

Magma viscosity and density

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1. Table 1 lists bulk rock compositions as measured for erupted lavas from various volcanoes.

Table 1: Dry, bulk rock compositions for various volcanoes (wt%). Two different lavas were erupted at Unzen in the 1991-1995 eruption.

Volcano	Eruption	SiO ₂	Al ₂ O ₃	TiO ₂	FeO	Fe ₂ O ₃	MnO	MgO	CaO	K ₂ O	Na ₂ O	P ₂ O ₅
Unzen ¹	1991-1995	64.27	16.13	0.71	0.00	5.10	0.10	2.47	4.69	2.54	3.81	0.17
Unzen ²	1991-1995	51.84	18.19	1.28	0.00	10.27	0.18	4.62	9.43	1.22	2.79	0.17
Kilauea	1992	49.78	12.79	2.37	0.00	12.40	0.17	8.75	10.59	0.41	2.14	0.22
Novarupta	1912	77.19	12.28	0.18	1.3	0.0	0.05	0.19	0.87	3.17	4.32	0.05

- (a) Use Figure 1 to classify the composition of the rocks in Table 1.
- (b) Create a new table converting the Novarupta composition shown in Table 1 to mol%. Make use of Fig. 2 and

$$X_i(\text{mol}\%) = \frac{100X_i(\text{wt}\%)}{M_i} \left(\sum_i \frac{X_i(\text{wt}\%)}{M_i} \right)^{-1}, \quad (1)$$

where M_i is the molar mass of component X_i .

Table 2 shows the liquidus temperatures of the magmas in Table 1 for both dry composition and 4 wt% water at 2000 MPa.

- (c) For the Novarupta volcano, determine the wet melt composition, with 4 wt% H₂O, (in mol%) at the wet liquidus temperature at 2000 MPa.

The volume V_m of 1 mol of melt of composition \mathbf{X} as a function of pressure P and temperature T is given by

$$V_m(T, P, \mathbf{X}) = \sum_i X_i(\text{mol}\%) \left[\bar{V}_i(T = T_R, P = P_R) + \frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \Big|_{T=T_R} (T - T_R) + \frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \Big|_{P=P_R} (P - P_R) \right], \quad (2)$$

where $T_R = 1673$ K and $P_R = 10^{-4}$ GPa. Values for the red, blue and green quantities can be found in Table 3. The density can then be calculated from

$$\rho_m = \frac{1}{V_m} \sum_i X_i(\text{mol}\%) M_i. \quad (3)$$

- (d) For the Novarupta volcano, determine the density of the **dry** melt composition at the **dry** liquidus temperature, at 2000 MPa. Take care with units and show your method clearly (it may be helpful to draw a table).
- (e) For the same volcano, determine the density of the **wet** melt composition at the **dry** liquidus temperature, at 2000 MPa.
- (f) Again for Novarupta, determine the density of the **wet** melt composition at the **wet** liquidus temperature, at 2000 MPa.

