Finger formation at the base of ash clouds: A linear stability analysis

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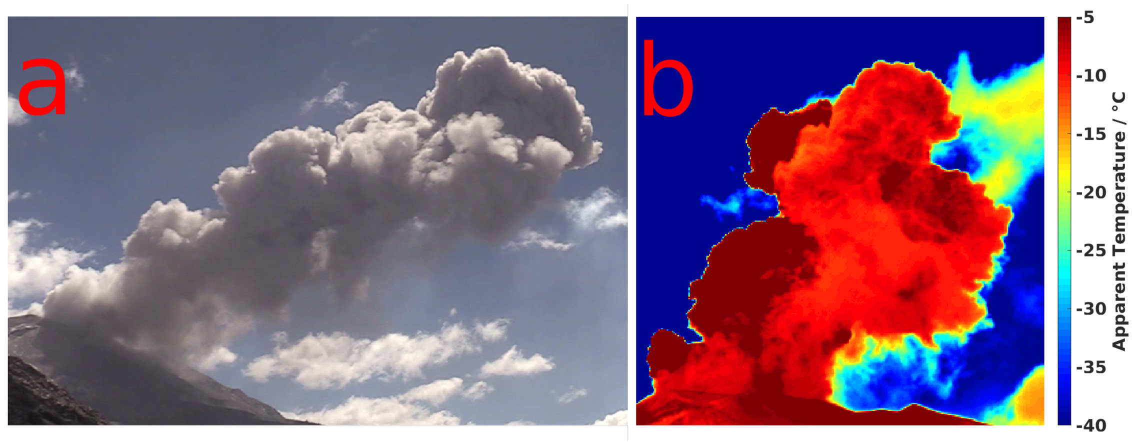

Figure 1. a) Visible and b) infrared images of ash clouds from Vulcanian explosions of Sabancaya volcano, Peru. Fingers can be seen in both images.

Introduction

Volcanic ash presents a hazard for infrastructure and human health. Understanding ash settling is therefore crucial for assessing the associated risk. Downard-propagating fingers(Figure 1), hypothesised to form from a gravitational instability at the base of the ash cloud (Figure 2) have been seen at many volcanoes. Here we present a linear stability analysis, that predicits the initial growth rate of the fingers.

