Phase Transition In The Random Field Ising Model

Seminary for the course of magnetism and superconductivity

Order, Broken Simmetry and Critical Exponents

Physics Engineering

Author: D. Patricelli

Professor G. Ghiringhelli

Model

The **Random Field Ising Model (RFIM)** is an extension of the Ising model, where each spin is coupled not only to an adjacent spin but also to a site-dependent field:

$$\mathcal{H} = -J \sum_{<i,j>}^{n} S_i S_j - \sum_{i}^{n} h_i S_i$$

Where:

- J is considered positive, homogeneous, isotropic, and short-range (NN)
- The spin has dimensionality $D = 1$ so $S_i = \pm 1$
- The random field is fixed and distributed according to a Gaussian with zero mean and variance Δ **(Randomness)**

$$h_i \sim \mathcal{N}(0, \Delta)$$

Why?

"…if you can understand the physics of some simple (even ridiculously over-idealized) model and show that it's in the same universality class as a system of interest, then you win"

- John McGreevy-

University of California

Why?

In this spirit, RFIM is a powerful prototype for the study of long range order and phase transtion in systems where defects are present, namely **real mateials**.

With a strong interplay between:

Theory: "Playground" » for powerful tools in statistichal mechanics (Replica method, Renormaization Group, Perturbation theory).

Experiments: Diluted antiferromagnets in a homogeneous external field are well described by FM RFIM [1]

$$\mathcal{H} = -\sum_{<i,j>}^{n} J_{ij}\varepsilon_i\varepsilon_j S_i\, S_j - \sum_{i}^{n} H\varepsilon_i\, S_i$$

Overview of Phase Transition

How randomness affect the critical dimension for phase transtion?

Remark: always consider weak randomness ($\Delta \ll J$)

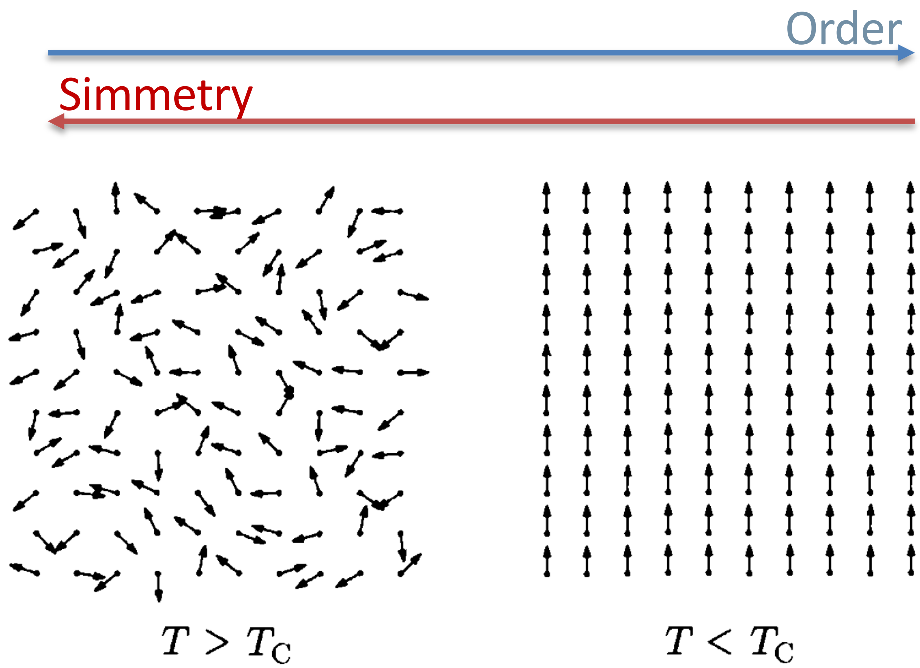
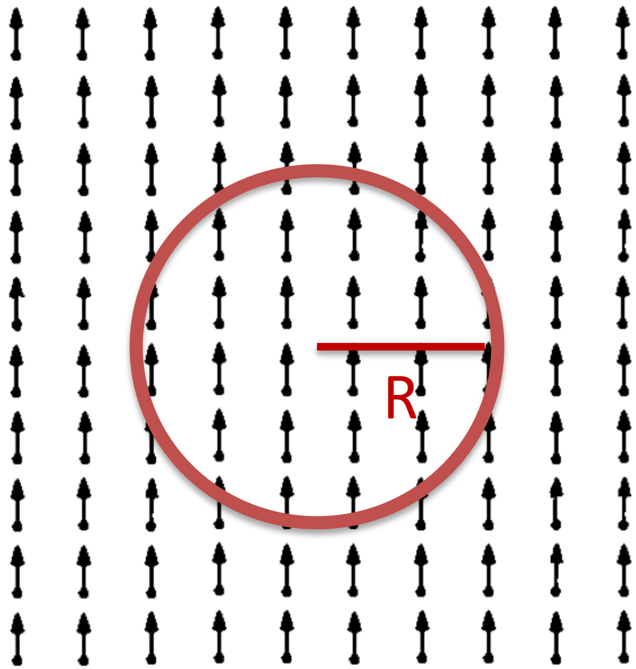


