Applying Hooke's Law to Multiple Bungee Cords

Introduction

Hooke's Law declares that the force exerted on a spring is proportional to the amount of stretch or compression on the spring, is always directed toward the equilibrium position, and is independent of the mass attached to the spring. These characteristics also apply to the stretch of a bungee cord and are useful when assessing the necessary parameters for a successful bungee jump. The purpose of this experiment is to determine how the spring constant, k is altered when multiple bungee cords are used together to support the weight of a hanging mass. We will use the following equations as the central dogma of this experiment to demonstrate how k changes with the number of bungee cords used.

Equation 1: $F = kx$
Where F is the total force exerted on the spring in Newtons, k is the spring constant in Newtons per meter, and x is the displacement of the spring in meters.

Equation 2: $F = ma$
Where F is the forced exerted on the spring in Newtons, m is the mass of the hanging weight attached to the bungee system, and a is the acceleration of the system.

Methods

For this experiment, we used vertically hanging bungee cords and different masses to determine the displacement of the cord system and to calculate the spring constant, k. We operated under the assumption that the k value for the system was independent of the length of the bungee cord, so length was not kept constant. The implications of this assumption will be discussed later in the report.
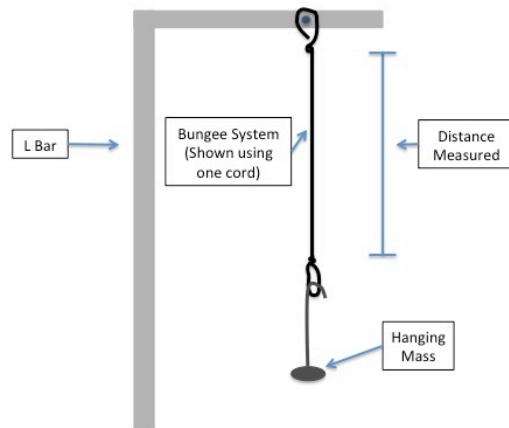

Diagram 1: Experiment Set-Up. This diagram demonstrates the overall set-up for the experiment and the distance that was measured during each trial.

We first assessed a single bungee cord. A loop was knotted at the top and bottom of the cord. The top loop was hung on the notch in the L bar, and the bottom loop was attached to the hanging weights. After hanging the specific weight on the bottom loop, the cord was brought to equilibrium and the distance from the bottom of the top knot to the top of the bottom knot was measured (Diagram 1). This process was repeated for hanging weights of 0-250g at 50g increments. The same method was applied to a system that used two bungee cords and a system of three bungee cords. Displacement of the system was measured to determine how much the bungee system stretched with increasing force exerted on the system (increasing hanging mass). This measurement was made by subtracting the measured length of the cord (from knot to knot) when there was no mass attached to the system from the measured length of the cord (from knot to knot) after the hanging mass was placed on the system and the system was brought to equilibrium. To create the system of multiple bungee cords, the single cord available was folded over itself and a looped knot was made at the top and bottom of the cords so that the number of cords in the system was also the number of cords in the loop of the knot (Diagram 2).
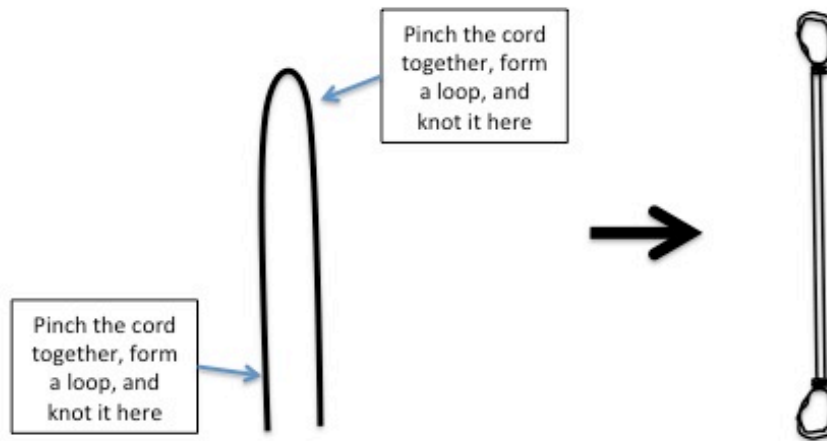

