide which information is important to keep from the tanh output.

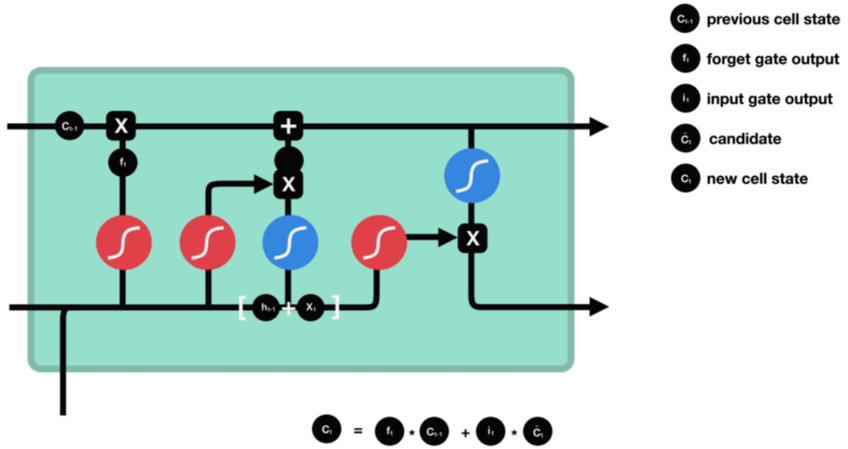


Input gate operations

https://medium.com/data-science/illustrated-guide-to-lstms-and-gru-s-a-step-by-step-explanation-44e9eb85bf21

Cell State

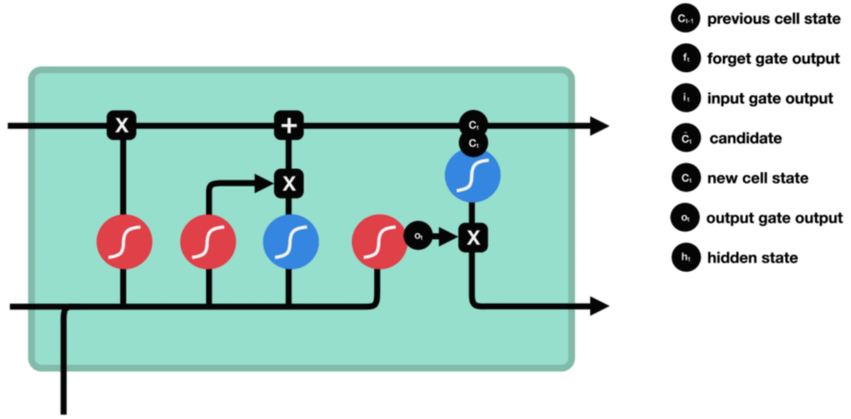
Now we should have enough information to calculate the cell state. First, the cell state gets pointwise multiplied by the forget vector. This has a possibility of dropping values in the cell state if it gets multiplied by values near 0. Then we take the output from the input gate and do a pointwise addition which updates the cell state to new values that the neural network finds relevant. That gives us our new cell state.



Calculating cell state

Output Gate

Last we have the output gate. The output gate decides what the next hidden state should be. Remember that the hidden state contains information on previous inputs. The hidden state is also used for predictions. First, we pass the previous hidden state and the current input into a sigmoid function. Then we pass the newly modified cell state to the tanh function. We multiply the tanh output with the sigmoid output to decide what information the hidden state should carry. The output is the hidden state. The new cell state and the new hidden is then carried over to the next time step.



output gate operations

https://medium.com/data-science/illustrated-guide-to-lstms-and-gru-s-a-step-by-step-explanation-44e9eb85bf21

How does LSTM solve vanishing gradients?

- The LSTM architecture makes it much easier for an RNN to preserve information over many timesteps
  - e.g., if the forget gate is set to 1 for a cell dimension and the input gate set to 0, then the information of that cell is preserved indefinitely.
  - In contrast, it's harder for a vanilla RNN to learn a recurrent weight matrix W_h that preserves info in the hidden state
  - In practice, you get about 100 timesteps rather than about 7

- However, there are alternative ways of creating more direct and linear pass-through connections in models for long distance dependencies

Is vanishing/exploding gradient just an RNN problem?

