

Analiza numeryczna

Wykład 5 i 6. Interpolacja wielomianowa

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Przykłady

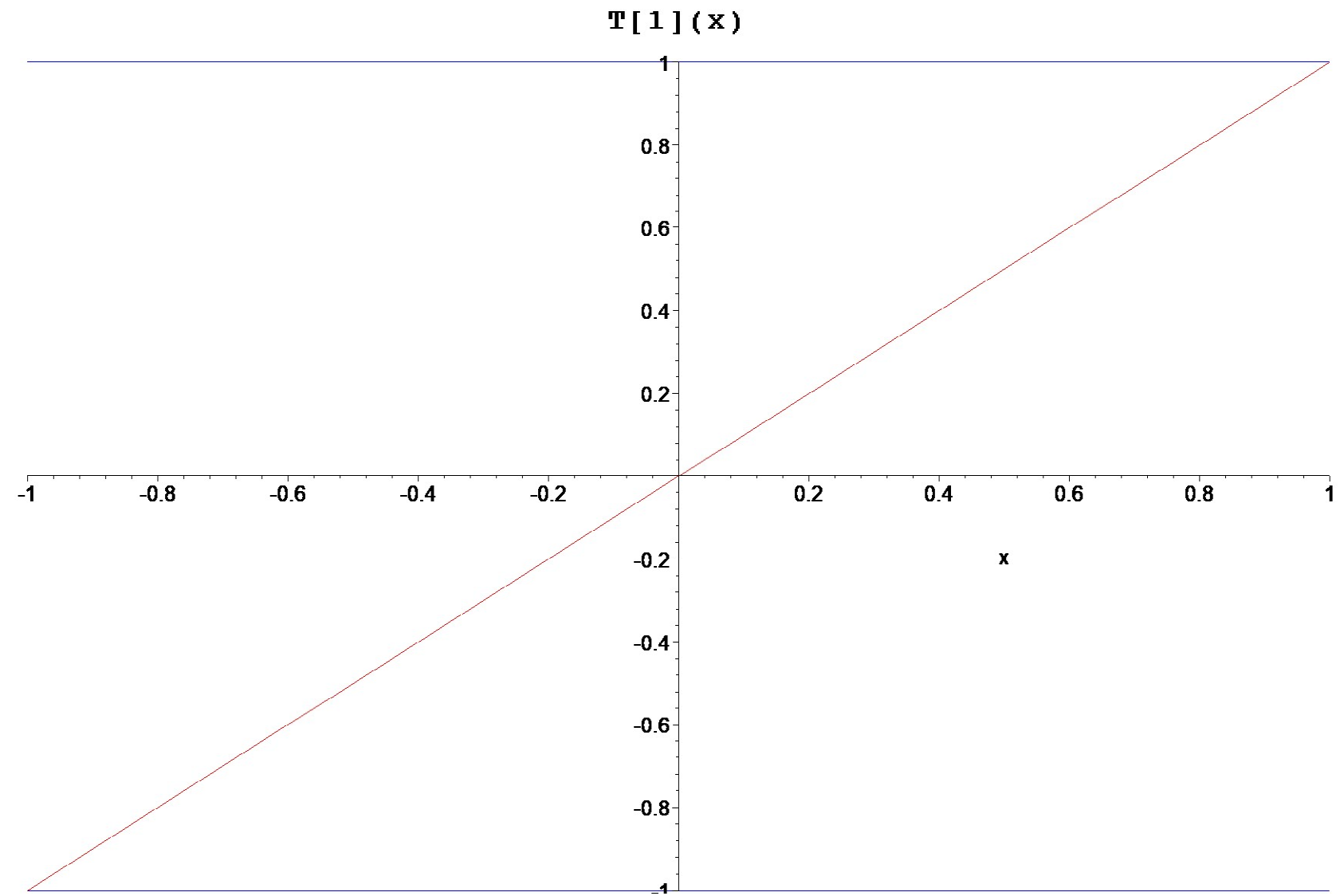
1. Wielomiany Czebyszewa

```
>  
>  
> restart:  
>  
> with(orthopoly):  
>  
> seq(print(T(n,x)),n=0..15);  
>
```

$$\begin{aligned} &1 \\ &x \\ &-1 + 2x^2 \\ &4x^3 - 3x \\ &1 + 8x^4 - 8x^2 \\ &16x^5 - 20x^3 + 5x \\ &-1 + 32x^6 - 48x^4 + 18x^2 \\ &64x^7 - 112x^5 + 56x^3 - 7x \\ &1 + 128x^8 - 256x^6 + 160x^4 - 32x^2 \\ &256x^9 - 576x^7 + 432x^5 - 120x^3 + 9x \\ &-1 + 512x^{10} - 1280x^8 + 1120x^6 - 400x^4 + 50x^2 \\ &1024x^{11} - 2816x^9 + 2816x^7 - 1232x^5 + 220x^3 - 11x \\ &1 + 2048x^{12} - 6144x^{10} + 6912x^8 - 3584x^6 + 840x^4 - 72x^2 \\ &4096x^{13} - 13312x^{11} + 16640x^9 - 9984x^7 + 2912x^5 - 364x^3 + 13x \\ &-1 + 8192x^{14} - 28672x^{12} + 39424x^{10} - 26880x^8 + 9408x^6 - 1568x^4 + 98x^2 \\ &16384x^{15} - 61440x^{13} + 92160x^{11} - 70400x^9 + 28800x^7 - 6048x^5 + 560x^3 - 15x \end{aligned}$$

```
>  
>  
> N:=25:  
>  
> asymp:=plot([-1,1],x=-1..1,color=blue):  
>  
> for k from 1 to N
```

```
do
  opis:="T[|k|](x)":
  wykres:=plot(T(k,x),x=-1..1,color=red):
  klatka[k]:=plots[display](wykres,asyp,title=opis,titlefont=[COURIER,BOLD,15])
od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
```



