IAD-wzory

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$$f(x) = w_0 + \sum_{i=1}^{c} w_i k(||x - c_i||, \sigma_i) \tag{1}$$

w_i jest i-tą wagą

c_i jest i-tym centrum

$$k(d, \sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{d^2}{2\sigma^2}\right) \tag{2}$$

$$Q(P) = \frac{1}{2P} \sum_{i=0}^{P} (f(x_i) - y_i)^2 \tag{3}$$

$$w_n(t+1) = w_n(t) - \alpha \frac{\partial}{\partial w_n} Q(w(t)) \tag{4}$$

1 Metoda najmniejszego spadku dla wag

$$\frac{\partial}{\partial w_n} Q(P) = \frac{\partial}{\partial w_n} \frac{1}{2P} \sum_{i=0}^{P} \left(w_0 + \sum_{j=1}^{c} w_i k(||x_i - c_j||, \sigma_j) - y_i\right)^2$$

ze wzoru na pochodną funkcji złożonej $(f(g(x)))' = f'(g(x))g'(x)$ dla $n \neq 0$

$$\frac{\partial}{\partial w_n} Q(P) = \frac{1}{P} \sum_{i=0}^{P} \left(w_0 + \sum_{j=1}^{c} w_i k(||x_i - c_j||, \sigma_j) - y_i\right) \cdot w_n$$

zatem

$$\frac{\partial}{\partial w_n} Q(P) =$$

$$\begin{cases} w_n(t+1) = w_n(t) - \alpha \frac{1}{P} \sum_{i=0}^{P} \left(w_0 + \sum_{j=1}^{c} w_i k(||x_i - c_j||, \sigma_j) - y_i\right) \cdot w_n & \text{dla } n \in \{1, 2 \ldots j\} \\ w_n(t+1) = w_n(t) - \alpha \frac{1}{P} \sum_{i=0}^{P} \left(w_0 + \sum_{j=1}^{c} w_i k(||x_i - c_j||, \sigma_j) - y_i\right) & \text{dla } n = 0 \end{cases}$$

2 Metoda najmniejszego spadku dla promienia sąsiedztwa

$$\sigma_n(t+1) = \sigma_n(t) - \alpha \frac{\partial}{\partial \sigma_n} Q(\sigma(t)) \tag{5}$$

$$\frac{\partial}{\partial \sigma_n} Q(P) = \frac{\partial}{\partial \sigma_n} \frac{1}{2P} \sum_{i=0}^{P} \left(w_0 + \sum_{j=1}^{c} w_i k(||x_i - c_j||, \sigma_j) - y_i \right)^2 =$$

$$\frac{1}{P} \sum_{i=0}^{P} \left(w_0 + \sum_{j=1}^{c} (w_i k(||x_i - c_j||, \sigma_j) - y_i) \cdot \frac{\partial}{\partial \sigma_n} k(||x_i - c_j||, \sigma_j) \right)$$

$$\frac{\partial}{\partial \sigma} k(d, \sigma) = \frac{\partial}{\partial \sigma} \frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{d^2}{2\sigma^2} \right) =$$

ze wzoru na iloczyn funkcji $(f(x)g(x))' = f'(x)g(x) + f(x)g'(x)$

$$\frac{\partial}{\partial \sigma} k(d, \sigma) = \frac{1}{\sqrt{2\pi}} \left(-\sigma^{-2} \right) \exp\left(-\frac{d^2}{2\sigma^2} \right) + \frac{1}{\sqrt{2\pi} \cdot \sigma} \exp\left(-\frac{d^2}{2\sigma^2} \right) (-2\sigma)^{-3} =$$

$$-\frac{1}{\sqrt{2\pi} \cdot \sigma} \cdot \exp\left(-\frac{d^2}{2\sigma^2} \right) \cdot \left(\frac{1}{\sqrt{2\pi} \cdot \sigma} + (2\sigma)^{-3} \right) = -k(d, \sigma) \left(\frac{1}{\sqrt{2\pi} \cdot \sigma} + (2\sigma)^{-3} \right)$$

stąd

$$\frac{\partial}{\partial \sigma_n} Q(P) = \frac{1}{P} \sum_{i=0}^{P} \left(\left[w_0 - \sum_{j=1}^{c} (w_i k(||x_i - c_j||, \sigma_j) - y_i) \right] \left[k(||x_i - c_n||, \sigma_n) \left(\frac{1}{\sqrt{2\pi} \cdot \sigma_n} + (2\sigma_n)^{-3} \right) \right] \right)$$