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Gravity currents

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Volcanic flows

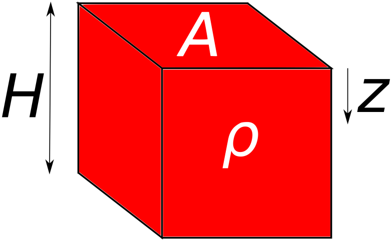
Lava flows

Cloud spreading

Pyroclastic density currents (PDCs)

Lahars

Hydrostatic gradients



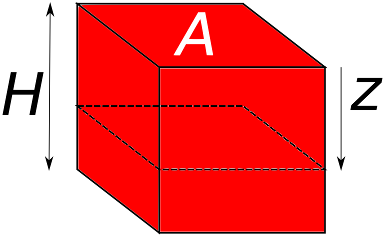
Consider a volume of fluid of :

- Density ρ
- Height H
- Horizontal cross section A

$z =$ Negative vertical coordinate (depth below top surface)

Consider horizontal plane at depth z
What are the forces acting on this plane?

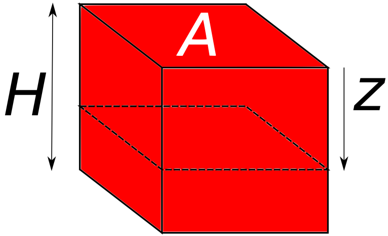
- **Weight** of overlying fluid
$$W = \rho A z g$$
- Balanced by **hydrostatic pressure**
$$F_{\mathrm{p}} = PA$$

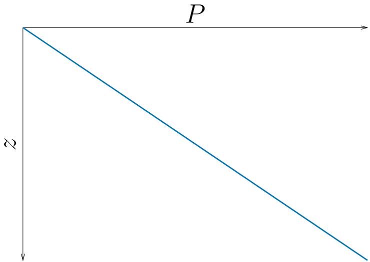


Nothing is moving $\implies$ **Mechanical equilibrium**

$$W = F_\mathrm{p}$$

$$P = \rho g z$$



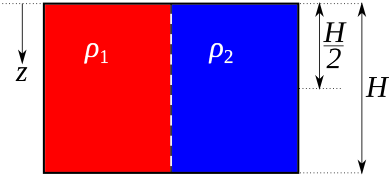


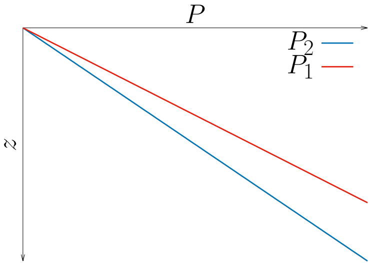
P increases linearly with z

Hydrostatic gradient:

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \rho g$$

Gravity current - A horizontal flow in a gravitational field that is driven by a density difference





Consider two fluids (densities ρ_1 and ρ_2, $\rho_1 > \rho_2$) initially side-by-side and separated by a vertical barrier

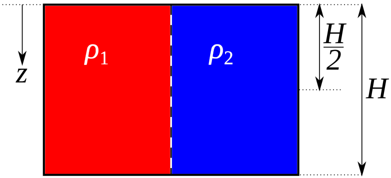
Pressure difference:

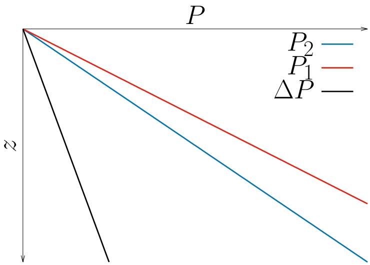
$$P_1 = \rho_1 g z$$

$$P_2 = \rho_2 g z$$

$$\Delta P = P_2 - P_1 = (\rho_2 - \rho_1) g z$$

Gravity currents - Horizontal force balance



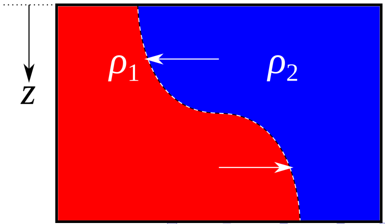


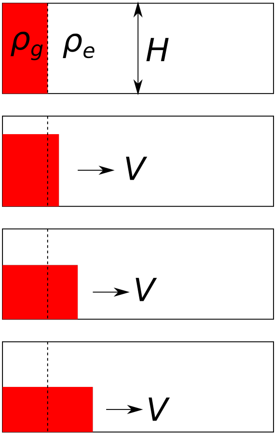
Remove barrier, and consider pressure difference ΔP across line