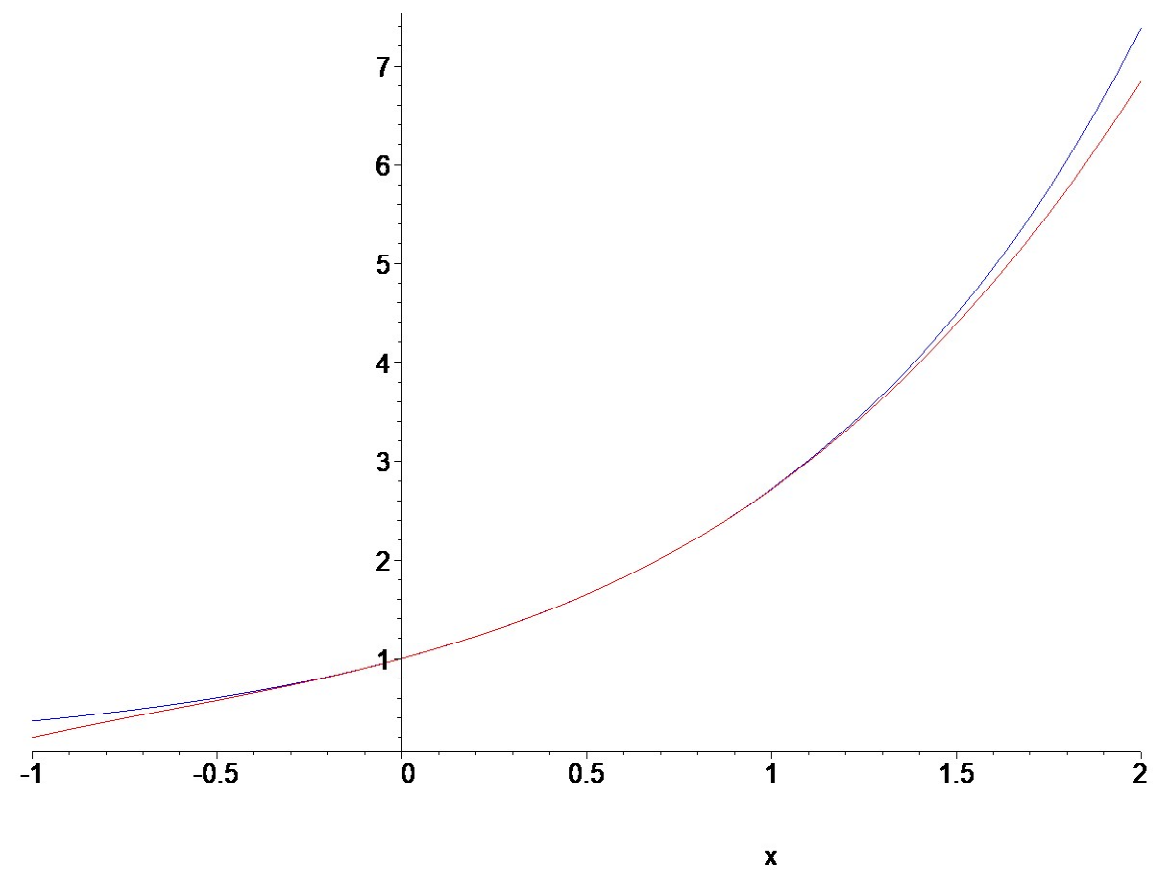
```
[ >
```

2. Interpolacja wielomianowa

2.1. Przykład

```
[ >
>
> restart:
>
> L[3]:=unapply(interp([0.0,0.2,0.6,0.8],[exp(0.0),exp(0.2),exp(0.6),exp(0.8)],x),x);
>
> plot([L[3](x),exp(x)],x=-1..2,color=[red,blue],legend=["L[3](x)","exp(x)"]);
>
```

$$L_3 := x \rightarrow 0.2511337917 x^3 + 0.4570534918 x^2 + 1.005557740 x + 1.$$



— $L[3](x)$
 — $\exp(x)$

[>

2.2. Przykład

[>

>

> restart:

>

> N:=14:

>

> f:=x->sin(x):

>

> wykres_f:=plot(f(x),x=0..2*Pi,color=blue,axes=frame):