- No! It can be a problem for all neural architectures (including feed-forward and convolutional neural networks), especially very deep ones.
  - Due to chain rule / choice of nonlinearity function, gradient can become vanishingly small as it backpropagates
  - Thus, lower layers are learned very slowly (i.e., are hard to train)
- Another solution: lots of new deep feedforward/convolutional architectures add more direct connections (thus allowing the gradient to flow)

For example:

- Residual connections aka "ResNet"
- Also known as skip-connections
- The identity connection preserves information by default
- This makes deep networks much easier to train

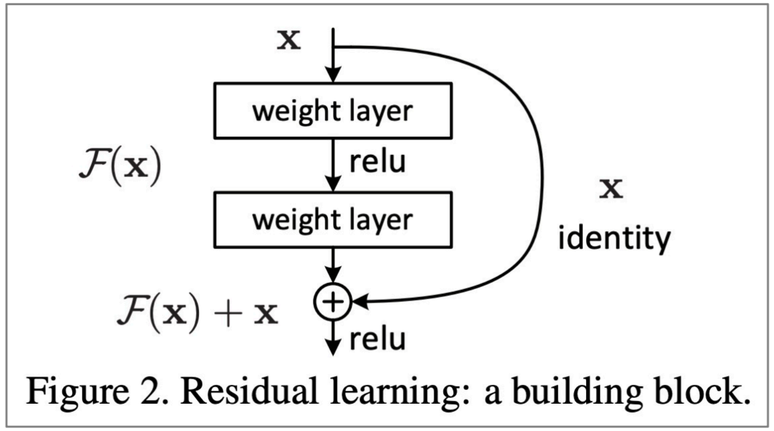


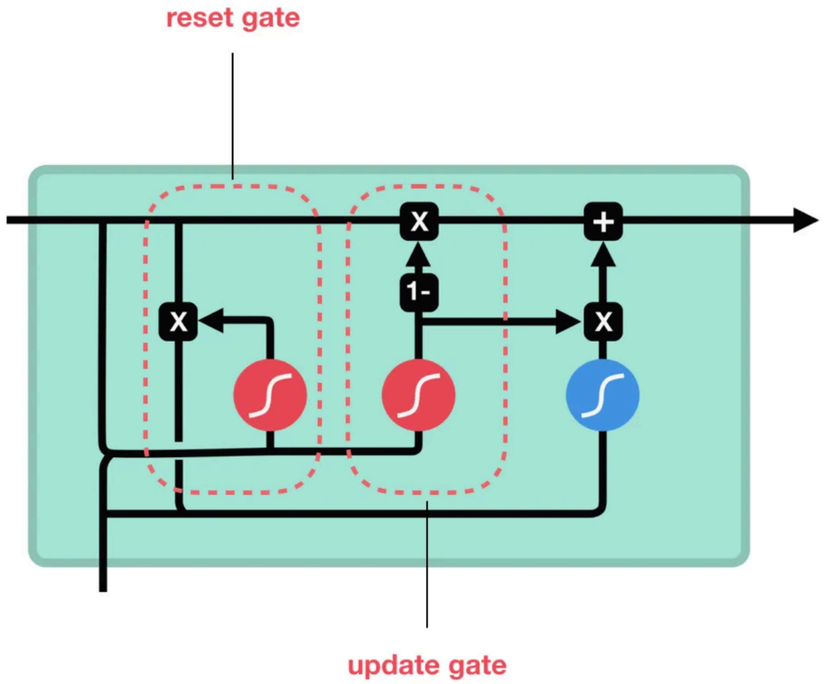
Figure 2. Residual learning: a building block.

"Deep Residual Learning for Image Recognition", He et al, 2015. https://arxiv.org/pdf/1512.03385.pdf

GRU

1

So now we know how an LSTM work, let's briefly look at the GRU. The GRU is the newer generation of Recurrent Neural networks and is pretty similar to an LSTM. GRU's got rid of the cell state and used the hidden state to transfer information. It also only has two gates, a reset gate and update gate.



