Analiza numeryczna

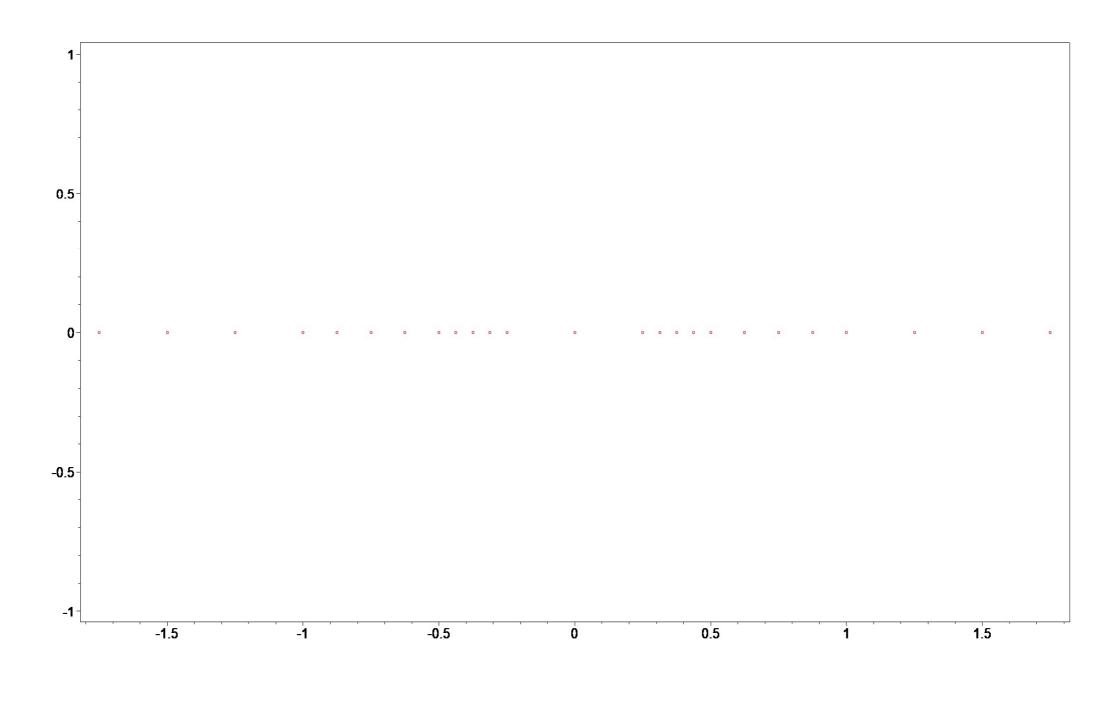
Wyklad 2 i 3. Podstawowe pojecia teorii bledow

Pawel Wozny

Wroclaw, 11 i 18 pazdziernika 2023 r.

