Magma density and viscosity

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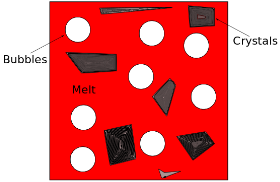
Magma density



Bulk density depends on volume fraction of crystals and bubbles

$$\rho = \rho_{\mathrm{m}} \left(1 - \sum_i \phi_i\right) + \sum_i \rho_i \phi_i$$

ρ_{m} = Melt density

- Depends on $T, P, \mathbf{X}$

i = quartz, hornblende, plagioclase etc. and H_2O, CO_2 bubbles etc.

ρ_i = Density of phase i

- Depends on T, P for bubbles
- Depends on composition for crystals

ϕ_i = Volume fraction of phase i

Melt density

$$\rho_m = \sum_i X_i M_i \left(\sum_i X_i \bar{V}_i(T, P) \right)^{-1}$$

M_i = Molar mass of component i

- Mass of 1 mol of i
- $M_{SiO_2} = 28 \text{ g mol}^{-1} + 2 \times 16 \text{ g mol}^{-1} = 60 \text{ g mol}^{-1}$
- $M_{H_2O} = 2 \times 1 \text{ g mol}^{-1} + 16 \text{ g mol}^{-1} = 18 \text{ g mol}^{-1}$

$\bar{V}_i$ = Partial molar volume of component i

- Change in mixture volume when 1 mol of i is added

Need to determine $\bar{V}_i(T, P)$ empirically

Relationship between **pressure**, **volume (density)** and **temperature**
Experiments - measure volume of a sample of **X** at a different P and T.
Find **empirical** EoS

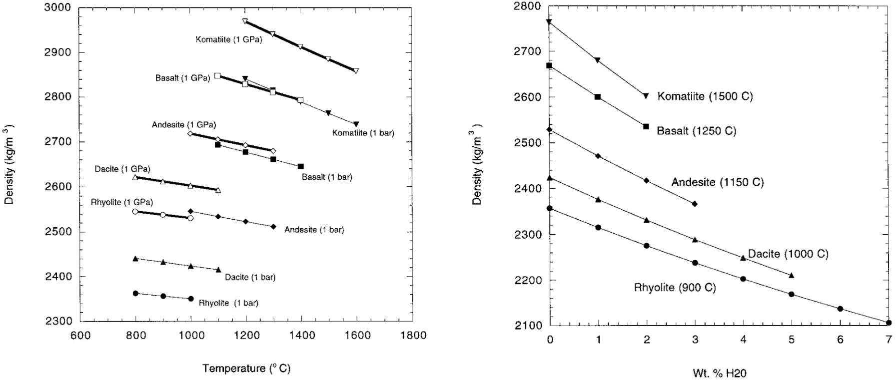
$$V_m(T,P,\mathbf{X}) = \sum_i X_i \left[\bar{V}_i(T=T_\mathrm{R}, P=P_\mathrm{R}) + \left.\frac{\partial \bar{V}_i(T,P=P_\mathrm{R})}{\partial T}\right|_{T=T_\mathrm{R}} (T-T_\mathrm{R}) + \left.\frac{\partial \bar{V}_i(T=T_\mathrm{R},P)}{\partial P}\right|_{P=P_R} (P-P_\mathrm{R}) \right]$$

T_R = Reference temperature = 1673 K
P_R = Reference pressure = 10^{-4} GPa

Lange & Carmichael (1990)
Lange (1997)
Ochs & Lange (1997)

	$\bar{V}_i(T=T_\mathrm{R}, P=P_\mathrm{R})$ $/10^{-6}$ m^3 mol^{-1}	$\left.\frac{\partial \bar{V}_i(T,P=P_\mathrm{R})}{\partial T}\right\|_{T=T_\mathrm{R}}$ $/10^{-9}$ m^3 mol^{-1} K	$\left.\frac{\partial \bar{V}_i(T=T_\mathrm{R},P)}{\partial P}\right\|_{P=P_R}$ $/10^{-6}$ m^3 mol^{-1} GPa
SiO_2	26.86	0.0	-1.89
TiO_2	23.16	7.24	-2.31
Al_2O_3	37.42	0.0	-2.31
Fe_2O_3	42.13	9.09	-2.53
FeO	13.65	2.92	-0.45
MgO	11.69	3.27	0.27
CaO	16.53	3.74	0.34
Na_2O	28.88	7.68	-2.40
K_2O	45.07	12.08	-6.75
Li_2O	16.85	5.25	-1.02
H_2O	26.27	9.46	-3.15
CO_2	33.0	0.0	0.0