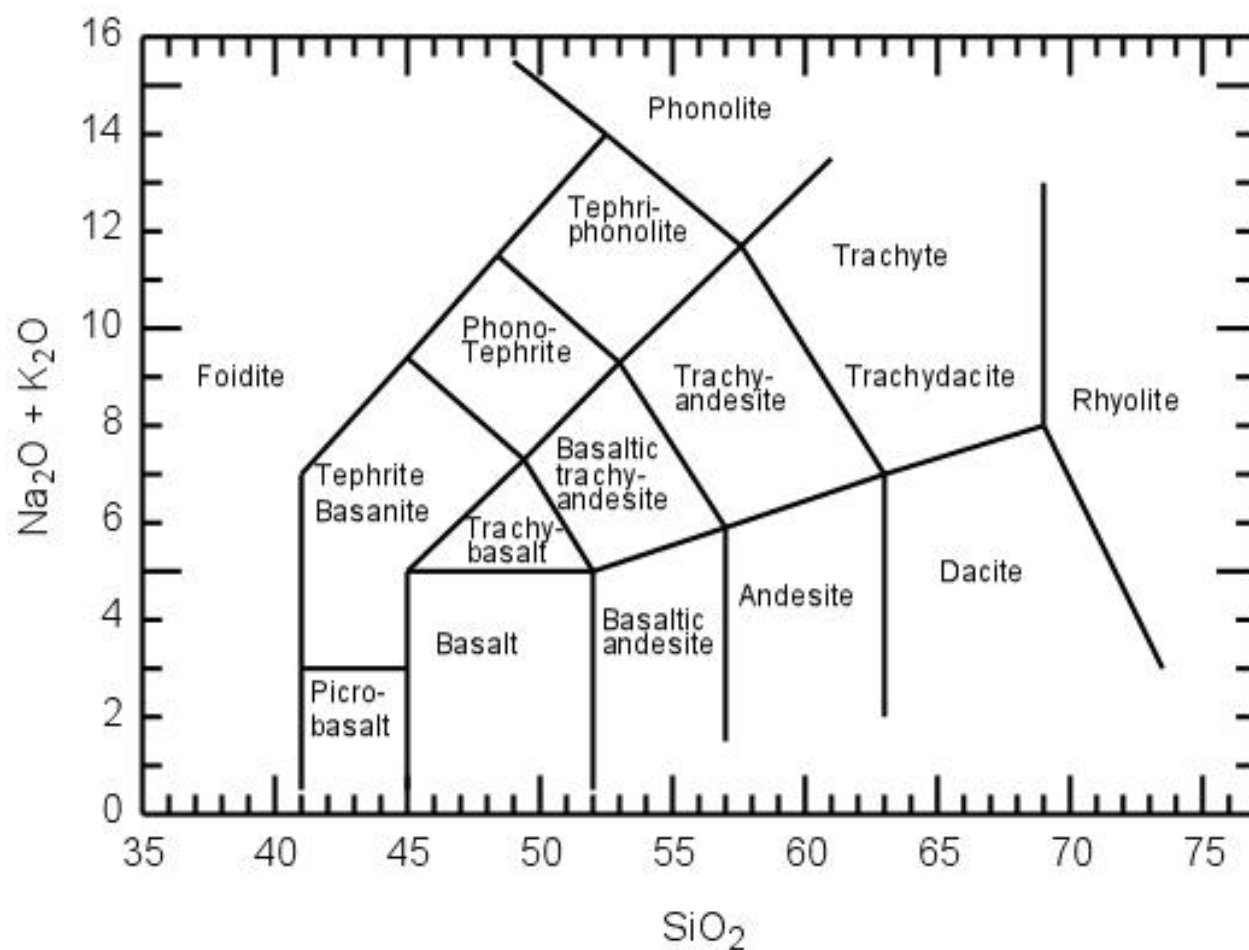


Figure 1: Total Alkali-Silica (TAS) diagram for characterisation of volcanic rocks.

Table 2: Liquidus temperatures for rocks shown in Table 1 at 2000 MPa for dry and wet (4 wt% water) compositions.

Volcano	Eruption	Dry Liquidus T /°C	Wet Liquidus T /°C
Unzen ¹	1991-1995	1148	1043
Unzen ²	1991-1995	1226	1079
Kilauea	1992	1237	1172
MORB	N/A	1215	1145
Novarupta	1912	1123	827