Image from the book: magnetism in condesed matter. Author: Stephen Blundel. Oxford University Press.

Imri and Ma Argument

First argument was presented in 1975 by Imry and Ma showing that there is no ordered phase for $d \leq 2$ [2]

Suppose to have a perfect ordered phase and evaluate, at 0 Temperature the energy change associated to the flip of a droplet U of radious R:



Exchange Energy:

$$\delta E_{ex} \sim J|\partial U| \sim JR^{d-1} > 0$$

Random field Energy: even in <h> = 0, local fluctuation are allowed:

$$\overline{\delta E_{RF}^2} \sim \Delta^2 |U| \sim \Delta^2 R^d \quad \Rightarrow \quad \overline{\delta E_{RF}} \sim \pm \Delta R^{\frac{d}{2}}$$

Imri and Ma Argument

In the Thermodynamic limit there will be favourable domain in which:

$$\delta E_{ex} + \delta E_{RF} \sim JR^{d-1} - \Delta R^{\frac{d}{2}}$$

- $\boldsymbol{d} = \mathbf{1}$**:** There will always be a suffiviently large domain such that:

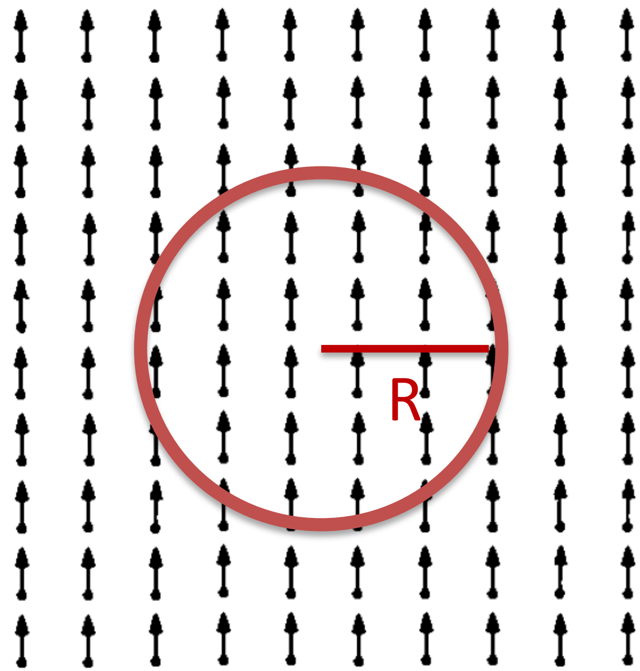
$$\delta E_{ex} + \delta E_{RF} \sim J - \Delta R^{\frac{1}{2}} < 0$$

- $\boldsymbol{d} = \mathbf{2}$**:** there is always a small but finite probability of finding a domain of any size R where local fluctuation allows:

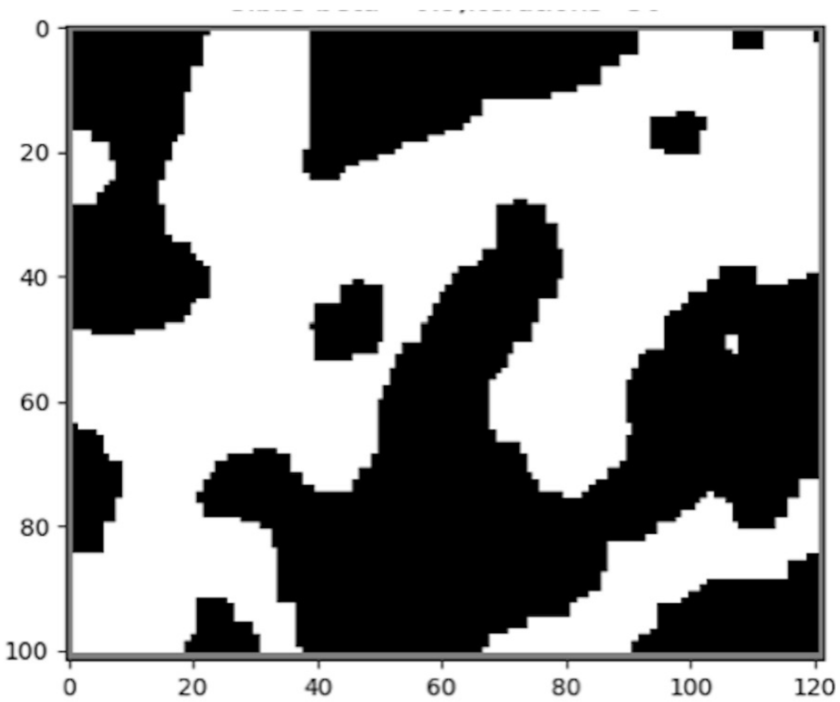
$$\delta E_{ex} + \delta E_{RF} \sim (J - \Delta_{loc})R < 0$$

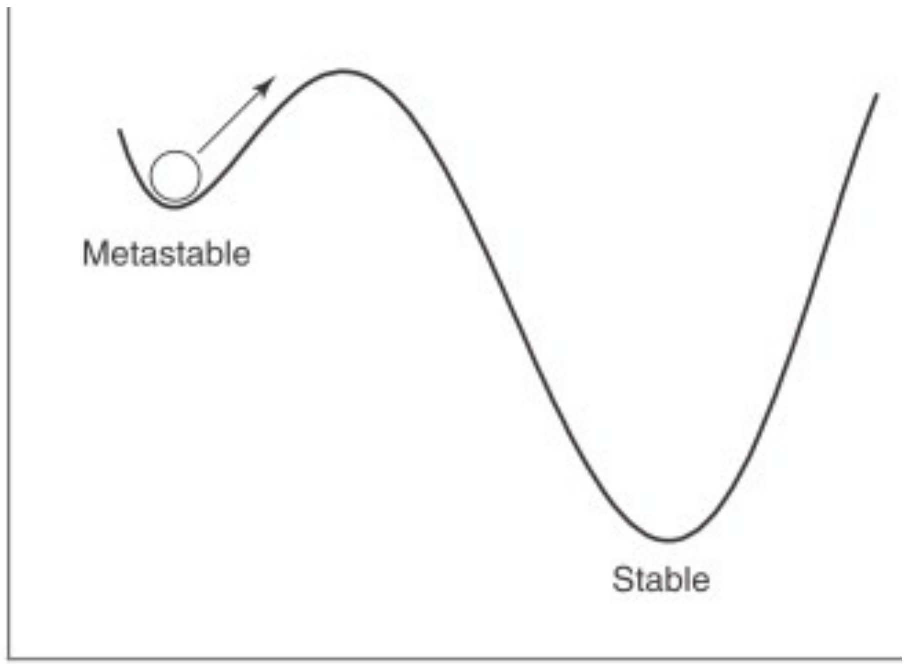
- $\boldsymbol{d} = \mathbf{3}$**:** for sufficiently large R, flipping is never favourble:

$$\delta E_{ex} + \delta E_{RF} \sim \left(JR^{1/2} - \Delta\right)R^{3/2} > 0$$



Intermezzo: Metastable States





Back To Phase Transition

While **intriguing**, Imry and Ma's argument is not entirely rigorous (for instance there could be additional terms due to domain roughness)