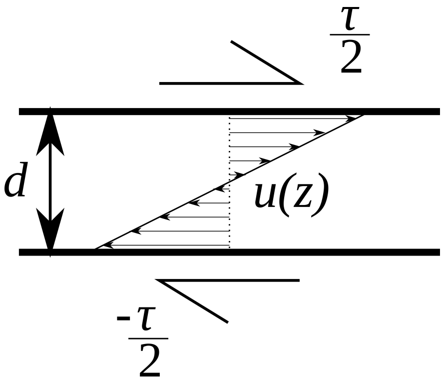
Melt density - Effect of T and X_{H_2O}



Temperature has small effect on density, particularly for high-silica melts

Water content has much stronger effect

Viscosity - A measure of a substance's resistance to flow (deformation). It relates an applied shear stress to the velocity gradient.



Consider fluid between two sheared parallel plates

τ = Applied shear stress
d = Separation between plates
$u(z)$ = Fluid velocity in gap

Viscosity η is defined as:

$$\tau = \eta \frac{\mathrm{d}U}{\mathrm{d}z} = \eta \dot{\epsilon}$$

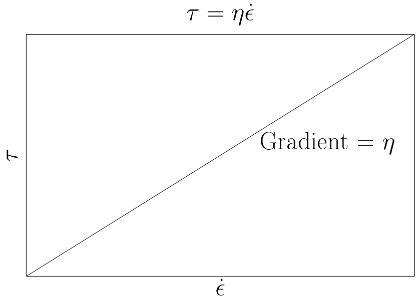
where $\dot{\epsilon} = \mathrm{d}U/\mathrm{d}z$ = **strain rate**

Magma viscosity - Newtonian fluids

Generally, viscosity is a function of strain rate: $\eta = \eta(\dot{\epsilon})$

Therefore, impossible to assign a single value of η to a material

Newtonian fluids - Special case where η is independent of $\dot{\epsilon}$



Flow curve - Relationship between τ and $\dot{\epsilon}$

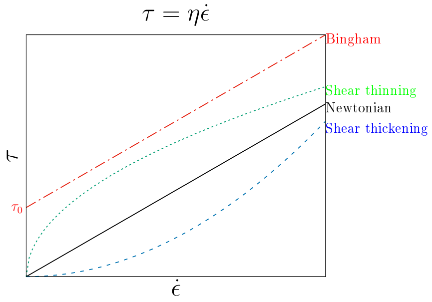
$$\tau = \eta(\dot{\epsilon})\dot{\epsilon}$$

For Newtonian fluid, $\eta =$ constant

$\implies$ flow curve is straight line

$\implies$ gradient $= \eta$

Magma viscosity - Rheological materials



Newtonian

- Constant η
- e.g. water, magmatic melt

Shear thickening

- η increases with $\dot{\epsilon}$
- e.g. cornstarch

Shear thinning

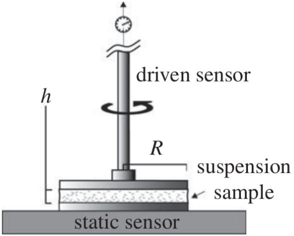
- η decreases with $\dot{\epsilon}$
- e.g. butter

Bingham

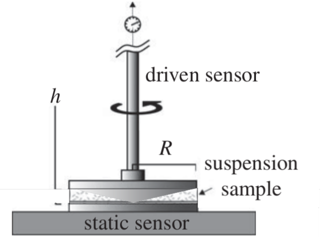
- Fluid has **yield stress** τ_0
- For $\tau < \tau_0$ fluid does not flow ($\dot{\epsilon} = 0$)
- e.g. Mayonaise

Measuring rheological properties

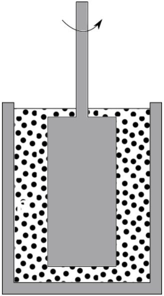
Parallel plate



Cone and plate



Concentric cylinder



Apply torque $M \implies$ stress τ
Measure angular velocity $\Omega \implies$ strain rate $\dot{\gamma}$
Determine flow curve $\tau = \eta(\dot{\gamma})\dot{\gamma}$

Magma viscosity - Melt as a Newtonian fluid

Silica melt is almost perfectly Newtonian.