Periodic Table of the Elements

1																	18				
1A																	8A				
2																	2				
3	4											10	11	12	13	14	15	16	17	18	
IA	IIA											IIIA	IVA	VA	VIA	VIIA	VIIIA	VIIIA	VIIIA	VIIIA	
1	2											3	4	5	6	7	8	9	10	11	12
Hydrogen	Helium											Lithium	Beryllium	Boron	Carbon	Nitrogen	Oxygen	Fluorine	Neon	Sodium	Magnesium
1.008	4.003											6.941	9.012	10.811	12.011	14.007	15.999	18.998	20.180	22.990	24.305
3	4											5	6	7	8	9	10	11	12	13	14
Li	Be											B	C	N	O	F	Ne	Al	Si	P	S
Lithium	Beryllium											Boron	Carbon	Nitrogen	Oxygen	Fluorine	Neon	Aluminum	Silicon	Phosphorus	Sulfur
6.941	9.012											10.811	12.011	14.007	15.999	18.998	20.180	26.982	28.086	30.974	32.066
11	12											13	14	15	16	17	18	19	20	21	22
Na	Mg											Al	Si	P	S	Cl	Ar	K	Ca	Sc	Ti
Sodium	Magnesium											Aluminum	Silicon	Phosphorus	Sulfur	Chlorine	Argon	Potassium	Calcium	Scandium	Titanium
22.990	24.305											26.982	28.086	30.974	32.066	35.453	39.948	40.078	44.956	47.867	
19	20											21	22	23	24	25	26	27	28	29	30
K	Ca											Sc	Ti	V	Cr	Mn	Fe	Co	Ni	Cu	Zn
Potassium	Calcium											Scandium	Titanium	Vanadium	Chromium	Manganese	Iron	Cobalt	Nickel	Copper	Zinc
39.098	40.078											44.956	47.867	50.942	51.996	54.938	55.845	58.933	58.693	63.546	65.38
37	38											39	40	41	42	43	44	45	46	47	48
Rb	Sr											Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd
Rubidium	Strontium											Yttrium	Zirconium	Niobium	Molybdenum	Technetium	Ruthenium	Rhodium	Palladium	Silver	Cadmium
85.468	87.62											88.906	91.224	92.906	95.94	98.907	101.07	102.906	106.42	107.868	112.414
55	56											57-71	72	73	74	75	76	77	78	79	80
Cs	Ba												Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg
Cesium	Barium												Hafnium	Tantalum	Tungsten	Rhenium	Osmium	Iridium	Platinum	Gold	Mercury
132.905	137.328												178.49	180.948	183.84	186.207	190.23	192.217	195.085	196.967	200.592
87	88											89-103	104	105	106	107	108	109	110	111	112
Fr	Ra												Rf	Db	Sg	Bh	Hs	Mt	Ds	Rg	Cn
Francium	Radium												Rutherfordium	Dubnium	Seaborgium	Bohrium	Hassium	Meitnerium	Darmstadtium	Roentgenium	Copernicium
223.020	226.025												[261]	[262]	[266]	[264]	[269]	[278]	[281]	[280]	[285]
Lanthanide Series																					
57	58	59	60	61	62	63	64	65	66	67	68	69	70	71							
La	Ce	Pr	Nd	Pm	Sm	Eu	Gd	Tb	Dy	Ho	Er	Tm	Yb	Lu							
Lanthanum	Cerium	Praseodymium	Neodymium	Promethium	Samarium	Europium	Gadolinium	Terbium	Dysprosium	Holmium	Erbium	Thulium	Ytterbium	Lutetium							
138.905	140.116	140.908	144.243	144.913	150.36	151.964	157.25	158.925	162.500	164.930	167.259	168.934	173.055	174.967							
Actinide Series															103						
89	90	91	92	93	94	95	96	97	98	99	100	101	102	103							
Ac	Th	Pa	U	Np	Pu	Am	Cm	Bk	Cf	Es	Fm	Md	No	Lr							
Actinium	Thorium	Protactinium	Uranium	Neptunium	Plutonium	Americium	Curium	Berkelium	Californium	Einsteinium	Fermium	Mendelevium	Nobelium	Lawrencium							
227.028	232.038	231.036	238.029	237.048	244.064	243.061	247.070	247.070	251.080	[254]	[257]	[258]	[259]	[262]							

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Figure 2: Periodic table of elements.

Table 3: Partial molar volume, thermal expansions and compressibilities of oxide components

	$\bar{V}_i(T = T_R, P = P_R)$ /10 ⁻⁶ m ³ mol ⁻¹	$\left. \frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \right _{T=T_R}$ /10 ⁻⁹ m ³ mol ⁻¹ K ⁻¹	$\left. \frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \right _{P=P_R}$ /10 ⁻⁶ m ³ mol ⁻¹ GPa ⁻¹
SiO ₂	26.86	0.0	-1.89
TiO ₂	23.16	7.24	-2.31
Al ₂ O ₃	37.42	0.0	-2.31
Fe ₂ O ₃	42.13	9.09	-2.53
FeO	13.65	2.92	-0.45
MgO	11.69	3.27	0.27
CaO	16.53	3.74	0.34
Na ₂ O	28.88	7.68	-2.40
K ₂ O	45.07	12.08	-6.75
Li ₂ O	16.85	5.25	-1.02
H ₂ O	26.27	9.46	-3.15
CO ₂	33.0	0.0	0.0

