Problem Set 6 - QM

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1 Initial values and exogenous process.

1.1 Initial guess.

We have to come up with initial guess for different value functions of the type $v(k, \epsilon; \bar{k}, z_s)$ for $\epsilon = \{0, 1\}$ and $z = \{good, bad\}$ assuming that:

- Agents expect that the individual and aggregate states $(\bar{k}, \epsilon, z)$ will not change in the future.
- Agents' policy is such that $g(k, \epsilon; \bar{k}, z) =$ k

The value function boils down to the following expression:

$$v(k, \epsilon; \bar{k}, z_s) = \max_{\{c, k'\}} \frac{c^{1-\gamma} - 1}{1-\gamma} + \beta \mathbb{E}\big[v(k', \epsilon'; \bar{k}', z_s')\big] \tag{1}$$

where

$$c = w(\bar{k}, z_s)\epsilon + (1 + r(\bar{k}, z_s) - \delta)k - g(\bar{k}, z_s) \tag{2}$$

The technology is given by a Cobb-Douglas aggregate production function:

$$y = z\bar{k}^{\alpha} l^{(1-\alpha)} \tag{3}$$

Then, from (3) we can recover factor prices:

$$w(\bar{k}, z_s) = (1-\alpha) z_s \big(\frac{\bar{k}}{l}\big)^{\alpha} \tag{4}$$

$$r(\bar{k}, z_s) = \alpha z_s \big(\frac{l}{\bar{k}}\big)^{1-\alpha} \tag{5}$$

Now, plug (4) and (5) in (2) and (2) in (1):

$$v(k, \epsilon; \bar{k}, z_s) = \max_{\{k'\}} \frac{\Big[\big((1-\alpha) z_s \big(\frac{\bar{k}}{l}\big)^{\alpha}\big)\epsilon + \big(1 + \alpha z_s \big(\frac{l}{\bar{k}}\big)^{1-\alpha} - \delta\big)k - g(\bar{k}, z_s)\Big]^{1-\gamma} - 1}{1-\gamma} + \beta \mathbb{E}\big[v(k', \epsilon'; \bar{k}', z_s')\big] \tag{6}$$

What's next? Given our assumptions:

- $g(.) = k$.
- $\mathbb{E}\big[v(k', \epsilon'; \bar{k}', z_s')\big] = v(k, \epsilon; \bar{k}, z_s)$

(6) reduces to:

$$v(k, \epsilon; \bar{k}, z_s) = \frac{\Big[\big((1-\alpha) z_s \big(\frac{\bar{k}}{l}\big)^{\alpha}\big)\epsilon + \big(\alpha z_s \big(\frac{l}{\bar{k}}\big)^{1-\alpha} - \delta\big)k\Big]^{1-\gamma} - 1}{(1-\gamma)(1-\beta)} \tag{7}$$

From (7) we can derive expressions for the value functions depending on both the idiosyncratic and aggregate shock:

$$v(k, 1; \bar{k}, z_g) = \frac{\Big[\big((1-\alpha) z_g \big(\frac{\bar{k}}{1-u_g}\big)^{\alpha}\big) + \big(\alpha z_g \big(\frac{1-u_g}{\bar{k}}\big)^{1-\alpha} - \delta\big)k\Big]^{1-\gamma} - 1}{(1-\gamma)(1-\beta)} \tag{8}$$

$$v(k, 0; \bar{k}, z_g) = \frac{\Big[\big(\alpha z_g \big(\frac{1-u_b}{\bar{k}}\big)^{1-\alpha} - \delta\big)k\Big]^{1-\gamma} - 1}{(1-\gamma)(1-\beta)} \tag{9}$$

$$v(k, 1; \bar{k}, z_b) = \frac{\Big[\big((1-\alpha) z_b \big(\frac{\bar{k}}{1-u_g}\big)^{\alpha}\big) + \big(\alpha z_b \big(\frac{1-u_g}{\bar{k}}\big)^{1-\alpha} - \delta\big)k\Big]^{1-\gamma} - 1}{(1-\gamma)(1-\beta)} \tag{10}$$

$$v(k, 0; \bar{k}, z_b) = \frac{\Big[\big(\alpha z_b \big(\frac{1-u_b}{\bar{k}}\big)^{1-\alpha} - \delta\big)k\Big]^{1-\gamma} - 1}{(1-\gamma)(1-\beta)} \tag{11}$$

