UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic processes: volcanic plumes

Dr. Eduardo Rossi
Prof. Costanza Bonadonna

Physical volcanology group
University of Geneva

Outline of the lesson

What can we learn from the observation of Volcanic plumes in Nature?

The momentum-driven region

The buoyancy-driven region

Exercise 1: win a duck using Archimedes' law!

Empirical relations for MER vs. H

Modelling volcanic plumes

1D vs. 3D models

An example of 1D steady-state model

Exercise 2: roll up your sleeves! It is time for Matlab

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Volcanic plumes in Nature

A volcanic eruption: what you were used to see before this lesson

Photo credits: The Atlantic

A volcanic eruption: what you will see from now on!

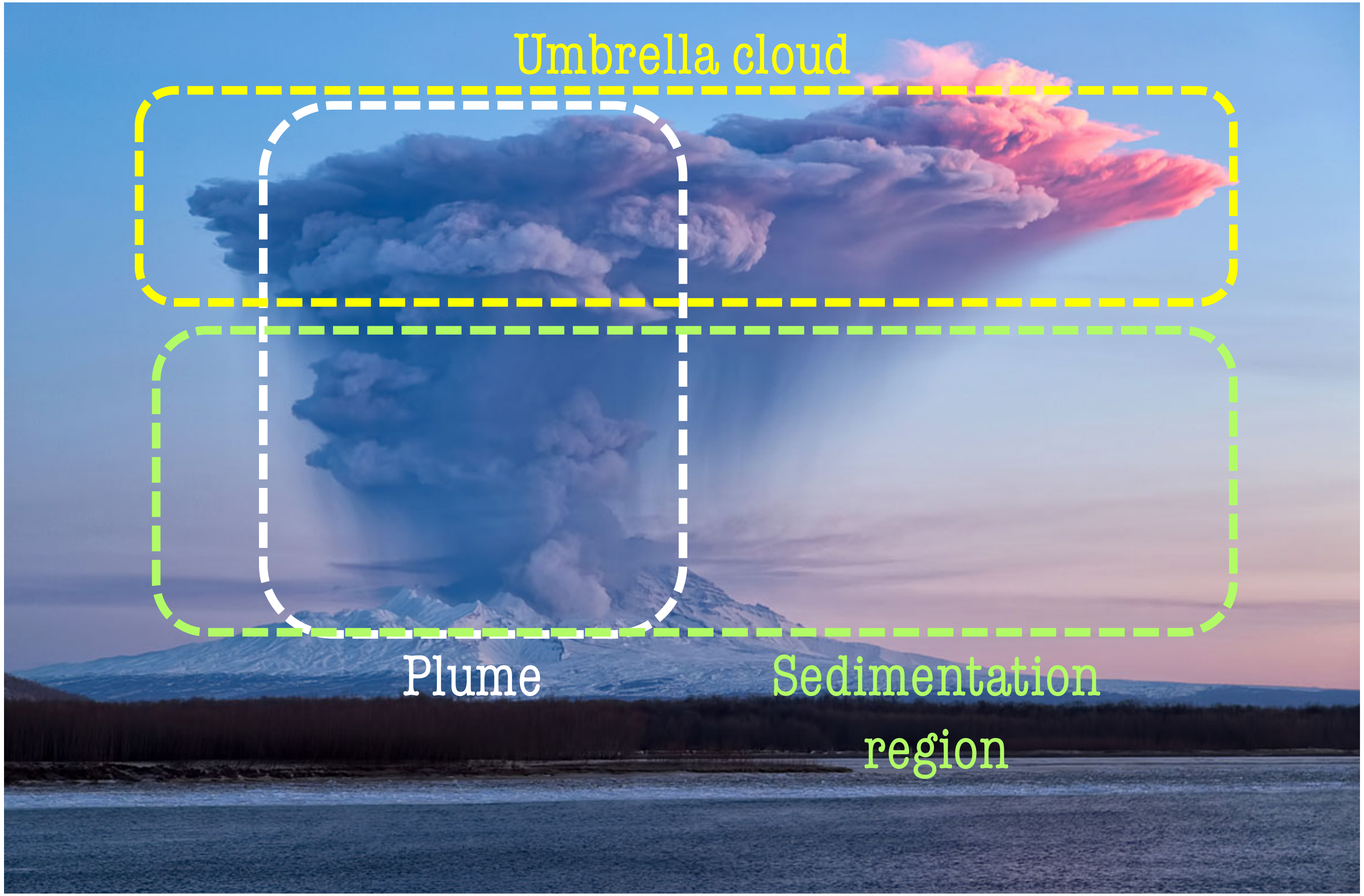


Photo credits: The Atlantic

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

The focus of the present lesson: volcanic plumes

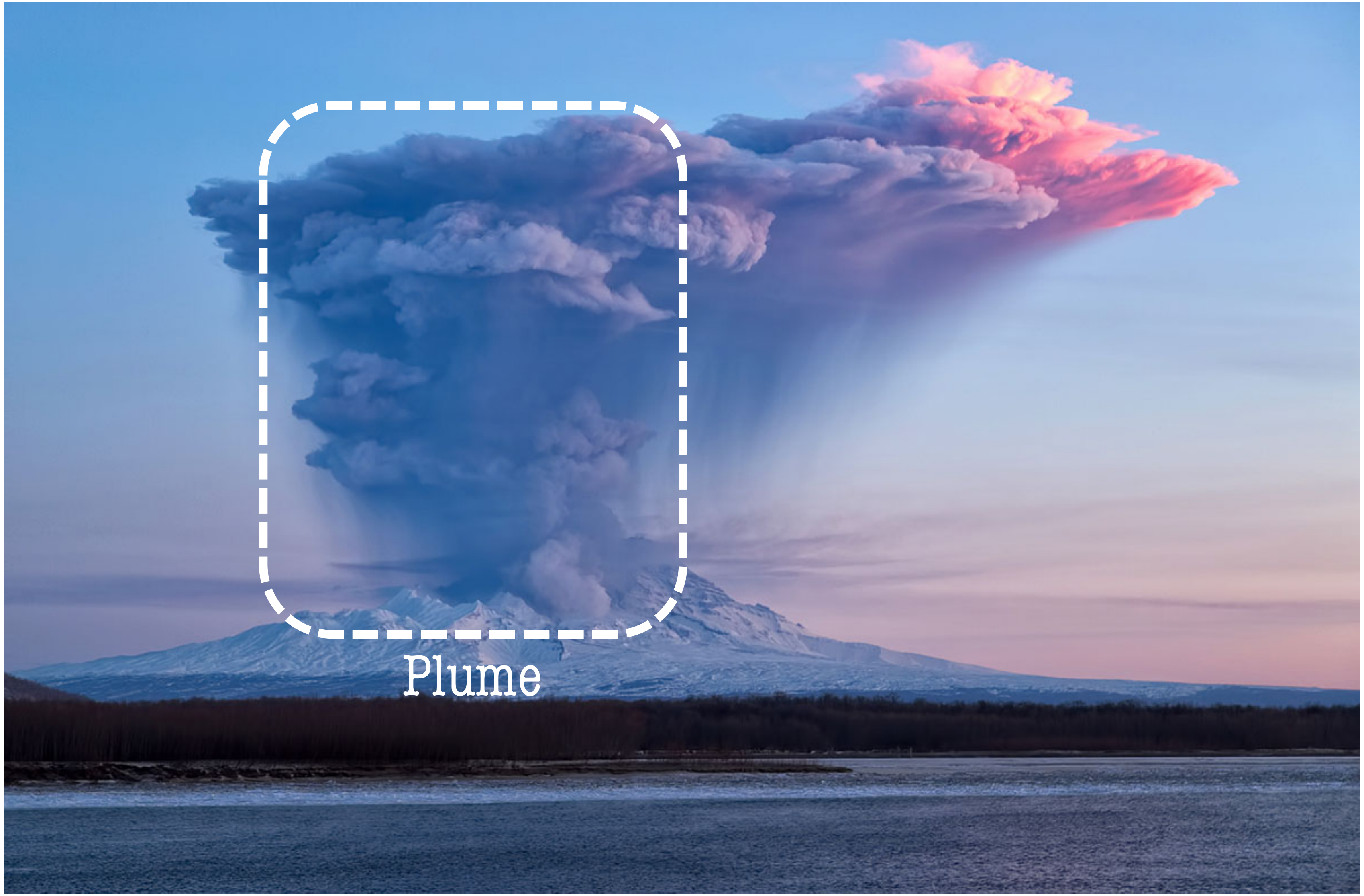


Photo credits: The Atlantic

Three main regions: gas thrust, convective region, overshooting

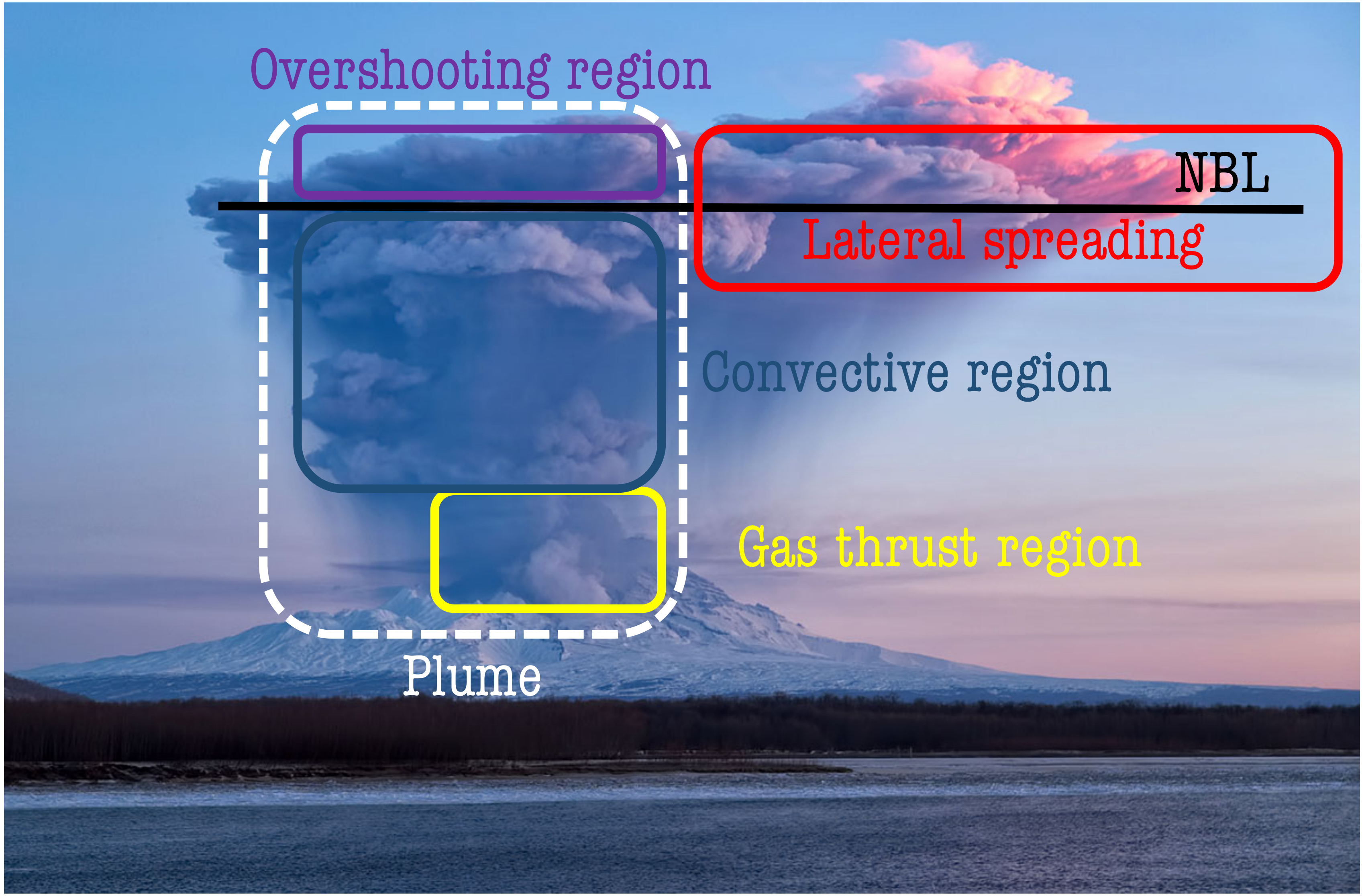
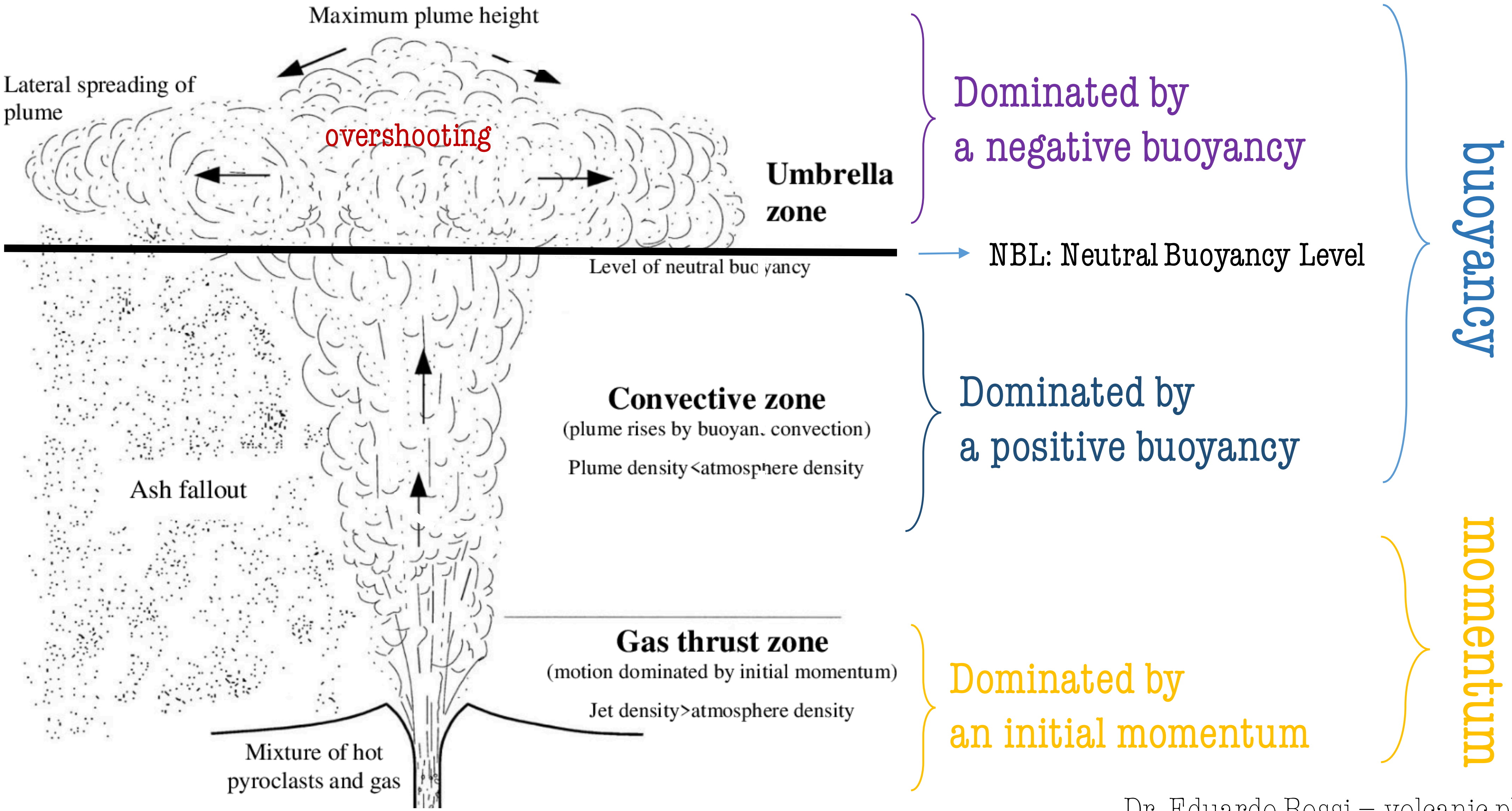