Diagram 2: Bungee System Preparation. This diagram demonstrates the method used to convert the single bungee cord into a system of two bungee cords. The same method was used to create a system of three bungee cords except the single cord was folded over itself twice.
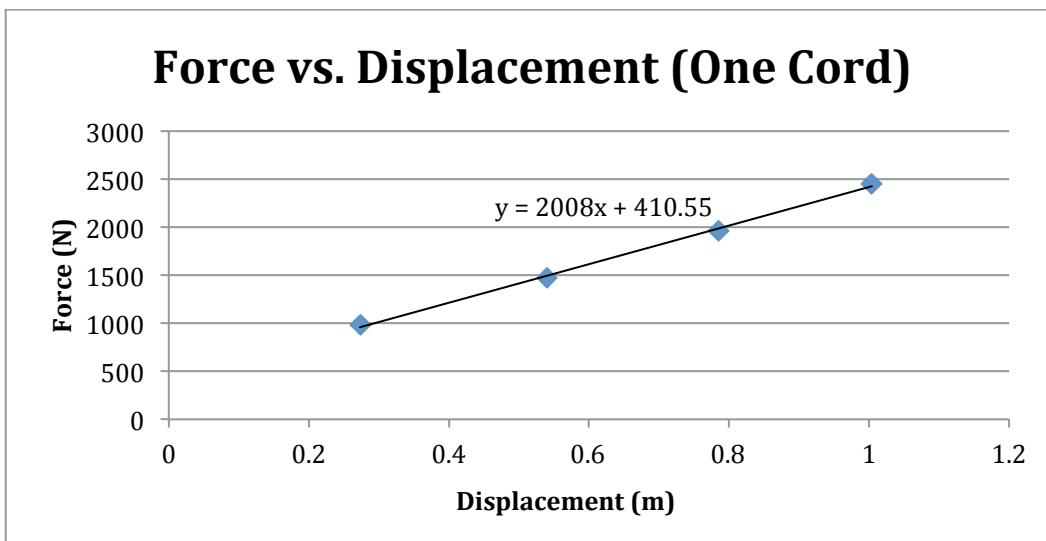
Results

We found that as the number of cords in the bungee system increased, the spring constant, k also increased. This means that there is a direct relationship

between the number of cords in the system and the "stiffness", or resistance, of the bungee system.

Mass of hanging weight (g)	Force exerted on the Bungee system (N)	Displacement (m)	Displacement per unit length
100	981	0.27	0.73
150	1472	0.54	1.44
200	1962	0.79	2.09
250	2452	1.00	2.68

Table 1: Measurements for a System of One Bungee Cord. This table contains the data collected from executing the experiment with a bungee system of one cord.

Table 1 displays measurements for each hanging mass that was attached to the bottom of the bungee system of one cord. We calculated force exerted on the bungee system by using Equation 2 (F=ma), where m is the mass of the hanging weight and a is the acceleration due to gravity (9.81 m/s^2). The displacement of the bungee system was measured for each hanging weight as described in the methods section, and the displacement per unit length was calculated by dividing the displacement of the bungee system by the length of the bungee system.

