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Introduction to Reinsurance

Reinsurance is a practice in the insurance industry where insurance companies transfer a portion of their risks to another insurer known as a reinsurer. It helps insurance companies manage their exposure to large or catastrophic losses by sharing the risk and financial burden with reinsurers. Reinsurance provides a layer of protection to insurance companies, allowing them to underwrite policies with higher coverage limits and reducing their potential for insolvency due to large claim payouts.

Excess of Loss (XL) Reinsurance

An excess of loss contract, often abbreviated as "XL," is a type of reinsurance agreement that specifically covers losses exceeding a certain threshold or deductible. In an XL contract, the reinsurer agrees to indemnify the insurance company for losses that surpass the defined deductible, up to a certain limit. This type of contract is commonly used to protect against infrequent but severe losses that go beyond the insurance company's risk appetite or capacity.

An excess of loss reinsurance contract with an annual aggregate limit (AAL) and deductible (AAD), works as follows:

1. Define the contract parameters:
 - **Deductible:** This is the amount that is to be paid by the insurer. The exceedance is incurred by the reinsurer. Hence the name 'Excess of Loss'.
 - **Limit of coverage:** This is the maximal amount the reinsurer will pay on a per claim basis.

- Annual Aggregate Deductible: This is the amount of losses that the primary insurer (ceding company) must absorb before the reinsurer's coverage begins.
- Annual Aggregate Limit: This is the maximum amount the reinsurer will pay for losses in a given year. It represents the threshold beyond which the reinsurer's liability ends.

2. Calculate the net loss:

- When a loss occurs, the primary insurer first determines the net loss, which is the loss amount minus the deductible. If the from ground up loss is below the deductible, the primary insurer retains the full responsibility for the loss.

"From ground-up losses" (FGU) refers to the calculation of losses starting from the original or individual loss events without considering any deductions or reimbursements. It represents the total losses incurred by an insurer or reinsurer before any deductibles, limits, or coverage adjustments are taken into account.

3. Check the annual aggregate limit:

- If the net loss exceeds the deductible, the reinsurer's coverage comes into play. The primary insurer submits the net loss to the reinsurer for reimbursement.
- The reinsurer reviews the accumulated losses for the year. If the total of the net losses reaches or surpasses the annual aggregate limit, the reinsurer will not pay any additional claims beyond this limit.

4. Reimbursement calculation:

- If the total net losses for the year are below the annual aggregate limit, the reinsurer calculates the reimbursement amount based on the excess of loss coverage.
- The reimbursement is equal to the net loss minus the deductible, up to the limit of coverage specified by the excess of loss contract.

5. Examples:

Example 1:

- FGU Loss: \$2 million
- Deductible: \$1 million
- Limit of coverage: \$5 million

In this case, the FGU Loss exceeds the deductible, so the reinsurer's coverage applies. The reimbursement calculation would be: $\text{Reimbursement} = \text{FGU Loss} - \text{Deductible} = \$2 \text{ million} - \$1 \text{ million} = \1 million

Since the reimbursement amount is below the limit of coverage, the reinsurer would reimburse \$1 million to the primary insurer.

Example 2:

- FGU Loss: \$10 million
- Deductible: \$2 million
- Limit of coverage: \$5 million

Here, the FGU Loss surpasses both the deductible and the limit of coverage. The reimbursement calculation would be: $\text{Reimbursement} = \text{Limit of coverage} = \5 million

Since the total net losses have reached the annual aggregate limit, the reinsurer would reimburse the maximum limit of coverage, which is \$5 million, to the primary insurer.

Example 3:

- FGU Loss: \$500,000
- Deductible: \$1 million
- Limit of coverage: \$2 million

In this scenario, the FGU Loss is below the deductible, so the primary insurer retains full responsibility for the loss. The reimbursement calculation would be: $\text{Reimbursement} = 0$ (No reimbursement)

Since the FGU Loss does not exceed the deductible, the reinsurer does not provide any reimbursement in this case.

Example 4:

- FGU Loss: \$1.5 million
- Deductible: \$500,000
- Annual Aggregate Deductible: \$3 million
- Limit of coverage: \$10 million

In this case, the net loss exceeds the deductible, but it does not exceed the annual aggregate deductible. The reimbursement calculation would be: $\text{Reimbursement} = \text{FGU Loss} - \text{Deductible} - \text{Annual Aggregate Deductible} = \$1.5 \text{ million} - \$500,000 - \$3 \text{ million} = 0$ (No reimbursement)

Since the total net losses for the year have not reached the annual aggregate deductible, the reinsurer would reimburse \$1 million to the primary insurer.

