UNIVERSITÉ DE GENÈVE

Magma transport processes

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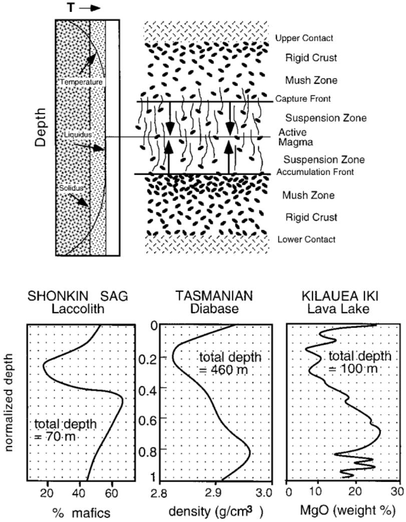
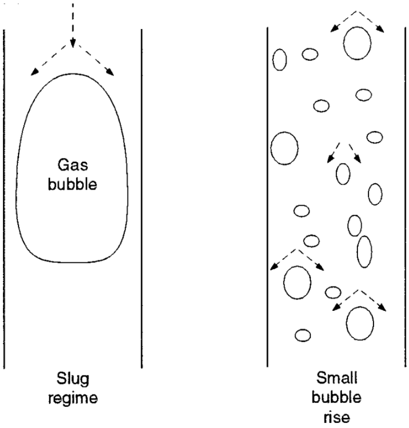
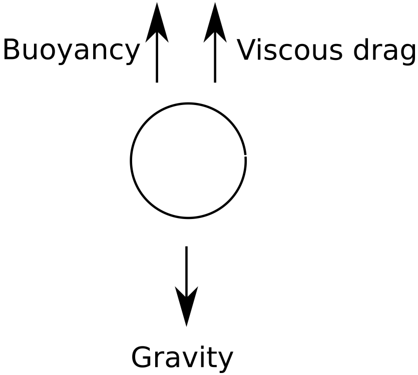
Viscosity and density control how magma is transported within the Earth's crust
Can consider transport of bulk magma, or **fractionation** of individual phases





Fractionation by crystal settling

Sills can contain **cumulates** - dense regions of crystals which have settled to the base of a chamber



In viscous fluid, three forces act on sphere:

- **Gravity** $F_{\mathrm{g}} = 4\pi\rho_{\mathrm{c}} r^3 g/3$
- **Buoyancy** $F_{\mathrm{b}} = 4\pi\rho_{\mathrm{m}} r^3 g/3$
- **Viscous drag** $F_{\mathrm{v}} = 6\pi\eta_{\mathrm{m}} r v_{\mathrm{s}}$

where r = radius, v_{s} = settling speed

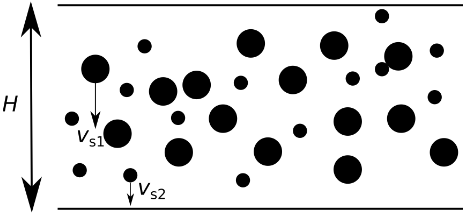
In equilibrium $F_{\mathrm{g}} = F_{\mathrm{b}} + F_{\mathrm{v}} \implies$

$$v_{\mathrm{s}} = \frac{2(\rho_{\mathrm{c}} - \rho_{\mathrm{m}}) g r^2}{9\eta_{\mathrm{m}}}$$

Simple models of cumulate formation: Convecting or static magma

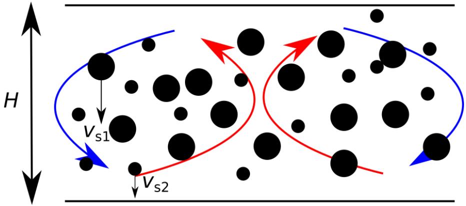
Partially-molten sill, 2 populations of crystals size d_1 and d_2 where $d_1 > d_2$ $\implies v_{s,1} > v_{s,2}$