GRU cell and it's gates

Fully gated unit [edit]

Initially, for $t = 0$, the output vector is $h_0 = 0$.

$$
\begin{aligned}
z_t &= \sigma(W_z x_t + U_z h_{t-1} + b_z) \\
r_t &= \sigma(W_r x_t + U_r h_{t-1} + b_r) \\
\hat{h}_t &= \phi(W_h x_t + U_h (r_t \odot h_{t-1}) + b_h) \\
h_t &= (1 - z_t) \odot h_{t-1} + z_t \odot \hat{h}_t
\end{aligned}
$$

Variables (d denotes the number of input features and e the number of output features):

- $x_t \in \mathbb{R}^d$: input vector
- $h_t \in \mathbb{R}^e$: output vector
- $\hat{h}_t \in \mathbb{R}^e$: candidate activation vector
- $z_t \in (0, 1)^e$: update gate vector
- $r_t \in (0, 1)^e$: reset gate vector
- $W \in \mathbb{R}^{e \times d}$, $U \in \mathbb{R}^{e \times e}$ and $b \in \mathbb{R}^e$: parameter matrices and vector which need to b

Activation functions

- σ: The original is a logistic function.
- ϕ: The original is a hyperbolic tangent.

10.2. Gated Recurrent Units (GRU)

SAGEMAKER STUDIO LAB COLAB [PYTORCH]

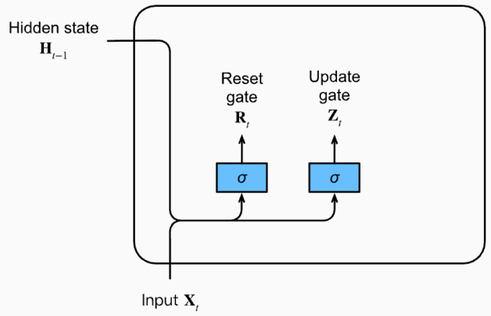
As RNNs and particularly the LSTM architecture (Section 10.1) rapidly gained popularity during the 2010s, a number of researchers began to experiment with simplified architectures in hopes of retaining the key idea of incorporating an internal state and multiplicative gating mechanisms but with the aim of speeding up computation. The gated recurrent unit (GRU) (Cho *et al.*, 2014) offered a streamlined version of the LSTM memory cell that often achieves comparable performance but with the advantage of being faster to compute (Chung *et al.*, 2014).

PYTORCH MXNET JAX TENSORFLOW

```
import torch
from torch import nn
from d2l import torch as d2l
```

10.2.1. Reset Gate and Update Gate

Here, the LSTM's three gates are replaced by two: the *reset gate* and the *update gate*. As with LSTMs, these gates are given sigmoid activations, forcing their values to lie in the interval $(0, 1)$. Intuitively, the reset gate controls how much of the previous state we might still want to remember. Likewise, an update gate would allow us to control how much of the new state is just a copy of the old one. Fig. 10.2.1 illustrates the inputs for both the reset and update gates in a GRU, given the input of the current time step and the hidden state of the previous time step. The outputs of the gates are given by two fully connected layers with a sigmoid activation function.



```python
# Parameters
vocab_size = 10     # size of input vocabulary
embed_size = 16     # size of word embeddings
hidden_size = 32    # size of RNN hidden state
output_size = 1     # binary classification output
seq_len = 5         # sequence length
batch_size = 4
num_epochs = 20
use_lstm = True     # Set False to use GRU instead
```

```python
# Dummy dataset: random integer sequences + binary labels
def generate_dummy_data(num_samples):
    X = torch.randint(0, vocab_size, (num_samples, seq_len))
    y = torch.randint(0, 2, (num_samples, 1)).float()
    return X, y

X, y = generate_dummy_data(100)
```

```python
# RNN Model
class RNNModel(nn.Module):
    def __init__(self, use_lstm=True):
        super(RNNModel, self).__init__()
        self.embedding = nn.Embedding(vocab_size, embed_size)
        if use_lstm:
            self.rnn = nn.LSTM(embed_size, hidden_size, batch_first=True)
        else:
            self.rnn = nn.GRU(embed_size, hidden_size, batch_first=True)
        self.fc = nn.Linear(hidden_size, output_size)
        self.sigmoid = nn.Sigmoid()

    def forward(self, x):
        x = self.embedding(x)
        out, _ = self.rnn(x)
        out = out[:, -1, :]  # Take last hidden state
        out = self.fc(out)
        return self.sigmoid(out)
```

```python
# Initialize model
model = RNNModel(use_lstm=use_lstm)
criterion = nn.BCELoss()
optimizer = optim.Adam(model.parameters(), lr=0.01)

# Training loop
for epoch in range(num_epochs):
    permutation = torch.randperm(X.size(0))
    for i in range(0, X.size(0), batch_size):
        indices = permutation[i:i+batch_size]
        batch_x, batch_y = X[indices], y[indices]

        outputs = model(batch_x)
        loss = criterion(outputs, batch_y)

        optimizer.zero_grad()
        loss.backward()
        optimizer.step()

    print(f"Epoch [{epoch+1}/{num_epochs}], Loss: {loss.item():.4f}")
```