Example 5:

- FGU Loss: \$5 million
- Deductible: \$1 million
- Annual Aggregate Deductible: \$3 million
- Limit of coverage: \$10 million

Here, the net loss exceeds both the deductible and the annual aggregate deductible. The reimbursement calculation would be: $\text{Reimbursement} = \text{FGU Loss} - \text{Deductible} - \text{Annual Aggregate Deductible} = \$5 \text{ million} - \$1 \text{ million} - \$3 \text{ million} = \$1 \text{ million}$

Note that the 'Annual Aggregate Deductible' is completely used up by this loss and the next one occurring in the same year will not be further reduced by the AAL.

Example 6:

Loss 1:

- FGU Loss: \$20 million
- Deductible: \$1 million

- Annual Aggregate Limit: \$15 million
- Limit of coverage: \$10 million

Here, the net loss exceeds deductible. The reimbursement calculation would be: $\text{Reimbursement} = \min(\text{FGU Loss} - \text{Deductible}, \text{Limit of coverage}) = \min(\$20 \text{ million} - \$1 \text{ million}, \$10 \text{ million}) = \$10 \text{ million}$. So the the total reimbursement so far amounts to \$10 million. Let's introduce one extra loss of the same size:

Loss 2:

- FGU Loss: \$20 million
- Deductible: \$1 million
- Total reimbursement so far: \$10 million
- Annual Aggregate Limit: \$15 million
- Limit of coverage: \$10 million

Here, the net loss exceeds deductible but should be limited to the AAL. The reimbursement calculation would be: $\text{Reimbursement} = \min(\min(\text{FGU Loss} - \text{Deductible}, \text{Limit of coverage}), \text{Annual Aggregate Limit} - \text{Total reimbursement so far}) = \min(\min(\$20 \text{ million} - \$1 \text{ million}, \$10 \text{ million}), \$15 \text{ million} - \$10 \text{ million}) = \$5 \text{ million}$. Notice that we're capping the total annual reimbursement to \$15 million. Meaning that any further loss will not generate additional reimbursements.

These examples illustrate how the excess of loss reinsurance contract with both an annual aggregate deductible and limit operates.

This type of contract is commonly used for risks with low frequency but high severity, such as natural disasters or catastrophic events. It allows insurance companies to mitigate their exposure to extreme losses by sharing the risk with a reinsurer. The annual aggregate limit ensures that the reinsurer's liability is capped for a specific period, while the deductible provides a retention level for the primary insurer.

By implementing this type of reinsurance contract, both the primary insurer and the reinsurer can manage their risk exposure more effectively and ensure financial stability in the face of large and infrequent losses.

Monte Carlo Simulation for Reinsurance

Given that catastrophic loss events are infrequent, we have a very limited amount of historical data to work with. Therefore we rely on simulating losses using statistical techniques like the Monte Carlo simulation.

Monte Carlo simulation is a computational technique used to model and analyze the behavior of complex systems or processes through random sampling. It takes its name from the famous Monte Carlo casino in Monaco, known for its games of chance and randomness.

In a Monte Carlo simulation, a problem or system is simulated multiple times using random inputs or variables, often following specific probability distributions. The simulation generates a large number of possible outcomes, which are collectively used to estimate probabilities, assess risks, or make predictions about the system's behavior.

To price an excess of loss contract using Monte Carlo simulation, the following steps can be taken:

1. Define the underlying loss model: Develop a model that describes the probability distributions for the frequency and severity of losses covered by the XL contract.
2. Generate random scenarios: Utilize the Monte Carlo simulation technique to generate a large number of random scenarios representing different combinations of loss frequencies and severities. This involves sampling from the specified distributions for each scenario.
3. Calculate loss amounts: For each generated scenario, calculate the total loss amount by summing up the individual losses exceeding the deductible.
4. Aggregate losses: Aggregate the calculated loss amounts across all the scenarios to obtain a distribution of total losses covered by the XL contract.
5. Assess contract parameters: Analyze the generated distribution of total losses to evaluate the contract's parameters, such as the average loss and the Value at Risk (VaR) at a specified confidence level (e.g., VaR 99%). The average loss represents the expected payout under the contract, while VaR provides a measure of the potential maximum loss beyond which the insurance company is not protected.
6. Price the contract: Based on the analysis of the generated loss distribution and the desired risk profile, determine the appropriate premium or pricing for the excess of loss contract. This pricing process involves considering the expected losses, potential tail risks, risk appetite, and other relevant factors.

By utilizing Monte Carlo simulation, insurance companies can gain insights into the potential losses and risks associated with excess of loss contracts. This information enables them to make informed decisions about pricing, risk management, and portfolio optimization in the reinsurance market.

Generating Losses using Monte Carlo Simulation

In this case, we use the Poisson distribution for frequency and the Pareto distribution for severity.

