The Targeted Depletion Benchmark (TDB): A Constraint-Optimization Model for Labor Cessation

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Abstract

Traditional personal finance literature predominantly focuses on capital preservation models, such as the Safe Withdrawal Rate, to mitigate longevity risk. While financially robust, these models often result in significant "over-saving," yielding high terminal wealth values that represent unconsumed labor utility. This paper introduces the **Targeted Depletion Benchmark (TDB)**, a framework that synthesizes the Life-Cycle Hypothesis with "Coast" accumulation mechanics. We define a mathematical formula to identify the minimum critical capital threshold required to cease savings contributions early. By relying on compound growth to fund a risk-adjusted depletion strategy, the TDB model demonstrates that rational agents can achieve financial security significantly earlier than traditional heuristics suggest.

1 Introduction

The prevailing dogma in personal finance suggests that individuals should accumulate a net worth sufficient to support a withdrawal rate (typically 4%) indefinitely. While this "Endowment Model" minimizes the risk of ruin (Bengen, 1994), it maximizes the "inefficiency of labor." From the perspective of the Life-Cycle Hypothesis (Ando and Modigliani, 1963), the goal of a rational agent is to smooth consumption over a lifetime rather than to maximize wealth at the time of death.

However, recent empirical evidence suggests a disconnect between this theoretical smoothing and actual behavior. Olafsson and Pagel (2018) document a "retirement-consumption puzzle" where individuals not only drop consumption upon retirement but also increase liquid savings and deleverage, contradicting the standard smoothing model. Similarly, Battistin et al. (2009) find a significant drop in consumption expenditures at retirement that cannot be fully explained by work-related expenses. These findings suggest that agents systematically over-accumulate or under-consume due to an inability to solve the complex optimization problem of depletion.

This paper proposes a formal mathematical framework to address this inefficiency: the **Targeted Depletion Benchmark (TDB)**. The TDB identifies the specific point at which an agent has accumulated enough capital to stop contributing to savings, relying instead on the "coasting" effect of compound interest to reach a target that is designed to deplete exactly at the end of life.

2 Literature Review and Theoretical Basis

2.1 The Life-Cycle and Uncertain Lifetimes

The theoretical foundation of consumption smoothing rests on the Life-Cycle Hypothesis, which posits that individuals plan their consumption and savings behavior over their long-term life cycle (Ando and Modigliani, 1963). Yaari (1965) extended this to uncertain lifetimes, arguing that if an agent has no bequest motive, the optimal strategy involves annuitization to manage longevity risk. Leung (1994) further characterizes this, noting that savings should theoretically be depleted before the maximum possible lifetime if utility is to be maximized.

2.2 The Problem of Over-Accumulation

Despite these theoretical underpinnings, traditional advice often relies on heuristic rules derived from historical worst-case scenarios, such as the 4 percent rule proposed by Bengen (1994). While Clare et al. (2020) highlight the critical importance of sequence of returns risk in determining these withdrawal rates, the focus remains on portfolio survival rather than utility maximization. This defensive posture contributes to the empirical phenomena observed by Olafsson and Pagel (2018), where retirees continue to accumulate wealth rather than decumulating it, representing a sub-optimal use of life energy and labor. Hakansson (1969) provides a framework for optimal investment under risk, but the practical application for the average saver remains computationally prohibitive without a simplified benchmark like the TDB.

3 Methodology and Model Formulation

To derive the TDB, we distinguish between three phases: the *Accumulation Phase* (working and saving), the *Coasting Phase* (working without saving), and the *Decumulation Phase* (retirement).

3.1 Variable Definitions

We define the following scalar variables for the model. For simplicity, the model assumes all values are in real terms (inflation-adjusted) and withdrawals occur at the end of each period.

- t_{now}: Current Age.
- t_{ret}: Target Retirement Age.
- t_{end}: Target Depletion Age (Estimated lifespan).
- C: Annual Consumption Requirement.
- r_{acc}: Real Rate of Return during Accumulation/Coasting.
- r_{dec}: Real Rate of Return during Decumulation (Conservative).
- λ: Risk Coefficient (Safety Buffer, e.g., 1.1).
- P_{ins}: Longevity Insurance Premium (Cost of Deferred Income Annuity).

