

Low-Reynolds number settling of spheres through fluid interfaces

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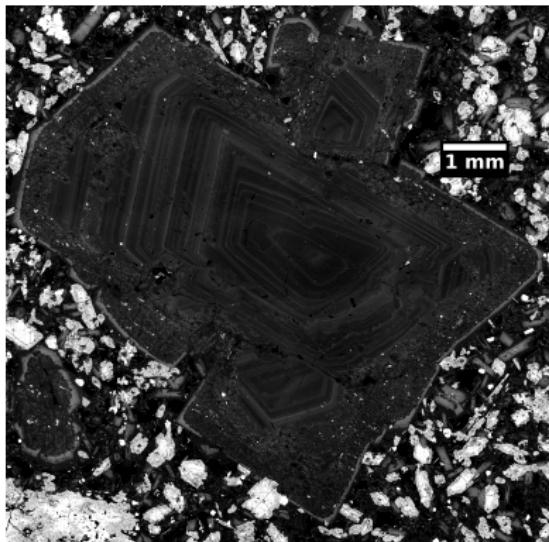
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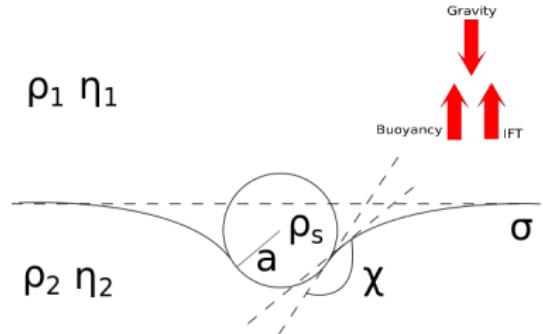
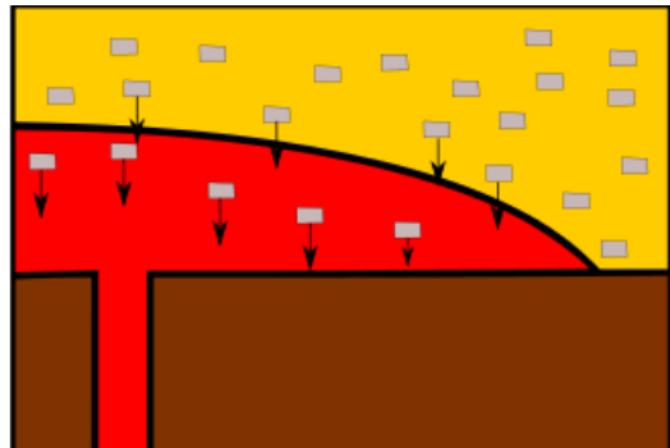
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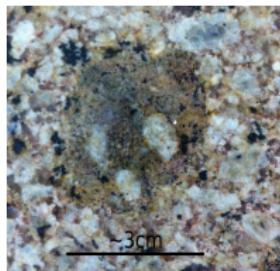
Motivation: Magmatic xenocrysts



Gravitational Settling



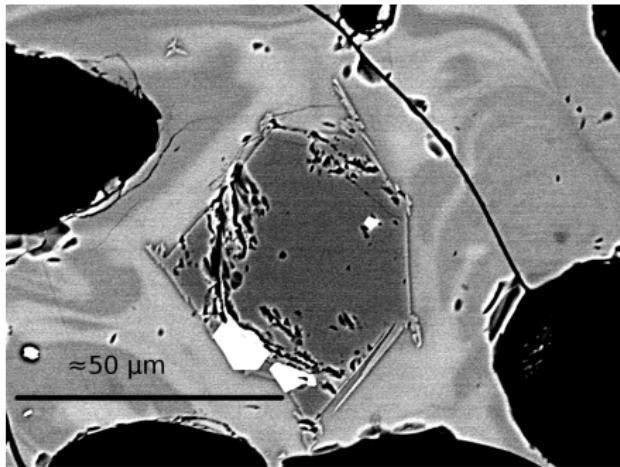
- $\rho_{1(2)}$ = Density of fluid 1(2)
- $\eta_{1(2)}$ = Viscosity of fluid 1(2)
- σ = Interfacial tension (IFT)
- a = Radius
- ρ_s = Particle density
- χ = Contact angle