- (g) Describe the effect of both temperature and water content on the density of the magmatic melt at Novarupta. Be as quantitative as you can.
2. At a temperature of 767 °C and pressure of 2000 MPa, the equilibrium crystal assemblage of the wet Novarupta magma is feldspar ($\phi_{\text{fld}} = 0.5, \rho_{\text{fld}} = 2.5 \text{ g cm}^{-3}$), quartz ($\phi_{\text{qz}} = 0.3, \rho_{\text{qz}} = 2.5 \text{ g cm}^{-3}$), spinel ($\phi_{\text{sp}} = 0.007, \rho_{\text{sp}} = 4.9 \text{ g cm}^{-3}$) and biotite ($\phi_{\text{bio}} = 0.5, \rho_{\text{bio}} = 2.75 \text{ g cm}^{-3}$) whilst the melt density is 2.21 g cm^{-3} and the melt viscosity is 147910.8 Pa s . Additionally, the gravitational settling velocity of a crystal of size d and density ρ_c in melt of density ρ_m and viscosity η_m is given by

$$v_s = \frac{(\rho_c - \rho_m)gd^2}{18\eta_m}, \quad (4)$$

where $g = 9.81 \text{ m s}^{-2}$ is the gravitational acceleration.

- (a) Assuming the crystals are $500 \mu\text{m}$ in size, calculate the settling velocities of each crystal phase in the Novarupta magma.

For the wet Kilauea composition, the equilibrium assemblage at the same pressure is olivine ($\phi_{\text{ol}} = 0.03, \rho_{\text{ol}} = 3.37 \text{ g cm}^{-3}$), orthopyroxene ($\phi_{\text{opx}} = 0.006, \rho_{\text{opx}} = 3.28 \text{ g cm}^{-3}$), clinopyroxene ($\phi_{\text{cpx}} = 8.21, \rho_{\text{cpx}} = 3.29 \text{ g cm}^{-3}$) and spinel ($\phi_{\text{sp}} = 0.02, \rho_{\text{sp}} = 4.9 \text{ g cm}^{-3}$), with a melt density of 2.31 g cm^{-3} and a melt viscosity of 28.1 Pa s .

- (b) Calculate the settling velocities of each crystal phase in the Kilauea magma.
- (c) It has been suggested that crystal cumulates can form at the base of magma chambers of sills by gravitational settling of crystals. Consider 10 m thick sills of both the Novarupta and Kilauea compositions. If the sills were maintained at constant temperatures, how long would it take for such a cumulate to form in each case? What crystals would they contain?
- (d) What are the most important parameters in determining whether such cumulates can form? Also consider processes which have been neglected here.
3. The 1991-1995 eruption of Mt. Unzen, Japan, produced a series of lava domes. A sample of this lava can be seen in Figure 3. The white rock (Unzen¹ in Table 1) forms most of the lava, whilst the dark rock (Unzen²) is present as a many discrete enclaves existing within the white rock. These enclaves are interpreted to have formed prior to eruption, when a hotter magma was injected into a cooler magma storage region.

An empirical model for the viscosity of melt η_m as a function of temperature T and composition \mathbf{X} is (Giordano et al., 2008)



Figure 3: Sample of dome lava erupted from Unzen in the 1991-1995 eruption. The bulk of the lava is the white rock (Unzen¹ in Table 1), whilst the dark rock (Unzen²) exists as discrete enclaves.

Table 4: The coefficients b_i, b_{1j}, c_i and c_{11} and expressions for M_i, M_{1j}, N_i and N_{11} as used in the melt viscosity model of Giordano et al. (2008). All X_i are in mol%.

$b_1 = 159.6$	$M_1 = X_{\text{SiO}_2} + X_{\text{TiO}_2}$	$c_1 = 2.75$	$N_1 = X_{\text{SiO}_2}$
$b_2 = -173.3$	$M_2 = X_{\text{Al}_2\text{O}_3}$	$c_2 = 15.7$	$N_2 = X_{\text{TiO}_2} + X_{\text{Al}_2\text{O}_3}$
$b_3 = -72.1$	$M_3 = X_{\text{Fe}} + X_{\text{MnO}} + X_{\text{P}_2\text{O}_5}$	$c_3 = 8.3$	$N_3 = X_{\text{Fe}} + X_{\text{MnO}} + X_{\text{MgO}}$
$b_4 = -75.7$	$M_4 = X_{\text{MgO}}$	$c_4 = 10.2$	$N_4 = X_{\text{CaO}}$
$b_5 = -39.9$	$M_5 = X_{\text{CaO}}$	$c_5 = -12.3$	$N_5 = X_{\text{Na}_2\text{O}} + X_{\text{K}_2\text{O}}$
$b_6 = -84.1$	$M_6 = X_{\text{Na}_2\text{O}} + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}}$	$c_6 = -99.1$	$N_6 = \ln(1 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$
$b_7 = -141.5$	$M_7 = X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}} + \ln(1 + X_{\text{H}_2\text{O}})$		
$b_{11} = -2.43$	$M_{11} = M_1 N_3$	$c_{11} = 0.3$	$N_{11} = (M_2 + N_3 + N_4 -$
$b_{12} = -0.91$	$M_{12} = (N_1 + N_2 + X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}})$		$X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$
$b_{13} = 17.6$	$M_{13} = M_2 N_5$		

