Low-Reynolds number settling of spheres through fluid interfaces

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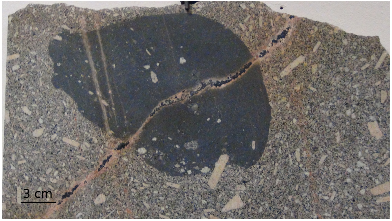
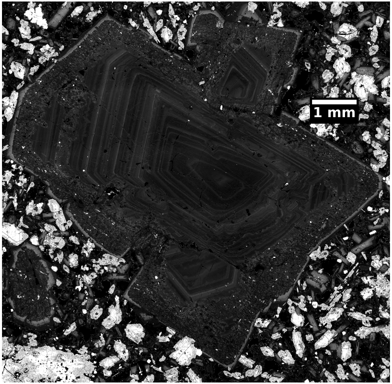
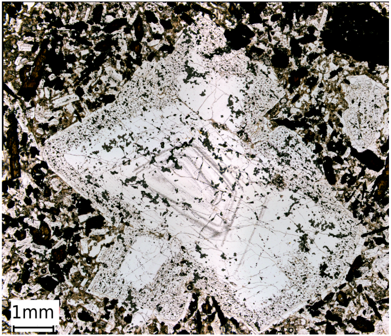
Motivation: Magmatic xenocrysts





Implications of xenocrysts

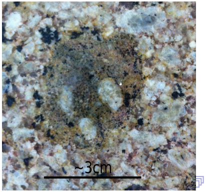
Mt Unzen - Porphyritic enclaves with hornblende and plagioclase xenocrysts



Thermal models $\implies$ equilibration ≈ 1 hour (Jaeger 1968)

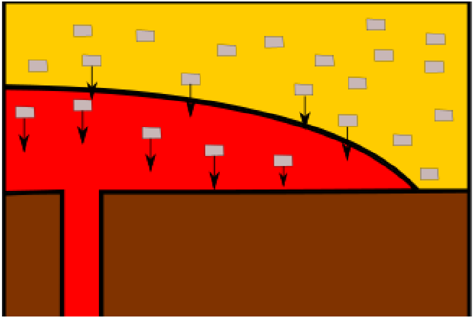
Experiments $\implies$ resorption zones require 0.3 - 8 days to form (Nakamura and Shimakita 1998)

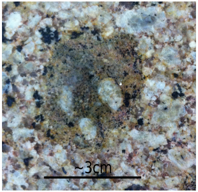
Crystals encorporated prior to enclave formation (Browne et al. 2006)



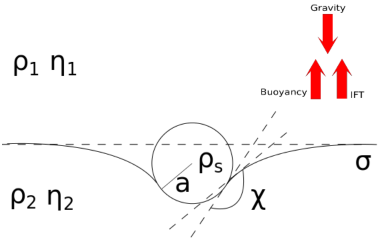
Even smaller enclaves contain xenocrysts

Gravitational Settling





Credit Jon Blundy

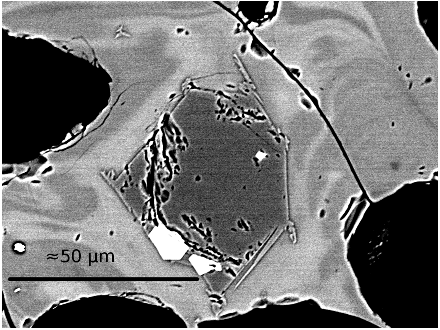


- $\rho_{1(2)}$ = Density of fluid 1(2)
- $\eta_{1(2)}$ = Viscosity of fluid 1(2)
- σ = Interfacial tension (IFT)
- a = Radius
- ρ_s = Particle density
- χ = Contact angle

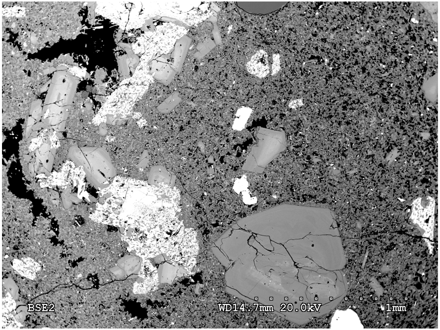
Questions

- For what conditions is sinking assciated with entrainment of upper phase fluid?
  Consequences for magma hybridisation