Static magma



$$\phi_1, \phi_2 = \phi_1(\mathbf{x}, t), \phi_2(\mathbf{x}, t)$$

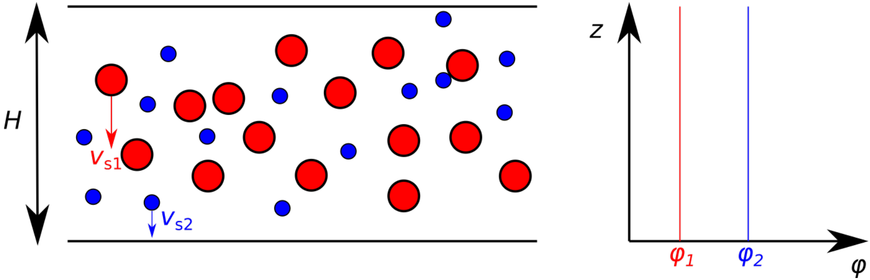
Convecting magma



$$\phi_1, \phi_2 = \phi_1(t), \phi_2(t)$$

Cumulate crystal size distribution: Static magma

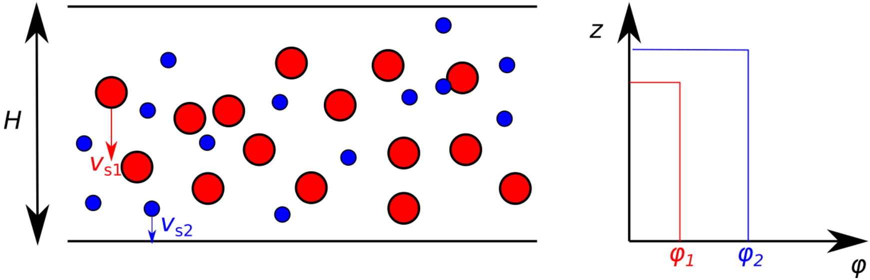
$$t = 0$$



Both populations are homogeneously dispersed throughout the sill

Cumulate crystal size distribution: Static magma

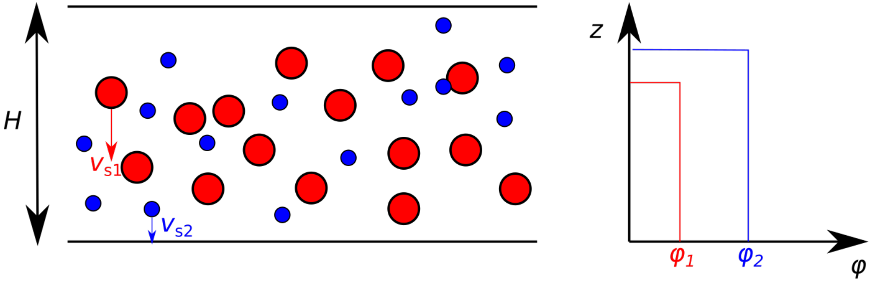
$$t = \mathrm{d}t$$



Populations settle at different speeds

Cumulate crystal size distribution: Static magma

$$t = \mathrm{d}t$$



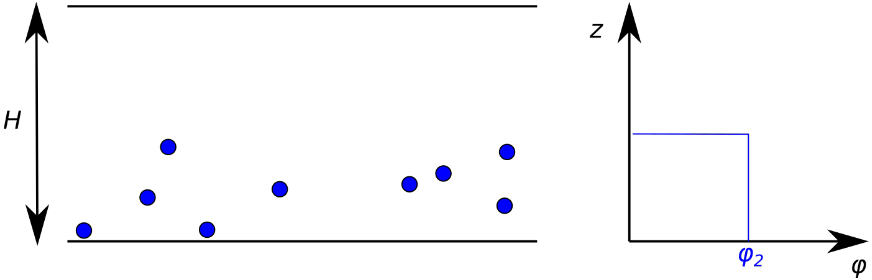
Volume of settling particles per unit area:

$$\phi_1 v_{s,1} \mathrm{d}t \qquad \phi_2 v_{s,2} \mathrm{d}t$$

$$\text{Ratio of population volumes} = \frac{\phi_1 v_{s,1}}{\phi_2 v_{s,2}}$$

Cumulate crystal size distribution: Static magma

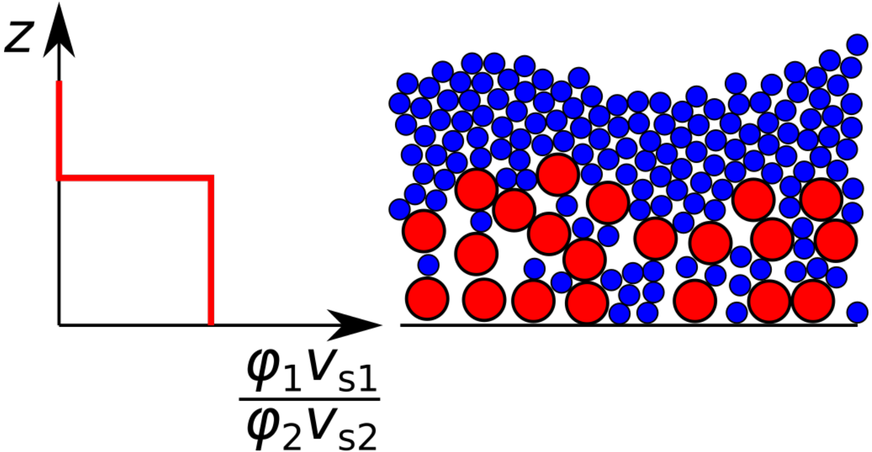
$$t > H/v_{s,1}$$



All of the coarse population has settled.

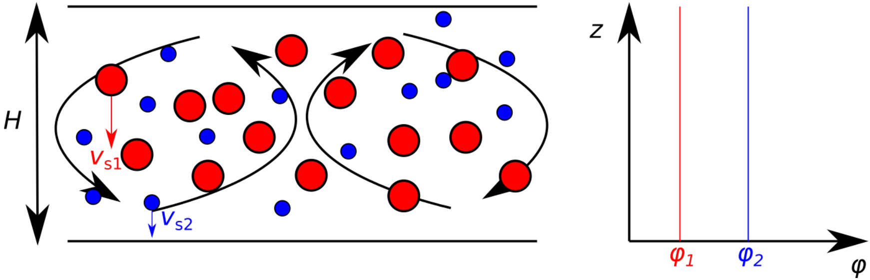
Cumulate crystal size distribution: Static magma

What does the cumulate look like?



Cumulate crystal size distribution: Convecting magma

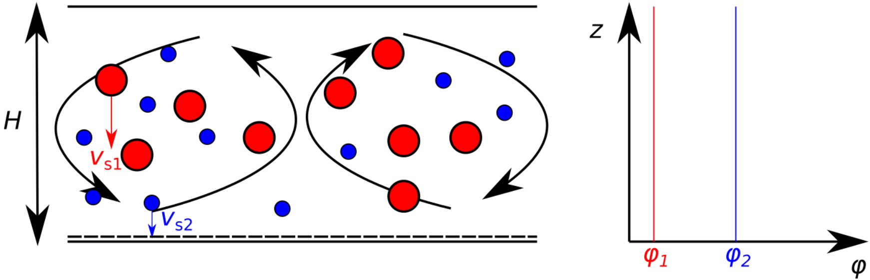
$t = 0$



Both populations are homogeneously dispersed throughout the sill

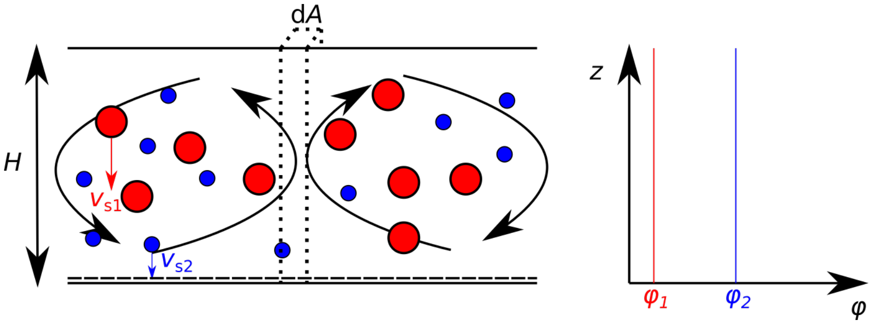
Cumulate crystal size distribution: Convecting magma

$$t = \mathrm{d}t$$



Convection keeps both populations homogeneously dispersed
Viscous boundary layer at base of sill from which crystals cannot escape

