Burns-Mitchell Framework:

1.1 Quarterly GDP growth rates:

$$Y_t = \log(GDP_t) - \log(GDP_{t-1})$$

$$Y_t = \log\left(\frac{GDP_t}{GDP_{t-1}}\right)$$

$$Y_t = \log\left(1 + \frac{GDP_t}{GDP_{t-1}} - 1\right)$$

$$Y_t = \log\left(1 + \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}\right)$$

$$\Rightarrow Y_t \approx \frac{GDP_t - GDP_{t-1}}{GDP_{t-1}}$$

1.2 Defining a Peak through the growth-rates:

$$p \in \{0, 1, \dots, m\} \quad ; \quad \{Y_p\}_{p=0}^{m}$$

$$P = \{p \in \{1, \dots, m-2\} \mid Y_{p-1} > 0, Y_p > 0, Y_{p+1} < 0, Y_{p+2} < 0\}$$

$$I_p = \{Y_{p+s} \mid s \in \{-10, \dots, 10\}, 0 \leq p+s \leq m\}, \; p \in P$$

$$\overline{Y}_s = \frac{1}{|P_s = \{p \in P \mid 0 \leq p+s \leq m\}|} \sum_{p \in P_s} Y_{p+s} \qquad , s \in \{-10, \dots, 10\}$$

$$\overline{\mu} = \{\overline{Y}_s \mid s \in \{-10, \dots, 10\}\} \in \mathbb{R}^{21}$$

$$\overline{Y} = \frac{1}{|P_s = \{p \in P \mid 0 \leq p+s \leq m\}|} \cdot \frac{1}{|s|} \sum_{p \in P} \sum_{s=-10}^{10} Y_{p+s} \in \mathbb{R}^1$$

$$\Delta_s = \overline{Y}_s - \overline{Y} \quad , s \in \{-10, \dots, 10\}$$

$$\Delta = \{\Delta_s \mid s \in \{-10, \dots, 10\}\}$$

1.3 Normal and Lévy Fitting on the GDP growth rates:

$$Y_t \sim N(\mu, \sigma^2) \;\wedge\; Y_t \sim L(\alpha, \beta, \mu, \gamma)$$

$$f_N(x \mid \mu, \sigma) = \frac{1}{\sqrt{2\pi} \cdot \sigma} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

$$\varphi(q) = \begin{cases} \exp\left(i\mu q - \gamma |q|^\alpha \left[1 - i\beta \frac{q}{|q|} \tan\left(\frac{\pi}{2}\alpha\right)\right]\right) & , \alpha \neq 1 \\ \exp\left(i\mu q - \gamma |q|^\alpha \left[1 + i\beta \frac{q}{|q|} \frac{2}{\pi} \ln(|q|)\right]\right) & , \alpha = 1 \end{cases}$$

$$\mathcal{L}(\theta) = \prod_{t=1}^{n} f(Y_t \mid \theta)$$

$$\ell(\theta) = \sum_{t=1}^{n} \log(f(Y_t \mid \theta))$$

$$\Rightarrow \hat{\theta}_{MLE} = \arg\max_{\theta} \ell(\theta)$$

1.4 Assessing statistical validity / significance of $Y_p - \bar{Y}$, $\forall p \in P$ with Bootstrapping:

$$\delta_p = \bar{Y}_p - \bar{Y}, \forall p \in P$$

$$\delta_p \in \mathbb{R}^{|P_s = \{p \in P \mid 0 \leq p+s \leq n\}|}$$

$$\bar{Y} = \frac{1}{|P_s = \{p \in P \mid 0 \leq p+s \leq n\}|} \cdot \frac{1}{|s|} \sum_{p \in P} \sum_{s=-10}^{10} Y_{p+s} \in \mathbb{R}^1$$

$$\bar{Y}_p = \frac{1}{|P_s = \{p \in P \mid 0 \leq p+s \leq n\}|} \cdot \sum_{p \in P} (Y_p - \bar{Y})$$

$$b = \{1, \dots, |P_s| \exp(|P_s|)\}$$

$$\hat{\theta}_b^* = \frac{1}{|P_s = \{p \in P \mid 0 \leq p+s \leq n\}|} \cdot \sum_{p \in P}^{|P_s|} \delta_{p^*}$$