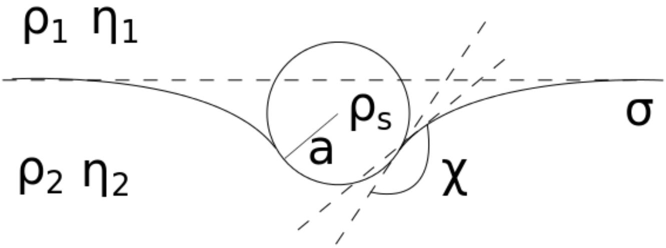
- What is the timescale of sinking?
  Important if other timescales matter e.g. solidifcation



Credit: Geoff Kilgour



Dimensionless Parameters



Bond Number

$$Bo = \frac{(\rho_2 - \rho_1)ga^2}{\sigma}$$

Modified Density Ratio

$$D = \frac{\rho_s - \rho_1}{\rho_2 - \rho_1}$$

Viscosity Ratio

$$\lambda = \frac{\eta_2}{\eta_1}$$

Length- and time-scales

Capillary length

$$l_c = \left(\frac{\sigma}{(\rho_2 - \rho_1)g} \right)^{1/2}$$

For $a < l_c$, $Bo < 1$, expect σ to control the interfacial response

For $a < l_c$, $Bo > 1$, expect density contrast to control the interfacial response

Multiple possible timescales e.g.

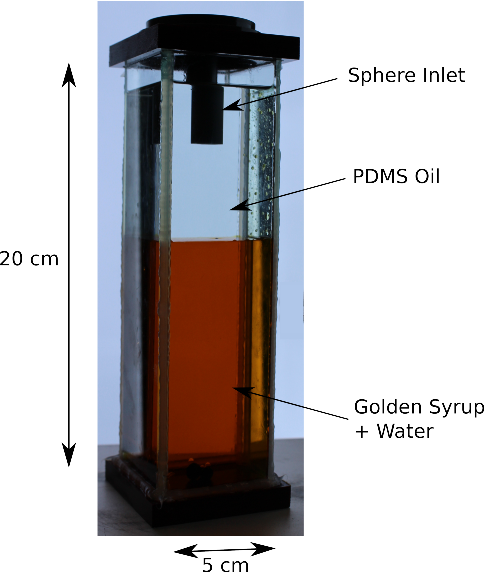
$$t = \frac{\eta_n l_c}{\sigma}, \quad t = \frac{\eta_n}{(\rho_s - \rho_n)ga},$$

$$n = 1, 2$$

Choose to non-dimensionalise with

$$t_i = \frac{\eta_1 a}{\sigma}$$

Experiments



Glass spheres of various radii 2-10 mm

Water content in syrup from 0-5%

Grade of PDMS oil

Temperature from 0-32°C

Can achieve ranges:

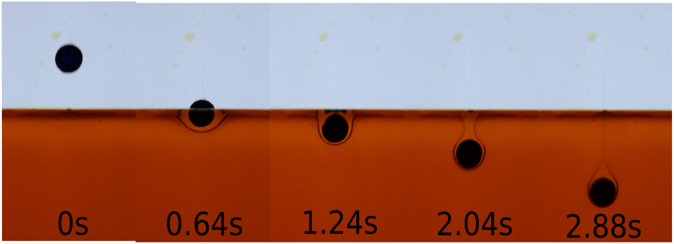
$$0.1 \leq Bo \leq 5$$

$$10^{-2} \leq \lambda \leq 10^{3}$$

$$D \approx 3.3$$

Modes of Transfer

Tailing Mode

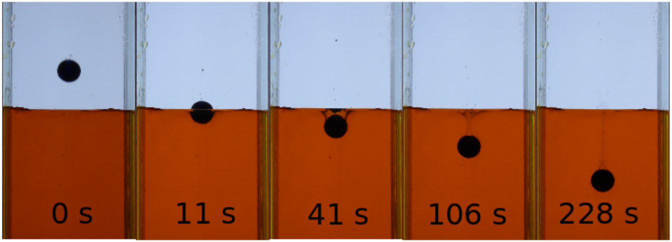


$Bo = 2.39 \pm 0.2$

$D = 3.4777 \pm 0.007$

$\lambda = 3.9 \pm 0.4$

Film Drainage Mode



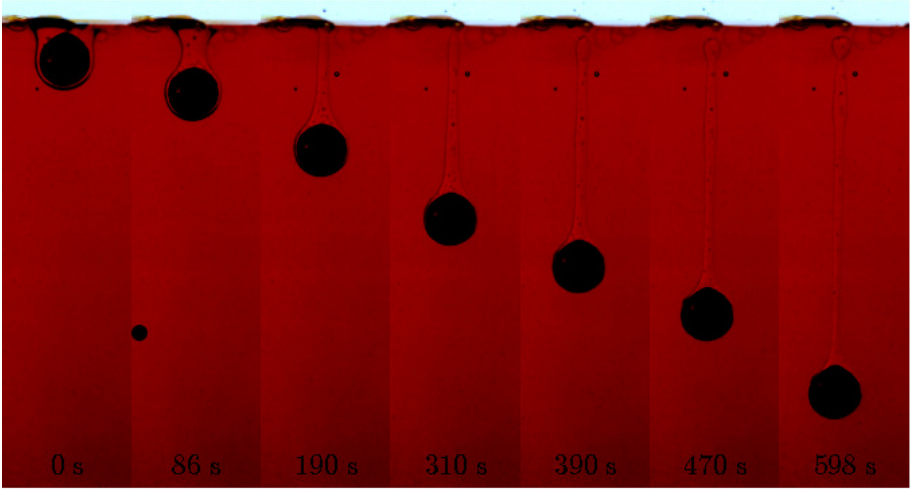
$Bo = 4.0 \pm 0.1$

$D = 3.248 \pm 0.005$

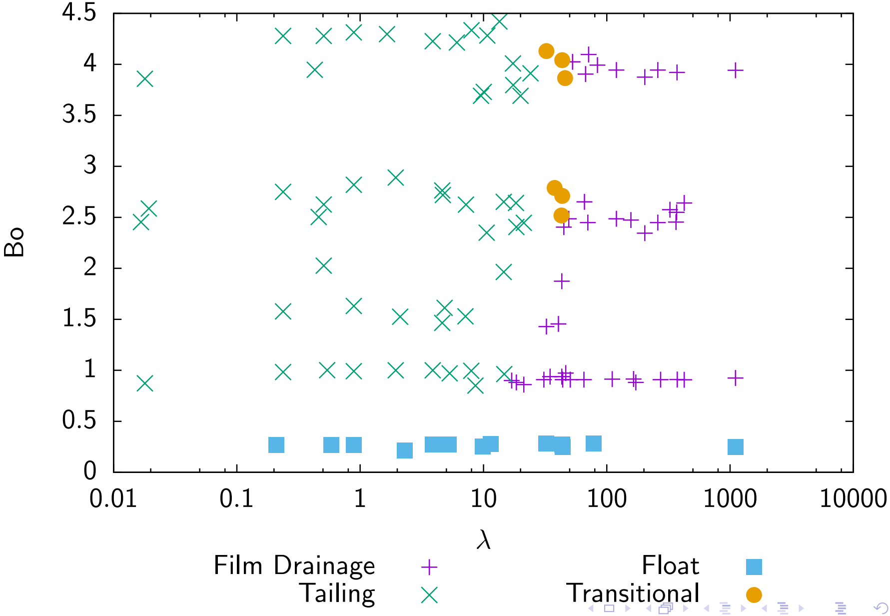
$\lambda = 259 \pm 6$

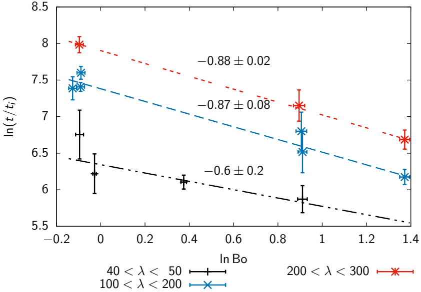
The Tailing Mode

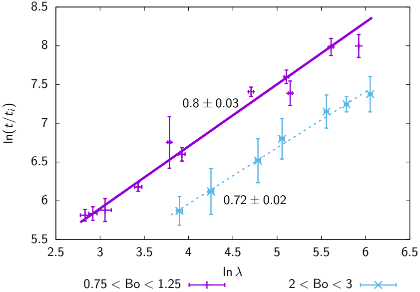
$Bo = 4.30 \pm 0.05 \qquad D = 3.22 \pm 0.01 \qquad \lambda = 1.65 \pm 0.06$