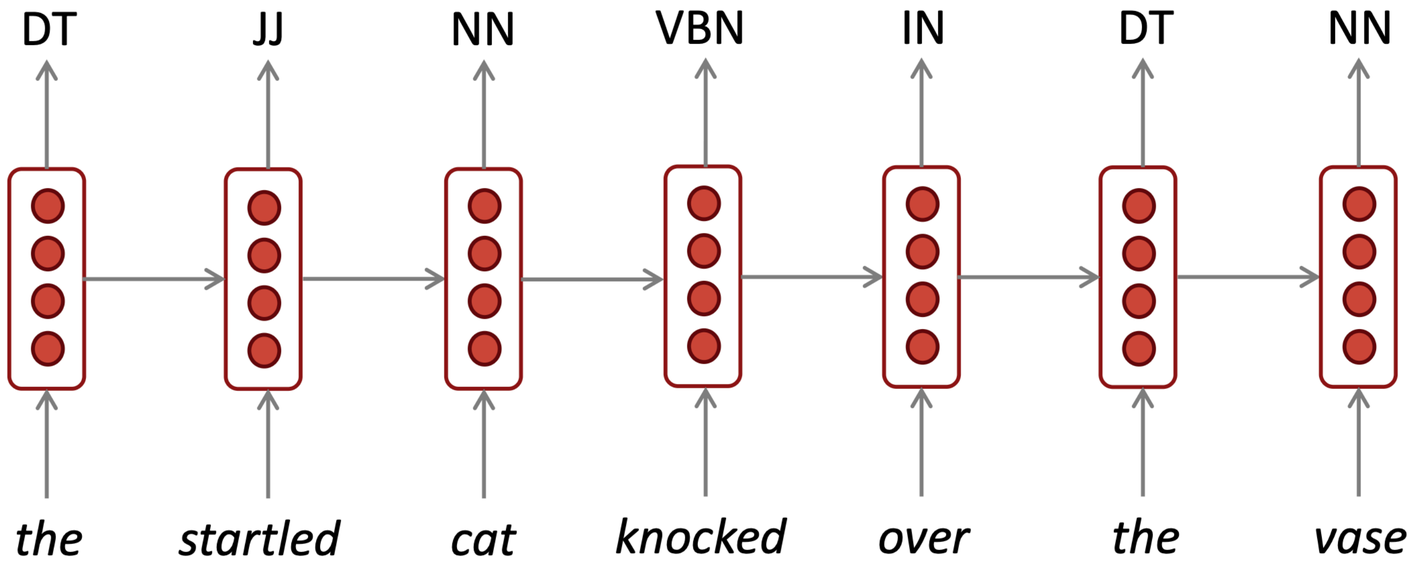
GRU vs LSTM

GRUs are more computationally efficient because they combine the forget and input gates into a single update gate. GRUs do not maintain an internal cell state as LSTMs do, instead they store information directly in the hidden state making them simpler and faster.