Cumulate crystal size distribution: Convecting magma



Depth-integrated volume of suspended particles:

$$H\phi_1 \mathrm{d}A \qquad\qquad H\phi_2 \mathrm{d}A$$

Volume of particles settling into viscous boundary layer, and therefore sedimenting, in interval $\mathrm{d}t$

$$v_{s,1}\phi_1 \mathrm{d}t\mathrm{d}A \qquad\qquad v_{s,2}\phi_2 \mathrm{d}t\mathrm{d}A$$

Cumulate crystal size distribution: Convecting magma

$$\mathrm{d}(H\phi_1 \mathrm{d}A) = -v_{s,1}\phi_1 \mathrm{d}t\mathrm{d}A \qquad \mathrm{d}(H\phi_2 \mathrm{d}A) = -v_{s,2}\phi_2 \mathrm{d}t\mathrm{d}A$$

$$H\mathrm{d}A\mathrm{d}\phi_1 = -v_{s,1}\phi_1 \mathrm{d}t\mathrm{d}A \qquad H\mathrm{d}A\mathrm{d}\phi_2 = -v_{s,2}\phi_2 \mathrm{d}t\mathrm{d}A$$

$$\frac{\mathrm{d}\phi_1}{\mathrm{d}t} = -\frac{v_{s,1}\phi_1}{H} \qquad \frac{\mathrm{d}\phi_2}{\mathrm{d}t} = -\frac{v_{s,2}\phi_2}{H}$$

Linear, first-order, ordinary differential equation $\implies$ exponential solution

$$\phi_1 = \alpha_1 \exp\left(-\frac{v_{s,1}t}{H}\right) + \beta_1 \qquad \phi_2 = \alpha_2 \exp\left(-\frac{v_{s,2}t}{H}\right) + \beta_2$$

Boundary conditions:

$$\phi_1(t \to \infty) \to 0 \implies \beta_1 = 0 \qquad \phi_2(t \to \infty) \to 0 \implies \beta_2 = 0$$

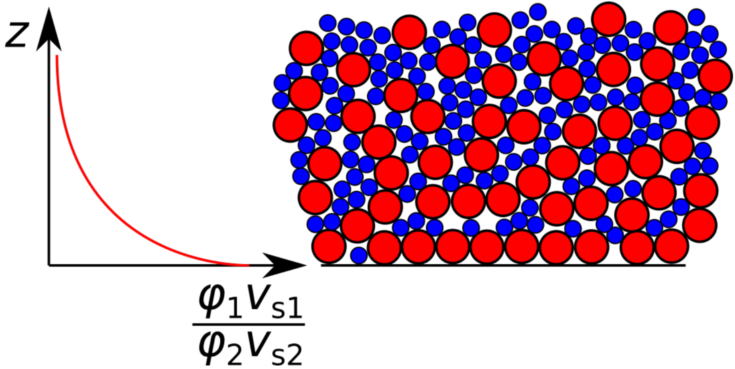
$$\phi_1(t = 0) = \phi_{1,0} \implies \alpha_1 = \phi_{1,0} \qquad \phi_2(t = 0) = \phi_{2,0} \implies \alpha_2 = \phi_{2,0}$$

Cumulate crystal size distribution: Convecting magma

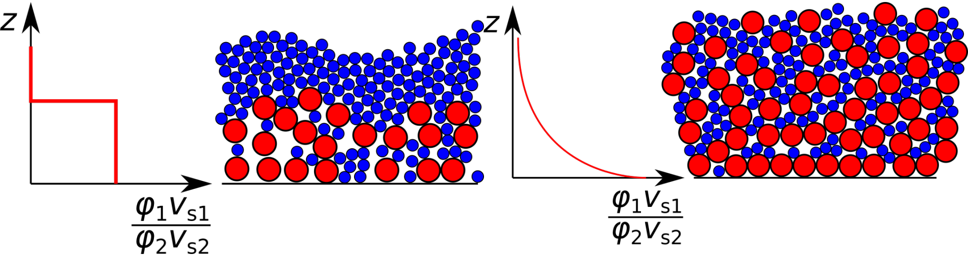
Ratio of population volumes:

$$\frac{\phi_1 v_{s,1}}{\phi_2 v_{s,2}} = \frac{\phi_{1,0} v_{s,1}}{\phi_{2,0} v_{s,2}} \exp\left(-\frac{(v_{s,1} - v_{s,2})t}{H}\right)$$

What does the cumulate look like?