ΔP increases with depth

Flow follows a pressure gradient - but horizontal pressure gradient is greatest at the depth

This initiates from from high to low pressure at the base, which is compensated by return flow at the top





Assume:

- Volume of current is conserved
- Current is always rectangular
- Energy is always conserved*

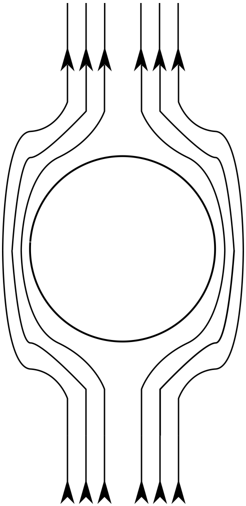
Energy balance between kinetic energy and change in GPE shows

$$V = \left(\frac{(\rho_c - \rho_e) g H}{2\rho_c} \right)^{1/2}$$

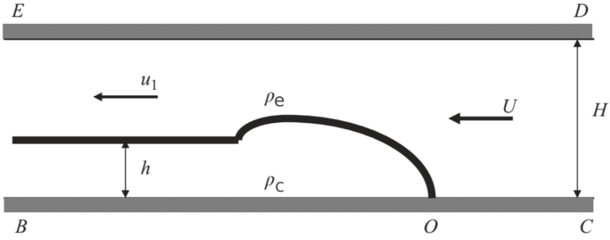
* Means assume energy only exists as kinetic or GPE. What other types of energy are there?

Fundamental fluid dynamics: Streamlines

Streamlines - Curves which are instantaneously tangent to the velocity field



Gravity currents - Benjamin's model



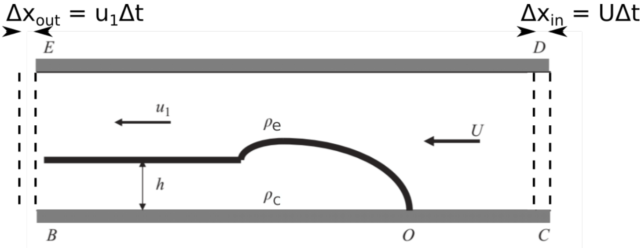
Modified from Shin et al. (2004)

Assumptions:

- Hydrostatic conditions upstream and and downstream of head
- No relative flow within current

Consider a reference frame moving with the head $\implies$ velocity inside current equals zero

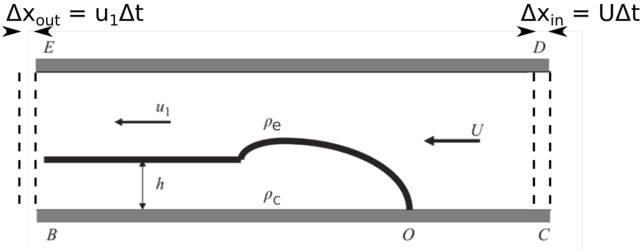
Gravity currents - Benjamin's model



Modified from Shin et al. (2004)

Let W = Width of channel
During time interval Δt, volume of fluid:

- entering domain $= V_{\text{in}} = \Delta x_{\text{in}} WH = U \Delta t WH$
- leaving domain $= V_{\text{out}} = \Delta x_{\text{out}} W(H-h) = u_1 \Delta t W(H-h)$