However, at extremely high shear rates, all materials start to undergo shear-thinning

Critical shear rate is given by

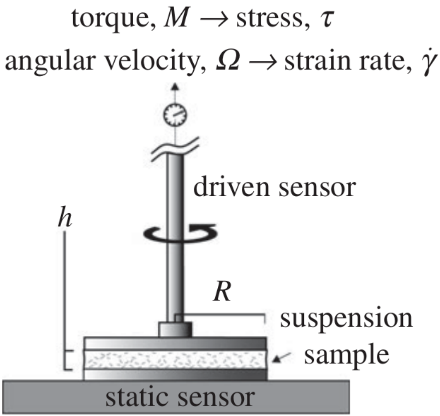
$$\dot{\epsilon_c} \approx \frac{10^{-3} G}{\eta_m}$$

G = Shear modulus $\approx 10^{10}$ Pa

At even higher shear rates, the melt cannot deform as a fluid and snaps in a brittle manner.

Magma viscosity - Melt viscosity

Melt is modelled according to the Vogel-Fulchner-Tammann (VFT) equation (Giordano et al., 2008)



$$\log \eta_{\mathrm{m}} = A + \frac{B(\mathbf{X})}{T - C(\mathbf{X})}$$

$$\eta_{\mathrm{m}} = 10^{A + B(\mathbf{X})/[T - C(\mathbf{X})]}$$

Measurements of the viscosity of samples of different $\mathbf{X}$ at different T are used to determine $A, B(\mathbf{X}), C(\mathbf{X})$

N.B: This is a fitted, purely empirical equation - it is not dimensionally consistent!

Magma viscosity - Fitted parameters

$$A = \text{constant} = \text{-4.55}$$

$$B = \sum_{i=1}^{7} b_i M_i + \sum_{j=1}^{3} b_{1j} M_{1j}$$

$$C = \sum_{i=1}^{6} c_i N_i + c_{11} N_{11}$$

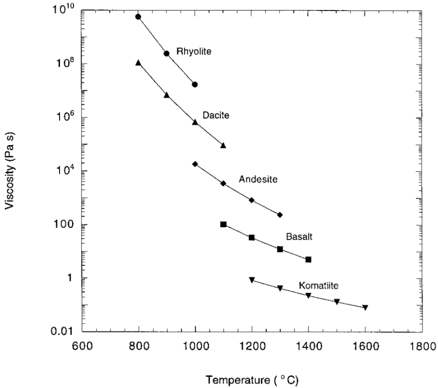
$$\log \eta_{\mathrm{m}} = A + \frac{B(\mathbf{X})}{T - C(\mathbf{X})}$$

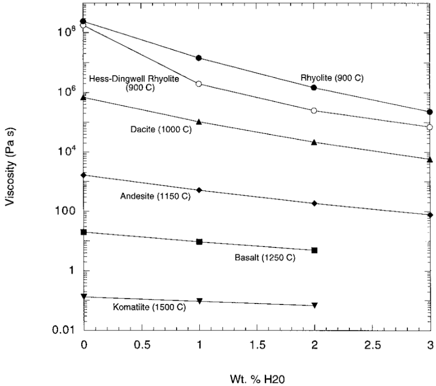
$$\eta_{\mathrm{m}} = 10^{A+B(\mathbf{X})/[T-C(\mathbf{X})]}$$