Can use crystal size distributions to distinguish if cumulates formed from static or convecting magmas

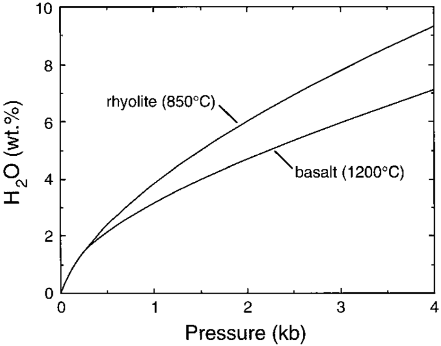


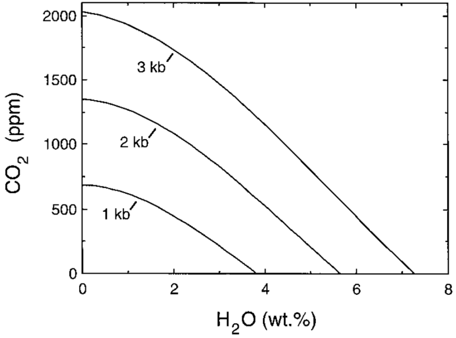
Model has many simplifying assumptions, e.g.:

- Two crystal populations
- No crystallisation during settling
- No cooling of the sill

As magma rises, pressure falls and bubble solubility decreases

Solubility - Amount of substance that can be dissolved in a mixture

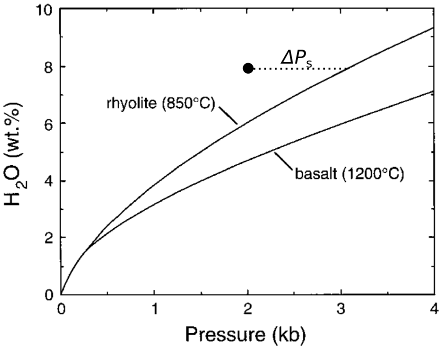




If volatile concentrations exceed solubility, then magma is **supersaturated**

Bubble formation - Supersaturation

Supersaturation - Difference between actual pressure, and that at which concentration of dissolved volatiles would be in equilibrium



Nucleation - Process by which bubbles initially form

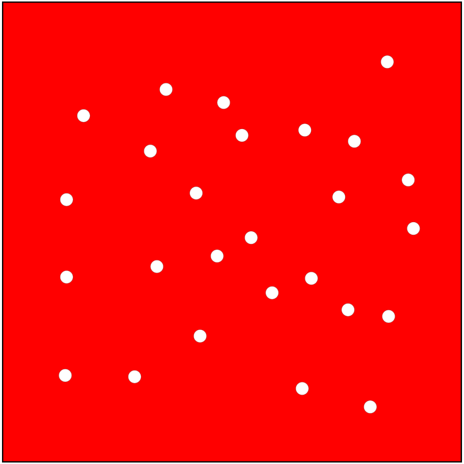
Nucleation creates an interface between melt and volatile

Interfacial tension - Energy created to create an interface between two substances

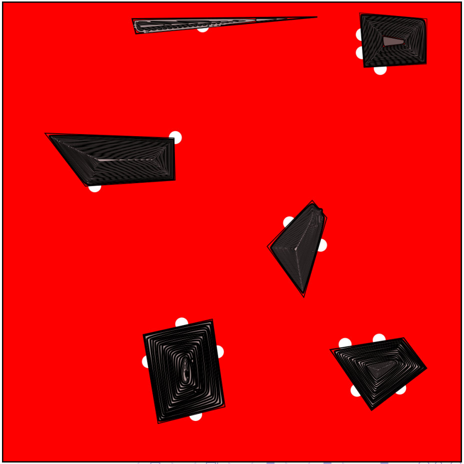
Required amount of supersaturation corresponds to energy needed