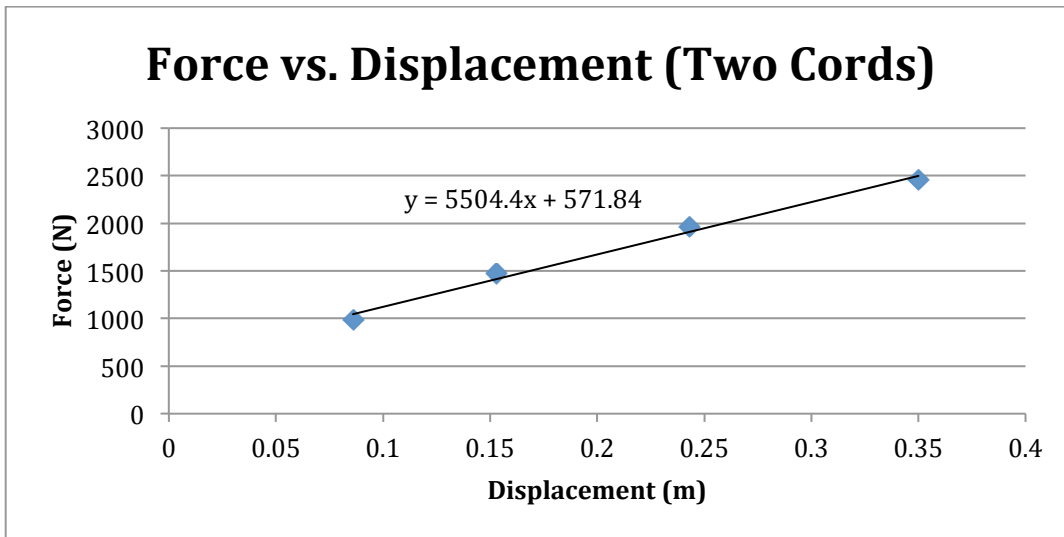**Force vs. Displacement (One Cord)**

$y = 2008x + 410.55$

Graph 1: Force vs. Displacement for A Bungee System of One Cord. This graph shows the relationship between the force placed on the system and the resulting displacement of the bungee cord.

Graph 1 displays the linear correlation between the force exerted on the bungee system and the displacement of the system, using the data from Table 1. The variables of force and displacement are directly proportionate, with the slope of the linear regression (2008 N/m) proportionate to the spring constant for this bungee system, with an uncertainty of ± 60 N/m.

Mass of hanging weight (g)	Force exerted on the Bungee system (N)	Displacement (m)	Displacement per unit length
100	981	0.09	0.24
150	1472	0.15	0.43
200	1962	0.24	0.68
250	2453	0.35	0.98

Table 2: Measurements for a System of Two Bungee Cords. This table contains the data collected from executing the experiment with a bungee system of two cords.

Table 2 displays measurements for each hanging mass that was attached to the bottom of the bungee system of two cords. We calculated force, displacement of the bungee system, and displacement per unit length of the system in the same manner that was described for the bungee system of one cord.
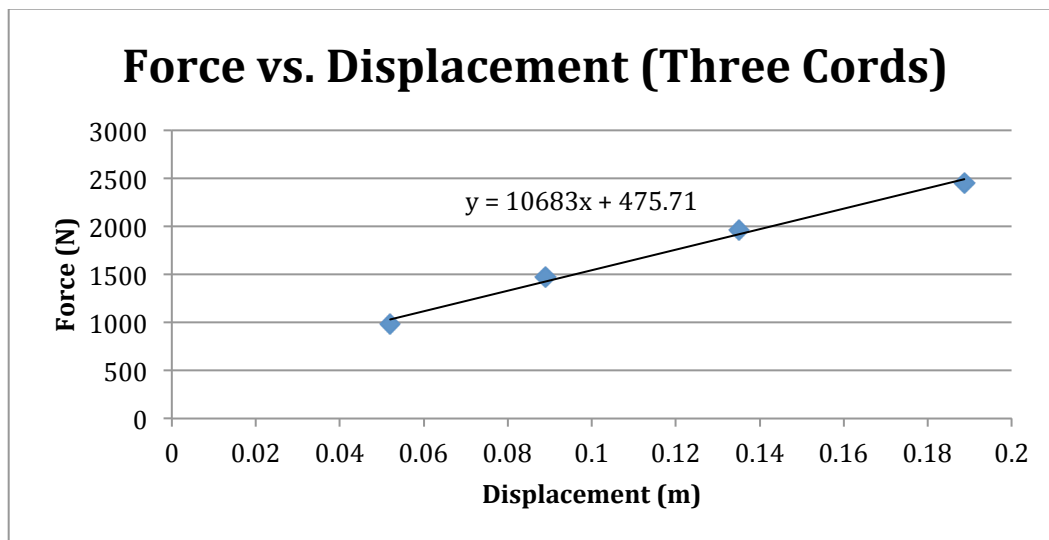

Graph 2: Force vs. Displacement for A Bungee System of Two Cords. This graph shows the relationship between the force placed on the system and the resulting displacement of the bungee cords.