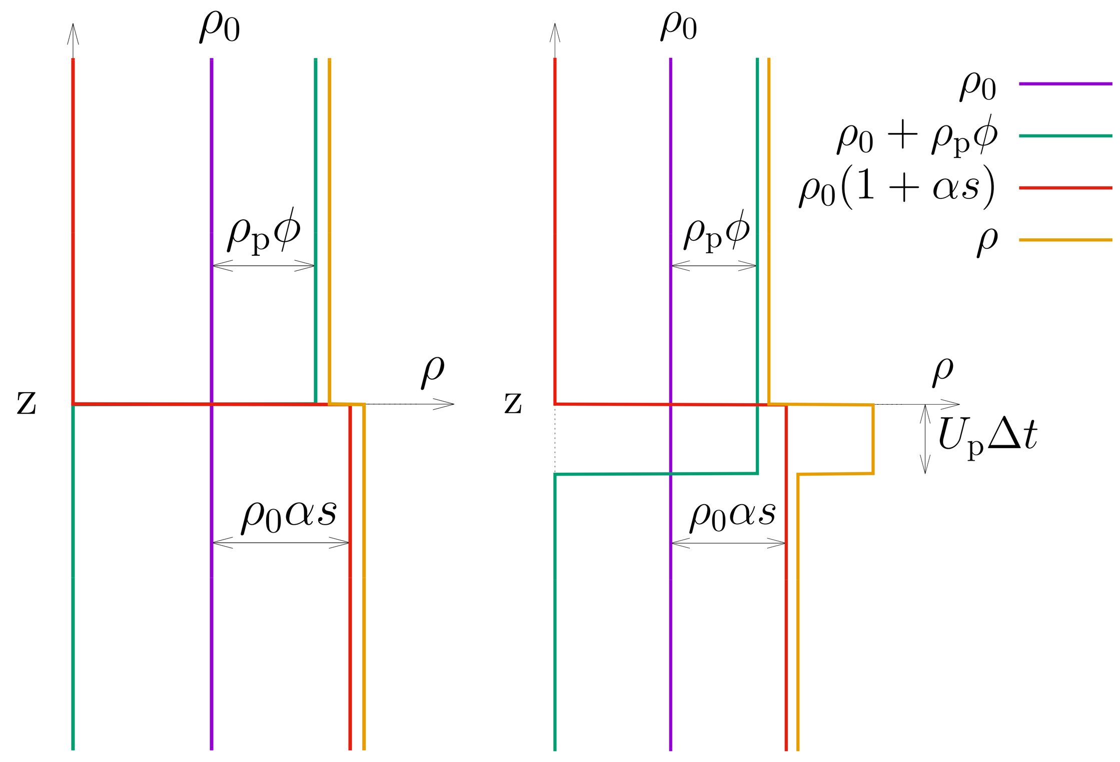

Figure 2. Despite an initial stable configuration, ash settling leads to the formation of a gravitationally unstable **particle boundary layer**.

Equations of motion

Conservation of mass

$$\nabla \cdot \mathbf{u}(\mathbf{x},t)$$

Conservation of momentum

$$\frac{\partial \mathbf{u}(\mathbf{x},t)}{\partial t} + [\mathbf{u}(\mathbf{x},t)] \cdot \nabla]\mathbf{u}(\mathbf{x},t) = \frac{\nabla P(\mathbf{x},t)}{\rho_0} + \nu \nabla^2 \mathbf{u}(\mathbf{x},t) - g'\hat{\mathbf{z}}$$

Conservation of particles

$$\frac{\partial \phi(\mathbf{x},t)}{\partial t} + [\mathbf{u}(\mathbf{x},t)] - \mathbf{U}_{\mathsf{p}}] \cdot \nabla \phi(\mathbf{x},t) = D_{\mathsf{p}} \nabla^2 \phi(\mathbf{x},t)$$

Conservation of solute

$$\frac{\partial s(\mathbf{x},t)}{\partial t} + \mathbf{u}(\mathbf{x},t)x \cdot \nabla s(\mathbf{x},t) = D_{\mathsf{s}} \nabla^2 s(\mathbf{x},t)$$

Reduced gravity

$$g' = g\left([1+\alpha s(\mathbf{x},t)][1-\phi(\mathbf{x},t)] + \frac{\rho_p \phi(\mathbf{x},t)}{\rho_0}\right)$$

Unknowns		Parameters	
$\mathbf{u}(\mathbf{x},t)$	Fluid velocity	ρ_0	Reference fluid density
$P(\mathbf{x},t)$	Pressure	ν	Kinematic viscosity
$\phi(\mathbf{x},t)$	Particle volume	$\mathbf{U}_{\mathsf{p}}$	Particle settling velocity
$s(\mathbf{x},t)$	Solute concentration	ρ_{p}	Particle density
Variables		D_{p}	Particle diffusivity
$\mathbf{x}$	Position vector	D_{s}	Solute diffusivity
t	time	g	Gravity
$\hat{\mathbf{z}}$	Vertical unit vector	α	Expansivity

Expansion: Base state + Perturbation

$$\mathbf{u}(\mathbf{x},t) = \mathbf{u}^{(0)}(\mathbf{x},t) + \hat{\mathbf{u}}(z)e^{i(k_x x + k_y y - \omega t)}, \quad \hat{\mathbf{u}} \ll \mathbf{u}^{(0)}$$

$$P(\mathbf{x},t) = P^{(0)}(\mathbf{x},t) + \hat{P}(z)e^{i(k_x x + k_y y - \omega t)}, \quad \hat{P} \ll P^{(0)}$$

$$\phi(\mathbf{x},t) = \phi^{(0)}(\mathbf{x},t) + \hat{\phi}(z)e^{i(k_x x + k_y y - \omega t)}, \quad \hat{\phi} \ll \phi^{(0)}$$

$$s(\mathbf{x},t) = s^{(0)}(\mathbf{x},t) + \hat{s}(z)e^{i(k_x x + k_y y - \omega t)}, \quad \hat{s} \ll s^{(0)}$$