$b_1 = 159.6$	$M_1 = X_{SiO_2} + X_{TiO_2}$	$c_1 = 2.75$	$N_1 = X_{SiO_2}$
$b_2 = -173.3$	$M_2 = X_{Al_2O_3}$	$c_2 = 15.7$	$N_2 = X_{TiO_2} + X_{Al_2O_3}$
$b_3 = -72.1$	$M_3 = X_{FeO} + X_{MnO} + X_{P_2O_5}$	$c_3 = 8.3$	$N_3 = X_{FeO} + X_{MnO} + X_{MgO}$
$b_4 = -75.7$	$M_4 = X_{MgO}$	$c_4 = 10.2$	$N_4 = X_{CaO}$
$b_5 = -39.9$	$M_5 = X_{CaO}$	$c_5 = -12.3$	$N_5 = X_{Na_2O} + X_{K_2O}$
$b_6 = -84.1$	$M_6 = X_{Na_2O} + X_{H_2O} + X_{F_2O}$	$c_6 = -99.1$	$N_6 = \ln(1 + X_{H_2O} + X_{F_2O})$
$b_7 = -141.5$	$M_7 = X_{H_2O} + X_{F_2O} + \ln(1 + X_{H_2O})$		
$b_{11} = -2.43$	$M_{11} = M_1 N_3$	$c_{11} = 0.3$	$N_{11} = (M_2 + N_3 + N_4 -$
$b_{12} = -0.91$	$M_{12} = (N_1 + N_2 + X_{P_2O_5})(N_5 + X_{H_2O})$		$X_{P_2O_5})(N_5 + X_{H_2O} + X_{F_2O_{-1}})$
$b_{13} = 17.6$	$M_{13} = M_2 N_5$		

For a given melt composition, can evaluate η

Melt viscosity - Effect of T and X_{H_2O}





Increase in T and X_{H_2O} can reduce melt viscosity by orders of magnitude

Magma viscosity - Effect of crystals

Particles suspended in a fluid increase the viscosity of the medium

Suspension - A mixture of particles and fluid

ϕ_c = **Volume fraction** of crystals

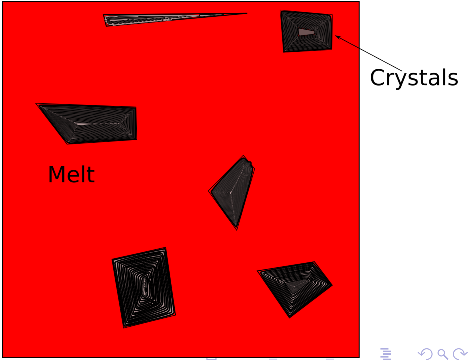
Define **relative viscosity** $\eta_r(\phi_c)$ to contain effect of ϕ_c

$$\eta = \eta_m \eta_r(\phi_c)$$

$\phi_c \lesssim 0.01$ - **Dilute**

$0.01 \lesssim \phi_c \lesssim 0.25$ - **Semi-dilute**

$\phi_c \gtrsim 0.25$ - **Concentrated**



Magma viscosity - Dilute particle suspensions

Dilute suspension: $\phi_\mathrm{c} \lesssim 0.01$

Interactions between particles are weak and effect on viscosity is small

Viscosity depends linearly on ϕ_c (Einstein, 1906; 1911)

$$\eta_\mathrm{r} = 1 + B\phi_\mathrm{c}$$

Experiments suggest that $B \approx 2.5$

Suspension rheology models are only strictly valid for suspensions on **spherical** particles.

Developing models for suspensions of differently shaped particles is difficult
- particularly for magmas where crystals have many different and irregular shapes

However, for dilute and semi-dilute suspensions, models work well.

Magma viscosity - Semi-dilute suspensions

Semi-dilute suspension: $0.01 \lesssim \phi_c \lesssim 0.25$

Particles interact with each other, affecting the viscosity

Viscosity depends non-linearly on ϕ_c (Guth & Gold, 1938; Vand, 1948; Manley & Mason, 1955)

Can model the viscosity with a **polynomial** expression

$$\eta_r = 1 + B\phi_c + B_1\phi_c^2 + ...$$

Experiments suggest that $7.35 \lesssim B_1 \lesssim 14.1$

Predictions from polynomial models get worse as ϕ_c increases.
- This is because as particles begin to touch the viscosity rises very fast

Magma viscosity - Maximum packings

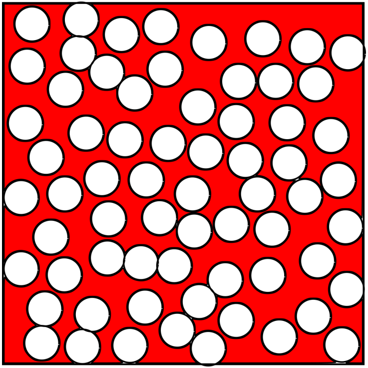
Concentrated suspension: $\phi_c \gtrsim 0.25$