>

> for i from 1 to N

do

L:=unapply(interp([seq(evalf(2*Pi*k/i),k=0..i)],
 [seq(evalf(f(2*Pi*k/i)),k=0..i)],x),x):

blad:=max(seq(abs(evalf(L(2*Pi*j/100)-f(2*Pi*j/100))),j=0..100):

blads:=convert(evalf(blad,5),string):

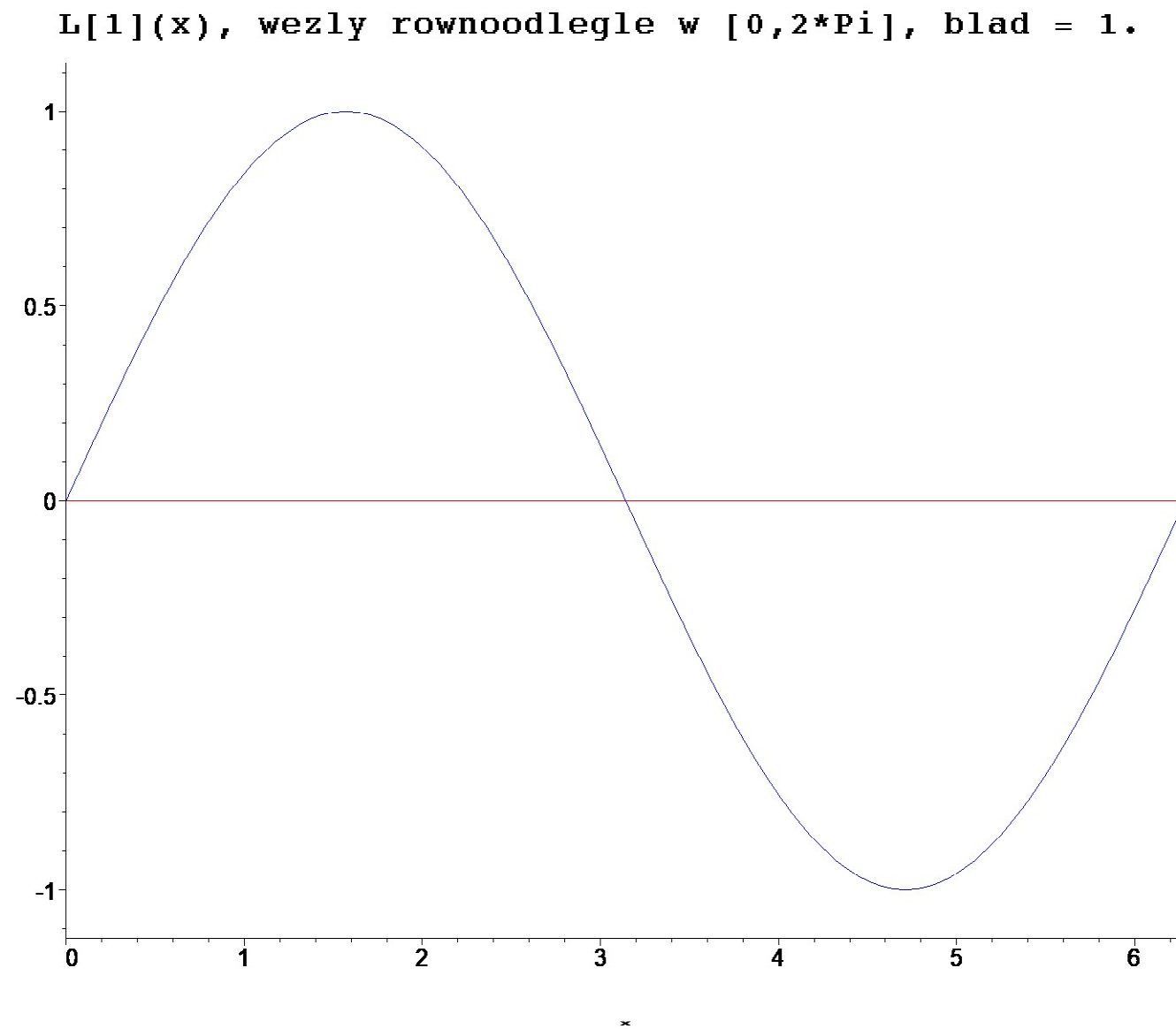
opis:="L["||i||"](x), wezly rownoodlegle w [0,2*Pi], blad = "||blads:

wykres_L:=plot(L(x),x=0..2*Pi,color=red,axes=frame):

klatka[i]:=plots[display](wykres_f,wykres_L,title=opis,titlefont=[COURIER,BOLD,15])

od:

```
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
```



```
[ >
```

