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Gravity currents

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Volcanic flows

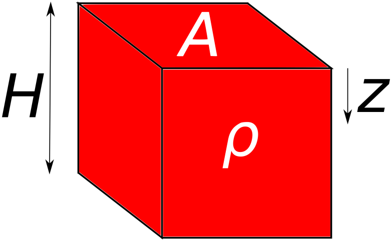
Lava flows

Cloud spreading

Pyroclastic density currents (PDCs)

Lahars

Hydrostatic gradients



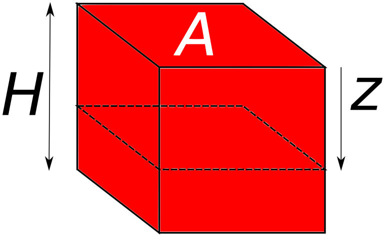
Consider a volume of fluid of :

- Density ρ
- Height H
- Horizontal cross section A

z = Negative vertical coordinate (depth below top surface)

Consider horizontal plane at depth z
What are the forces acting on this plane?

- **Weight** of overlying fluid
$$W = \rho A z g$$
- Balanced by **hydrostatic pressure**
$$F_{\mathrm{p}} = PA$$

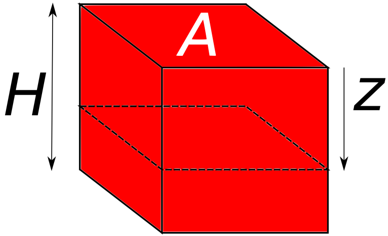


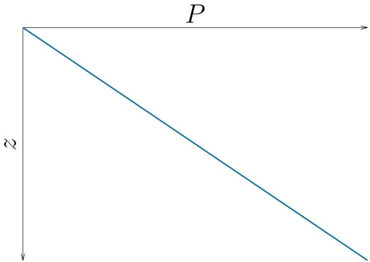
Hydrostatic gradients

Nothing is moving $\implies$ **Mechanical equilibrium**

$$W = F_{\mathrm{p}}$$

$$P = \rho g z$$



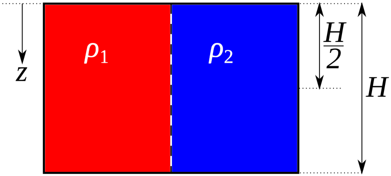


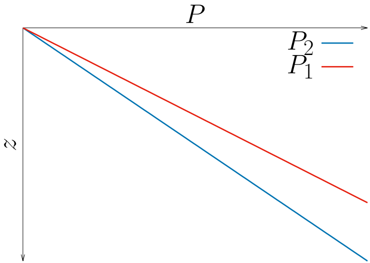
P increases linearly with z

Hydrostatic gradient:

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \rho g$$

Gravity current - A horizontal flow in a gravitational field that is driven by a density difference





Consider two fluids (densities ρ_1 and ρ_2, $\rho_1 > \rho_2$) initially side-by-side and separated by a vertical barrier

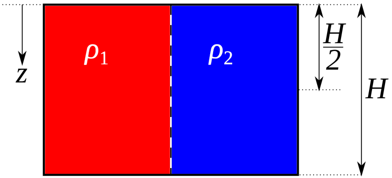
Pressure difference:

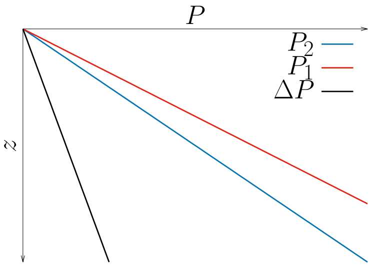
$$P_1 = \rho_1 g z$$

$$P_2 = \rho_2 g z$$

$$\Delta P = P_2 - P_1 = (\rho_2 - \rho_1) g z$$

Gravity currents - Horizontal force balance



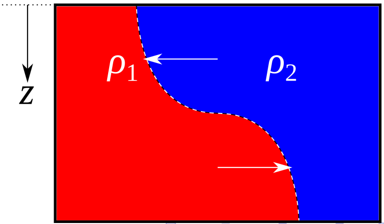


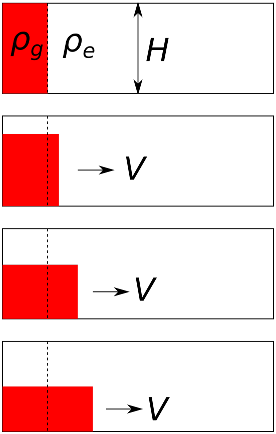
Remove barrier, and consider pressure difference ΔP across line