Photo credits: The Atlantic

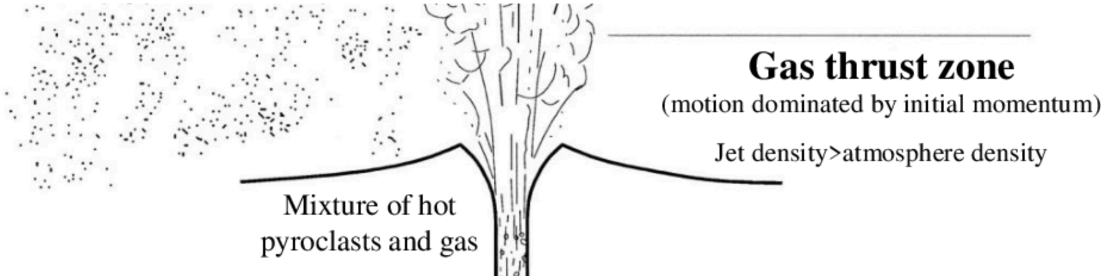
UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Three main regions: gas thrust, convective region, overshooting

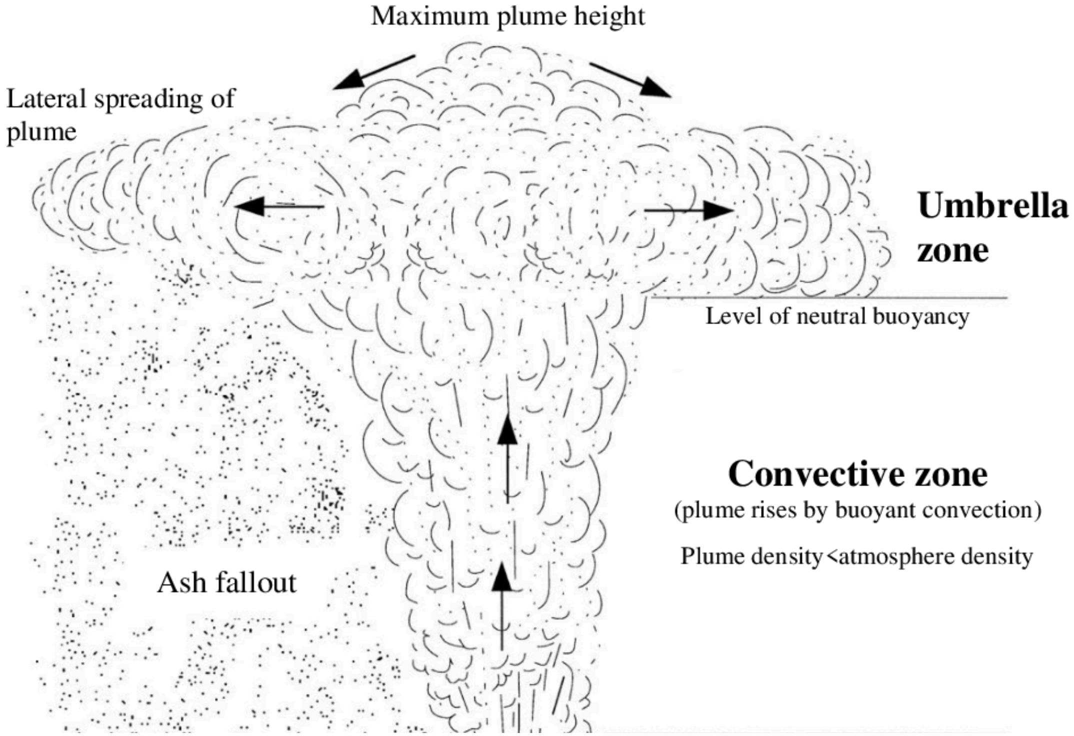


Credits: modified by G.M. Wilson, (2011) from Carey and Bursik, (2000)

Momentum



Buoyancy



Momentum

Let me give you a practical example of what « momentum » is...

Momentum

Let me give you a practical example of what « momentum » is...