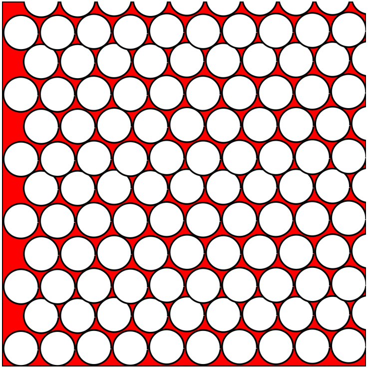
Random close packing - Maximum volume fraction of crystals that can be obtained in a disordered system

For spheres: $\phi_m \approx 0.64$

Densest regular packing - Maximum possible packing if crystals are arranged orderly

For spheres: $\phi_r \approx 0.74$





Magma viscosity - Concentrated suspensions

Concentrated suspension: $\phi_c \gtrsim 0.25$

Once $\phi_c = \phi_m$, then further deformation is impossible
- the material is **rheologically locked**
- $\eta_r \to \infty$

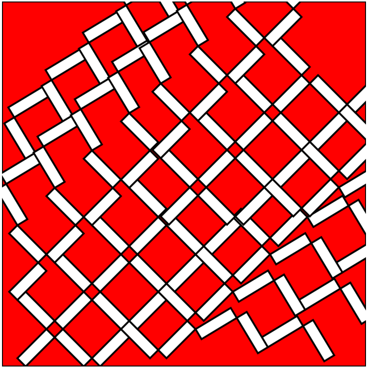
For magmas, ϕ_m depends on crystal shape and size distribution
- Very difficult to accurately predict
- Many experiments, find $\phi_m \approx 0.4$

Multiple models describe rheology of concentrated suspensions, but commonly used model is **Krieger & Dougherty (1959)**

$$\eta_r = \left(1 - \frac{\phi_c}{\phi_m}\right)^{-B\phi_m}$$

Magma viscosity - Yield stress

Once a touching network of crystals exists, then the magma has a **yield stress**



Experiments show magmas with ϕ_c as small as 0.2 can have a yield stress

ϕ_y = Minimum value of ϕ_c at which yield stress exists

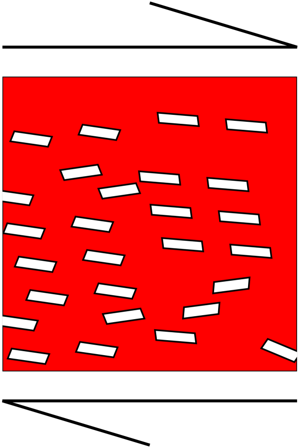
$$\tau_0 = 6.9 \left(\frac{\phi_c}{\phi_y} - 1 \right) \left(1 - \frac{\phi_c}{\phi_m} \right)^{-1}$$

(Saar et al., 2001; Andrews & Manga, 2014)

Magma viscosity - Non-Newtonian effects

Definition of η_r assumes Newtonian or Bingham rheology
i.e. η is independent of $\dot{\epsilon}$

In reality:



Elongated particles align with the flow

Longest axis becomes parallel to flow lines

The greater the strain rate the larger the reduction in viscosity $\implies$ **shear thinning**

More complicated models exist to describe suspensions of elongate particles e.g.

$$\eta_r = \left(1 - \frac{\phi_c}{\phi_m}\right)^{-2} \dot{\epsilon}^{0.2 r_p (\phi_c/\phi_m)^4}$$

r_p = Crystal **aspect ratio**

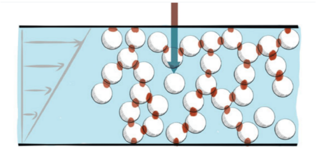
Magma rheology - Suspension or granular media

Classically, model mixtures of particles and fluid as **suspensions**:

- No relative motion between fluid and particles
- ϕ is control variable
- System described by equations of **fluid dynamics** with **effective viscosity**

In reality, particularly for dense suspensions:

- **Phase segregation** can be important
- ϕ can depend on particle pressure P_{p}
- System is described by **Granular mechanics** with **interstitial fluid**



Guazzelli and Pouliquen (2018)

How to describe these systems remains highly contentious

Extremely difficult to model theoretically, numerically or experimentally

Magma viscosity - Effect of bubbles

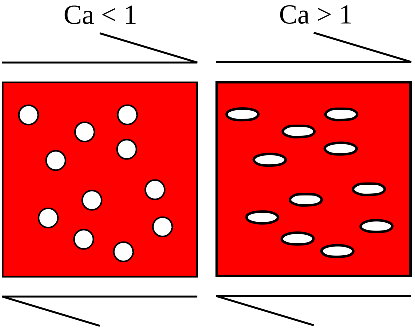
Rheological behaviour is determined by the **capillary** number

$$\mathrm{Ca} = \frac{\eta_\mathrm{m}\dot{\epsilon}r_\mathrm{b}}{\sigma}$$

η_m = Melt viscosity
$\dot{\epsilon}$ = Shear rate
r_b = Bubble radius
σ = Surface tension

Ca is a balance between:

- **Deforming viscous stress** $\eta_\mathrm{m}\dot{\epsilon}$
- **Restoring surface tension stress** σ / r_b



If Ca $\lesssim$ 1, surface tension stress dominates and bubbles remain spherical
Bubbles behave like solid particles
$\implies$ increase in viscosity

If Ca $\gtrsim$ 1, deforming stress dominates and bubbles can have irregular shapes
Bubbles streamline with shear
$\implies$ Decrease in viscosity and shear thinning

Magma viscosity - Modeling the effect of bubbles

Bubble deformation makes it difficult to model effect of bubbles on rheology

Relaxation time - Characteristic time for a deformed bubble to return to equilibrium state

$$\lambda = \frac{\eta_\mathsf{m} r_\mathsf{b}}{\sigma}$$

Compare with timescale for changing shear conditions $\dot{\gamma}/\ddot{\gamma}$

Define **Dynamic capillary number:**

$$\mathsf{Cd} = \frac{\lambda\ddot{\gamma}}{\dot{\gamma}}$$

$\mathsf{Cd} < 1 \implies$ Steady flow $\implies$ Rheology controlled by Ca

$\mathsf{Cd} > 1 \implies$ Unsteady flow $\implies$ Bubbles never in equilibrium

Viscosity is less than bubble-free fluid

Magma viscosity - Modeling the effect of bubbles

Whether bubbles increase or decrease viscosity depends on $\dot{\gamma}$ and $\ddot{\gamma}$
Semi-empirical model of Mader et al. (2013):

$$\eta_{\mathrm{r}} = \eta_{\mathrm{r},\infty} + \frac{\eta_{\mathrm{r},0} - \eta_{\mathrm{r},\infty}}{1 + \mathrm{Cx}^m}$$

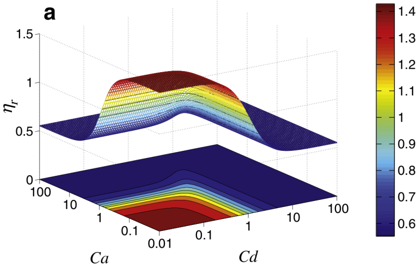
where

$$\eta_{\mathrm{r},\infty} = 1 - \frac{5\phi_{\mathrm{b}}}{3}$$

$$\eta_{\mathrm{r},0} = 1 + \phi_{\mathrm{b}}$$

$$\mathrm{Cx} = \sqrt{\mathrm{Ca}^2 + \mathrm{Cd}^2}$$

m depends on size distribution
$= 2$ for mono-disperse bubbles



Mader et al. (2013)

Experimentally tested for $\phi_{\mathrm{b}} < 0.46$

For $\phi_{\mathrm{b}} \geq 0.74$ a foam forms

Magma density and viscosity - conclusions

- Empirical and semi-empirical models parameterise magma density and viscosity
- Both depend on volume fraction of solid and gas phases
- Melt density is described by an empirical equation of state and depends strongly on water content
- Melt viscosity is described by an empirical VFT equation and depends on temperature and composition
- Magma viscosity strongly depends on crystal and bubble content