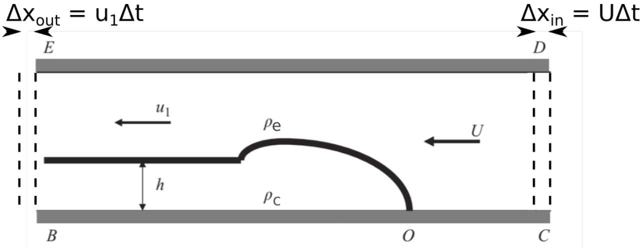
Modified from Shin et al. (2004)

During time interval Δt, mass of fluid

- entering domain $= m_{\text{in}} = \rho_{\text{e}} V_{\text{in}} = \rho_{\text{e}} U \Delta t W H$
- leaving domain $= m_{\text{out}} = \rho_{\text{e}} V_{\text{out}} = \rho_{\text{e}} u_1 \Delta t W (H - h)$

Mass conservation $\implies m_{\text{in}} = m_{\text{out}} \implies UH = u_1(H - h)$

Gravity currents - Benjamin's model



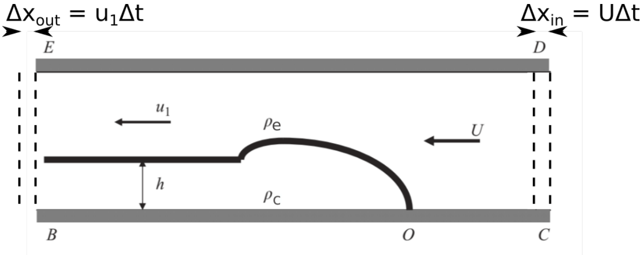
Modified from Shin et al. (2004)

Assumptions:

During time interval Δt, momentum of fluid:

- entering domain $= p_{\text{in}} = m_{\text{in}} U = \rho_{\text{e}} U^2 \Delta t W H$
- leaving domain $= p_{\text{out}} = m_{\text{out}} u_1 = \rho_{\text{e}} u_1^2 \Delta t W (H - h)$

Change in momentum $= \Delta p = p_{\text{in}} = p_{\text{out}} = [U^2 H - u_1^2 (H - h)] \rho_{\text{e}} \Delta t W$



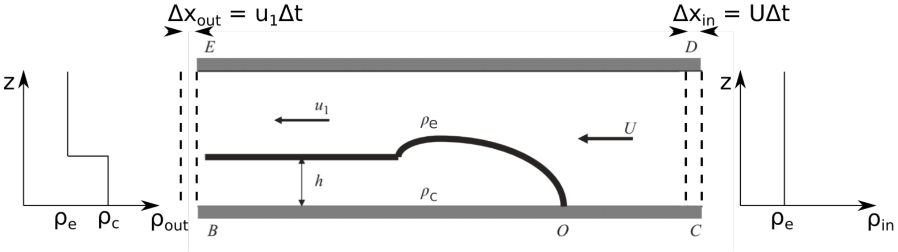
Modified from Shin et al. (2004)

Newton's second law: Force acting on substance = Rate of change of momentum

$$F = \frac{\Delta p}{\Delta t} = [U^2 H - u_1^2 (H - h)]\rho_e W$$

Force F depends on pressures at the inlet and outlet

Gravity currents - Benjamin's model



Modified from Shin et al. (2004)

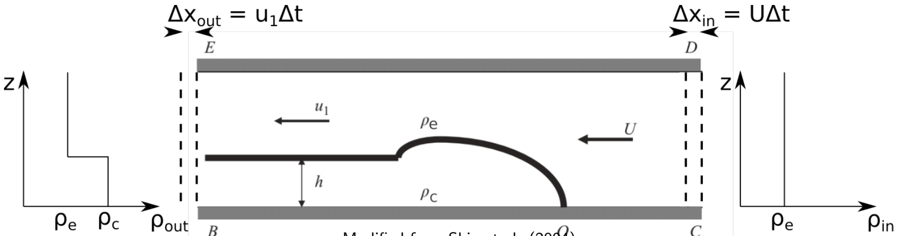
$$\frac{\mathrm{d}P_{\text{out}}}{\mathrm{d}z} = -g\rho_{\text{out}}$$

$$\frac{\mathrm{d}P_{\text{in}}}{\mathrm{d}z} = -g\rho_{\text{in}}$$

$$\rho_{\text{out}} = \begin{cases} \rho_{\text{c}} & z < h \\ \rho_{\text{e}} & z > h \end{cases}$$

$$\rho_{\text{in}} = \rho_{\text{e}}$$

Gravity currents - Benjamin's model



Modified from Shin et al. (2004)