Here's an explanation of each distribution:

1. Poisson Distribution:

The Poisson distribution is commonly used to model the occurrence of rare events or incidents in a given time period. In our case, it represents the frequency of losses happening during the cover period of the reinsurance contract. The key parameter of the Poisson distribution is λ (lambda), which represents the average rate or intensity of event occurrences. For example, in the Monte Carlo simulation, we can generate random samples from the Poisson distribution with $\lambda=1$, indicating an average frequency of one loss occurrence during the cover period. Each randomly generated sample represents the number of losses that would occur in a given simulation run.

For more information see: https://en.wikipedia.org/wiki/Poisson_distribution

By using the Poisson distribution, we capture the randomness and variability in the occurrence of losses, allowing us to simulate different scenarios with varying frequencies of loss events.

2. Pareto Distribution:

The Pareto distribution is commonly used to model the distribution of extreme or rare events, such as large-scale losses in the insurance context. It is characterized by a heavy-tailed shape, meaning that it allows for the occurrence of very large values with low probabilities. In our case, the Pareto distribution is used to model the severity or size of individual losses (ground-up losses). The distribution is defined

by two parameters: a shape parameter, denoted as ' α ', and a scale parameter, denoted as ' θ '. The shape parameter determines the tail behavior of the distribution, while the scale parameter sets the minimum value for the losses.

For more information see: https://en.wikipedia.org/wiki/Pareto_distribution

In the Monte Carlo simulation, we can generate random samples from the Pareto distribution with parameters $\alpha=1.2$ and $\theta=2m$. These samples represent the sizes of individual losses in each simulation run. These parameters would lead to an average loss of $\frac{(1.2 \cdot 2m)}{(1.2-1)} = 12m$. The Pareto distribution allows for the possibility of rare but significant losses, which is essential for accurately modeling the extreme events that reinsurance aims to cover.

By incorporating the Poisson and Pareto distributions into the Monte Carlo simulation, we can capture the stochastic nature of loss occurrences and their varying severity, enabling us to estimate average losses and assess risk measures like Value at Risk (VaR) or Tail Value at Risk (TVaR).

Sampling from Distributions using Inverse CDF (Quantile) Method

In this section, we will dive deeper into the process of sampling from probability distributions using the Inverse CDF (Cumulative Distribution Function) method, also known as the quantile method. Sampling from distributions is a fundamental step in Monte Carlo simulations and plays a crucial role in generating random values that adhere to specific probability distributions.

The Inverse CDF method leverages the relationship between the cumulative distribution function and the quantile function to obtain random samples from a given distribution. By using this method, we can generate random values that follow the distribution's shape and characteristics.

We will explore the mathematical concept behind the Inverse CDF method and its practical implementation in Python. This method provides a powerful and widely used approach to sampling from various probability distributions, including the Poisson and Pareto distributions mentioned earlier.

The Inverse CDF

The Inverse CDF (Cumulative Distribution Function) method, also known as the quantile method or inverse transform sampling, is a technique used to generate random samples from a probability distribution. It involves inverting the cumulative distribution function to obtain values that correspond to specific probabilities.

The cumulative distribution function (CDF) of a random variable provides the probability that the variable takes on a value less than or equal to a given value. The inverse CDF, also known as the quantile function or percent-point function, calculates the value corresponding to a specific probability.

To sample from a distribution using the inverse CDF method, we follow these steps:

1. Compute the inverse CDF or quantile function: Invert the cumulative distribution function to obtain the inverse CDF or quantile function. This function maps probabilities to corresponding values of the random variable.
2. Generate random samples: Generate random numbers uniformly distributed between 0 and 1. Then, use the inverse CDF or quantile function to map these uniform random numbers to values from the

target distribution. This ensures that the generated values follow the desired distribution.

By using the inverse CDF method, we can generate random samples that closely match the probability distribution of interest. This technique is widely used in statistical simulations, including Monte Carlo simulations, as it allows for sampling from a wide range of distributions, such as normal, exponential, pareto, beta, and many others.

In summary, the inverse CDF method is a powerful technique that leverages the relationship between cumulative distribution functions and quantile functions to generate random samples from probability distributions. It provides a flexible and efficient approach for simulating random variables that adhere to specific distributions.

Poisson

When it comes to sampling from the Poisson distribution, using the inverse CDF method is not as straightforward as it is for continuous distributions. The Poisson distribution is a discrete probability distribution that describes the number of events occurring within a fixed interval of time or space. Since it is a discrete distribution, it has a probability mass function (PMF) rather than a continuous cumulative distribution function (CDF).

The PMF of the Poisson distribution gives the probability of observing a specific number of events within the interval. However, obtaining the inverse CDF for the Poisson distribution is not possible in closed form due to its discontinuous nature. Therefore, directly applying the inverse CDF method to sample from the Poisson distribution is not feasible.

