Modeling the Motion of a Ball with Spring and Damper

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In the document, it will be present the mathematical development that was used for the work of analyzing the mathematical models.

Definitions and Initial Equations

Equation of motion of the ball:

$$m_b\ddot{y}_b + c_b(\dot{y}_b - \dot{y}_j) + k_b(y_b - y_j) = 0 \tag{1}$$

Where:

- m_b = mass of the ball
- y_b = vertical position of the ball
- y_j = vertical position of the string
- $\dot{y}_b$ = velocity of the ball
- $\dot{y}_j$ = velocity of the string
- $\ddot{y}_b$ = acceleration of the ball
- c_b = damping constant
- k_b = spring constant

Fundamental Laws

Newton's Second Law:

$$F = ma \tag{2}$$

Hooke's Law:

$$F_{\text{spring}} = -k\Delta x \tag{3}$$

Where Δx is replaced with:

$$\Delta y = y_b - y_j$$

Therefore:

$$F_{\text{spring}} = -k_b(y_b - y_j) \tag{3'}$$

Newton's Law of Viscosity

$$\tau = \mu \frac{du}{dy} \tag{5}$$

Shear Stress Definition

$$\tau = \frac{F}{A} \tag{6}$$

Equating equations (5) and (6):

$$\frac{F}{A} = \mu \frac{du}{dy} \Rightarrow F = \mu A \frac{du}{dy}$$

Approximation

Assume:

$$\frac{du}{dy} \approx \frac{\dot{y}_b - \dot{y}_j}{L} \tag{7}$$

Substitute into the previous equation:

$$F = \mu A \left(\frac{\dot{y}_b - \dot{y}_j}{L} \right)$$

Define the constant:

$$c = \frac{\mu A}{L}$$

And substitute:

$$F = c(\dot{y}_b - \dot{y}_j) \tag{1}$$

Linear Viscous Damper

Damping force:

$$F_{\text{damp}} = -c_b(\dot{y}_b - \dot{y}_j)$$

Total Forces

$$F_T = F_{\text{spring}} + F_{\text{damp}} \tag{4}$$

$$F_T = -k_b(y_b - y_j) - c_b(\dot{y}_b - \dot{y}_j)$$

Using equation (2):

$$m_b\ddot{y}_b = -k_b(y_b - y_j) - c_b(\dot{y}_b - \dot{y}_j)$$

Bringing everything to one side:

$$m_b\ddot{y}_b + c_b(\dot{y}_b - \dot{y}_j) + k_b(y_b - y_j) = 0 \tag{1, rewritten}$$