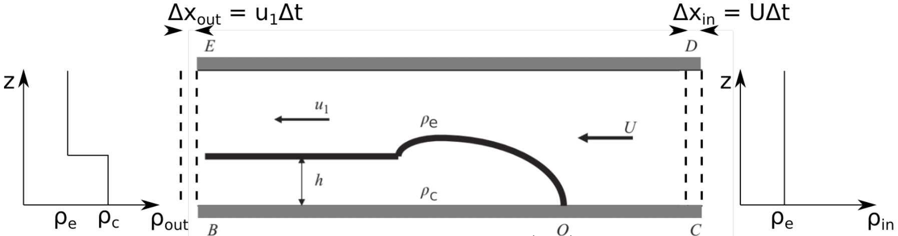
$$P_{\text{out}}(z) - P_{\text{B}} = -g\int_0^z \rho_{\text{out}}\mathrm{d}z$$

$$P_{\text{in}}(z) - P_{\text{C}} = -g\int_0^z \rho_{\text{in}}\mathrm{d}z$$

$$P_{\text{out}}(z) = P_{\text{B}} - g\begin{cases} \rho_{\text{c}}\int_0^z \mathrm{d}z & z < h \\ \rho_{\text{c}}\int_0^h \mathrm{d}z + \rho_{\text{e}}\int_h^z \mathrm{d}z & z > h \end{cases}$$

$$P_{\text{in}}(z) = P_{\text{C}} - g\rho_{\text{e}}z$$

$$P_{\text{out}}(z) = P_{\text{B}} - g\begin{cases} \rho_{\text{c}}z & z < h \\ \rho_{\text{c}}h + \rho_{\text{e}}(z-h) & z > h \end{cases}$$



Modified from Shin et al. (2004)

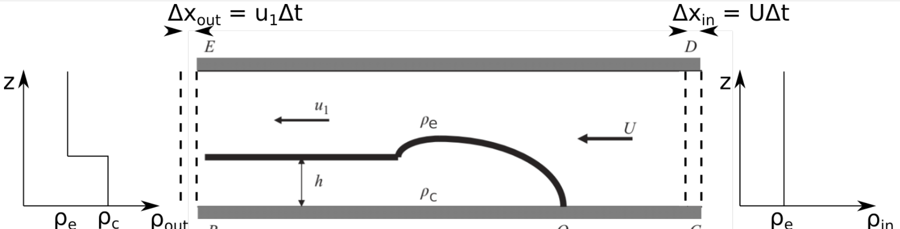
$$F_{\text{out}} = \int P_{\text{out}} \mathrm{d}A \qquad F_{\text{in}} = \int P_{\text{in}} \mathrm{d}A$$

$$\mathrm{d}A = W \mathrm{d}z$$

$$F = F_{\text{in}} - F_{\text{out}} = W \int_0^H (P_{\text{in}} - P_{\text{out}}) \mathrm{d}z$$

$$F = W \left(\int_0^h (P_{\text{in}} - P_{\text{out}}) \mathrm{d}z + \int_h^H (P_{\text{in}} - P_{\text{out}}) \mathrm{d}z \right)$$