To overcome this challenge, an alternative approach can be used. We can exploit the fact that the interval between two Poisson-distributed events follows an exponential distribution, which is continuous and amenable to the inverse CDF method. By simulating the time intervals between events, we can estimate the occurrence count within a given time frame.

To accomplish this, we can simulate the time intervals between Poisson-distributed events using the exponential distribution. We continue summing the time intervals until the total elapsed time reaches a predefined period, such as one year. At that point, we stop counting the occurrences and consider it as the final count.

This approach allows us to indirectly sample from the Poisson distribution.

The following code snippet implements two functions, `rExponential` and `rPoisson`, for generating random numbers from the exponential and Poisson distributions, respectively.

```
import math
import random

def rExponential(rate:float) -> float:
    # Inverse CDF of the exponential distribution
    return -math.log(1 - random.uniform(0, 1)) / rate

def rPoisson(rate:float, interval_length=1:float) -> int:
    # The time between two poisson distributed events has an #exponential
```

```

distribution with mean_exponential = 1/mean_poisson
# We stop when the event occurs outside the interval.
totalOccurrences = 0
timeOfOccurrence = 0.0
while timeOfOccurrence <= interval_length:
    timeOfOccurrence += rExponential(rate)
    if timeOfOccurrence <= interval_length:
        totalOccurrences += 1

return totalOccurrences

```

Code explanation generated by ChatGPT:

1. `rExponential(rate:float) -> float`:

- This function generates a random number from the exponential distribution with the given rate parameter.
- It uses the inverse CDF (quantile) method to generate the random value.
- The formula `-math.log(1 - random.uniform(0, 1)) / rate` calculates the inverse CDF of the exponential distribution.
- `math.log()` computes the natural logarithm, and `random.uniform(0, 1)` generates a random number between 0 and 1.

2. `rPoisson(rate, interval_length=1) -> int`:

- This function generates a random integer representing the number of events occurring within a specified interval.
- It uses the connection between the Poisson and exponential distributions to indirectly sample from the Poisson distribution.
- The variable `rate` corresponds to the average rate of event occurrences per unit of time.
- The `interval_length` parameter defines the duration of the interval.
- The variable `totalOccurrences` keeps track of the count of events, which is initialized to 0.
- The variable `timeOfOccurrence` tracks the accumulated time until an event occurs, initially set to 0.0.
- A while loop continues to accumulate time intervals until the accumulated time exceeds the interval length.
- Within each iteration, it adds a random exponential time interval generated by `rExponential(rate)` to `timeOfOccurrence`.
- If the event occurs within the interval (i.e., `timeOfOccurrence` is less than or equal to `interval_length`), it increments the `totalOccurrences` count.
- Finally, it returns the total count of events within the interval as an integer.

These functions can be used to generate random numbers following the exponential and Poisson distributions by providing appropriate rate parameters and interval lengths.

Pareto

The inverse CDF (Cumulative Distribution Function) method can also be utilized to generate random variables that follow the Pareto distribution. Unlike the Poisson distribution, the Pareto distribution is continuous,

allowing for the direct application of the inverse CDF method.

```
import math
import random

def rPareto(shape:float, scale:float) -> float:
    # The hint suggest to use r_pareto = scale/(1-r_uniform)**(1/shape) but it is
    # better to limit the number
    # of divisions to improve computational accuracy so we can write:
    if shape<=0:
        raise ValueError('The shape parameter must be strictly greater than 0')
    if scale<=0:
        raise ValueError('The scale parameter must be strictly greater than 0')

    return scale*(1-random.uniform(0, 1))**(-1/shape)
```

Code explanation generated by ChatGPT: This code snippet implements a function `rPareto(shape, scale) -> float` that generates random numbers from a Pareto distribution with given shape and scale parameters.

Here's a breakdown of how the code works:

1. The function takes two parameters: `shape` and `scale`, which represent the shape parameter and scale parameter of the Pareto distribution, respectively.
2. It uses the inverse CDF method to generate random numbers from the Pareto distribution.
3. The formula `return scale*(1-random.uniform(0, 1))**(-1/shape)` is used to calculate the random values.
 - `random.uniform(0, 1)` generates a random number uniformly distributed between 0 and 1.
 - `(1-random.uniform(0, 1))**(-1/shape)` raises the generated random number to the power of `(-1/shape)`. This step corresponds to the inversion of the cumulative distribution function (CDF) of the Pareto distribution.
 - Finally, the value is multiplied by the scale parameter `scale` to obtain the desired random number from the Pareto distribution.

It ensures that the generated values adhere to the characteristics of the Pareto distribution, such as the heavy-tailed behavior and the long tail of rare events.

Implementing Excess of Loss (XL) Reinsurance

An Excess of Loss (XL) contract can be understood as a 'state machine' that keeps track of several parameters. In modern programming, state machines can be represented by class instances, perfect for tracking the state of a process.

