Zadanie 4

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Treść

Bianrny cykliczny kod BCH $C_{BHC}(15,7)$ o zdolności poprawiania błędów $t=2$ ma wielomian generujący

$$g(x) = (1 \oplus x \oplus x^4)(1 \oplus x \oplus x^2 \oplus x^3 \oplus x^4) = 1 \oplus x^4 \oplus x^6 \oplus x^7 \oplus x^8$$

nad ciałem F_{2^4}. Zdekoduj sygnał $\mathbf{r} = (100000001000000)$ stosując alogrytm Euklidesa.

Rozwiązanie

$$C_{BCH}(15,7) \quad t=2 \quad GF(2^4) \quad p(x) = 1 \oplus x \oplus x^4$$
$$r(x) = 1 \oplus^8$$

$$\begin{aligned}
s_1 = r(\alpha) &= 1 \oplus \alpha^8 &&= \alpha^2 \\
s_2 = r(\alpha^2) &= 1 \oplus \alpha &&= \alpha^4 \\
s_3 = r(\alpha^3) &= 1 \oplus \alpha^9 &&= \alpha^4 \\
s_4 = r(\alpha^4) &= 1 \oplus \alpha^2 &&= \alpha^8
\end{aligned}$$

$$\begin{aligned}
\sigma^{(-1)}(x) &= 1 & l_{-1} &= 0 & d_{-1} &= -1 \\
\sigma^{(0)}(x) &= 1 & l_0 &= 0 & d_0 &= s_1
\end{aligned}$$

$$p = \max_{d_\mu \neq 0}(\mu - l_\mu)$$

$$d_\mu = s_\mu \oplus \sigma_1^{(\mu)} s_\mu \oplus_2^{(\mu)} s_{k-1} \oplus \cdots \oplus \sigma_{l_\mu}^{(\mu)} s_{\mu+1-l_\mu}$$

$$\sigma^{\mu+1}(x) = \begin{cases} \sigma^\mu(x) & \text{dla} \quad d_\mu = 0 \\ \sigma^\mu(x) \oplus d_\mu d_p^{-1} \sigma^{\mu-\rho} & \text{dla} \quad d_\mu = 0 \end{cases}$$

μ	$\sigma^{(\mu)}(x)$	l_μ	d_μ	$\mu - l_\mu$	ρ	$\mu - \rho$
-1	1	0	1	-1		
0	1	0	α^2	0	-1	1
1	$1 \oplus \alpha^2 x$	1	0	0	0	1
2	$1 \oplus \alpha^2 x$	1	$\alpha^1 0$	0	0	2
3	$1 \oplus \alpha^2 x \oplus \alpha^8 x^2$	2	0	1	2	1
4	$1 \oplus \alpha^2 x \oplus \alpha^8 x^2$					

$$\sigma(x) = 1 \oplus \alpha^2 x \oplus \alpha^8 x^2 = (1 \oplus x)(1 \oplus \alpha^8 x)$$

$$\beta_1^{-jq} = \alpha^0 \quad \sigma(1) = 1 \oplus \alpha^2 \alpha^8 x^2 = (1 \oplus x)(1 \oplus \alpha^8 x) \quad j_1 = 0$$

$$\sigma(x) = (1 + \beta_1 x)(1 + \beta_2 x)$$

$\beta_1 = 1 = \alpha^1$	$j_1 = 0$	$e_1 = 1$	
$\beta_2 = \alpha^8$	$j_2 = 8$	$e_8 = 1$	$e(x) = 1 \oplus x^8$

$$c(x) = r(x) \oplus e(x) = 1 \oplus x^8 \oplus 1 \oplus x^8 = 0$$

$$c = 000000000000000$$