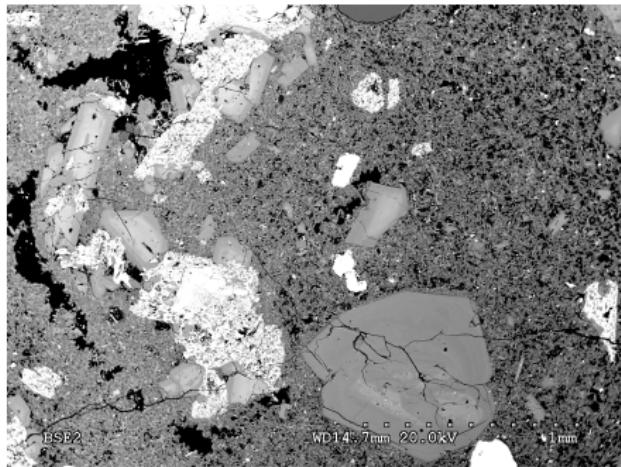
Credit Jon Blundy

Questions

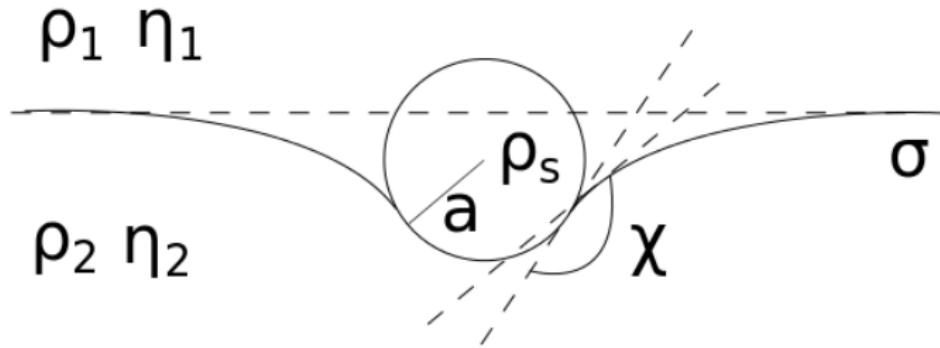
- For what conditions is sinking associated with entrainment of upper phase fluid?
Consequences for magma hybridisation
- What is the timescale of sinking?
Important if other timescales matter e.g. solidification



Credit: Geoff Kilgour



Dimensionless Parameters



Bond Number

Modified Density Ratio

Viscosity Ratio

$$Bo = \frac{(\rho_2 - \rho_1)ga^2}{\sigma}$$

$$D = \frac{\rho_s - \rho_1}{\rho_2 - \rho_1}$$

$$\lambda = \frac{\eta_2}{\eta_1}$$

Length- and time-scales

Capillary length

Multiple possible timescales e.g.