ΔP increases with depth

Flow follows a pressure gradient - but horizontal pressure gradient is greatest at the depth

This initiates from from high to low pressure at the base, which is compensated by return flow at the top





Assume:

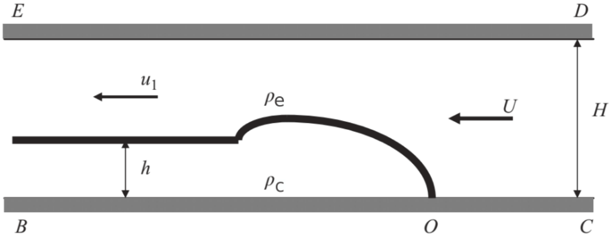
- Volume of current is conserved
- Current is always rectangular
- Energy is always conserved*

Energy balance between kinetic energy and change in GPE shows

$$V = \left(\frac{(\rho_c - \rho_e) g H}{2\rho_c} \right)^{1/2}$$

* Means assume energy only exists as kinetic or GPE. What other types of energy are there?

Gravity currents - Benjamin's model

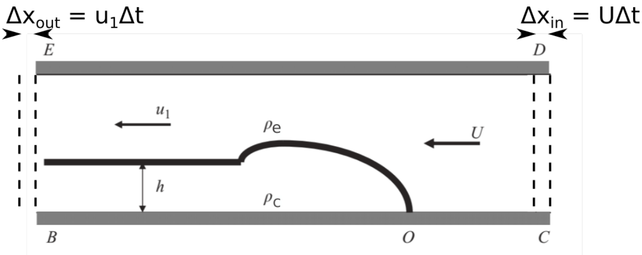


Modified from Shin et al. (2004)

Assumptions:

- Hydrostatic conditions upstream and and downstream of head
- No relative flow within current

Consider a reference frame moving with the head $\implies$ velocity inside current equals zero

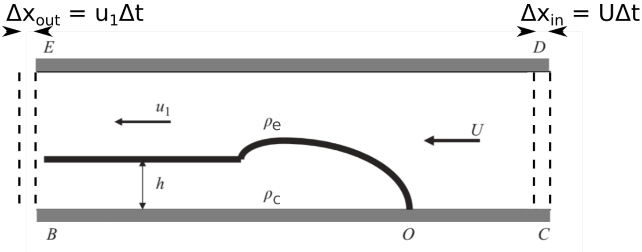


Modified from Shin et al. (2004)

Let W = Width of channel
During time interval Δt, volume of fluid:

- entering domain = $V_{in} = \Delta x_{in} WH = U \Delta t WH$
- leaving domain = $V_{out} = \Delta x_{out} W(H-h) = u_1 \Delta t W(H-h)$

Gravity currents - Benjamin's model



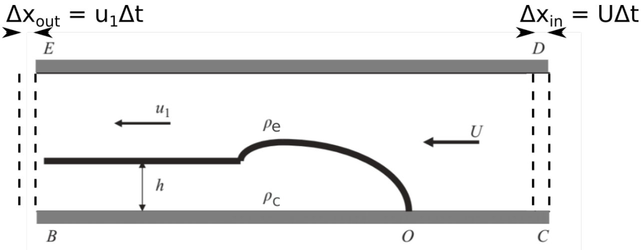
Modified from Shin et al. (2004)

During time interval Δt, mass of fluid

- entering domain $= m_{\text{in}} = \rho_{\text{e}} V_{\text{in}} = \rho_{\text{e}} U \Delta t W H$
- leaving domain $= m_{\text{out}} = \rho_{\text{e}} V_{\text{out}} = \rho_{\text{e}} u_1 \Delta t W (H - h)$