Graph 2 reveals the linear relationship between the force exerted on a bungee system of two cords and the displacement of this system. Force and displacement were calculated in the same manner as they were for the system of one bungee cord. The variables of force and displacement are directly proportionate, with the slope of the linear regression (5504.4 N/m) proportionate to the spring constant, with an uncertainty of ± 400 N/m.

| Mass of hanging weight (g)	Force exerted on the Bungee system (N)	Displacement (m)	Displacement per unit length
100	981	0.05	0.14
150	1472	0.09	0.23
200	1962	0.14	0.35
250	2453	0.19	0.49

Table 3: Measurements for a System of Three Bungee Cords. This table contains the data collected from executing the experiment with a bungee system of three cords.

Table 3 displays measurements for each hanging mass that was attached to the bottom of the bungee system of three cords. We calculated force, displacement of the bungee system, and displacement per unit length of the system in the same manner that was described for the bungee system of one cord.

Force vs. Displacement (Three Cords)

y = 10683x + 475.71

Graph 3: Force vs. Displacement for A System of Three Cords. This graph shows the relationship between the force placed on the system and the resulting displacement of the bungee cord.

Graph 3 reveals the linear relationship between the force exerted on a bungee system of three cords and the displacement of this system. Force and displacement were calculated in the same manner as they were for the bungee system of one cord. The variables force and displacement are directly proportionate, with the slope of the linear regression (10683 N/m) proportionate to the spring constant, with an uncertainty of ± 600 N/m.

Force vs. Displacement per Unit Length
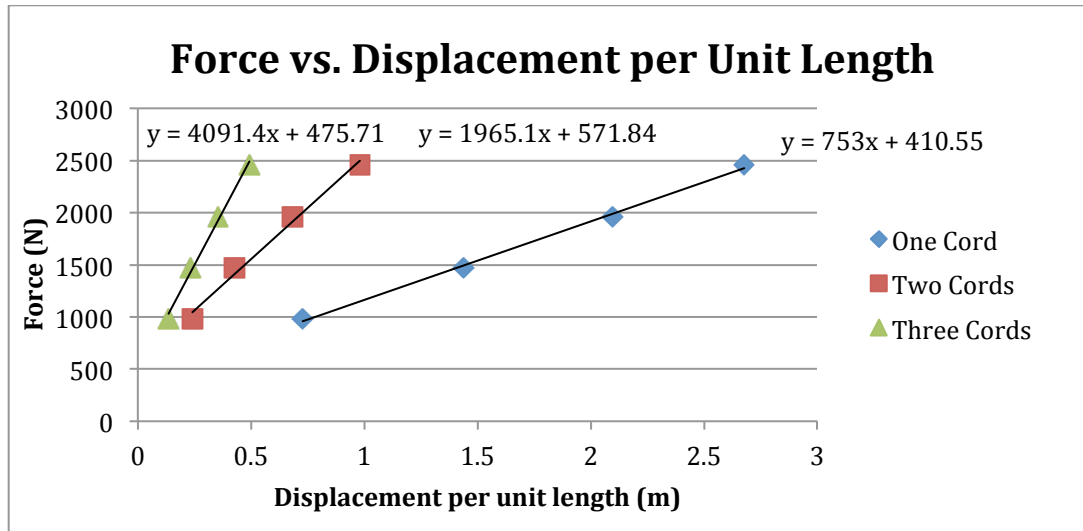
Graph 4: Force vs. Displacement per Unit Length. This graph shows the relationship between the force exerted on the system and the displacement of the bungee cord, normalized for the length of the cord.