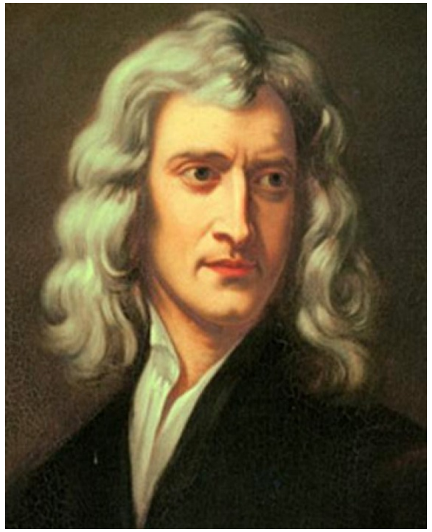
$$\vec{p} = m\,\vec{v}$$

UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Momentum

Let me give you a practical example of what « momentum » is...

$$\vec{p} = m\,\vec{v} \qquad \frac{d\vec{p}}{dt} = \vec{F}_e$$

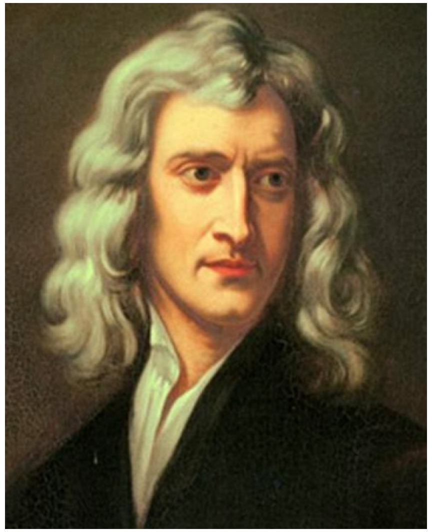


UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Momentum

Let me give you a practical example of what « momentum » is...

$$\vec{p} = m\,\vec{v} \qquad \frac{d\vec{p}}{dt} = \vec{F}_e \qquad \text{if } \vec{F}_e = 0 \rightarrow \frac{d\vec{p}}{dt} = 0 \rightarrow \vec{p} = \text{const}$$



$$m_{before}\,\vec{v}_{before} = m_{after}\,\vec{v}_{after}$$

Momentum

Something that will be important for volcanic plumes: what if the mass changes?

$$\vec{p} = m\,\vec{v} \qquad \frac{d\vec{p}}{dt} = \vec{F}_e$$

Some algebra and derivatives later...

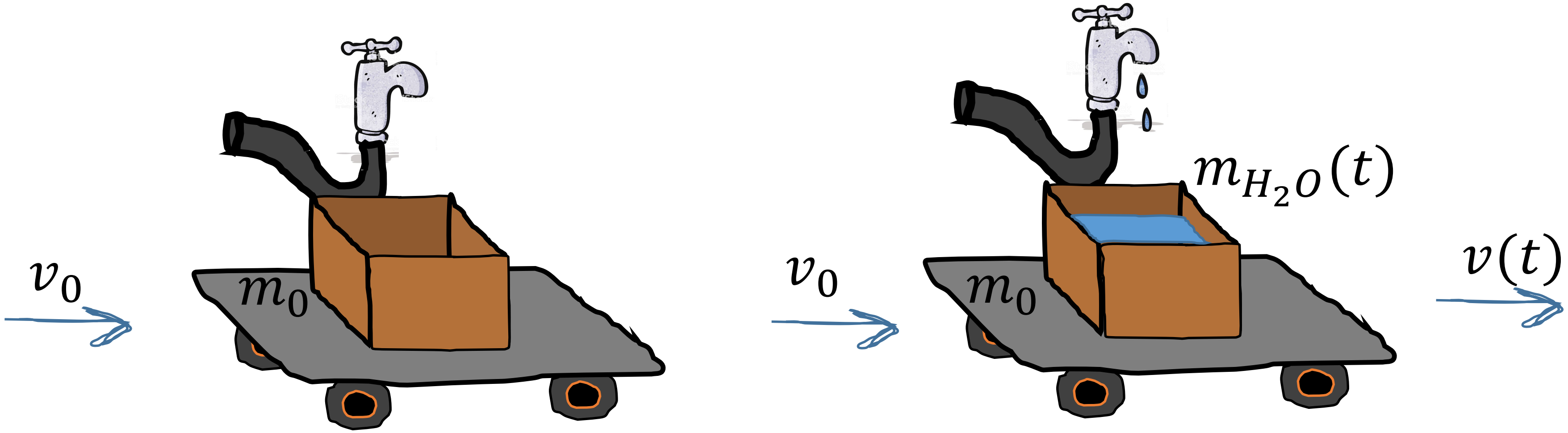
$$\frac{d\vec{p}}{dt} = \frac{d(m\,\vec{v})}{dt} = \frac{d(m\,\vec{v})}{dt} = \vec{v}\,\frac{d(m)}{dt} + m\,\frac{d(\vec{v})}{dt} = \vec{v}\,\frac{d(m)}{dt} + m\vec{a}$$

$$\frac{d\vec{p}}{dt} = \vec{F}_e \rightarrow \boxed{\vec{v}\,\frac{d(m)}{dt} + m\vec{a} = \vec{F}_e}$$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Momentum

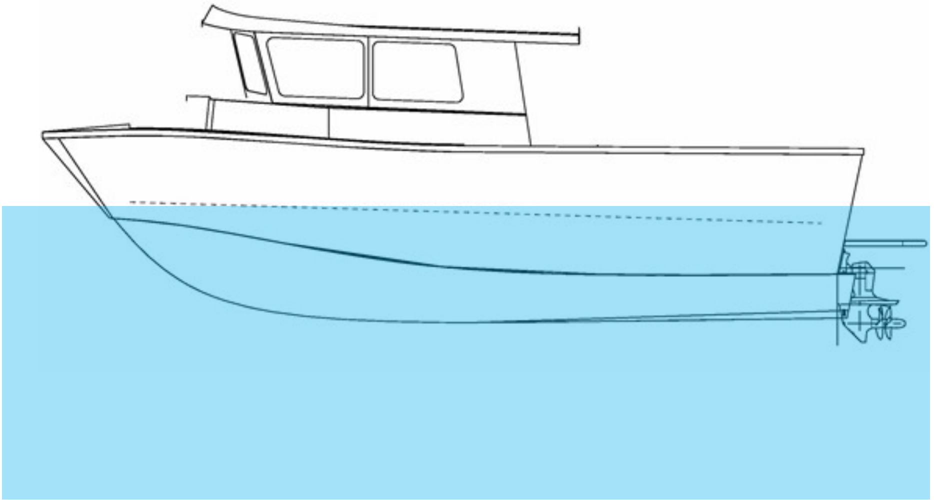
$$\frac{d\vec{p}}{dt} = \vec{F}_e \rightarrow \vec{v}\frac{d(m)}{dt} + m\vec{a} = \vec{F}_e$$