The provided code is an implementation of the excess of loss contract using a Python class called `ExcessOfLoss`.

```

from dataclasses import dataclass

@dataclass
class ExcessOfLoss:
    deductible: float
    limit: float
    aad: float = None
    aal: float = None

    def __post_init__(self):
        self.total_recoveries = 0.0
        self.claimAmountInExcess = 0.0
        self.available_aad = self.aad
        self.recoveries = 0.0

    def apply_treaty(self, claim_amount):
        self.claimAmountInExcess = max(claim_amount-self.deductible, 0)
        if self.claimAmountInExcess == 0:
            self.recoveries = 0
            return self

        if self.aad:
            aad = self.available_aad
            self.available_aad = max(aad - self.claimAmountInExcess, 0)
            self.claimAmountInExcess = max(self.claimAmountInExcess - aad, 0)

        recoveries = min(self.claimAmountInExcess, self.limit)

        if self.aal:
            available_recovery = self.aal - self.total_recoveries
            recoveries = min(recoveries, available_recovery)
            self.total_recoveries += recoveries

        self.recoveries = recoveries
        return self

```

Here is a breakdown of its features:

- The class is decorated with `@dataclass`, which is a Python module for automatically generating special methods such as `__init__` and `__repr__` based on the defined fields.
- The class has the following attributes:
 - `deductible`: Represents the deductible amount for the excess of loss contract.
 - `limit`: Represents the limit of coverage for the excess of loss contract.
 - `aad` (optional): Stands for Annual Aggregate Deductible. It represents the aggregate deductible for the contract.
 - `aal` (optional): Stands for Annual Aggregate Limit. It represents the aggregate limit for the contract.
- The `__post_init__` method is a special method that is automatically called after the initialization of the class. It initializes several attributes used for tracking the contract's recoveries and available coverage.

- The `apply_treaty` method is used to apply the excess of loss contract to a claim amount. It takes the claim amount as an input and calculates the recoveries based on the contract terms and conditions.
 - It first calculates the claim amount in excess of the deductible by subtracting the deductible from the claim amount.
 - If an Annual Aggregate Deductible (`aad`) is specified, it checks if there is any available coverage remaining (`available_aad`) and adjusts the claim amount in excess accordingly.
 - It then calculates the recoveries based on the claim amount in excess, limiting it to the contract's coverage limit (`limit`).
 - If an Annual Aggregate Limit (`aal`) is specified, it checks if there is any available recovery remaining (`available_recovery`) and further limits the recoveries.
 - The calculated recoveries are stored in the `recoveries` attribute, and the total recoveries for the contract are updated in the `total_recoveries` attribute.
- The method returns the updated `ExcessOfLoss` object with the recoveries information.

This implementation allows the user to create an instance of the `ExcessOfLoss` class, set the contract parameters, and apply the excess of loss contract to different claim amounts by invoking the `apply_treaty` method. It enables tracking the recoveries and available coverage based on the contract terms specified during initialization.

Pricing

When pricing an excess of loss reinsurance contract, simply relying on the average recoveries or "burning cost" may not provide a comprehensive assessment of the potential losses and associated risks. This is where risk measures like Value at Risk (VaR) and Tail Value at Risk (TVaR) come into play.

Pricing reinsurance contracts requires a thorough understanding of the potential loss scenarios and the associated probabilities. By incorporating VaR and TVaR, insurers and reinsurers can account for the tail risk and extreme events that have a low probability of occurrence but can result in substantial losses.

Using VaR and TVaR in pricing allows insurers and reinsurers to:

Capture Tail Risk: VaR and TVaR capture the potential losses beyond a specified threshold, which includes extreme events. By considering these tail risks, insurers and reinsurers can adequately price the coverage for such events, ensuring they are compensated for the higher exposure to catastrophic losses.

Reflect Risk Appetite: Different insurers and reinsurers have varying risk appetites and tolerances. VaR and TVaR enable pricing that aligns with the desired risk level. Insurers with a lower risk tolerance may price their excess of loss contracts to achieve a higher level of confidence (e.g., 99% VaR), ensuring a higher degree of protection against extreme losses.

Account for Capital Requirements: Insurers and reinsurers are subject to regulatory capital requirements, which are often based on risk measures like VaR and TVaR. Pricing the excess of loss contracts using these risk measures helps ensure that the pricing aligns with the capital needed to cover potential losses within the defined risk appetite.

Provide Tailored Coverage: VaR and TVaR allow insurers and reinsurers to offer tailored coverage that meets the specific needs of clients. By pricing based on risk measures, they can offer contracts with varying levels of

protection and define coverage limits that align with the client's risk tolerance and exposure to potential losses.