More formal studies led sometimes to controversial results:

- **The upper critical dimension is $d_{uc} = 6$** (Imry and Ma (1976))
- **Dimensional reduction (Contradiction??) critical exponents of the RFIM in $d = 6 - \varepsilon$ are the same as the Ising model in $d - 2$.** (Aharony, Imry, and Ma (1976), supersymmetry theory by Parisi and Sourlas (1979))

From that, following works confirmed the initial result of Imri and Ma and showed that the dimensional reduction holds from dimension 5.

Scaling Hypothesis and Power Laws

Near the critical point, thermodynamic quantities have a **regular** part and a **singular** part, proportional to a power of a parameter tending to zero:

$$t = \frac{T - T_c}{T_c}$$

A key quantity in critical phenomena is the correlation function, which describes how fluctuations of the order parameter are spatially correlated:

$$\Gamma(r) = \langle m(\vec{r})m(0)\rangle - \langle m(\vec{r})\rangle\langle m(0)\rangle \rightarrow_{t\rightarrow 0} r^{-p} e^{-\frac{r}{\xi}}$$

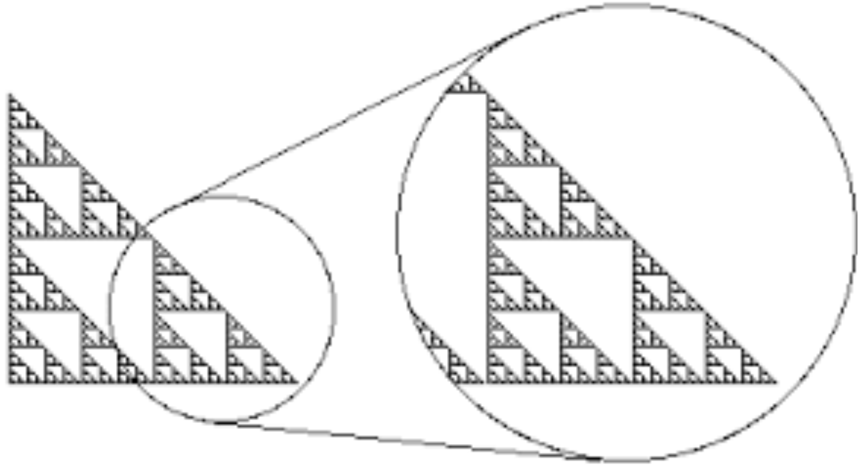
Where ξ is the **correlation length**.

Scaling Hypothesis:

Close to the critical point, the **correlation length** is the only **relevant parameter** for understanding the scaling behaviour of thermodynamic functions

Self-Similarity

In **Second Order Phase Transition** ξ follows also a **Power Law** approaching the critical point, where the correlation length diverges.



At this point, the system is **invariant under scale transformations**, thermodynamic functions vary regularly when the scale changes, remaining similar to themselves.

They are described by homogeneous functions:

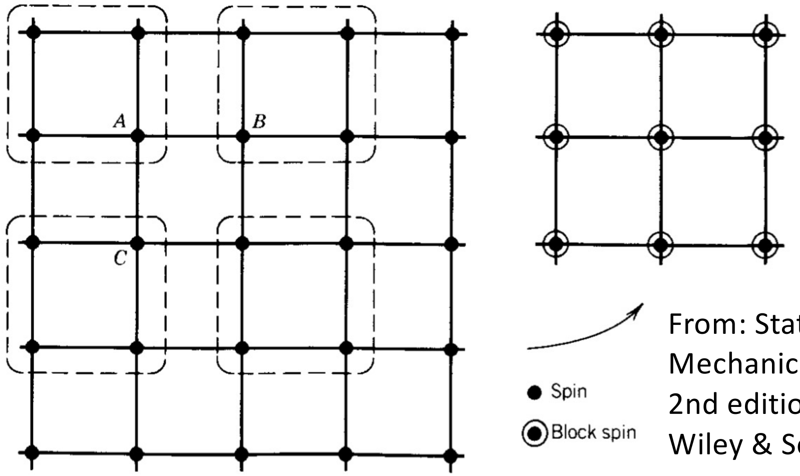
$$x \rightarrow x' = b^{D_b} x \Longrightarrow f(x') = b^{D_f} f(x)$$