Feature	LSTM (Long Short-Term Memory)	GRU (Gated Recurrent Unit)
Gates	3 (Input, Forget, Output)	2 (Update, Reset)
Cell State	Yes it has cell state	No (Hidden state only)
Training Speed	Slower due to complexity	Faster due to simpler architecture
Computational Load	Higher due to more gates and parameters	Lower due to fewer gates and parameters
Performance	Often better in tasks requiring long-term memory	Performs similarly in many tasks with less complexity

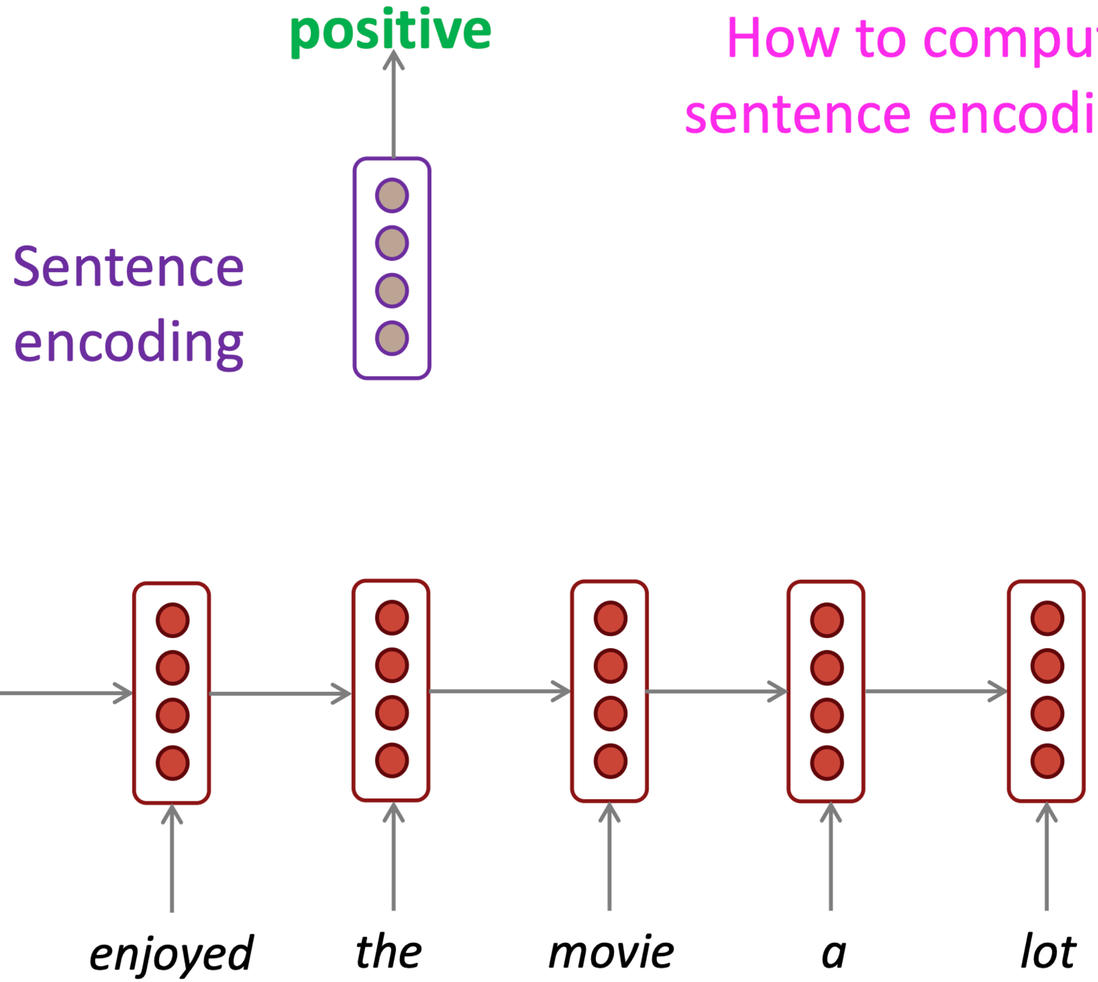
Other RNN uses: RNNs can be used for sequence tagging