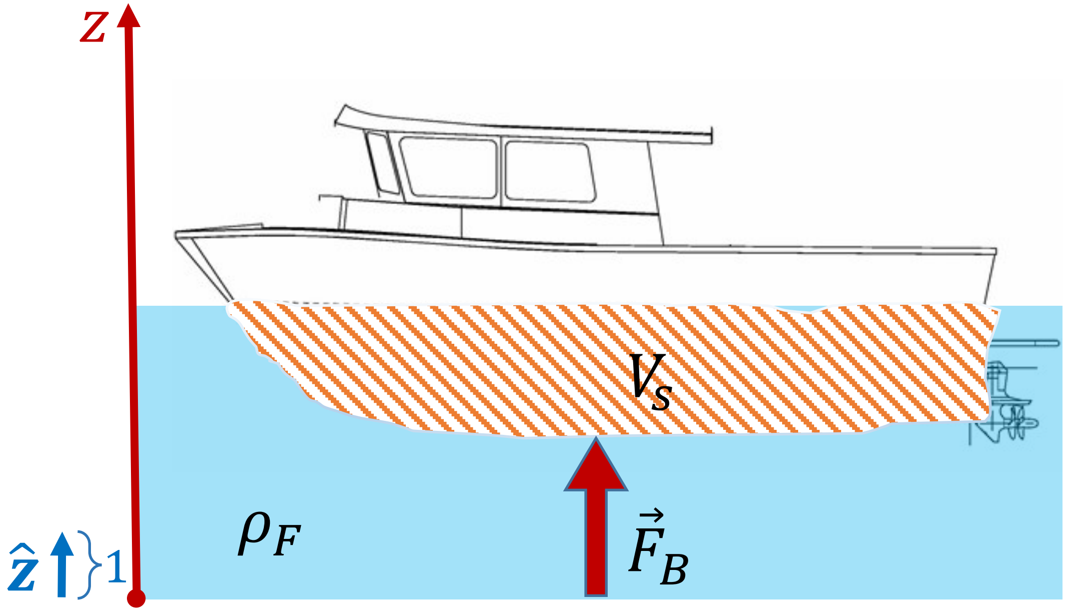
Let assume no friction on the wheels and a given law for the water flow (i.e. $\frac{d(m)}{dt} = \lambda$)

$$\vec{v}\frac{d(m)}{dt} + m\frac{d(\vec{v})}{dt} = 0 \rightarrow \frac{v}{v_0} = \frac{m_0}{(m_0 + \lambda \cdot t)}$$

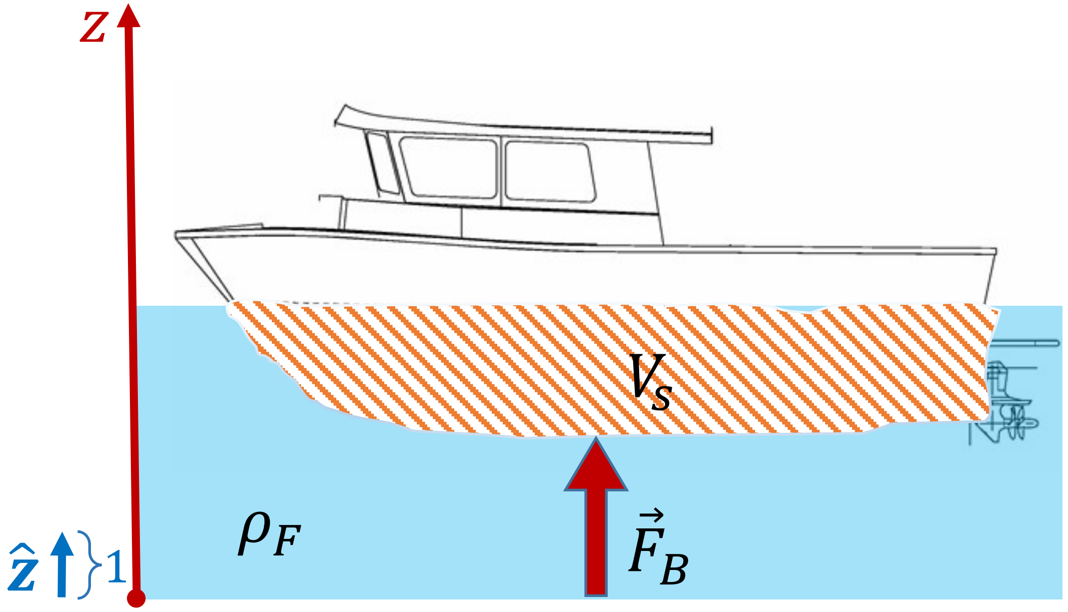
Buoyancy



Buoyancy



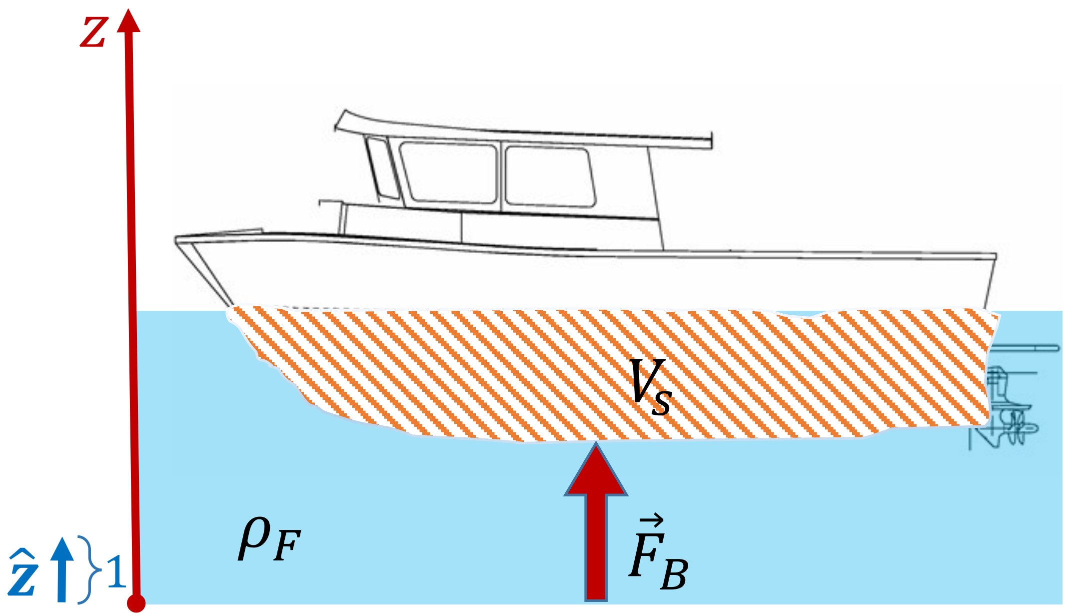
Buoyancy



Archimedes' principle

$$\vec{F}_B = F_B\,\hat{\mathbf{z}} = V_s\,\rho_F\,g\,\hat{\mathbf{z}}$$

Buoyancy



Archimedes' principle

$$\vec{F}_B = F_B\,\hat{\mathbf{z}} = V_s\,\rho_F\,g\,\hat{\mathbf{z}}$$

Buoyancy

$$a\,\hat{\mathbf{z}} = -g\left[1 - \frac{\rho_F}{\rho_p}\right]\hat{\mathbf{z}}$$

Buoyancy is the force related to a difference in density between the fluid and the surroundings. It can be upward or downward!

UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Buoyancy

Newton's law of motion (on a fluid element) → No acceleration within the fluid → **Hydrostatic pressure**

$$\frac{\partial P}{\partial z}\,\hat{\mathbf{z}} = -\rho_F(z)\cdot g\;\hat{\mathbf{z}}$$

→ **Barometric decay**

$\rho_F = f(P,T)$

(perfect gas)

→ **Stevin's law**

ρ_F is constant

Pressure gradient in the fluid $\Delta P = \rho_F \cdot g \cdot h$

Newton's law of motion (on an object) → The acceleration of the body is unknown → **Archimede's principle**

$$\vec{F}_B = F_B\,\hat{\mathbf{z}} = V_s\,\rho_F\,g\,\hat{\mathbf{z}}$$

→ **Buoyancy**

$$a\,\hat{\mathbf{z}} = -g\left[1-\frac{\rho_F}{\rho_p}\right]\hat{\mathbf{z}}$$

Exercise 1: raffling a duck

Buoyancy

Let me tell you a story…Do you know Banksy, the famous graffiti artist?

Credits: www.findingtheuniverse.com

UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

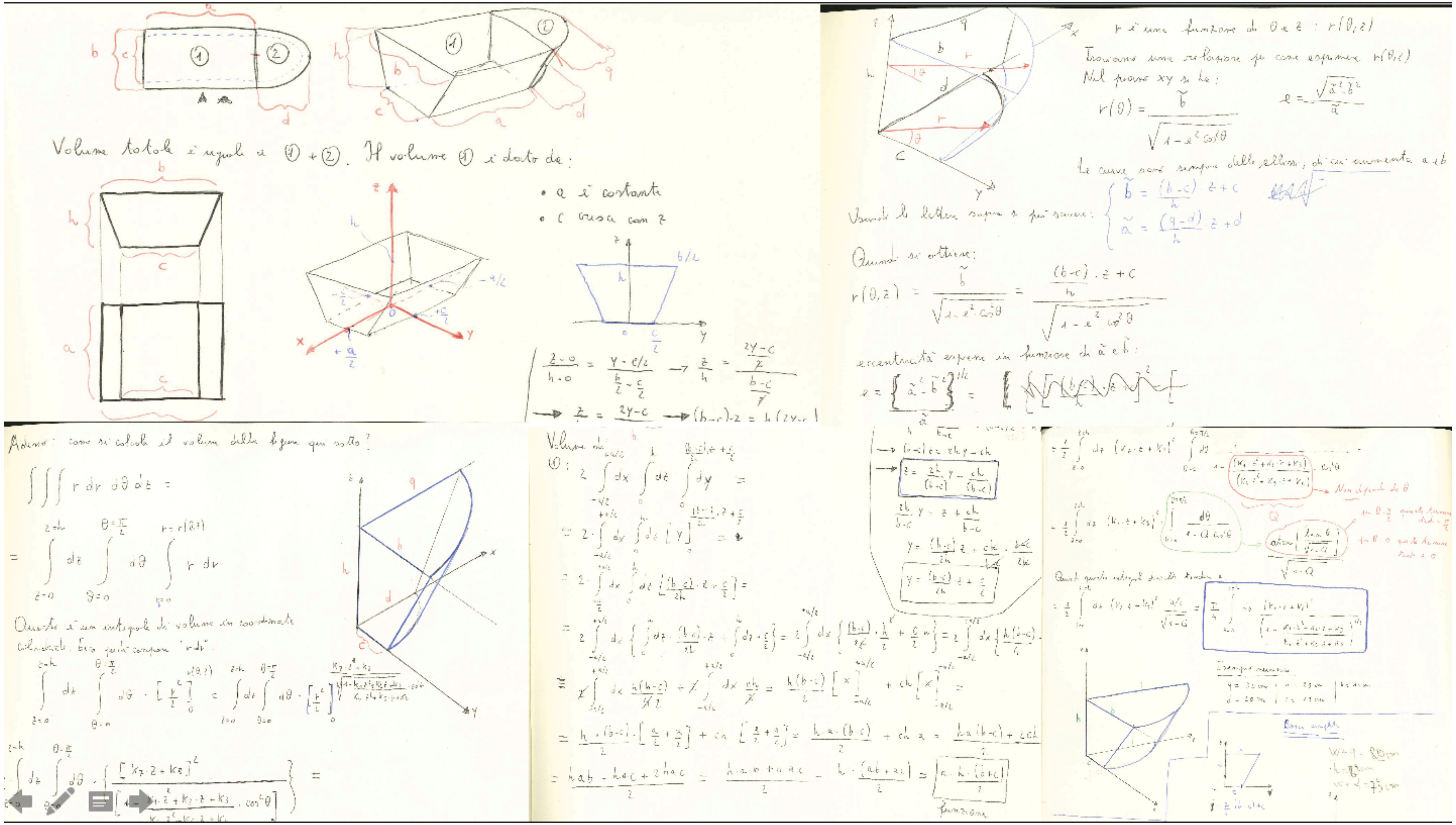
Buoyancy

The Banksy raffle for Christmas 2018: this masterpiece is yours if you guess its…weight!

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Buoyancy

The Banksy raffle for Christmas 2018: this masterpiece is yours if you guess its…weight!