In summary, using risk measures like VaR and TVaR in pricing excess of loss reinsurance contracts enables a more comprehensive assessment of potential losses, particularly extreme events. It accounts for tail risk, aligns with risk appetites, fulfills regulatory requirements, and allows for tailored coverage, ensuring a more accurate and appropriate pricing strategy in the reinsurance market.

Value at Risk (VaR) and Tail Value at Risk (TVaR)

Value at Risk (VaR) and Tail Value at Risk (TVaR) are both risk measures commonly used in the field of reinsurance to assess potential losses beyond a certain threshold. Here's an explanation of these concepts:

1. Value at Risk (VaR): Value at Risk is a statistical measure that quantifies the maximum potential loss, with a specified level of confidence, that an insurer or reinsurer may face within a given time frame. VaR provides an estimate of the potential losses at a particular confidence level, such as 95% or 99%. It helps organizations understand the magnitude of potential losses and assess their risk exposure.

In the context of reinsurance, VaR is often calculated based on the distribution of losses from various policies or contracts. By analyzing historical data or using simulation techniques like Monte Carlo simulations, the distribution of losses can be modeled. VaR is then computed by determining the loss amount corresponding to the specified confidence level. For example, a 99% VaR of \$10 million means that there is a 1% chance that losses will exceed \$10 million within the defined time frame.

2. Tail Value at Risk (TVaR) or Conditional Value at Risk (CVaR): Tail Value at Risk, also known as Conditional Value at Risk, goes beyond VaR by quantifying the expected loss in the tail of the loss distribution. The tail represents the extreme or rare events with high losses that occur beyond the VaR threshold.

TVaR measures the average loss magnitude that occurs when losses exceed the VaR threshold. It provides insight into the severity of extreme events and their potential impact on the insurer or reinsurer's financial position. TVaR is often expressed as a monetary value and is useful for evaluating the downside risk associated with extreme losses.

Similar to VaR, TVaR can be computed using historical data or simulation techniques. It involves estimating the conditional expected loss given that the loss exceeds the VaR level. By considering the tail of the loss distribution, TVaR provides a more comprehensive view of the potential losses under extreme scenarios.

Both VaR and TVaR play crucial roles in risk management for insurers and reinsurers. These measures assist in setting appropriate risk tolerances, determining capital reserves, pricing reinsurance contracts, and assessing the overall financial exposure to catastrophic or extreme events.

Pricing formula

In the context of reinsurance pricing, a specific formula is proposed to determine the premium for an excess of loss contract. The formula incorporates the average loss, the Tail Value at Risk (TVaR) loss at a 99% confidence level, and the cost of capital. Here's an explanation of the formula and its interpretation:

Premium = Average loss + (TVaR(99%) loss - Average loss) * Cost of Capital

1. **Average Loss:** The average loss represents the expected value of the losses incurred under the excess of loss contract. It is calculated based on historical data, actuarial analysis, or other relevant information. The average loss provides an estimate of the typical or expected loss magnitude.
2. **TVaR(99%) Loss:** The TVaR(99%) loss represents the expected loss in the tail of the loss distribution beyond the 99% confidence level. It quantifies the potential losses associated with extreme events that have a low probability of occurrence. TVaR(99%) loss provides a measure of the additional losses beyond the average loss that are considered in the pricing formula.
3. **Cost of Capital:** The cost of capital is the rate of return required by an insurer or reinsurer to compensate for the capital tied up in assuming the risk associated with the excess of loss contract. It represents the opportunity cost of deploying capital in this specific risk-bearing activity. The cost of capital reflects the risk appetite and desired return on investment.

Interpretation: The proposed pricing formula suggests that the premium for the excess of loss contract should be determined by adding the average loss to the product of the difference between the TVaR(99%) loss and the average loss, and the cost of capital.

- The term (TVaR(99%) loss - Average loss) represents the additional risk beyond the average loss that is captured by TVaR at the 99% confidence level. It signifies the compensation required for potential extreme losses.
- Multiplying this difference by the cost of capital accounts for the financial cost associated with assuming the additional risk. It reflects the premium charged to cover the capital tied up and the expected return on that capital.

By using this pricing formula, insurers and reinsurers aim to ensure that the premium charged for the excess of loss contract adequately accounts for the average loss, compensates for potential extreme losses, and factors in the cost of capital tied to assuming the risk. It provides a more comprehensive and risk-based approach to pricing, aligning with the underlying probability of losses and the desired return on investment.