$$l_c = \left(\frac{\sigma}{(\rho_2 - \rho_1)g} \right)^{1/2}$$

$$t = \frac{\eta_n l_c}{\sigma}, \quad t = \frac{\eta_n}{(\rho_s - \rho_n)ga},$$

$$n = 1, 2$$

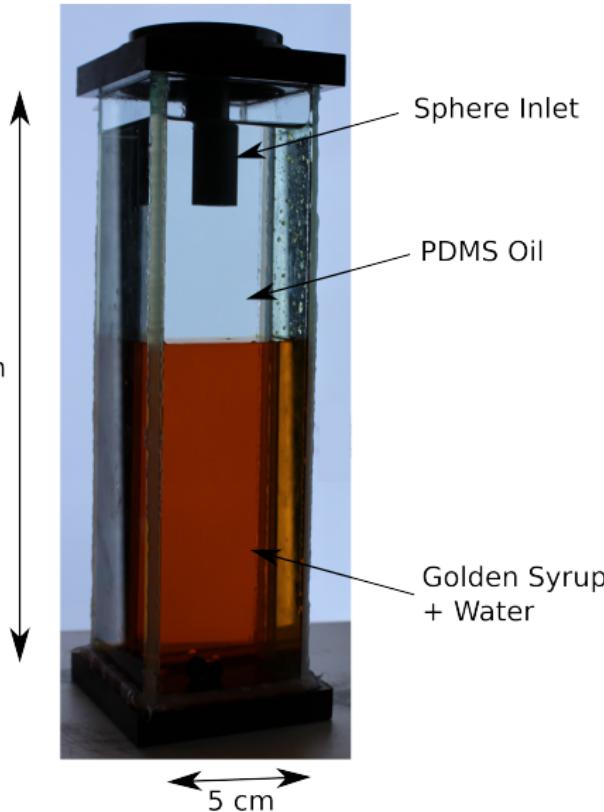
For $a < l_c$, $Bo < 1$, expect σ to control
the interfacial response

Choose to non-dimensionalise with

For $a < l_c$, $Bo > 1$, expect density
contrast to control the interfacial
response

$$t_i = \frac{\eta_1 a}{\sigma}$$

Experiments



Glass spheres of various radii 2-10 mm

Water content in syrup from 0-5%

Grade of PDMS oil

Temperature from 0-32°C

Can achieve ranges:

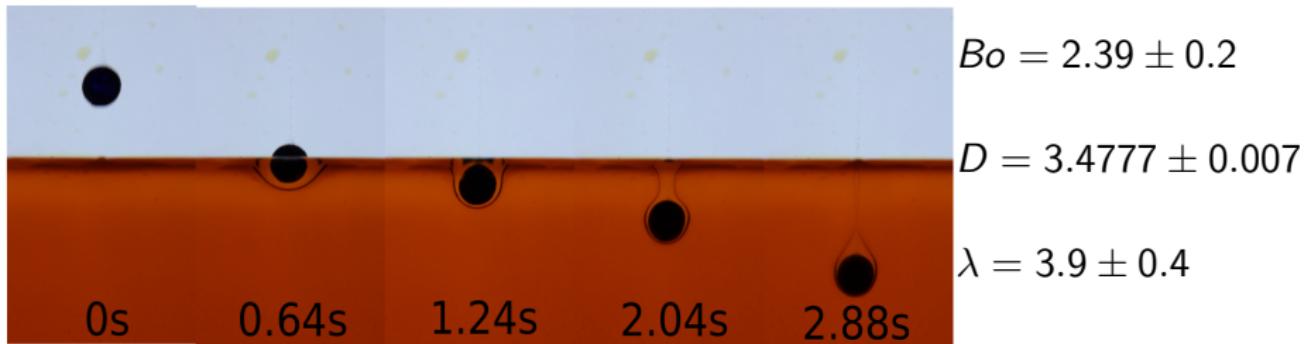
$$0.1 \leq Bo \leq 5$$

$$10^{-2} \leq \lambda \leq 10^3$$

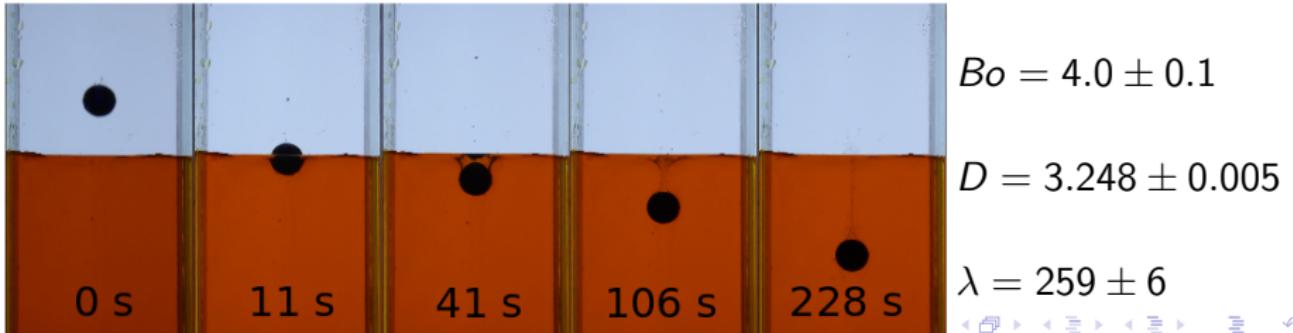
$$D \approx 3.3$$

Modes of Transfer

Tailing Mode



Film Drainage Mode

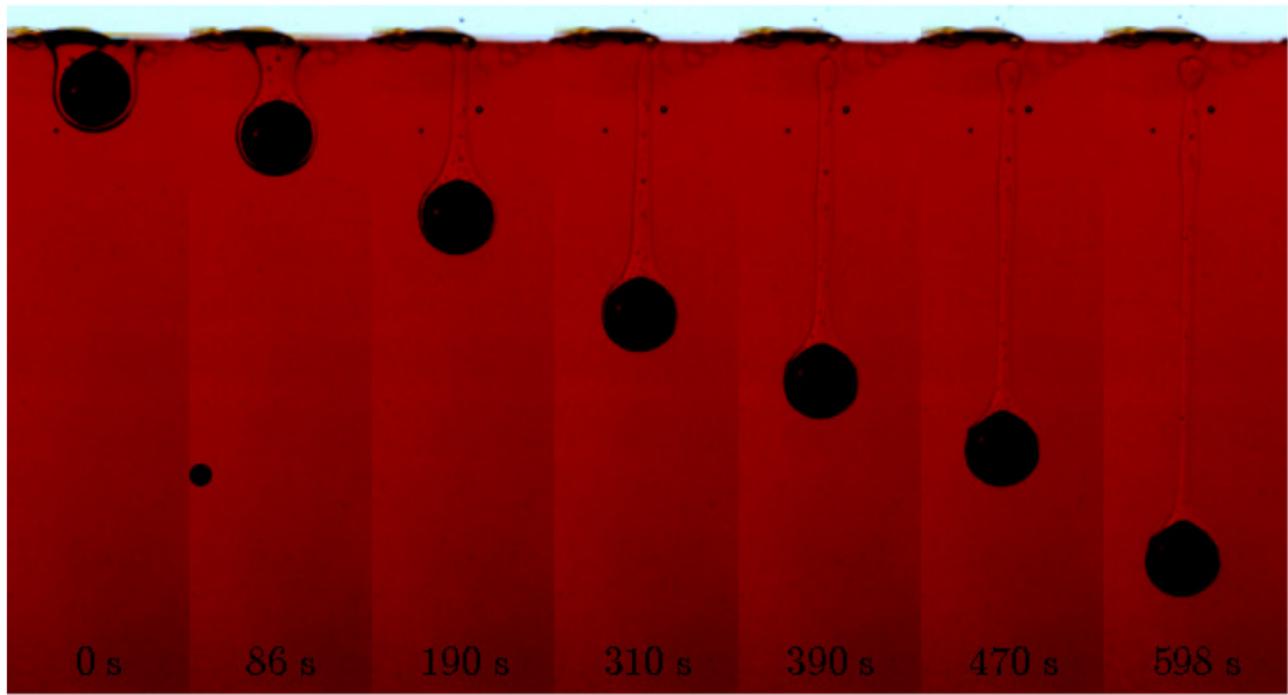