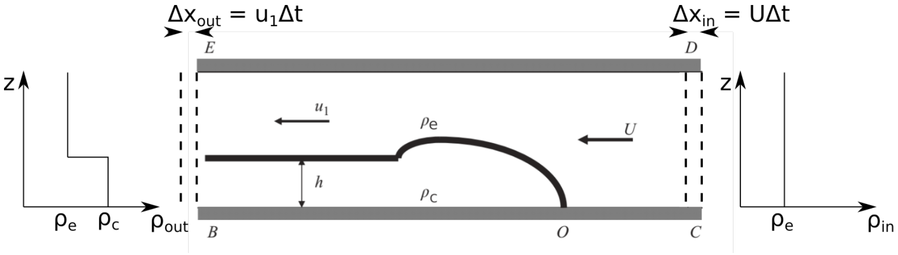
Gravity currents - Benjamin's model



Modified from Shin et al. (2004)

$$F = W\bigg(\int_0^h (P_C - g\rho_e z - P_B + g\rho_c z)\mathrm{d}z + \int_h^H [P_C - g\rho_e z - P_B + g\rho_c h + g\rho_e(z-h)]\mathrm{d}z\bigg)$$

$$F = W\bigg((P_C - P_B)h + \frac{g(\rho_c - \rho_e)h^2}{2} + [P_C - P_B + g(\rho_c - \rho_e)h](H-h)\bigg)$$

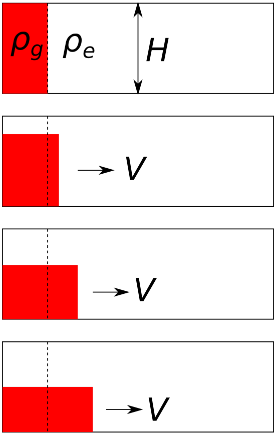


Modified from Shin et al. (2004)

$$F = W\left[(P_C - P_B)H + g(\rho_c - \rho_e)h\left(H - \frac{h}{2}\right)\right]$$

$$W\left[(P_C - P_B)H + g(\rho_c - \rho_e)h\left(H - \frac{h}{2}\right)\right] = [U^2H - u_1^2(H-h)]\rho_e W$$

Gravity currents - Reduced gravity



If envrionment is air, then often $\rho_\mathrm{e} \ll \rho_\mathrm{c}$

$$V = \left(\frac{(\rho_\mathrm{c} - \rho_\mathrm{e})gH}{2\rho_\mathrm{c}}\right)^{1/2} \approx \frac{(gH)^{1/2}}{2}$$

Therefore, we see that $(\rho_\mathrm{c} - \rho_\mathrm{e})/\rho_\mathrm{c}$ is factor by which envrionmental fluid reduced gravitational acceleration

Define **reduced gravity**:

$$g' = \frac{\rho_\mathrm{c} - \rho_\mathrm{e}}{\rho_\mathrm{c}}$$

So generally:

$$V = \frac{(g'H)^{1/2}}{2}$$

Gravity currents - Froude number

Froude number:

$$\mathrm{Fr} = \frac{V}{(g'H)^{1/2}}$$

Represents ratio of inertial (U) to buoyancy ($(g'H)^{1/2}$) forces

For an ideal gravity current $V = (g'H)^{1/2}/2$ so:

$$\mathrm{Fr} = \frac{1}{2}$$

Measuring Fr gives a means of testing model assumptions

Dimensionless numbers

Froude	Fr	$\frac{V}{(g'H)^{1/2}}$	Inertia and buoyancy forces
Capillary	Ca	$\frac{\eta Vr}{\sigma}$	Viscous and surface tension forces

Can use values of these quantities to define different fluid dynamical regimes

Reynolds number - Ratio of inertial and viscous forces within a fluid

$$\mathrm{Re} = \frac{\rho VL}{\eta}$$

ρ = Fluid density
V = Fluid velocity

L = Lengthscale
η = Viscosity

Reynolds number

$$\mathrm{Re} = \frac{\rho V L}{\eta}$$

$\mathrm{Re} \ll \mathrm{Re_c}$

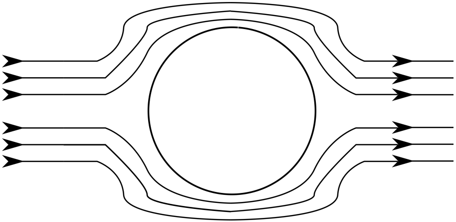
Laminar flow

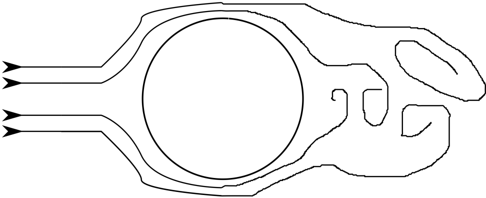
Fluid particles follow smooth paths in layers, with little or no mixing between different layers