1.2 Transition matrix

We have two idiosyncratic shocks $\epsilon = \{0, 1\}$ and two aggregate shocks $z = \{bad, good\}$. Thus, the economy can be shocked by 4 differents types of shocks $\{0b, 1b, 0g, 1g\}$. As a result, we need to build a 4x4 transition matrix. For doing so, we have used a system of 16 equations, coming from rows properties (equal to 1), the information about the duration of good and bad times, the duration of unemployment and so on. See the Python code for the details. In the end, we got the following transition matrix. Note that first row/column is for 0b, second for 1b, third 0g and fourth 1g. Lastly, notice that the higher probabilities are in the diagonal, meaning that there is a high degree of persistence:

$$\Pi = \begin{bmatrix} 0.6950 & 0.1800 & 0.0809 & 0.0441 \\ 0.0200 & 0.8550 & 0.0049 & 0.1200 \\ 0.1241 & 0.0009 & 0.7550 & 0.1200 \\ 0.0078 & 0.1171 & 0.0050 & 0.8700 \end{bmatrix}$$

2 Workers problem and simulation

2.1 Solution of the model by VFI.

We have tried to solve it with Python but we got flat policy functions. It seems that the budget constraint is not working properly, but we have not figured it out why yet. As an alternative, we have built a Matlab code (based on the sample code provided by the professor). By doing so, we have got the following policy functions.

Figure 1: Assets policy function for a good productivity shock.

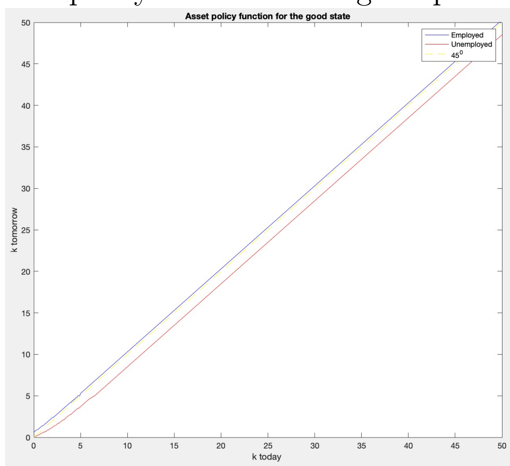
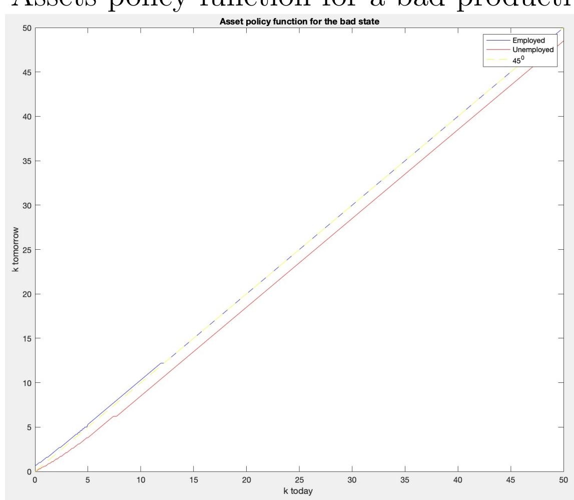


Figure 2: Assets policy function for a bad productivity shock.



When the economy is in good times, the employed agent is always accumulating assets (saving) and the unemployed guy is depleting assets. This agrees with the common sense: you save when employed and dis-save (or borrow) when unemployed. When bad times come, at some point the employed guy stops saving and just keep balance her budget. On the contrary, if a guy is shocked enough times with the unemployment, she will ends up having no assets (poverty trap), and will be borrowed constraint.

2.2 Simulation for 1000 agents and 2000 periods.

See the Matlab code.