3.2 Step 1: Solving for Terminal Capital (W_{ret})

We first calculate the capital required at age t_{ret} to fund consumption strictly until t_{end}. We use the formula for the Present Value of an Annuity, adjusted for risk:

$$W_{ret} = \left[C \cdot \frac{1 - (1 + r_{dec})^{-(t_{end} - t_{ret})}}{r_{dec}} \right] \cdot \lambda + P_{ins} \tag{1}$$

Here, λ provides a buffer against sequence risk as highlighted by Clare et al. (2020), and P_{ins} represents the purchase of a Deferred Income Annuity that begins payments at t_{end}, aligning with Yaari (1965).

3.3 Step 2: Solving for the Benchmark (W_{TDB})

The TDB is the wealth required **today** (t_{now}) to guarantee W_{ret} is achieved without further contribution. We discount W_{ret} using the aggressive growth rate (r_{acc}), as the portfolio is untouched (coasting) during this period.

$$W_{TDB} = \frac{W_{ret}}{(1 + r_{acc})^{(t_{ret} - t_{now})}} \tag{2}$$

Once an individual's net worth $\geq W_{TDB}$, the efficiency condition is met. Future income should be directed toward current consumption rather than savings.

4 Simulation and Results

To illustrate the efficiency of the TDB, we simulate a standard agent scenario to test the hypothesis that "the first \$100k" is a sufficient stop-loss for savings.

4.1 Scenario Parameters

- **Profile:** Age 30 (t_{now}), Retiring at 65 (t_{ret}), Deceasing at 95 (t_{end}).
- **Financials:** Desired Income $C = \$40,000$.
- **Market Assumptions:** Coast Growth $r_{acc} = 7\%$, Retirement Growth $r_{dec} = 4\%$.
- **Risk Controls:** Buffer $\lambda = 1.10$ (10% over-funding), Insurance $P_{ins} = \$50,000$.

4.2 Calculation

First, we calculate the capital required at age 65 (W_{ret}). Using the annuity formula for 30 years (65 to 95) at 4%:

$$\text{Base Annuity} = 40,000 \times 17.292 \approx \$691,680$$

Applying the safety buffer and insurance cost:

$$W_{ret} = (691,680 \times 1.10) + 50,000 = \$\mathbf{810,848}$$

Next, we calculate the TDB at age 30 using Equation 2. The growth period is 35 years at 7%:

$$W_{TDB} = \frac{810,848}{(1.07)^{35}} = \frac{810,848}{10.677} \approx \$\mathbf{75,943}$$

4.3 Comparison to Traditional Advice

Under the Traditional Model (Equation 1), the target is $40,000/0.04 = \$1,000,000$. To reach this by age 65 (starting from 0 at age 30), the agent must save approximately **\$7,200 annually** for the entire 35-year duration.

Metric	Traditional Model	TDB Model
Target Wealth at 65	\$1,000,000	\$810,848
Required Savings at Age 30	**\$0 (Must save annually)**	**\$75,943 (Done)**
Terminal Wealth at Age 95	\$1,000,000	\$0

Table 1: Comparison of Capital Requirements

5 Discussion: The Life Energy Metric

The disparity between the models can be quantified in "Life Energy," defined here as years of labor required to fund the strategy.

If the agent in our simulation is capable of saving \$15,000 per year:

1. **Traditional Path:** They must save for roughly **22 years** to be on track for \$1M.
2. **TDB Path:** They must save for only **4.5 years** to reach the \$75,943 benchmark.

The TDB model saves the agent **17.5 years** of savings-burdened labor. Once the \$75,943 benchmark is reached, the agent enters the "Coast" phase. During this phase, 100% of their income can be utilized for current utility. This significantly improves lifestyle during their prime health years (Age 35 to 65).

6 Conclusion

This paper formalizes the Targeted Depletion Benchmark. We demonstrated that by accepting a terminal wealth of zero (mitigated by longevity insurance) and leveraging the coasting phase of compound interest, individuals can achieve financial security with a fraction of the capital required by traditional models.

The widespread intuition that "the first $100k is the hardest" is mathematically validated here. For many agents, the first $100k is not just the hardest step, but potentially the *only* mandatory accumulation step required to achieve a baseline of financial freedom.

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