2.3. Przykład

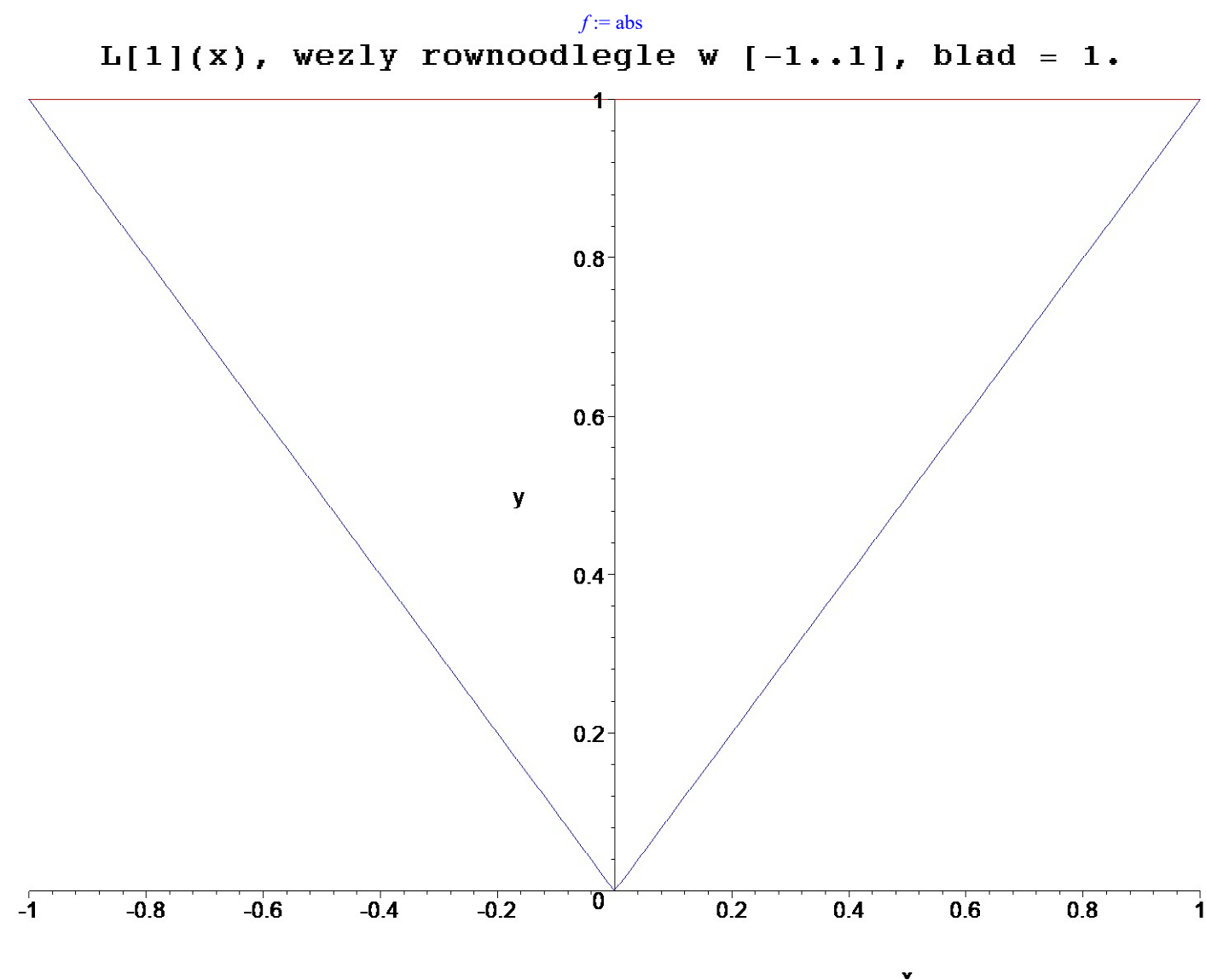
```
[ >
>
> restart:
>
> N:=20:
>
> f:=x->abs(x);
>
> wykres_f:=plot(f(x),x=-1..1,y=0..1,color=blue):
>
> for i from 1 to N
do
  L:=unapply(interp([seq(evalf(-1+2*k/i),k=0..i)],
                    [seq(evalf(f(-1+2*k/i)),k=0..i)],x),x):

  blad:=max(seq(abs(evalf(L(-1+2*j/200))-f(-1+2*j/200)),j=0..200)):
  blads:=convert(evalf(blad,5),string):
```

```

    opis:="L["||i||"](x), wezly rownoodlegle w [-1..1], blad = "||blads:
    wykres_L:=plot(L(x),x=-1..1,y=0..1,color=red,numpoints=150):
    klatka[i]:=plots[display](wykres_f,wykres_L,title=opis,titlefont=[COURIER,BOLD,15])
od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
>
>
>

```



[>

2.4. Przykład

```

[ >
>
> restart:
>
> Digits:=16:
>
> N:=40:
>
> f:=x->x^6;
>
> wykres_f:=plot(f(x),x=-2..2,y=-16..64,color=blue):
>