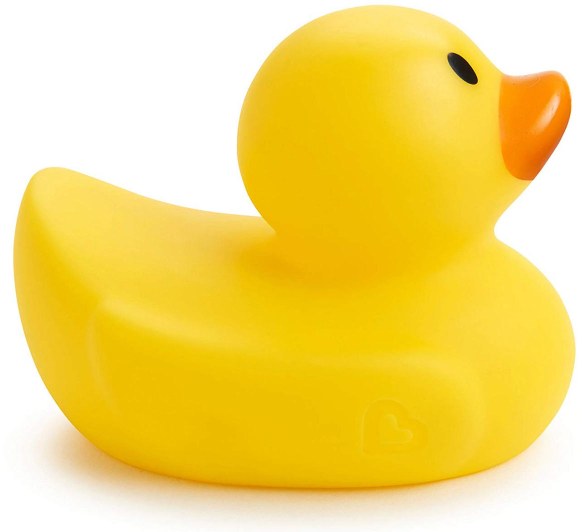
Actual mass 11.7 kg

My guess 13.2 kg

I lost ☹

Buoyancy

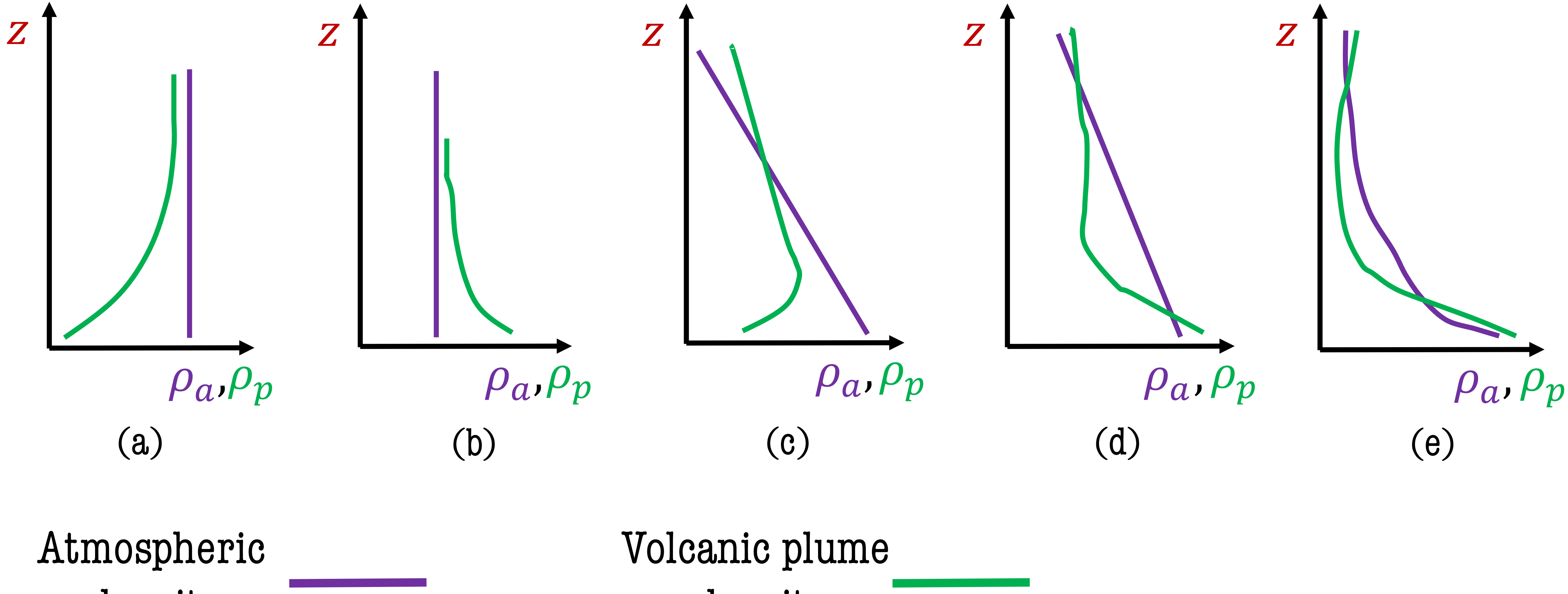
The “Rossi raffle for Christmas 2019”: this floating duck is yours if you guess its…total weight!



m_{duck}?

Plume density vs. atmospheric density

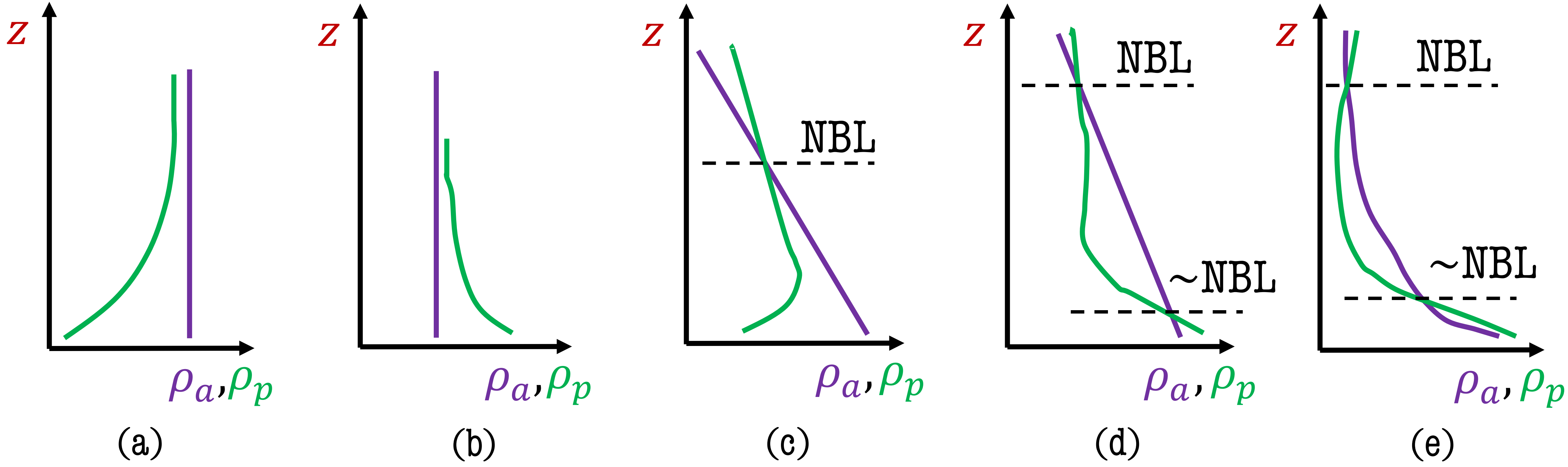
If you understood buoyancy…what can you say about these five profiles?



UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Plume density vs. atmospheric density

If you understood buoyancy...what can you say about these five profiles?