e.g., **part-of-speech tagging**, named entity recognition



RNNs can be used for sentence classification

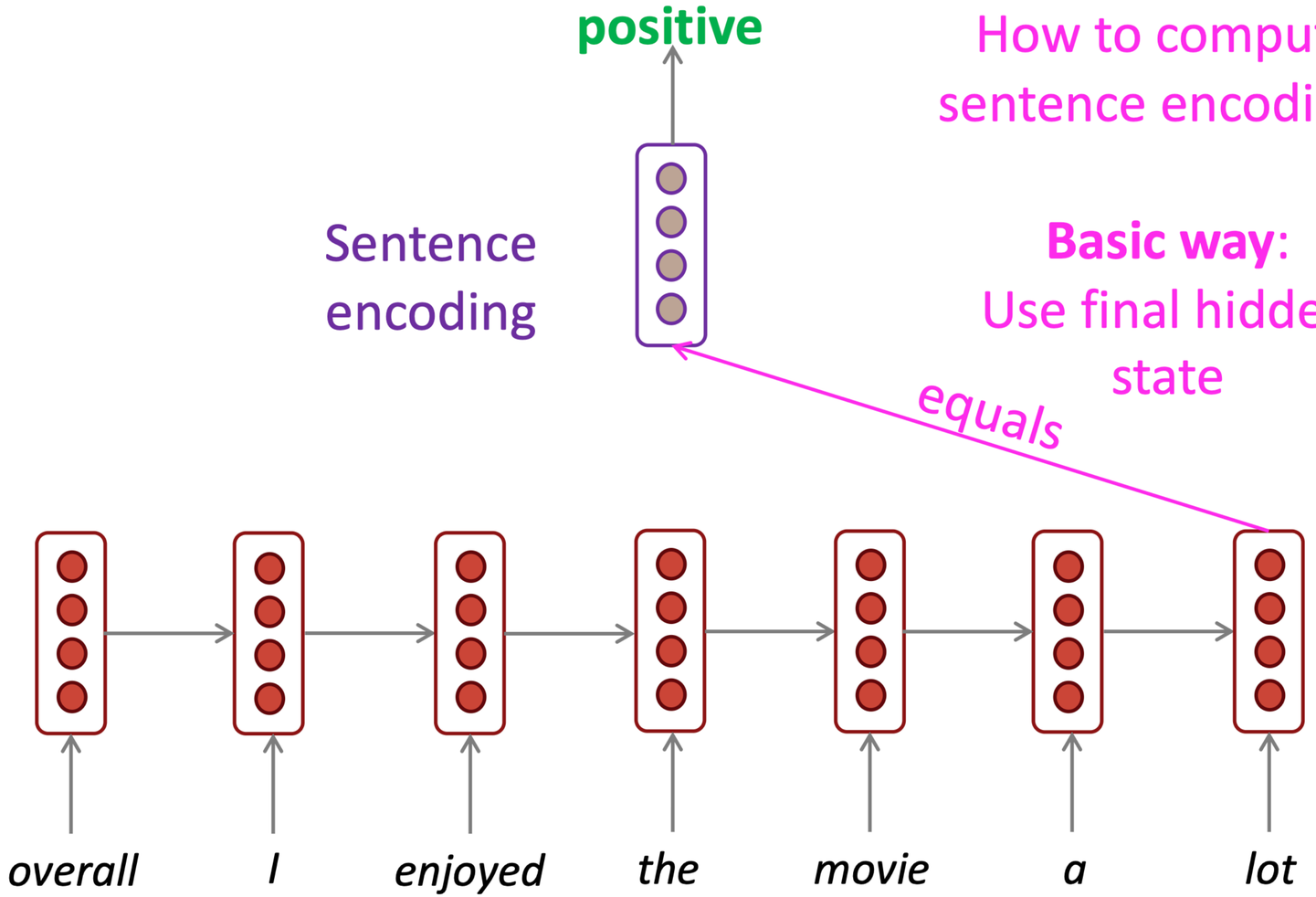
e.g., sentiment classification



How to compute sentence encoding?