Two types of nucleation:

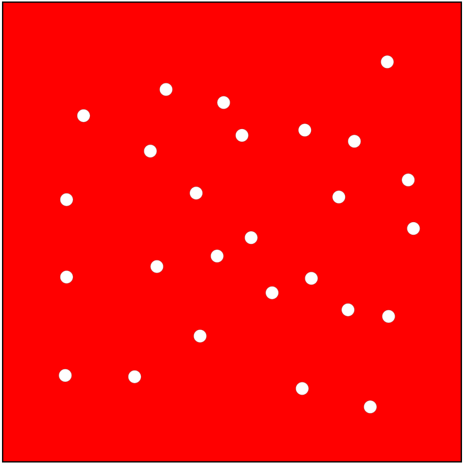
Homogeneous



Heterogeneous



Bubble formation - Homogenous nucleation

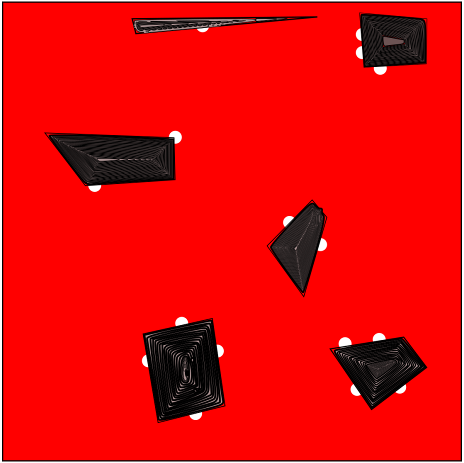


Occurs in the absence of crystals

Bubbles nucleate in the melt

Requires supersaturation of $\sim$ 10-100 MPa

Bubble formation - Heterogeneous nucleation



Interfacial energy between vapour and crystal less than that between vapour and melt

Bubbles nucleate on crystals

Requires supersaturation of $\sim$ 1-10 MPa

$\Longrightarrow$ in presence of crystals, nucleation will almost always be heterogeneous

Bubble growth - Mass and momentum conservation

As pressure decreases, bubbles grow due to:

- Direct decompression
- Increasing supersaturation

Model bubble growth according using conservation of momentum and mass:

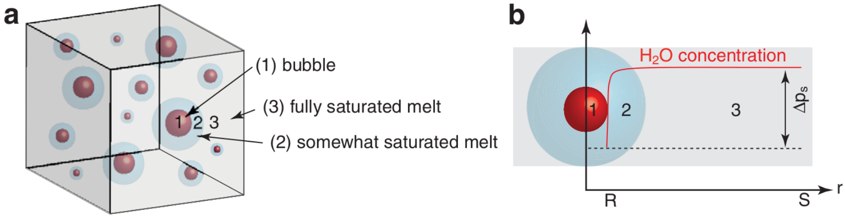
$$\frac{4\eta_{\mathrm{m}}}{r_{\mathrm{b}}}\frac{\mathrm{d}r_{\mathrm{b}}}{\mathrm{d}t} = P_{\mathrm{b}} - P_{\mathrm{m}} - \frac{2\sigma}{r_{\mathrm{b}}}$$

$$\frac{\mathrm{d}(\rho_{\mathrm{b}} r_{\mathrm{b}}^3)}{\mathrm{d}t} = 4 r_{\mathrm{b}}^2 \rho_{\mathrm{m}} \sum_i D_i \left.\frac{\partial C_i}{\partial r}\right|_{r=r_{\mathrm{b}}}$$

$P_{\mathrm{m(b)}}$ = Pressure in the melt (bubble)
σ = Interfacial tension
C_i = Concentration of volatile species in melt, e.g., X_{H_2O}, X_{CO_2}
D_i = Diffusion coefficient of each species