Claims development over time (IBNR/IBNER)

In the context of reinsurance and insurance, IBNR (Incurred But Not Reported) and IBNER (Incurred But Not Enough Reserved) are important concepts related to estimating and managing potential future claim liabilities. Here's an explanation of these concepts and the logic behind the development patterns of claim amounts:

1. **IBNR (Incurred But Not Reported):** IBNR refers to claims that have occurred but have not yet been reported to the insurer or reinsurer. These are typically claims that have been incurred but not yet communicated or filed by the policyholders. IBNR estimates are essential for insurers and reinsurers to accurately assess their potential future liabilities and reserve adequate funds to cover those claims.

The development of IBNR typically follows a pattern. Initially, when a loss event occurs, there may be a delay before the policyholder reports the claim. This delay can be influenced by factors such as the policyholder's awareness of the loss, administrative processes, or contractual requirements for claim reporting. As time progresses, the claims gradually get reported, and the IBNR estimate decreases until it reaches zero.

2. **IBNER (Incurred But Not Enough Reserved):** IBNER refers to claims that have been reported but for which the reserves set aside initially were not sufficient to cover the full expected cost of the claim. In

other words, the reserves were insufficient to meet the actual claim payments. This situation can arise due to various reasons, such as inaccurate initial reserving, unforeseen developments in the claim, or changes in the cost of settlements.

The development pattern of IBNER involves adjusting the reserve estimates as additional information becomes available. Insurers and reinsurers continually reassess the claim reserves based on new data, expert analysis, and evolving circumstances related to the claim. As more information is gathered, the IBNER estimate is refined, and the reserves are adjusted accordingly.

The logic behind development patterns of claim amounts: The development patterns of claim amounts, including IBNR and IBNER, are influenced by several factors:

1. **Reporting Delays:** There is typically a delay between the occurrence of a loss event and the reporting of the claim. This delay can vary depending on factors such as the policy type, complexity of the claim, reporting requirements, and the policyholder's awareness of the loss.
2. **Claim Settlement Process:** The time required to settle a claim can vary based on the complexity of the claim, the need for investigation or negotiation, legal processes, and other factors. Claims may undergo multiple stages, including assessment, evaluation, negotiation, and eventual settlement.
3. **Data Availability:** The availability and accuracy of data play a crucial role in estimating and developing claim amounts. As more data becomes available over time, insurers and reinsurers can refine their estimates and make adjustments to the reserves.
4. **External Factors:** External factors such as changes in legal regulations, economic conditions, healthcare practices, or industry trends can influence the development patterns of claim amounts. These factors may impact the severity and frequency of claims, leading to adjustments in reserve estimates.

Understanding the development patterns of claim amounts, including the dynamics of IBNR and IBNER, helps insurers and reinsurers in proper risk assessment, setting adequate reserves, and managing their financial stability. Accurate estimation of these liabilities is essential for effective risk management, pricing of insurance policies, and maintaining financial solvency in the insurance industry.

Specificities in reinsurance

In the context of reinsurance, there are situations where claims that occurred several years ago suddenly deteriorate and exceed the deductible set in the excess of loss contract. This can lead to a scenario where the reinsurer is contacted to cover these old claims that have experienced a significant increase in their cost or severity. Here's an explanation of this phenomenon:

1. **Deteriorating Claims:** Deteriorating claims refer to situations where the initial estimate or reserve set for a claim turns out to be insufficient to cover the full cost of the claim as it develops over time. The factors contributing to claim deterioration can include unforeseen complications, changes in circumstances, new medical information, legal developments, or other factors that increase the estimated cost of the claim.
2. **Reporting Threshold:** In some cases, reinsurance contracts have a reporting threshold, which is a specified limit or threshold beyond which the insurer is required to notify the reinsurer about claims that have exceeded certain criteria. This reporting threshold is often set in terms of the total incurred losses for a specific period or the individual claim amounts.

3. **Exceeding the Deductible:** When a claim deteriorates and its cost increases over time, it may eventually exceed the deductible specified in the excess of loss contract. The deductible represents the amount below which the reinsurer is not liable to pay for a claim. Once the claim surpasses the deductible, the reinsurer's responsibility is triggered, and they may be contacted to cover the remaining costs of the claim.
4. **Financial Impact:** Deteriorating claims exceeding the deductible can have financial implications for both the insurer and the reinsurer. The reinsurer may need to pay for a claim that was initially the responsibility of the insurer, resulting in an unexpected financial burden. This highlights the importance of accurate claim estimation and reserving by insurers to prevent underestimating potential claim costs and the need for appropriate risk management strategies by reinsurers to account for potential claim deterioration.

In summary, the occurrence of deteriorating claims that exceed the deductible in an excess of loss contract can lead to the reinsurer being contacted to cover old claims that have experienced a significant increase in their cost or severity. This emphasizes the need for thorough claim estimation, effective risk management, and clear communication between insurers and reinsurers to address potential claim developments and ensure proper coverage under reinsurance agreements.