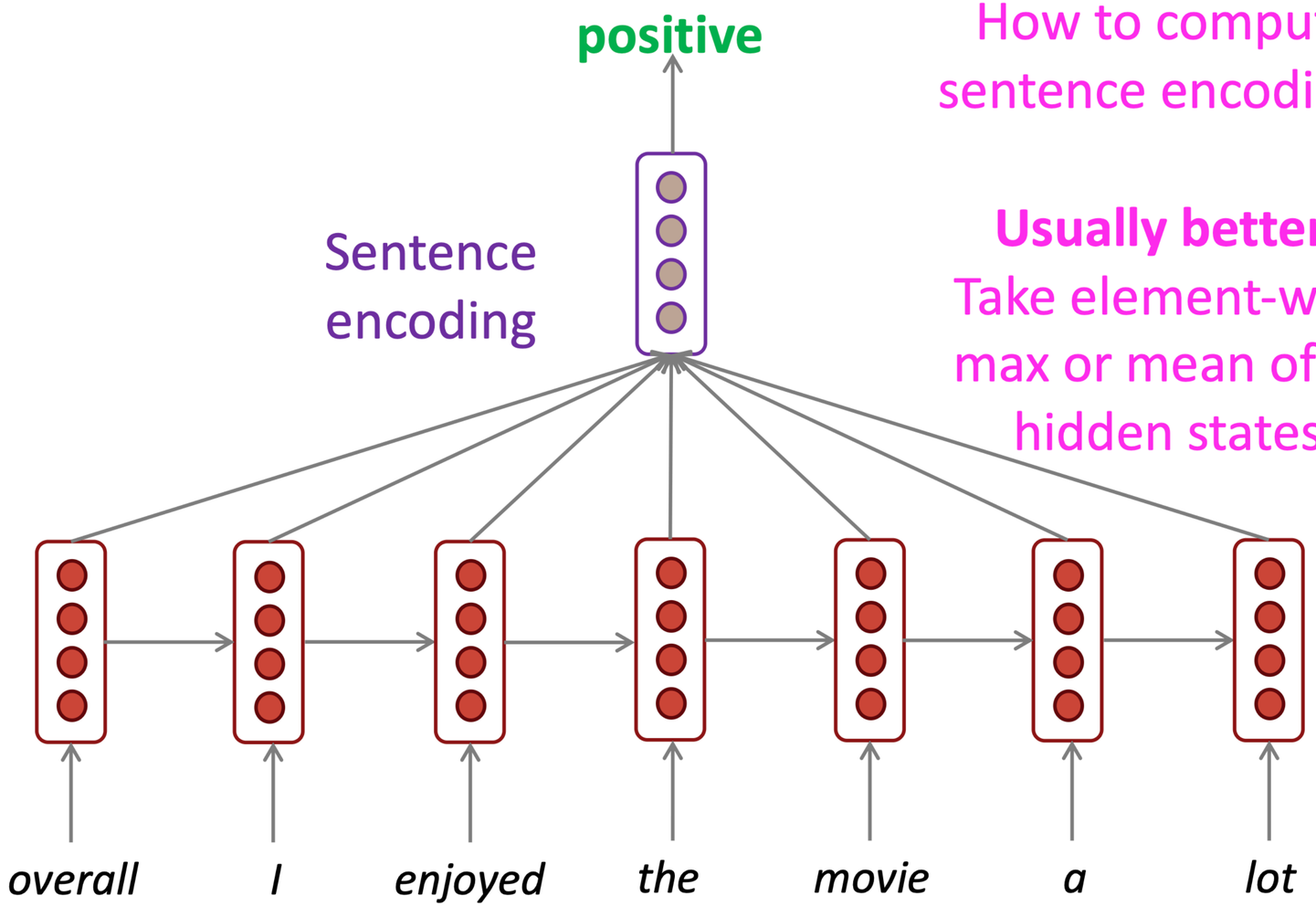
RNNs can be used for sentence classification

e.g., sentiment classification



RNNs can be used for sentence classification

e.g., sentiment classification



How to compute sentence encoding?

Usually better: Take element-wise max or mean of all hidden states

RNNs can be used as an encoder module

e.g., **question answering**, machine translation, *many other tasks!*

Here the RNN acts as an **encoder** for the Question (the hidden states represent the Question). The encoder is part of a larger neural system.