$$\eta_m = 10^{A+B(\mathbf{X})/[T-C(\mathbf{X})]}, \quad (5)$$

where $A = -4.55$, T is in K, and B and C are given by

$$B = \sum_{i=1}^7 b_i M_i + \sum_{j=1}^3 b_{1j} M_{1j}, \quad (6)$$

and

$$C = \sum_{i=1}^6 c_i N_i + c_{11} N_{11}. \quad (7)$$

Values for the coefficients b_i, b_{1j}, c_i and c_{11} , and the expressions for M_i, M_{1j}, N_i and N_{11} are given in Table 4.

Table 5: The melt compositions (mol%) of the Unzen magmas at their mixing temperatures and 2000 MPa.

Magma	Temperature /°C	SiO ₂	Al ₂ O ₃	TiO ₂	FeO	Fe ₂ O ₃	MnO	MgO	CaO	K ₂ O	Na ₂ O	P ₂ O ₅	H ₂ O
Unzen ¹	775	67.43	6.86	0.16	0.54	0.1	0.15	0.34	2.29	2.86	2.29	0.13	16.57
Unzen ²	1079	51.23	10.49	1.23	0.0	3.70	0.15	6.79	9.88	0.62	2.47	0.07	13.59

- (a) Assume that the enclave magma has 4 wt% water and at the time of enclave formation was at the wet liquidus temperature. Calculate the viscosity of this magma. You may find it helpful to draw a table.
 - (b) Assume that the host magma also has 4 wt% water, but is at 775°C. At this temperature, the equilibrium crystal assemblage is orthopyroxene ($\phi_{\text{opx}} = 0.07$, $\rho_{\text{opx}} = 3.52 \text{ g cm}^{-3}$), clinopyroxene ($\phi_{\text{cpx}} = 0.02$, $\rho_{\text{cpx}} = 3.35 \text{ g cm}^{-3}$) and feldspar ($\phi_{\text{fd}} = 0.30$, $\rho_{\text{fd}} = 2.62 \text{ g cm}^{-3}$) with minor amounts of spinel and oxides. The composition of the melt is given in Table 5. Also present is an exsolved gas phase with a volume fraction of $\phi_{\text{b}} = 0.07$ and density of 0.44 g cm^{-3} . Calculate the melt viscosity of this magma.
 - (c) Neglecting the gas phase and using the Krieger & Dougherty (1959) relation, estimate the total viscosity of the host magma, taking 0.4 as the maximum crystal volume fraction.
 - (d) Does the host magma have a yield stress? If so, calculate an estimate for it?
 - (e) Given the temperature difference between the magmas, discuss what you expect to happen once they come into contact in terms of processes such as heat transport, crystallisation and melting. How do you expect the rheological properties of the two magmas to change?
4. Consider two magmas, a basalt ($\eta = 1 \text{ Pa s}$, $\rho = 2800 \text{ kg m}^{-3}$) and rhyolite ($\eta = 10^6 \text{ Pa s}$, $\rho = 2600 \text{ kg m}^{-3}$), rising in a conduit. Assume a conduit radius of 10 m and 1 m for the rhyolite and basalt, respectively. Assume a pressure gradient driving magma ascent of 500 Pa m^{-1} . For both cases:
- (a) Calculate velocity as a function of radial position in the conduit
 - (b) Evaluate the strain rate at the conduit walls. Are the strain rates large enough that the melt might be shear-thinning or undergo structural failure?
 - (c) Calculate the capillary number for a 1 cm radius bubble. Assume the surface tension of melt is 10^{-3} N m^{-1} . Will the bubbles become deformed by the ascending magma?
 - (d) What effect do you think the bubbles will have on magma rheology?