$\mathrm{Re} \gg \mathrm{Re_c}$

Turbulent flow

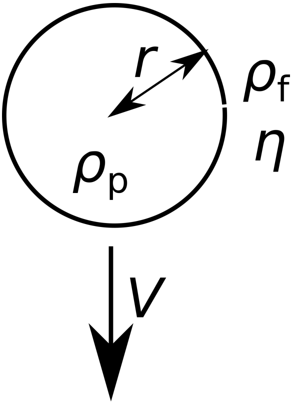
Chaotic changes in pressure and flow velocity, generating unsteady vortices



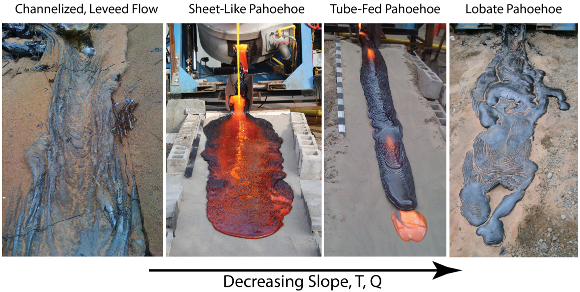


Reynolds number

What are the Reynolds numbers for these scenarios?



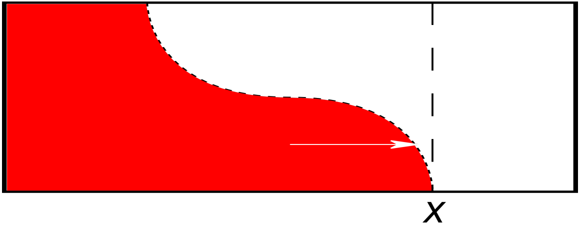
Low-Reynolds number gravity currents



For fixed release volume, viscous dissipation continuosly reduces V

Varying slope angle and flow rate produces different flow morphologies

For a 2D current:

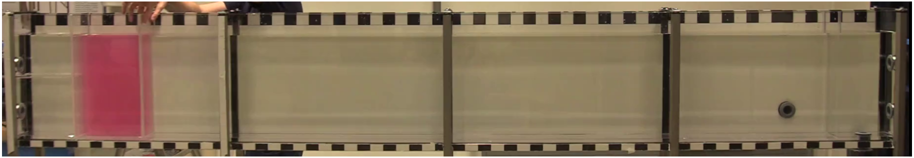


$$x \sim t^{4/5}$$

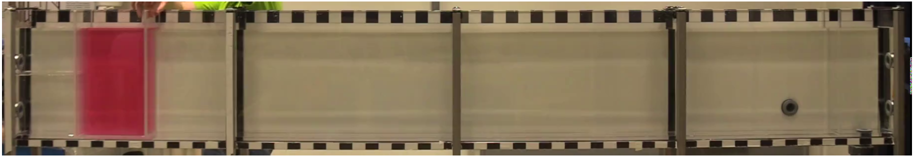
High-Reynolds number gravity currents

For Re $>$ 1000, Re has no effect on flow
Can neglect viscous dissipation $\implies V = (g'H)^{1/2}/2$