Bubble growth - Diffusion in the melt

Also need to model transport of volatiles to the bubbles



Gonnermann & Gardner (2013)

$$\frac{\partial C_i}{\partial t} + \mathbf{u} \cdot \nabla C_i = D_i \nabla^2 C_i$$

Exploit spherical symmetry:

$$\frac{\partial C_i}{\partial t} + u_r \frac{\partial C_i}{\partial r} = \frac{D_i}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial C_i}{\partial r} \right)$$

Boundary conditions:

$$C_i(r = r_\mathrm{b}) = C_{\mathrm{s},i}, \quad \left. \frac{\partial C_i}{\partial r} \right|_{r=S} = 0$$

Bubble growth: Viscosity-limited growth

Assumption: Only consider H_2O

$$\frac{4\eta_m}{r_b}\frac{\mathrm{d}r_b}{\mathrm{d}t} = P_b - P_m - \frac{2\sigma}{r_b}$$

$$\frac{\mathrm{d}(\rho_b r_b^3)}{\mathrm{d}t} = 4 r_b^2 \rho_m D_{H_2O} \left.\frac{\partial C_{H_2O}}{\partial r}\right|_{r=r_b}$$

$$\frac{\partial C_{H_2O}}{\partial t} + u_r \frac{\partial C_{H_2O}}{\partial r} = \frac{D_{H_2O}}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial C_{H_2O}}{\partial r}\right)$$

- Melt viscosity is sufficiently high to slow down bubble expansion
- Leads to large supersaturation and build up of over-pressure in bubbles (mechanical disequilibrium)
- Significant for $\eta_m \geq 10^9$ Pa s (silicic melts at shallow depths and low X_{H_2O}

Bubble growth: Diffusion-limited growth

Assumption: Only consider H_2O

$$\frac{4\eta_m}{r_b}\frac{\mathrm{d}r_b}{\mathrm{d}t} = P_b - P_m - \frac{2\sigma}{r_b}$$

$$\frac{\mathrm{d}(\rho_b r_b^3)}{\mathrm{d}t} = 4 r_b^2 \rho_m D_{H_2O} \left.\frac{\partial C_{H_2O}}{\partial r}\right|_{r=r_b}$$

$$\frac{\partial C_{H_2O}}{\partial t} + u_r \frac{\partial C_{H_2O}}{\partial r} = \frac{D_{H_2O}}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial C_{H_2O}}{\partial r}\right)$$

- Melt diffusivity is too low for oversaturated volatiles to diffuse to pre-existing bubbles (chemical disequilibrium)
- Leads to nucleation at the expense of growth
- Results in many small bubbles

Bubble growth: Solubility-limited growth

Assumption: Only consider H_2O

$$\frac{4\eta_\mathrm{m}}{r_\mathrm{b}}\frac{\mathrm{d}r_\mathrm{b}}{\mathrm{d}t} = P_\mathrm{b} - P_\mathrm{m} - \frac{2\sigma}{r_\mathrm{b}}$$

$$\frac{\mathrm{d}(\rho_\mathrm{b} r_\mathrm{b}^3)}{\mathrm{d}t} = 4 r_\mathrm{b}^2 \rho_\mathrm{m} D_{\mathrm{H_2O}} \left.\frac{\partial C_{\mathrm{H_2O}}}{\partial r}\right|_{r=r_\mathrm{b}}$$

$$\frac{\partial C_{\mathrm{H_2O}}}{\partial t} + u_r \frac{\partial C_{\mathrm{H_2O}}}{\partial r} = \frac{D_{\mathrm{H_2O}}}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial C_{\mathrm{H_2O}}}{\partial r}\right)$$

- Diffusivity high, and viscosity low, enough to allow mechanical and chemical equilibrium
- Bubbles can grow unhindered
- Favoured for low melt viscosity (hot, mafic) and low ascent rates

Bubble rise speed

Bubble rise speed can be estimated by assuming spherical shape and using Stokes law

$$v_{\mathrm{b}} = \frac{(\rho_{\mathrm{m}} - \rho_{\mathrm{b}}) g r_{\mathrm{b}}^2}{3\eta_{\mathrm{m}}}$$