Modes of Transfer







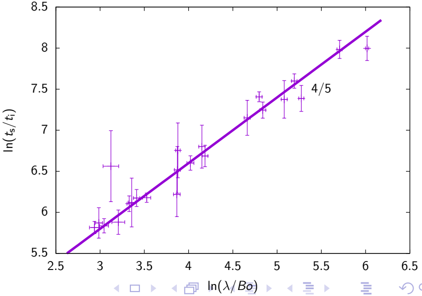
$$\frac{t}{t_{\mathrm{i}}} \sim Bo^{a}\lambda^{b}$$

$$\ln\left(\frac{t}{t_{\mathrm{i}}}\right) \sim a \ln Bo + b \ln \lambda$$

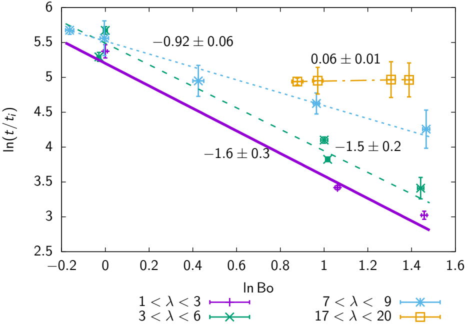
2D fitting $\implies a = -0.80 \pm 0.05$ and

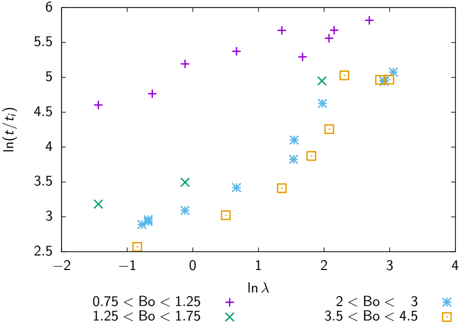
$$b = 0.80 \pm 0.02$$

$$\implies \frac{t}{t_{\mathrm{i}}} \sim \left(\frac{\lambda}{Bo}\right)^{4/5}$$



Sinking Timescales - Tailing





A power law dependence on Bond number

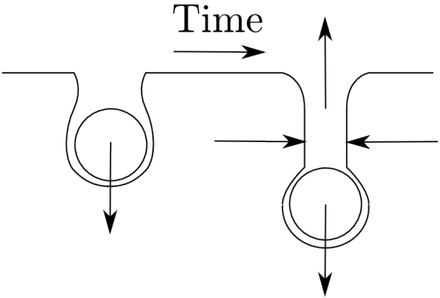
But exponenent depends on viscosity ratio

$$\frac{t}{t_i} \sim Bo^{a(\lambda)}\lambda^{b}$$

Clearly need more data

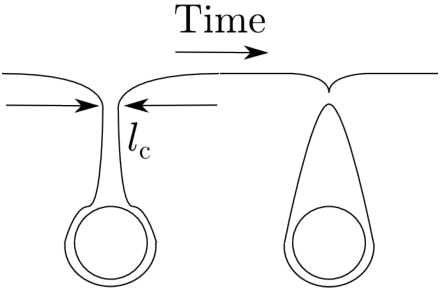
Hypothesis - Two stage sinking

Tail formation stretching and thinning



Tail thickness $> l_c$

Tail Rupture



Tail thickness $\sim l_c$

Each stage is associated with it's own timescale

When $Bo \lesssim 1$, $a \lesssim l_c$, sinking timescale dominated by rupture stage

Summary

Studied Low Reynolds number settling of spheres through interface between density-stratified, immiscible fluids

When does entrainment occur?

- For $\text{Bo} > 1$ and $\lambda < 30$ passage of sphere entrains fluid from upper phase
- As Bo decreases, value of λ at which transition occurs decreases

What is the timescale of sinking?

- For film drainage $t/t_\text{i} \sim (\lambda/\text{Bo})^{4/5}$
- Tailing mode - power law dependence on Bo but exponent depends on λ ?
- More data needed, but hypothesise two-stage sinking process