Mass conservation $\implies m_{\text{in}} = m_{\text{out}} \implies UH = u_1(H - h)$

Gravity currents - Benjamin's model



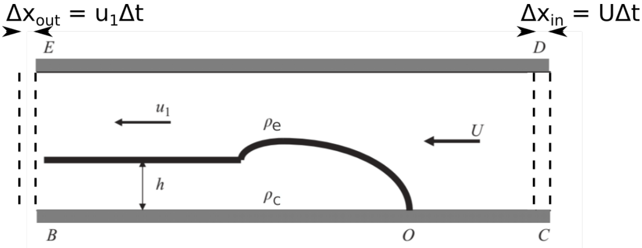
Modified from Shin et al. (2004)

Assumptions:
During time interval Δt, momentum of fluid:

- entering domain $= p_{\text{in}} = m_{\text{in}} U = \rho_{\text{e}} U^2 \Delta t W H$
- leaving domain $= p_{\text{out}} = m_{\text{out}} u_1 = \rho_{\text{e}} u_1^2 \Delta t W (H - h)$

Change in momentum $= \Delta p = p_{\text{in}} = p_{\text{out}} = [U^2 H - u_1^2 (H - h)] \rho_{\text{e}} \Delta t W$

Gravity currents - Benjamin's model

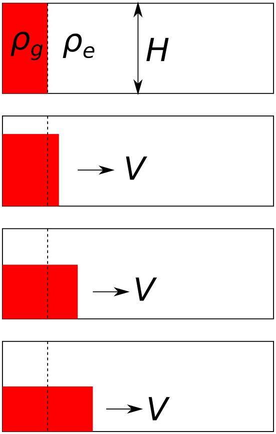


Modified from Shin et al. (2004)

Newton's second law: Force acting on substance = Rate of change of momentum

$$F = \frac{\Delta p}{\Delta t} = [U^2 H - u_1^2 (H - h)]\rho_e W$$

Force F depends on pressures at the inlet and outlet



If envrionment is air, then often $\rho_e \ll \rho_c$

$$V = \left(\frac{(\rho_c - \rho_e) g H}{2 \rho_c} \right)^{1/2} \approx \frac{(gH)^{1/2}}{2}$$

Therefore, we see that $(\rho_c - \rho_e)/\rho_c$ is factor by which envrionmental fluid reduced gravitational acceleration

Define **reduced gravity**:

$$g' = \frac{\rho_c - \rho_e}{\rho_c}$$

So generally:

$$V = \frac{(g' H)^{1/2}}{2}$$

Froude number:

$$\mathrm{Fr} = \frac{V}{(g'H)^{1/2}}$$

Represents ratio of inertial (U) to buoyancy ($(g'H)^{1/2}$) forces

For an ideal gravity current $V = (g'H)^{1/2}/2$ so:

$$\mathrm{Fr} = \frac{1}{2}$$

Measuring Fr gives a means of testing model assumptions

Dimensionless numbers

Froude	Fr	$\frac{V}{(g'H)^{1/2}}$	Inertia and buoyancy forces
Capillary	Ca	$\frac{\eta V r}{\sigma}$	Viscous and surface tension forces

Can use values of these quantities to define different fluid dynamical regimes

Reynolds number - Ratio of inertial and viscous forces within a fluid

$$\mathrm{Re} = \frac{\rho V L}{\eta}$$

$\rho =$ Fluid density
$V =$ Fluid velocity

$L =$ Lengthscale
$\eta =$ Viscosity

Reynolds number

$$\mathrm{Re} = \frac{\rho V L}{\eta}$$

$\mathrm{Re} \ll \mathrm{Re_c}$

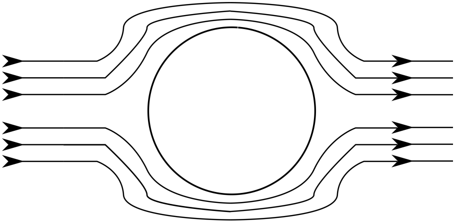
Laminar flow

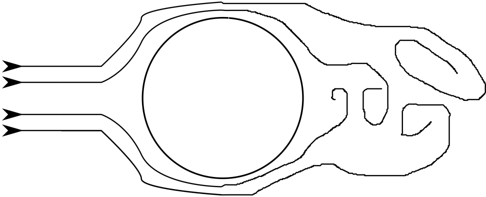
Fluid particles follow smooth paths in layers, with little or no mixing between different layers