Depends on:

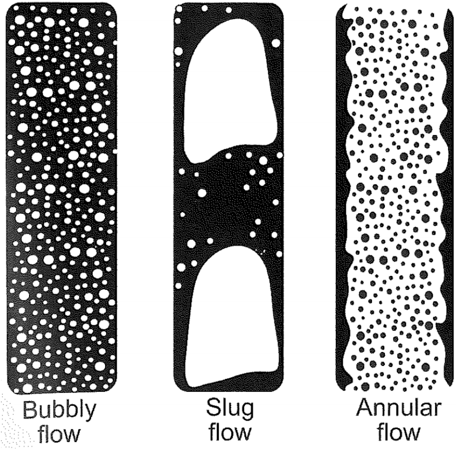
- ρ_{m} = Melt density
- ρ_{b} = Bubble density
- r_{b} = Bubble radius
- η_{m} = Melt viscosity

Other factors:

- Bubble shape
- Bubble concentration ϕ_{b}
- Crystal fraction ϕ_{c}

Bubble flow regimes

v_b = Bubble speed, v_m = Melt speed



If $v_b \ll v_m \implies$ **dispersed flow**:

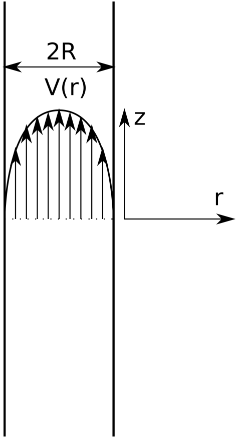
- **Bubbly flow**
- Bubbles dispersed
- Move as passive tracers

If $v_b \gtrsim v_m \implies$ **separated flow**

- $1 \lesssim v_b/v_m \lesssim 10 \implies$ **slug flow**
- $v_b/v_m \gtrsim 10 \implies$ **annular flow**

Flow regimes are observed for gas flow in a vertical pipe
Application to volcanic conduits remains debatable

Conduit flow



Flow driven by pressure gradient $\mathrm{d}P/\mathrm{d}z$
Velocity profile given by

$$\frac{\mathrm{d}V}{\mathrm{d}r} = \frac{r}{2\eta}\frac{\mathrm{d}P}{\mathrm{d}z}$$

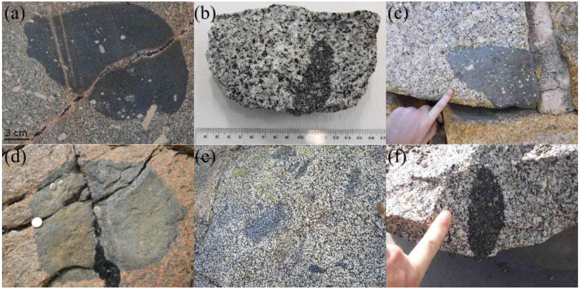
Friction with conduit walls means flow is fastest in centre
Model is valid if flow is NOT separated

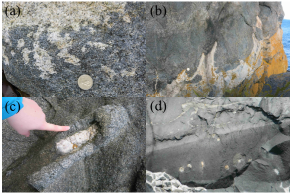
Fragmentation - During explosive eruptions, magma fragements to form **pyroclasts** - ash, lapilli, bombs

- Style of fragmentation depends on magma rheology
- In turn depends on $\phi_{\mathsf{c}}, \phi_{\mathsf{b}}, \eta_{\mathsf{m}}, \dot{\epsilon}$
- Controls style of eruption

Magma mixing and mingling - Magmas of different compositions juxtapose and interact

- Viscosity and density contrasts between magmas inhibit mixing
- Heat transfer fromm hot to cold magma associated with rheological changes
- Style of mixing changes with time





Summary

Magma transport processes can describe movement of bulk magma or individual phases

Various physical processes can be modeled, e.g.:

- Crystal fractionation
- Bubble formation, growth, rise
- Conduit flow
- Magma fragmentation
- Magma mixing and mingling

Models depend on properties and processes:

- Temperature
- Pressure
- Composition

- Fluid dynamics
- Heat transfer
- Phase equilibria