The Tailing Mode

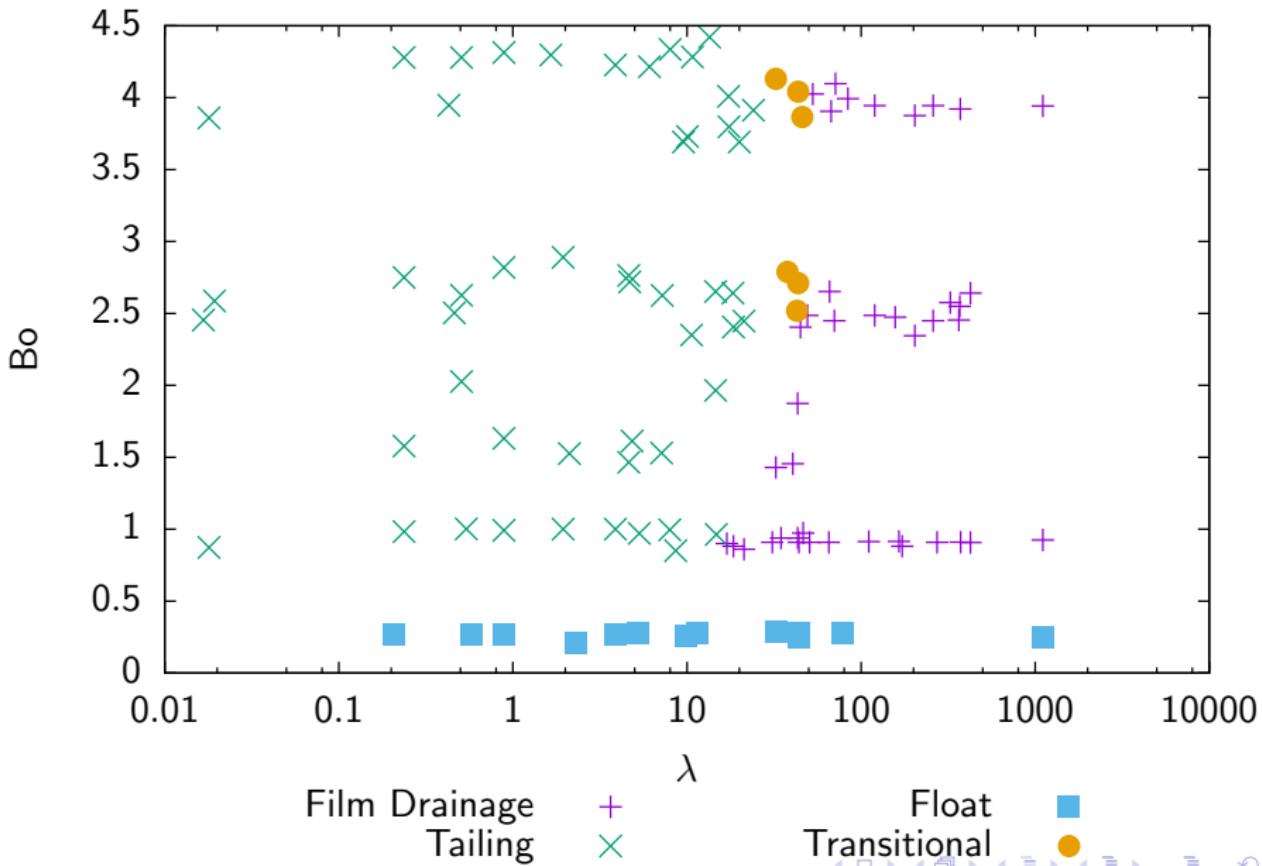
$$Bo = 4.30 \pm 0.05$$

$$D = 3.22 \pm 0.01$$

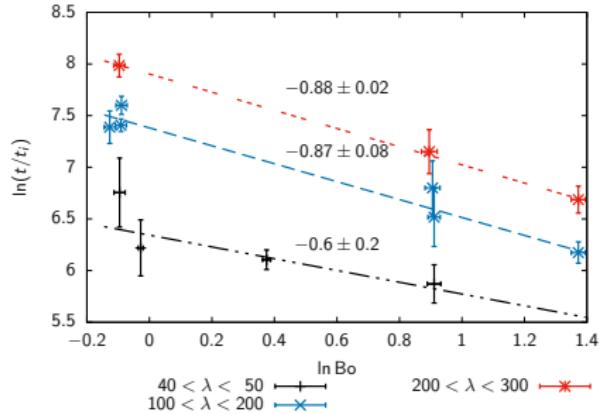
$$\lambda = 1.65 \pm 0.06$$



Modes of Transfer



Sinking Timescales - Film Drainage