Graph 4 demonstrates that as the bungee system increases in number of cords, the k value (proportionate to the slope of the linear trends in the graph) increases as well. For these linear trends, we normalized the displacement of the bungee cord system by dividing the measured displacement by the length of the bungee cord system from knot to knot. The variables of Force and displacement are directly proportionate through Hooke's Law (Equation 1), and the slope of the linear regression is proportionate to the k value for each system, with uncertainties of ± 20 N/m for one cord, ± 100 N/m for two cords, and ± 200 N/m for three cords.

Discussion

Assuming that the bungee system follows Hooke's law, our results indicate that there is something affecting the spring constant, k, other than the number of cords in the system. While graphing a plot of relevant forces exerted on the system versus the displacement of the bungee cords results in a linear regression, the linear equation is not equivalent to the expected form of F=kx (Equation 1). The y-intercept value is not zero, indicating that there are possibly other elements to consider when evaluating the spring constant of the bungee cord system. This is why the slopes of the linear regressions are proportionate to the spring constant, but not equal to the spring constant of the system. It is possible that the elasticity of

the bungee cords (ability to snap back to their original length) and the length of the bungee cords should have been considered in our model.

Based on Hooke's law, we would expect the spring constant for the system of one bungee cord to be half that of the system with two bungee cords and one-third that of the system with three bungee cords (ratio of 1:2:3). When evaluating the slope of the linear regressions of force vs. displacement for each bungee system, however, it becomes apparent that this is not the case. The ratio for these values proportionate to k is about 1:3:5. Based on these findings, we can conclude that we have not kept constant all of the factors affecting k.

This experiment was conducted under the assumption that the spring constant was independent of the length of the bungee system, but it appears that this assumption may be incorrect. In order to account for this variation as best as we could, we normalized the displacement of the bungee system by diving the calculated displacement by the length of the bungee system. These calculations are displayed in Graph 4, which shows new linear regressions relating force and normalized displacement for each bungee system. The ratio for these coefficients that are proportionate to k (1:2.5:5) still demonstrate that the execution of our system was not adequate to evaluate the true relationship between the number of cords in the system and the spring constant of the system.

Apart from conceptual errors that were not considered during the experiment, there is also a certain amount of error intrinsic to the system itself. The loops at the top and bottom of the bungee system, although not considered during the measuring of displacement, could have affected the bungee system by altering the stretch of the middle portion of the bungee cords. Additionally, as more force was exerted on the system, the knots at the top and bottom of the cord may have tightened and the bungee cords may have lost some of their ability to "snap back" to the original length, meaning the displacement of the cord for each force did not accurately reflect the stiffness of the bungee cord system. This possible error is reflected in the increasing percent uncertainty from a system of one bungee cord to systems that use more bungee cords (3.1% for one cord, 7.2% for two cords, and 5.8% for three cords). Since we experimented with the one-cord system, then the two-cord system, and then the three-cord system, our bungee cord may have been gradually loosing its stretch characteristic. Additionally, the looped knot of the multiple bungee cord system may have affected the stretch and displacement of the system for increasing forces by putting more tension on one cord than another or by inhibiting the stretch of the cords due to the large knots at the top and bottom of the system. In order to minimize these sources of error, we would need to reevaluate the method in which we created the systems with multiple cords, and also possibly reduce the forces exerted on the system so as to prevent the cord from over stretching.

Conclusions

Through this experiment, we found that as the number of cords in a bungee system increases, so does the k value of the system. The results reveal that more force must be exerted on a system of multiple bungee cords than on a system of a

single bungee cord in order to achieve the same displacement of the system. Still, based on the difference between our calculated linear regression modeling spring constant of the bungee cords and the ideal model of Hooke's Law, we can conclude that our system did not take into account all of the factors affecting the stiffness of the bungee cord system. The failure to keep length constant influenced the spring constant that was calculated based on the relationship between force exerted on the system and the displacement of the system. Additionally, the variation that results from knotting the bungee cord and reusing the same bungee cord for multiple trials led to high percent errors in the calculations of each bungee cord system's spring constant. Keeping the length of the system constant and only altering the number of cords in the system would improve the experiment and lead to a better quantification of the relationship between number of cords and spring constant. To further the investigation of the spring constant of a bungee cord system, an experiment could be conducted to assess the effects of bungee cord length on spring constant, k.