Przyklady

```
1. Pewien rozklad liczb
```



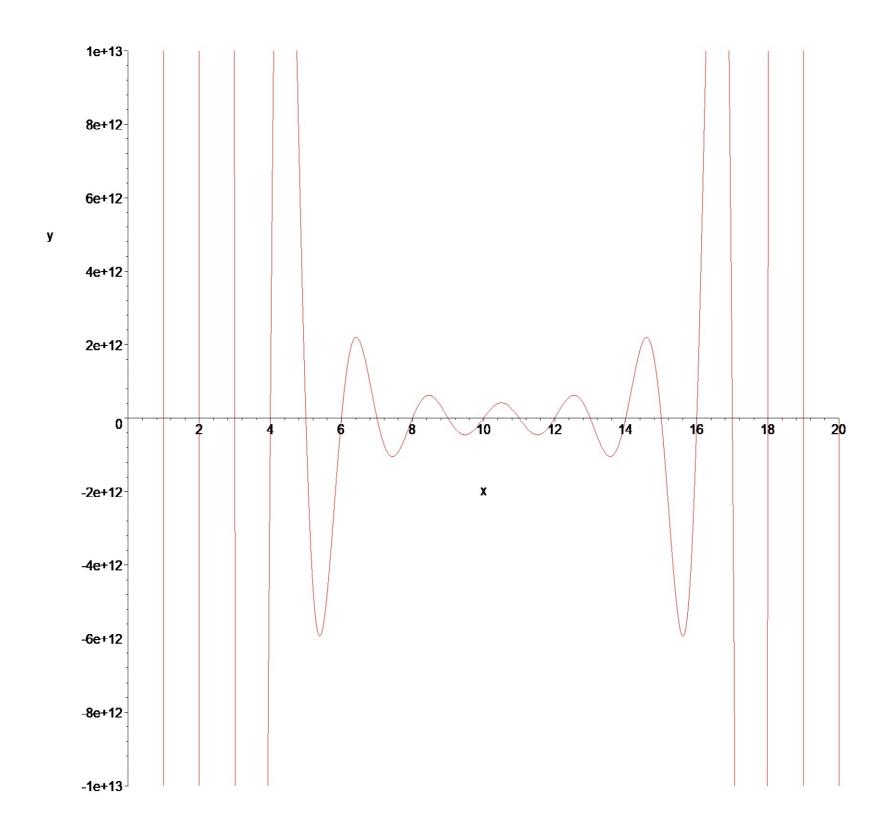
2. Utrata cyfr znaczacych

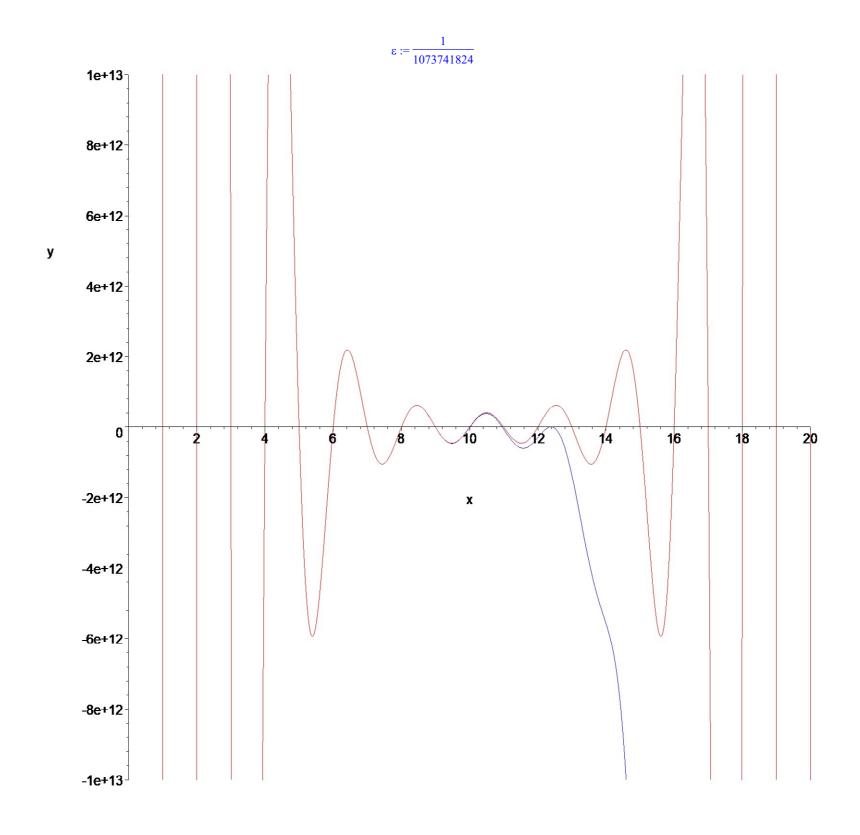
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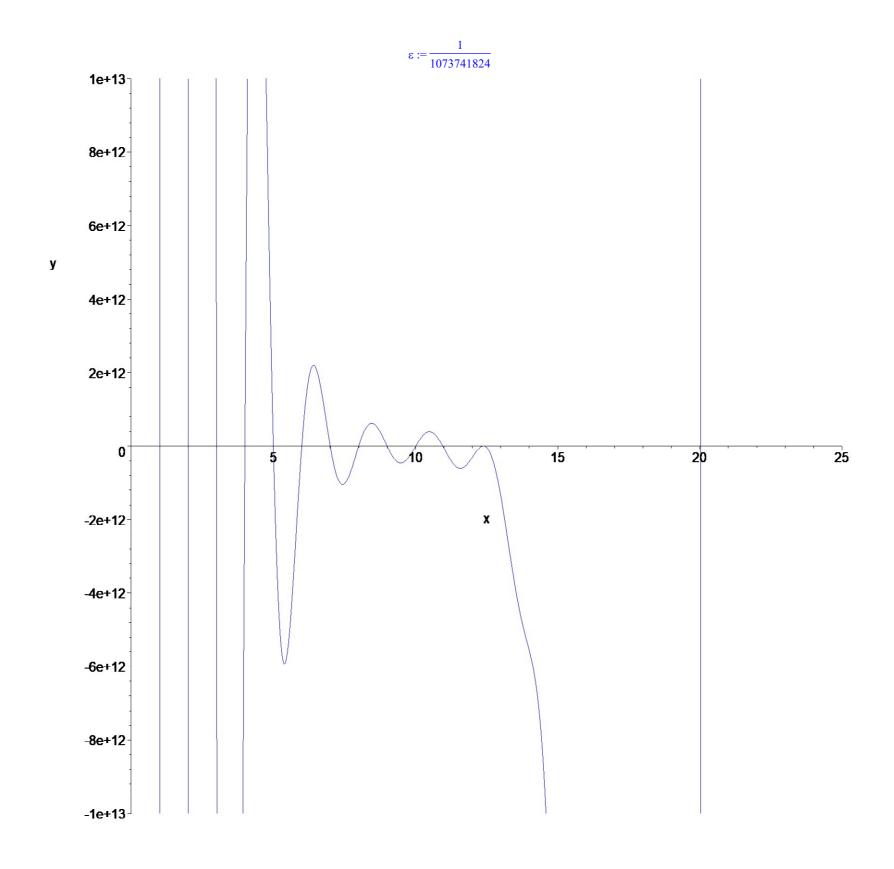
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> f1:=z->ln(z)-1;
                                                                                                   fl := z \rightarrow \ln(z) - 1
   [ >
    > x:=exp(1.0001):
    > printf("
                              x = %1.10e\n\n'',x);
                          f1(x) = %1.10e dla Digits:=%d\n ", f1(x), Digits);
    > printf("
    > printf(" Wynik dokladny = %1.30e\n\n",evalf(f1(x),64));
    >
                 x = 2.7185536700e+00
             f1(x) = 1.0000000000e-04 dla Digits:=10
     Wynik dokladny = 9.999991401555060459381913048956e-05
   [ >
    > f2:=z->ln(z/exp(1.0));
                                                                                                  f2 := z \to \ln\left(\frac{z}{e^{1.0}}\right)
   [ >
    > x:=exp(1.0001):
    > printf("
                              x = %1.10e\n\n'',x);
                          f2(x) = %1.10e dla Digits:=%d\n ",f2(x),Digits);
    > printf("
    > printf(" Wynik dokladny = %1.30e\n\n",evalf(f1(x),64));
    >
                 x = 2.7185536700e+00
              f2(x) = 9.99999999830e-05 dla Digits:=10
     Wynik dokladny = 9.999991401555060459381913048956e-05
   [ >
- Przykad 2
   [ >
    > Digits:=10:
    > f1:=z->z-sqrt(z^2+1);
                                                                                                 fI := z \to z - \sqrt{z^2 + 1}
   [ >
    > x:=1.23*10^20:
    > printf("
                              x = %1.10e\n\n",x);
    > printf("
                         f1(x) = %1.10e dla Digits:=%d\n ",f1(x),Digits);
    > printf(" Wynik dokladny = %1.30e\n\n",evalf(f1(x),64));
                 x = 1.2300000000e+20
              Wynik dokladny = -4.065040650406504065040700000000e-21
```

3. Perfidny wielomian Wilkinsona

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1.000000000
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                                                                                                             8.000055605
                                                                                                             8.999219785
                                                                                                             10.00717528
                                                                                                             10.95941175
                                                                                                             12.34999430
                                                                                                             12.42746890
                                                                                                     14.37766615 - 0.7489085572 I
                                                                                                     14.37766615 + 0.7489085572 I
                                                                                                     16.56965346 - 0.8579542081 I
                                                                                                     16.56965346 + 0.8579542081 I
                                                                                                     18.66274785 - 0.3242223997 I
                                                                                                     18.66274785 + 0.3242223997 I
                                                                                                             20.03654178
   [ >
     > plot([seq([Re(zera[i]),Im(zera[i])],i=1..20)],
             style=point,symbol=box,symbolsize=20,axes=boxed,scaling=unconstrained,labels=["Re","Im"]);
   _ >
   [ >
4. Macierzy Hilberta (też perfidna)
   [ >
    | >
     > restart:
     > with(linalg):
     > Digits:=8:
     Warning, the protected names norm and trace have been redefined and unprotected
   [ >
    ┌ >
    > n:=15;
     >
     > print(``);
     > H:=hilbert(n);
     > numeric_H:=evalf(evalm(H));
                                                                                                               n := 15
```