```

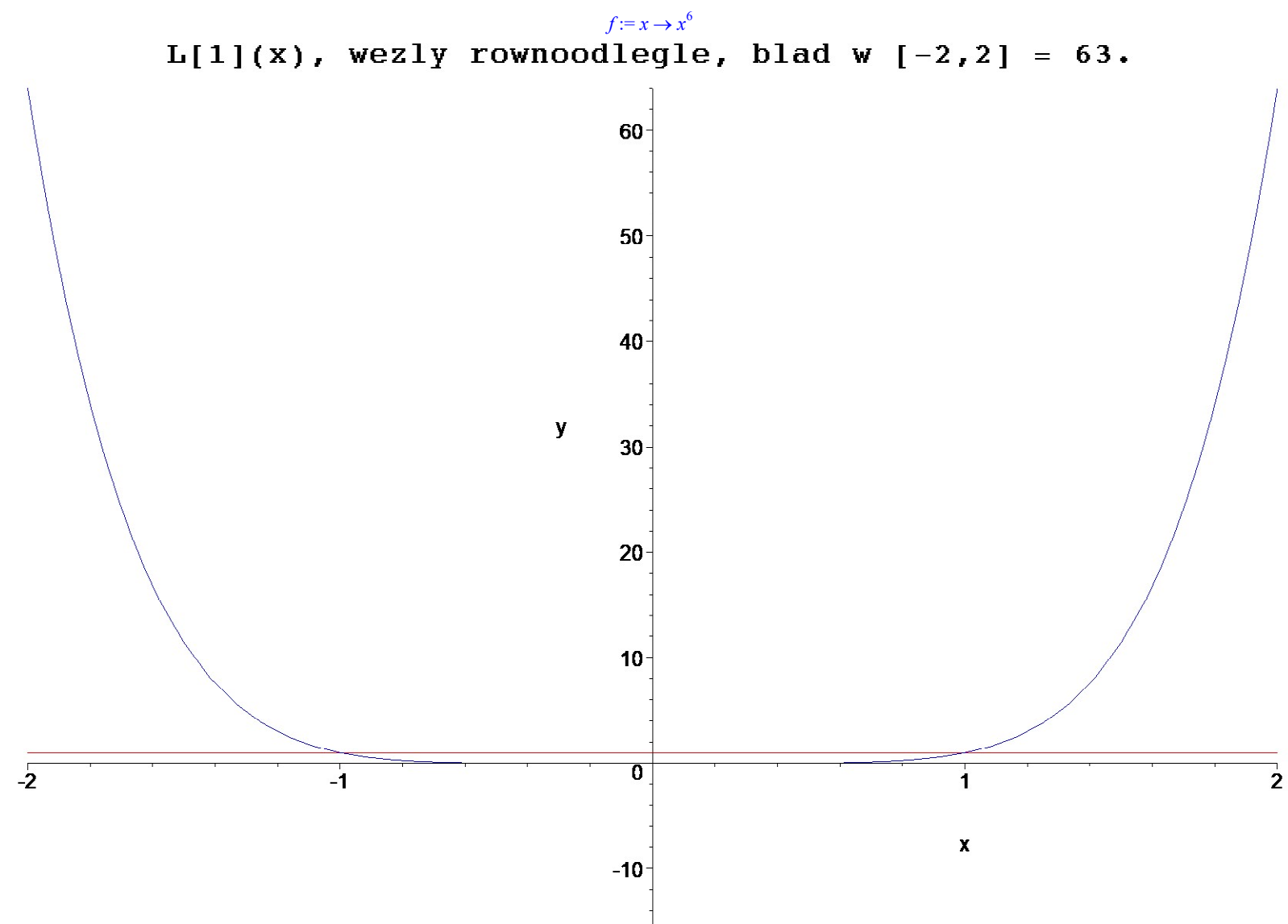
```

> for i from 1 to N
do
  L:=unapply(interp([seq(evalf(-1+2*k/i),k=0..i)],
    [seq(evalf(f(-1+2*k/i)),k=0..i)],x),x):

  blad:=max(seq(abs(evalf(L(-2+4*j/400))-f(-2+4*j/400)),j=0..400)):
  blads:=convert(evalf(blad,5),string):

  opis:="L["||i||"] (x), wezly rownoodlegle, blad w [-2,2] = "||blads:
  wykres_L:=plot(L(x),x=-2..2,y=-16..64,color=red,numpoints=150):
  klatka[i]:=plots[display](wykres_f,wykres_L,title=opis,titlefont=[COURIER,BOLD,15])
od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
>

```



```

[ >
>
> Digits:=16:
>
> f:=x->x^6;
>
> i:=6;
>

```

```
> printf("\n");
>
> L:=unapply(interp([seq(evalf(-1+2*k/i),k=0..i)],
                    [seq(evalf(f(-1+2*k/i)),k=0..i)],x),x);
>
>
>
```

$$f:=x \rightarrow x^6$$
$$i:=6$$

$$L:=x \rightarrow 0.9999999999999754 x^6 - 0.143 \cdot 10^{-13} x^5 + 0.303 \cdot 10^{-13} x^4 - 0.3 \cdot 10^{-14} x + 0.18 \cdot 10^{-13} x^3 - 0.5 \cdot 10^{-14} x^2$$