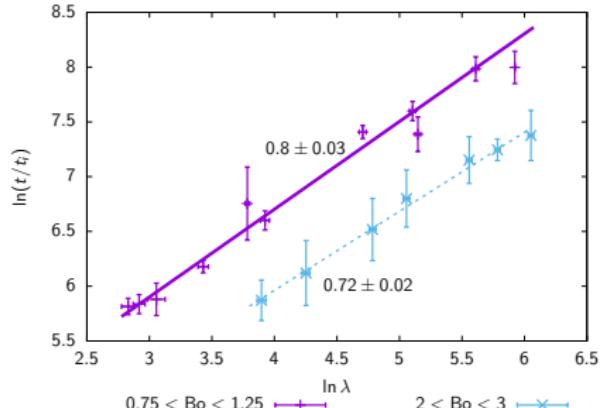
$40 < \lambda < 50$ $200 < \lambda < 300$
 $100 < \lambda < 200$ \star

$$\frac{t}{t_i} \sim Bo^a \lambda^b$$

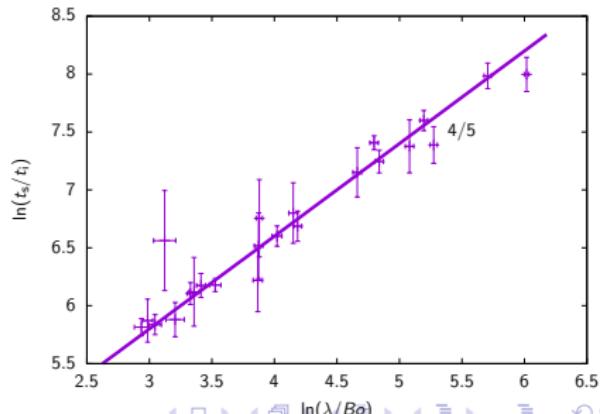
$$\ln\left(\frac{t}{t_i}\right) \sim a \ln Bo + b \ln \lambda$$