Estimation of payment patterns

Estimations of potential claim developments, including the occurrence of deteriorating claims, are often conducted by analyzing the historical payment pattern of claims. By examining past claim data and payment patterns, insurers and reinsurers can gain insights into the behavior of claims over time and make informed predictions about future claim developments. Here's an explanation of how historical payment patterns are analyzed for estimating claim developments:

1. **Historical Claim Data:** Insurers and reinsurers maintain comprehensive records of past claims, including details such as the date of occurrence, reported date, initial reserve, payment history, and any subsequent adjustments. This historical claim data serves as a valuable source for analyzing the payment patterns and understanding the trends and characteristics of claims.
2. **Payment Pattern Analysis:** Payment pattern analysis involves studying the sequence and timing of claim payments over the life cycle of a claim. This analysis focuses on factors such as the time between claim occurrence and reporting, the timing of reserve changes, and the actual payment timing. The goal is to identify patterns and trends in how claims evolve and develop in terms of their cost and severity.
3. **Development Triangles:** One common tool used in analyzing payment patterns is the development triangle or loss development triangle. A development triangle is a tabular representation that shows the historical development of claims over multiple periods, typically displayed in a triangular format. It tracks the changes in claim amounts or reserves over time, providing a visual representation of how claims have evolved and developed.
4. **Patterns and Trends:** By examining the development triangle and analyzing the historical payment patterns, insurers and reinsurers can identify patterns and trends in claim developments. This includes observing the average time between claim occurrence and reporting, the rate of reserve changes over time, and the overall development of claim costs.

5. Estimating Future Developments: Based on the insights gained from historical payment pattern analysis, insurers and reinsurers can make estimations and projections about future claim developments. This includes forecasting the potential impact of deteriorating claims, estimating the likelihood of claims exceeding deductibles or thresholds, and assessing the financial implications for both the insurer and the reinsurer.
6. Refining Estimations: It's important to note that estimations based on historical payment patterns are subject to ongoing refinement and adjustment as new data becomes available. As additional claim information and experience emerge, insurers and reinsurers update their estimations to account for any changes or shifts in claim development patterns.

Analyzing the historical payment pattern of claims provides valuable insights into how claims evolve over time and helps insurers and reinsurers make informed decisions about reserving, pricing, and risk management. It allows them to anticipate potential claim developments, including the occurrence of deteriorating claims, and take proactive measures to ensure adequate coverage and financial stability.

Payment patterns in the reinsurance pricing context

To accurately price reinsurance contracts, it is crucial to consider claims at ultimate. Claims at ultimate refer to the final settlement value of a claim, accounting for all its potential development and incurred costs over its entire lifecycle.

The reinsurer focuses on several aspects when dealing with large claims:

1. Complete Assessment of Claim Costs:

Claims can take time to fully develop, and their ultimate cost may differ from the initial estimate or reserve. By considering claims at ultimate, insurers and reinsurers can account for the potential deterioration or improvement in claim costs over time. This allows for a more comprehensive and accurate assessment of the overall claim costs that the reinsurance contract needs to cover.

2. Proper Risk Evaluation:

Reinsurance contracts involve transferring risk from the insurer to the reinsurer. To properly evaluate the risk and set appropriate pricing, the reinsurer needs to understand the potential magnitude of claims at ultimate. Failure to consider the ultimate claim costs may result in underestimating the risk exposure and pricing the reinsurance contract inadequately, leading to potential financial losses for the reinsurer.

3. Long-Tail Claims:

Certain types of insurance, such as liability insurance or certain medical malpractice policies, can involve long-tail claims. These claims have a prolonged development period, where the full extent of the claim may not be known for several years or even decades. By considering claims at ultimate, reinsurers can account for the extended development periods and the associated risks when pricing reinsurance contracts.

4. Reserving Adequacy:

Insurers set reserves to cover expected claim costs based on their assessments at a given point in time. However, these reserves may not fully reflect the ultimate claim costs. Reinsurers need to consider the

adequacy of the insurer's reserves and adjust their pricing accordingly. By factoring in claims at ultimate, reinsurers can ensure that the reinsurance contract adequately covers the potential shortfall between the insurer's reserves and the ultimate claim costs.

5. Financial Stability:

Accurate pricing of reinsurance contracts, considering claims at ultimate, contributes to the financial stability of both insurers and reinsurers. It helps reinsurers appropriately allocate capital, maintain solvency, and manage their overall risk exposure. Adequate pricing also supports insurers in fulfilling their obligations to policyholders by ensuring that sufficient coverage is available to handle claims that develop over time.

Therefore, considering claims at ultimate is crucial in the pricing of reinsurance contracts. It allows for a complete assessment of claim costs, proper risk evaluation, consideration of long-tail claims, evaluation of reserving adequacy, and contributes to the financial stability of insurers and reinsurers.