3. Wybor wezlow interpolacji

3.1. Wezly rownoodlegle

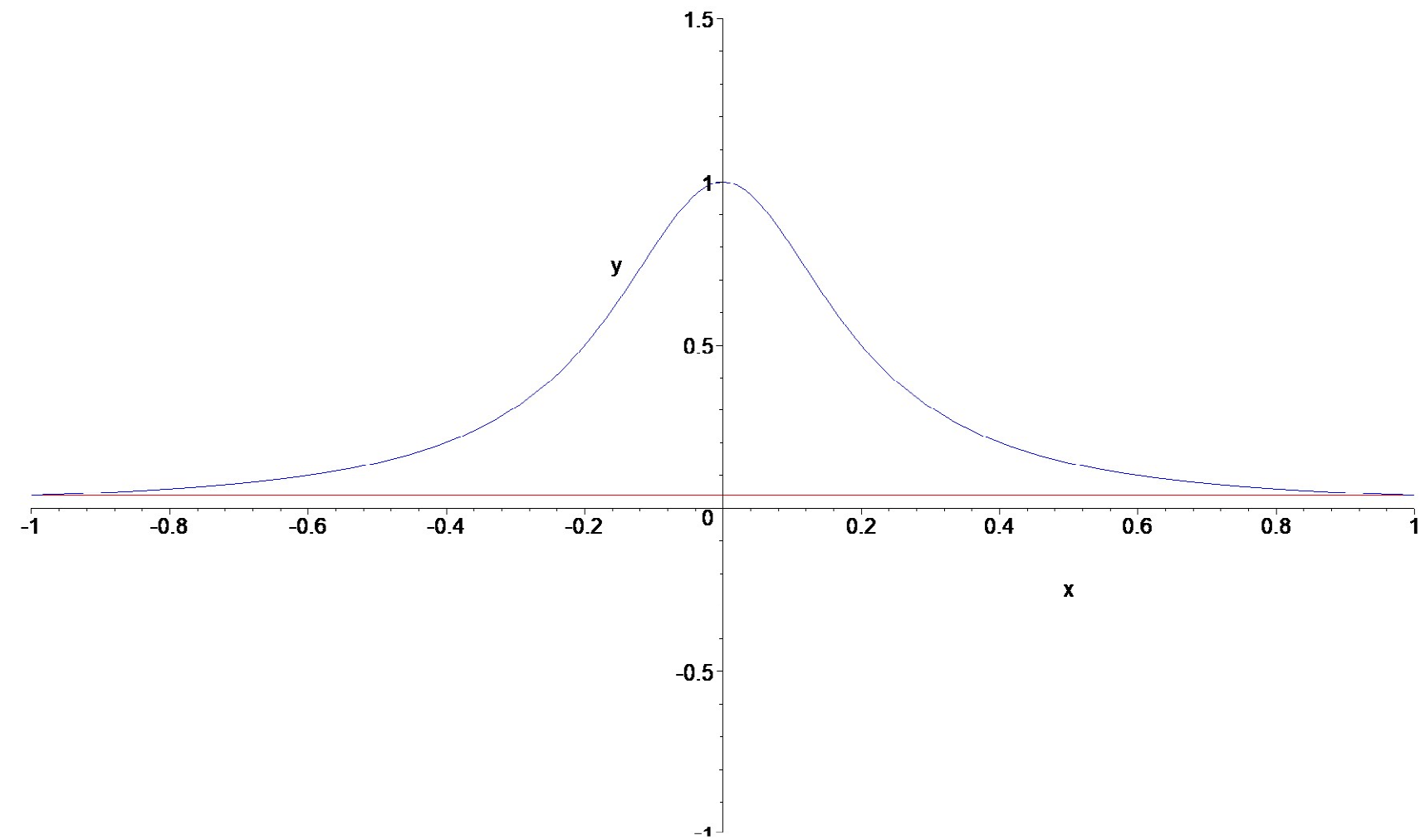
```
[ >
>
> restart:
>
> N:=20:
>
> f:=x->1/(25*x^2+1);
>
> wykres_f:=plot(f(x),x=-1..1,y=-1..1.5,color=blue):
>
> for i from 1 to N
do
    L:=unapply(interp([seq(evalf(-1+2*k/i),k=0..i)],
                      [seq(evalf(f(-1+2*k/i)),k=0..i)],x),x):

    blad:=max(seq(abs(evalf(L(-1+2*j/200))-f(-1+2*j/200)),j=0..200)):
    blads:=convert(evalf(blad,5),string):

    opis="L["||i||"](x), wezly rownoodlegle w [-1,1], blad = "||blads:
    wykres_L:=plot(L(x),x=-1..1,y=-1..1.5,color=red,numpoints=150):
    klatka[i]:=plots[display](wykres_f,wykres_L,title=opis,titlefont=[COURIER,BOLD,15])
od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
```