2D fitting $\implies a = -0.80 \pm 0.05$ and
 $b = 0.80 \pm 0.02$

$$\implies \frac{t}{t_i} \sim \left(\frac{\lambda}{Bo}\right)^{4/5}$$

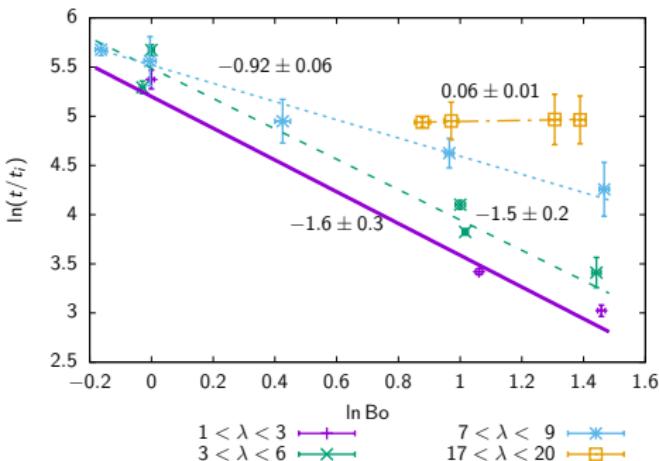


$0.75 < Bo < 1.25$ \star
 $2 < Bo < 3$ \times



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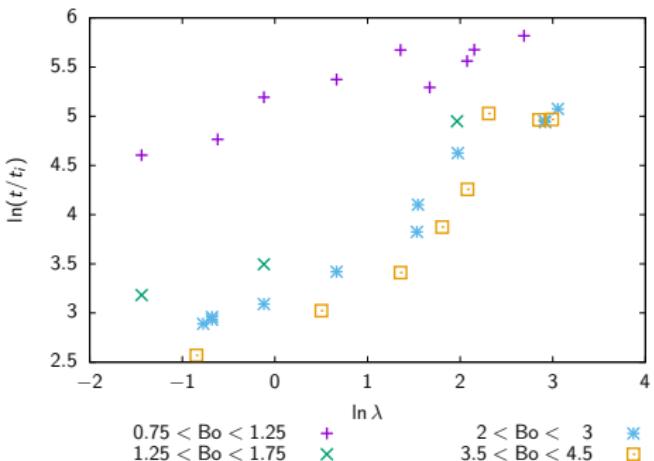
Sinking Timescales - Tailing



A power law dependence on Bond number

But exponent depends on viscosity ratio

$$\frac{t}{t_i} \sim Bo^{a(\lambda)} \lambda^b$$

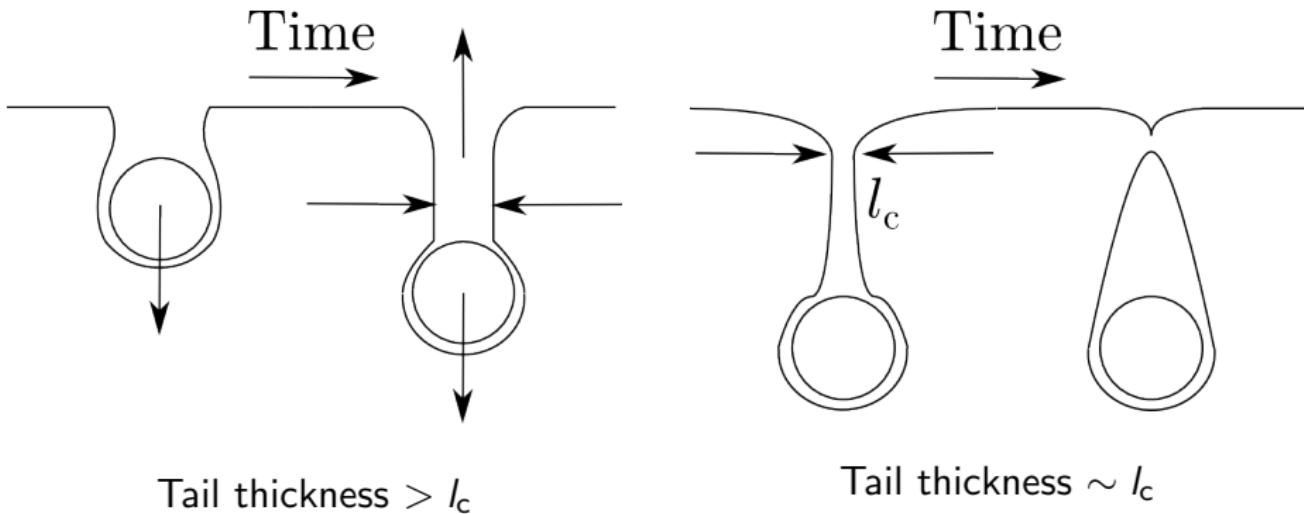


Clearly need more data

Hypothesis - Two stage sinking

Tail formation stretching and thinning

Tail Rupture



Each stage is associated with its own timescale

When $Bo \lesssim 1$, $a \lesssim l_c$, sinking timescale dominated by rupture stage

Summary

Studied Low Reynolds number settling of spheres through interface between density-stratified, immiscible fluids

When does entrainment occur?

- For $Bo > 1$ and $\lambda < 30$ passage of sphere entrains fluid from upper phase
- As Bo decreases, value of λ at which transition occurs decreases

What is the timescale of sinking?

- For film drainage $t/t_i \sim (\lambda/Bo)^{4/5}$
- Tailing mode - power law dependence on Bo but exponent depends on λ ?
- More data needed, but hypothesise two-stage sinking process