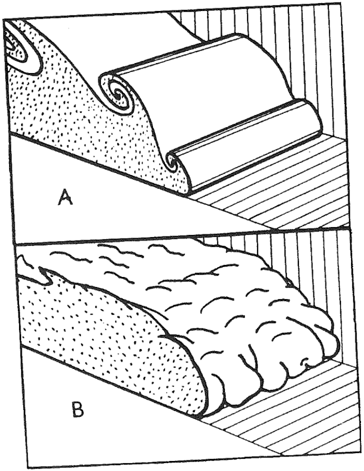
$g' = 0.06$



$g' = 0.16$



High-Reynolds number gravity currents - mixing

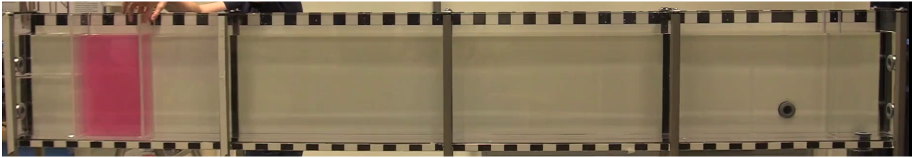


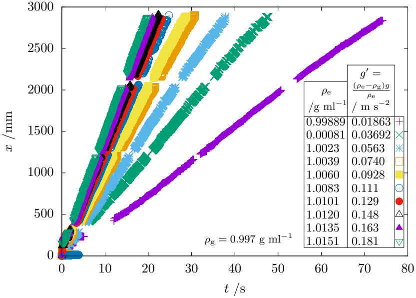
Mixing in fluid reduced density contrast

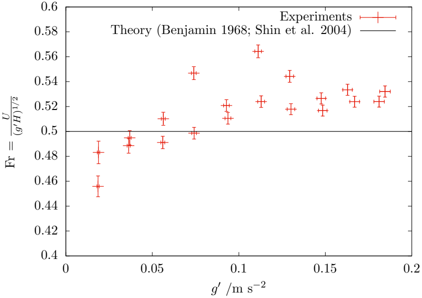
Two types of mixing:

- **Billowing** - Formed by shear between current and environment
- **Lobes and clefts** - Formed by interaction with ground at contact line

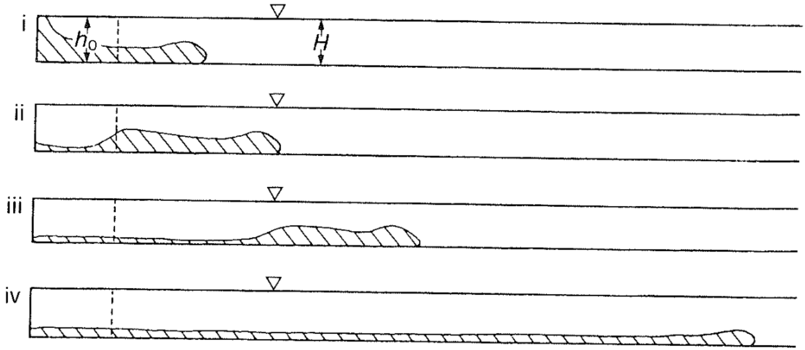
Constant volume release - Constant speed phase







Constant volume release - Self-similar phase

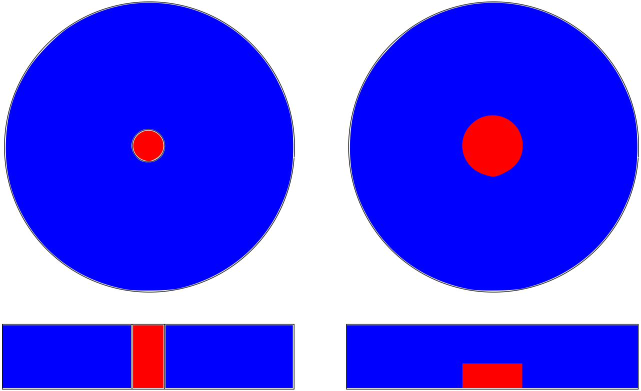


Constant speed | Self-similar | Viscous

$x \sim t$ | $x \sim t^{2/3}$ | $x \sim t^{1/5}$

Constant volume release - Radial collapse

$$r \sim t^{1/2}$$



Constant radial volume flow

$$r \sim Q^{1/3} t^{1/2}$$