$$f:=x \rightarrow \frac{1}{25 x^2 + 1}$$

$L[1](x)$, wezly rownoodlegle w $[-1,1]$, blad = .96154



[>

3.2. Wezly Czebyszewa

3.2.1 Przyklad

```
[ >
>
> restart:
>
> N:=20:
>
> f:=x->1/(25*x^2+1);
>
> wykres_f:=plot(f(x),x=-1..1,y=-1..1.5,color=blue):
>
> for i from 1 to N
do
  L:=unapply(interp([seq(evalf(cos((2*k+1)*Pi/(2*i+2))),k=0..i)],
    [seq(evalf(f(cos((2*k+1)*Pi/(2*i+2)))),k=0..i)],x),x):

  blad:=max(seq(abs(evalf(L(-1+2*j/200))-f(-1+2*j/200)),j=0..200)):
  blads:=convert(evalf(blad,5),string):

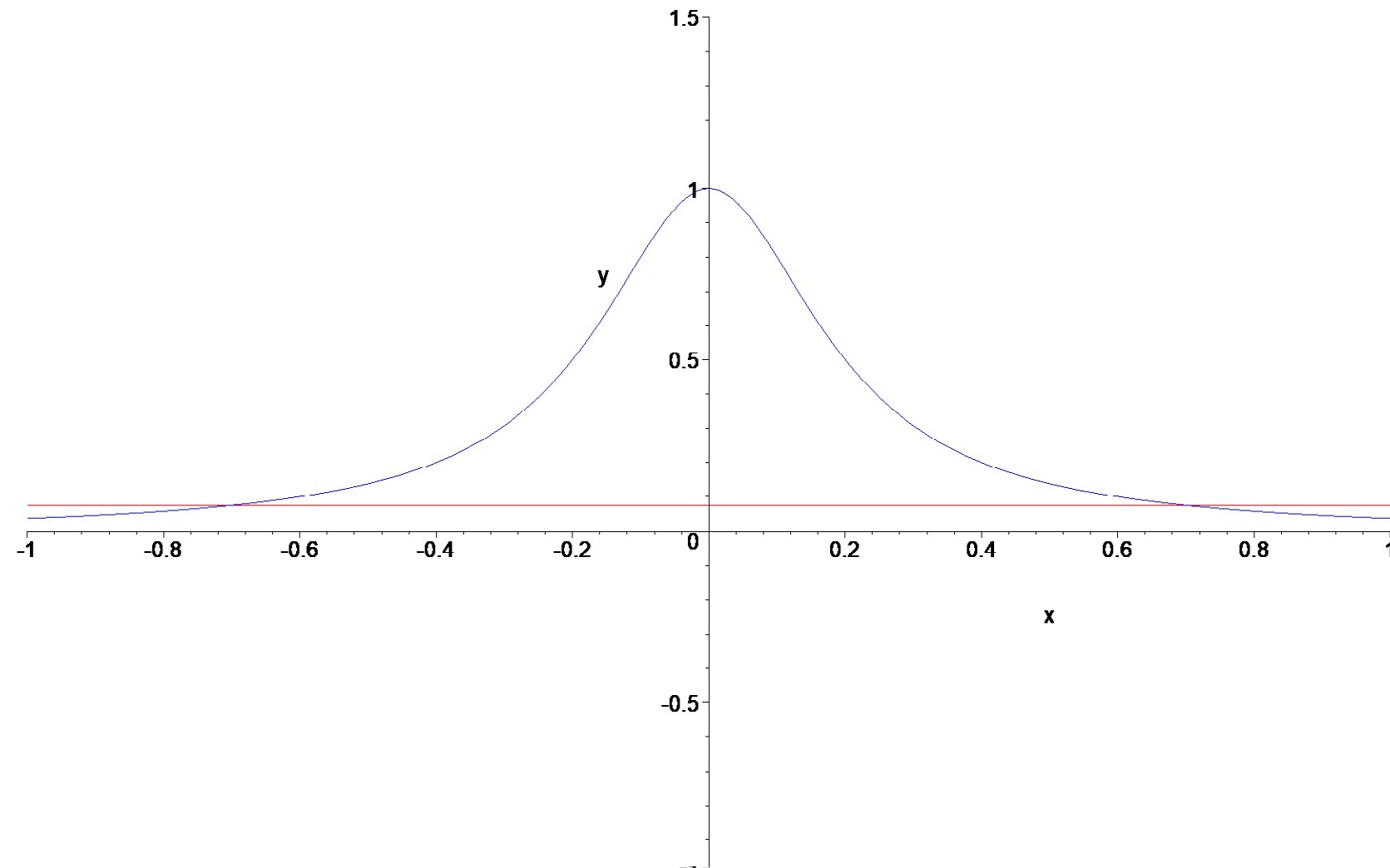
  opis:="L["||i||"](x), wezly Czebyszewa w [-1,1], blad = "||blads:
  wykres_L:=plot(L(x),x=-1..1,y=-1..1.5,color=red,numpoints=150):
  klatka[i]:=plots[display](wykres_f,wykres_L,title=opis,titlefont=[COURIER,BOLD,15])
od:
>
```



```
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
>
```