$\mathrm{Re} \gg \mathrm{Re_c}$

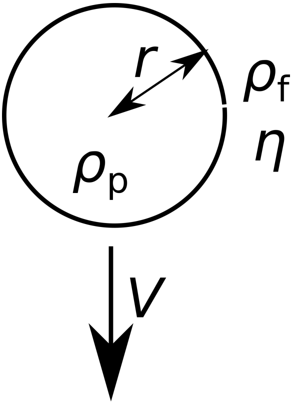
Turbulent flow

Chaotic changes in pressure and flow velocity, generating unsteady vortices

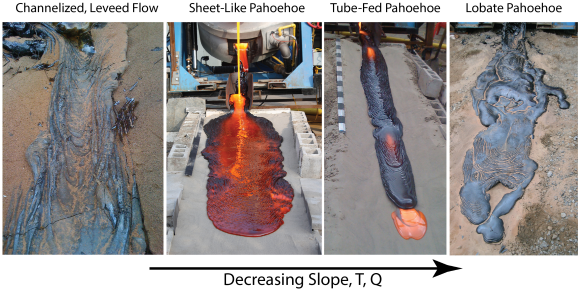




What are the Reynolds numbers for these scenarios?



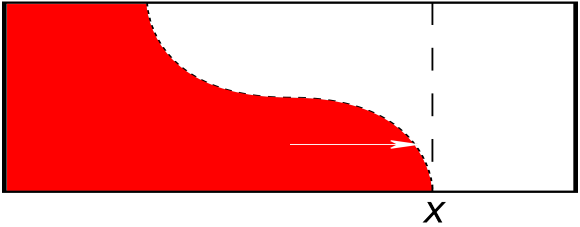
Low-Reynolds number gravity currents



For fixed release volume, viscous dissipation continuosly reduces V

Varying slope angle and flow rate produces different flow morphologies

For a 2D current:



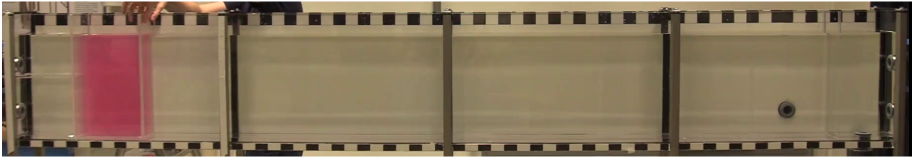
$$x \sim t^{4/5}$$

High-Reynolds number gravity currents

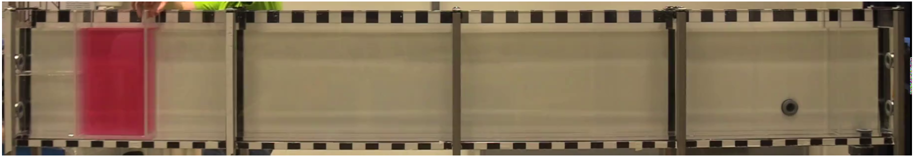
For Re > 1000, Re has no effect on flow

Can neglect viscous dissipation $\implies V = (g'H)^{1/2}/2$

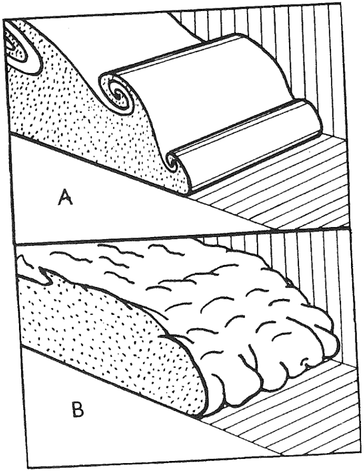
$$g' = 0.06$$



$$g' = 0.16$$



High-Reynolds number gravity currents - mixing

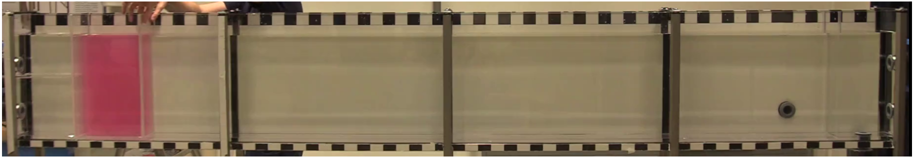


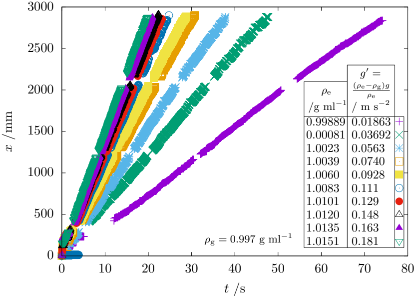
Mixing in fluid reduced density contrast

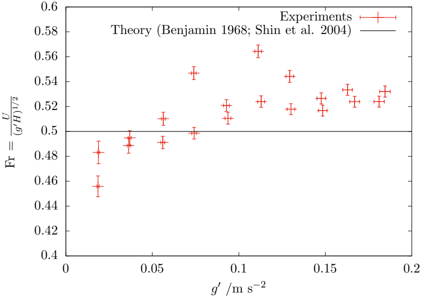
Two types of mixing:

- **Billowing** - Formed by shear between current and environment
- **Lobes and clefts** - Formed by interaction with ground at contact line

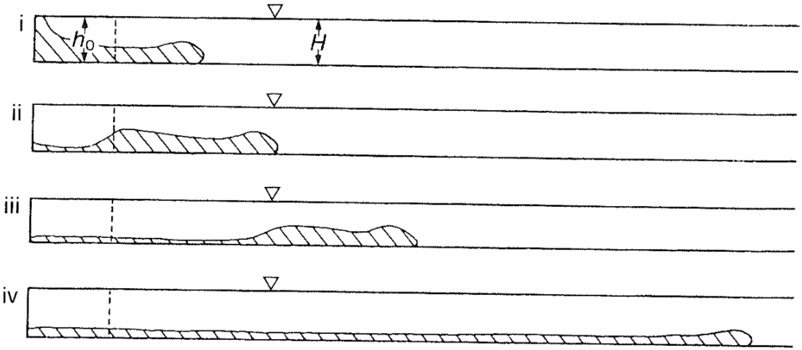
Constant volume release - Constant speed phase







Constant volume release - Self-similar phase



Constant speed

$x \sim t$

Self-similar

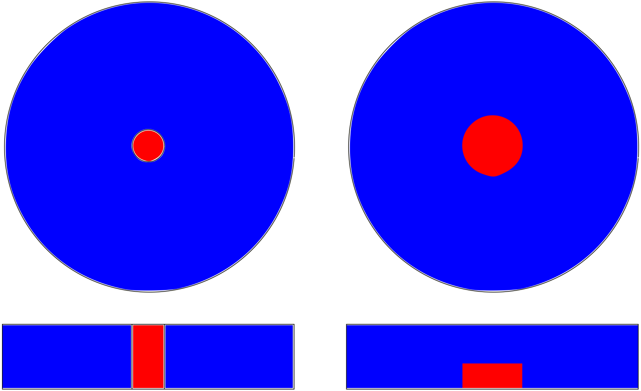
$x \sim t^{2/3}$

Viscous

$x \sim t^{1/5}$

Constant volume release - Radial collapse

$$r \sim t^{1/2}$$



Constant radial volume flow

$$r \sim Q^{1/3} t^{1/2}$$