Renormalization Group Approach

Idea: under scale invariance individual parts of the system behave similarly to the whole. In the case of a spin system, this allow to perform **block spin transformation**:

This is the key idea behind the **Renormalization Group** procedure, made by the iteration of two steps:

1. **Coarse-Graining**
2. **Unit Rescaling**



From: Statistical Mechanics K.Huang 2nd edition. Jonh Wiley & Sons

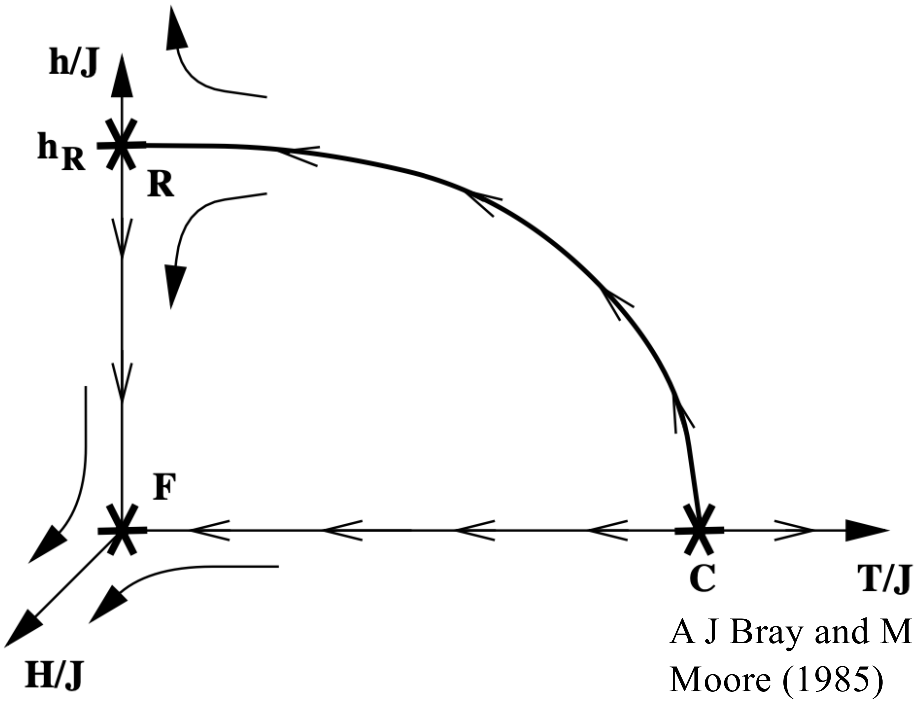
The RG transformation defines a discrete dynamical system R in the parameter space, which fixed point and their stability indicate the macroscopic behaviout of the system.

Renormalization Group Results

For the RFIM the map is : $(T^{n+1}, \Delta^{n+1}, J^{n+1}) = R(T^n, \Delta^n, J^n)$

A **Stability analysis** of the fixed points: $(T^*, \Delta^*, J^*) = R(T^*, \Delta^*, J^*)$

Confirms that in 3 dimensions there is a phase transition! [3]



A J Bray and M A Moore (1985)