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0.25000000

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0.14285714

0.11111111

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0.071428571 0.066666667

0.0666666667 0.062500000

0.50000000

0.33333333

0.25000000

0.20000000

0.16666667

numeric H:

[>

[>

[>

> W1:=det(H);

> W2:=det(numeric_H);

```
> evalf(W1); evalf(W2);
                                                                                                                 0.10585427 \ 10^{-123}
                                                                                                                  0.47591091\ 10^{-86}
[ >
 > evalf((W1-W2)/W2);
                                                                                                                    -1.0000000
[ >
 > n:=15: ### 7, 15
 > H:=hilbert(n):
  > numeric_H:=evalf(evalm(H)):
  > vec_b:=evalm(H&*matrix(n,1,[seq(1,i=1..n)])):
  > vec_x:=matrix(n,1,[seq(x[i],i=1..n)]):
  > lin_eqs:=evalm(H&*vec_x-vec_b):
 > solve({seq(lin_eqs[i,1],i=1..n)},{seq(x[i],i=1..n)});
                                                                          \{x_1 = 1, x_2 = 1, x_3 = 1, x_4 = 1, x_5 = 1, x_6 = 1, x_7 = 1, x_8 = 1, x_9 = 1, x_{10} = 1, x_{11} = 1, x_{12} = 1, x_{13} = 1, x_{14} = 1, x_{15} = 1\}
[ >
 > vec_numeric_b:=evalm(numeric_H&*matrix(n,1,[seq(1.0,i=1..n)])):
 > vec_numeric_x:=matrix(n,1,[seq(x[i],i=1..n)]):
  > numeric_lin_eqs:=evalm(numeric_H&*vec_numeric_x-vec_numeric_b):
  > solve({seq(numeric_lin_eqs[i,1],i=1..n)},{seq(x[i],i=1..n)});
 >
 >
             \{x_7 = 29.636238, x_6 = -10.159555, x_5 = 16.901975, x_4 = -10.455500, x_3 = 4.2058782, x_2 = 0.67145474, x_1 = 1.0077913, x_8 = -43.218415, x_9 = -4.4852584, x_{10} = 21.895480, x_{11} = 29.622289, x_{12} = -11.067398, x_{13} = 5.9439774, x_{15} = 30.329621, x_{14} = -45.821974\}
[ >
```

+ 5. Związki rekurencyjne