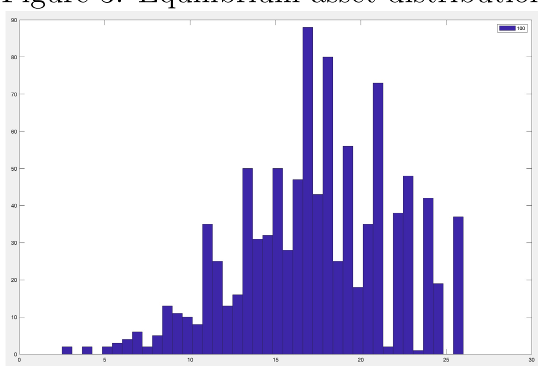
3 Solution of the model

3.1 Parameter values and distribution.

For the procedure to get convergence in β's see the Matlab code. Basically, we estimate the parametrized expectations parameters by using the time series for K and z got in the previous exercise; then we update the *beta* in the function H; simulate the model again; get time series for K and z; estimate β; keep doing that until the new β were almost equal to the previous one (convergence). The goodness of the estimation is high (R^2 higher than 0.9 for both states).

Now, let's take a look at the equilibrium asset distribution. Figure 3 plot it. The average capital, which in this setup is equal to the aggregate capital is 18.28. The standard deviation is 4.75, and the skewness is 0.01. It means that the inequality is quite high and that there are more people owning a high level of assets than poors.

Figure 3: Equilibrium asset distribution.



Now, we have run the experiment of comparing the evolution of the stationary distribution after 7 consecutive periods of being in a bad state (graph 4) and 7 consecutive periods of being in a good one (graph 5). For bad periods, the sd is 5.93 and the skewness is 1.45. For good periods, the sd is 5.09 and the skewness is 0.09. Then, for bad times the inequality is higher than for good, and also there are more people having more assets.

Figure 4: Equilibrium asset distribution after 7 bad periods.

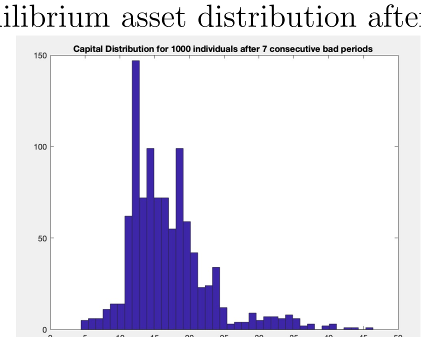
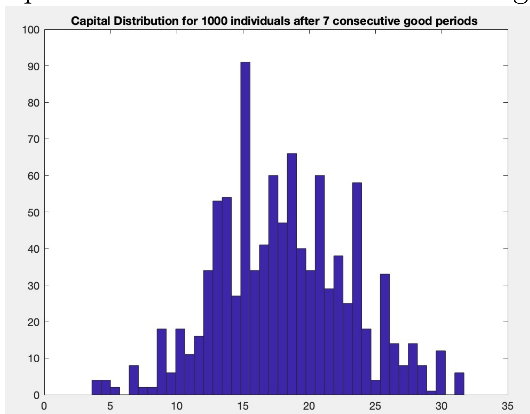


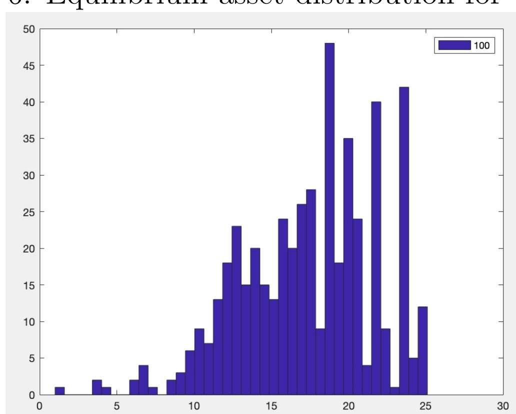
Figure 5: Equilibrium asset distribution after 7 good periods.



3.2 Heterogeneous expectations.

See the Matlab code. We took the assets distribution as an aggregate approximation of the welfare. Thus, figure 6 pictures the distribution for updaters and figure 7 for non-updaters.

Figure 6: Equilibrium asset distribution for updaters



The main differences are that (1) there are more poors between the non-updaters and (2) the average level of wealth is higher for updaters. Thus, the social welfare is going to be lower. Then there are no incentives for a non-updating behavior.

Figure 7: Equilibrium asset distribution for non-updaters