Experimental Validation: Diluite Antiferromagnets

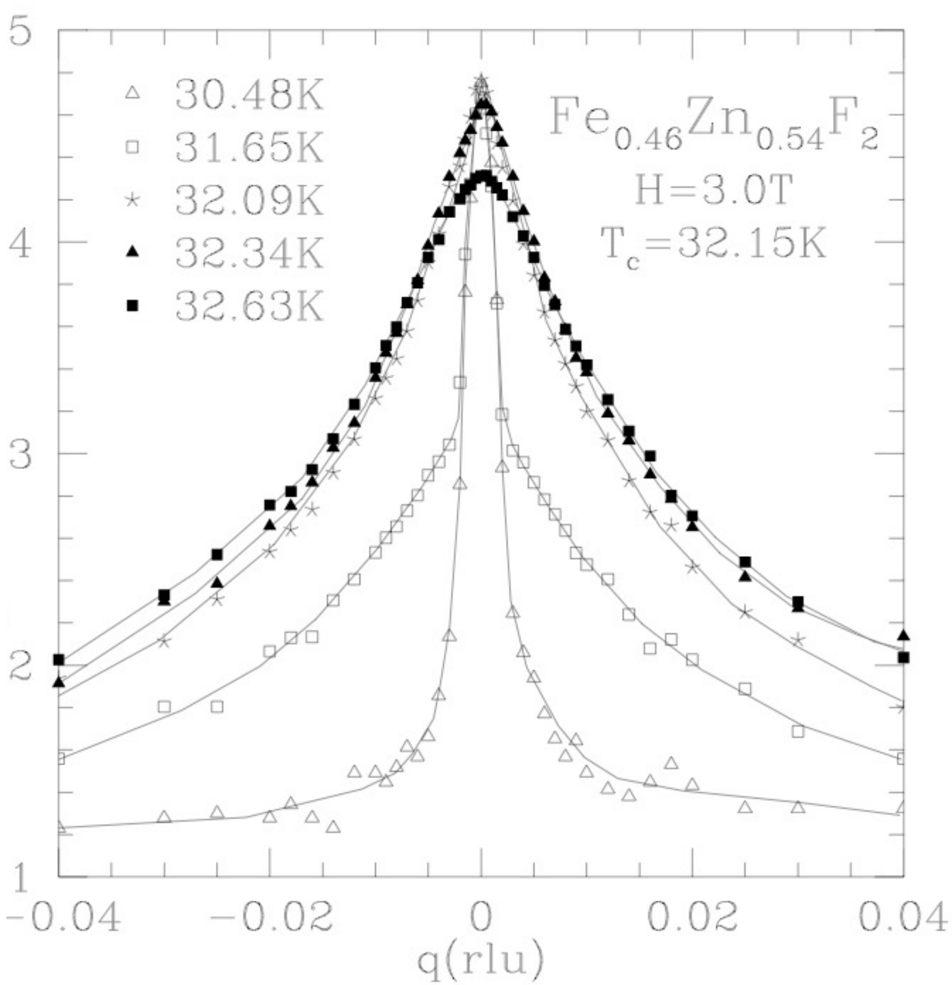


Image from [5], from the original work [4] (1987).

Bibliography and Further Reading

Text Books and Lecture notes:

- Statistical Mechanics Author: K.Huang 2nd edition. Jonh Wiley & Sons
- *Magnetism in Condesed Matter. Author: Stephen Blundel. Oxford University Press.*
- *Four Lectures on the Random Field Ising Model,Parisi-Sourlas Supersymmetry, and Dimensional Reduction. Author:* Slava Rychkov
- Physics 217: The Renormalization Group Fall 2018 Author: McGreevy

Papers and reviews:

1. S Fishman and A Aharony 1979 J. Phys. C: Solid State Phys. 12 L729
2. Y. Imry and S.-k. Ma,Phys. Rev. Lett. 35 (1975) 1399–1401
3. A J Bray and M A Moore 1985 J. Phys. C: Solid State Phys. 18 L927
4. D. P. Belanger, A. R. King, V. Jaccarino, and R. M. Nicklow, Phys. Rev. Lett. 59, 930 (1987)
5. Experiments On The Random Field Ising Model. Author: D.P. BELANGER

More on Montecarlo Simulations

1. M. E. J. Newman and G. T. Barkema, Phys. Rev. E 53, 393 (1996).