$$f:=x \rightarrow \frac{1}{25x^2+1}$$

L[1](x), wezly Czebyszewa w [-1,1], blad = .92593



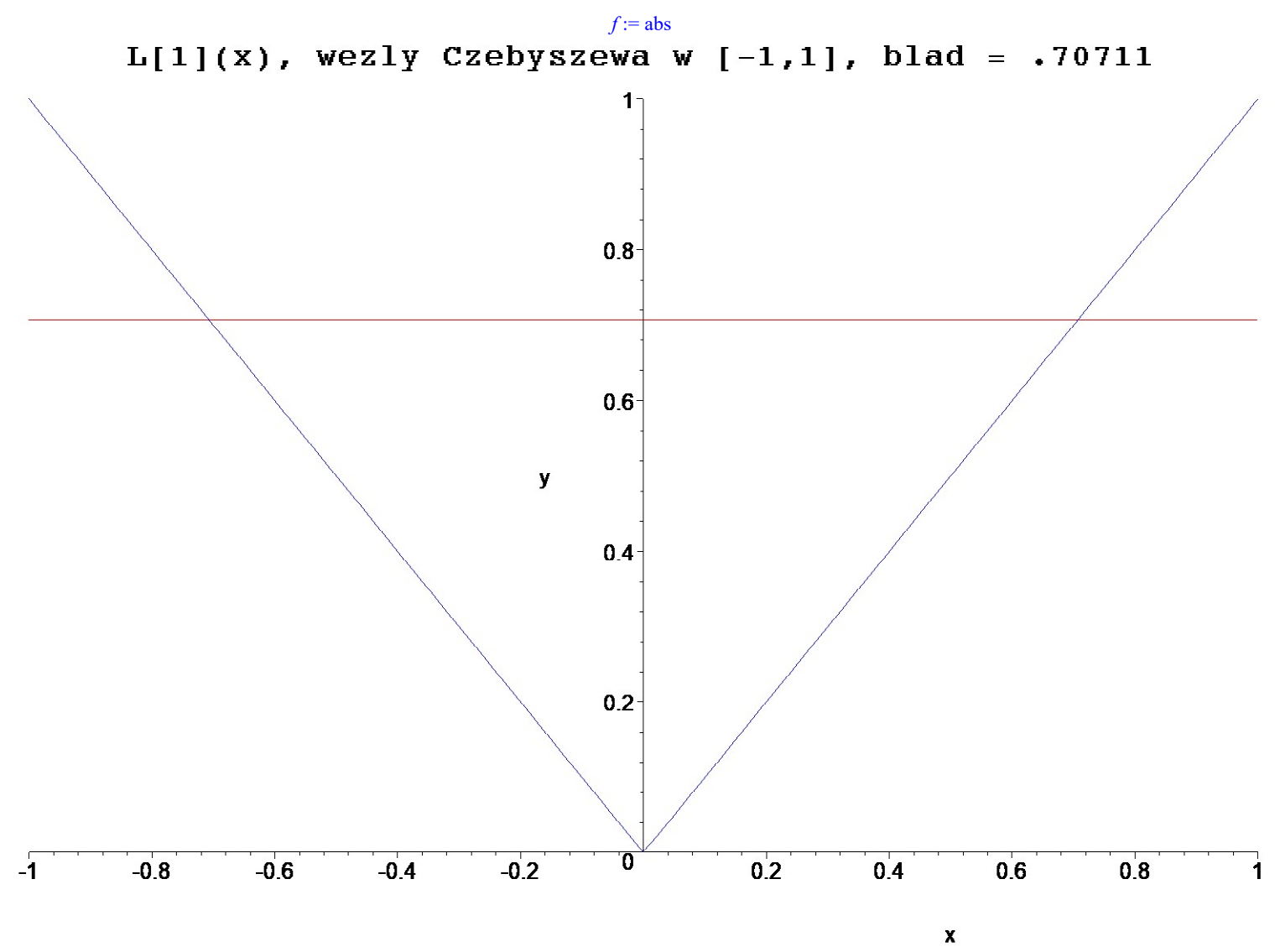
3.2.2. Przykład

```
>
>
> restart:
>
> N:=20:
>
> f:=x->abs(x);
>
> wykres_f:=plot(f(x),x=-1..1,y=0..1,color=blue):
>
> for i from 1 to N
do
  L:=unapply(interp([seq(evalf(cos((2*k+1)*Pi/(2*i+2))),k=0..i)],
    [seq(evalf(f(cos((2*k+1)*Pi/(2*i+2)))),k=0..i)],x),x):

  blad:=max(seq(abs(evalf(L(-1+2*j/200))-f(-1+2*j/200)),j=0..200)):
end do
```

```
blads:=convert(evalf(blad,5),string):

opis:="L["||i||"](x), wezly Czebyszewa w [-1,1], blad = "||blads:
wykres_L:=plot(L(x),x=-1..1,y=0..1,color=red,numpoints=150):
klatka[i]:=plots[display](wykres_f,wykres_L,title=opis,titlefont=[COURIER,BOLD,15])
od:
>
> plots[display]([seq(klatka[i],i=1..N)],insequence=true);
>
>
>
>
```



```
] >
] >
```