Atmospheric density

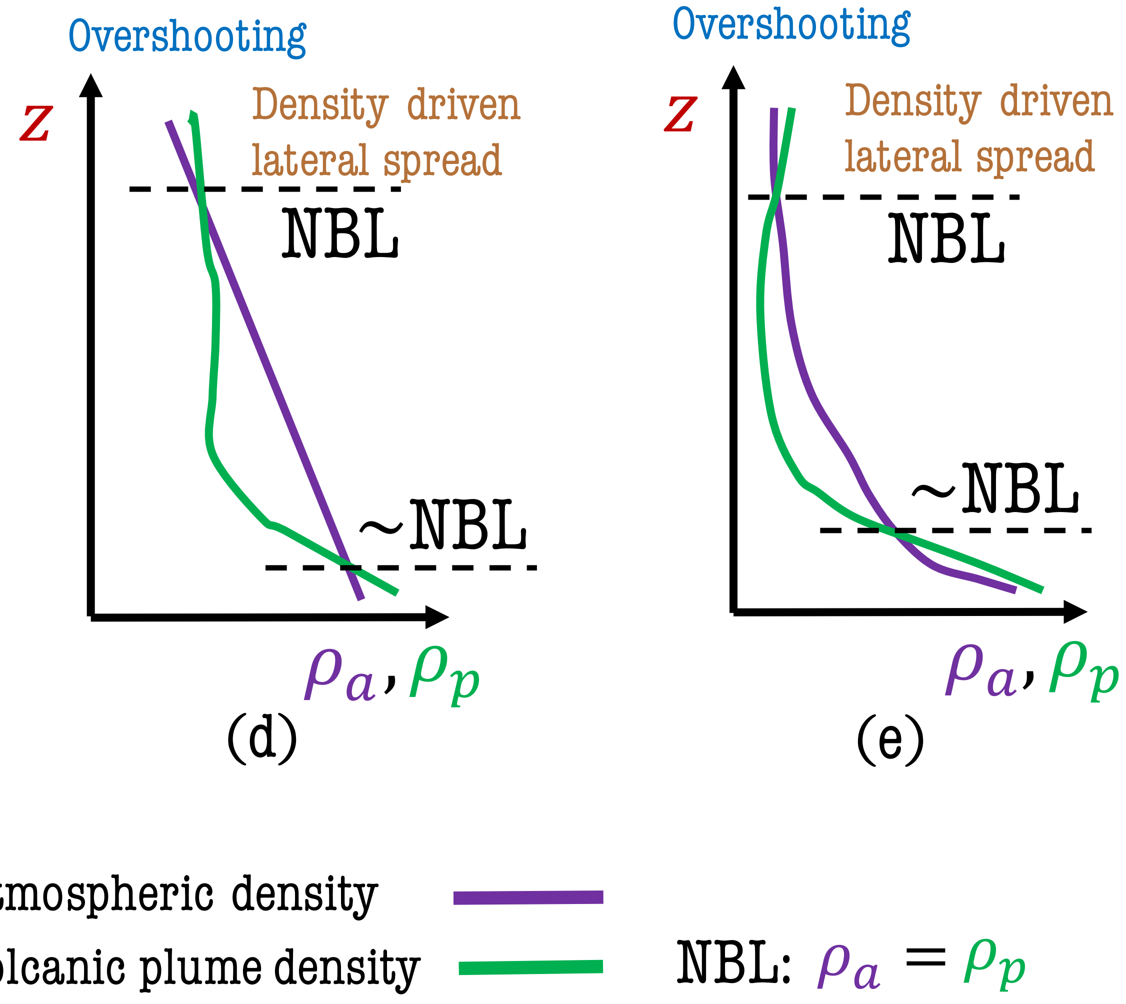
Volcanic plume density

NBL: $\rho_a = \rho_p$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Plume density vs. atmospheric density

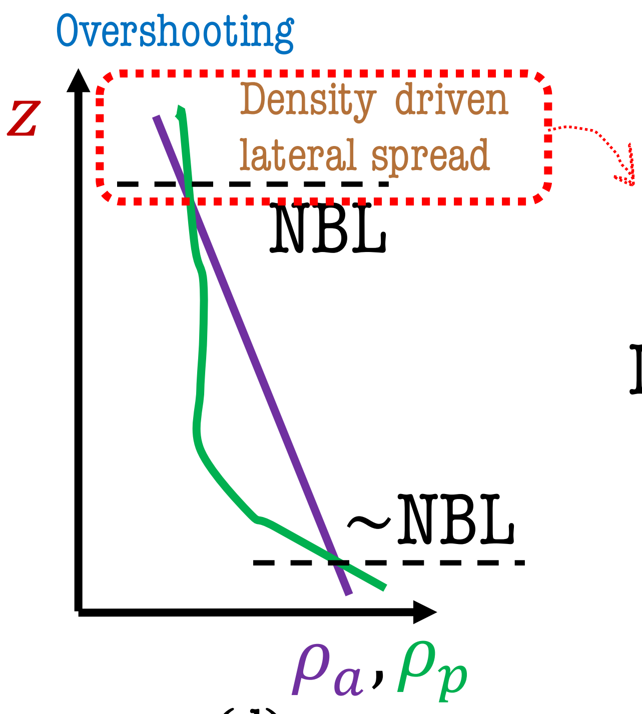
Overshooting and lateral spreading driven by density differences (cloud)



NBL: $\rho_a = \rho_p$

Plume density vs. atmospheric density

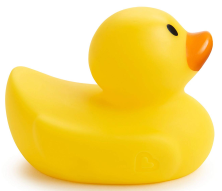
Overshooting and lateral spreading driven by density differences (cloud)



(d)

$\rho_p > \rho_a$

NBL - NBL



$\rho_p < \rho_a$

Atmospheric density

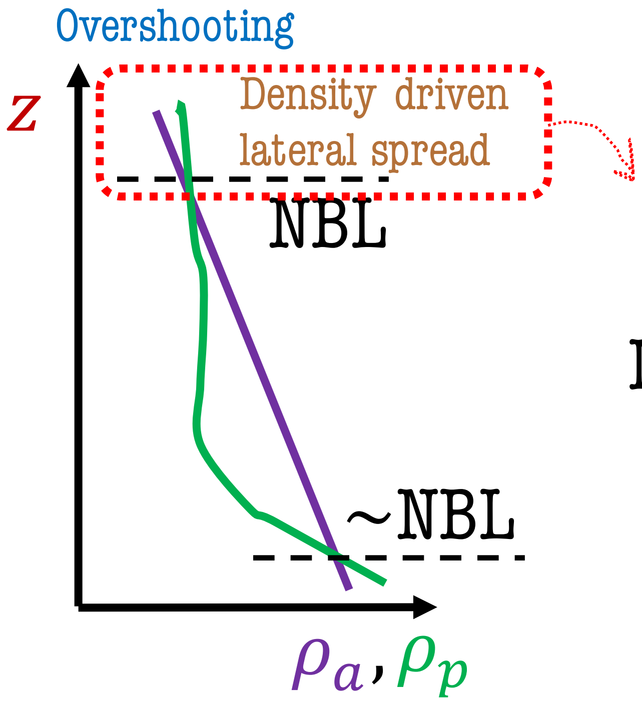
Volcanic plume density

NBL: $\rho_a = \rho_p$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

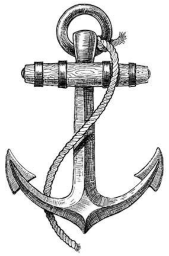
Plume density vs. atmospheric density

Overshooting and lateral spreading driven by density differences (cloud)



(d)

$\rho_p > \rho_a$



NBL - NBL

$\rho_p < \rho_a$

Continuous
injection
of mass

Atmospheric density

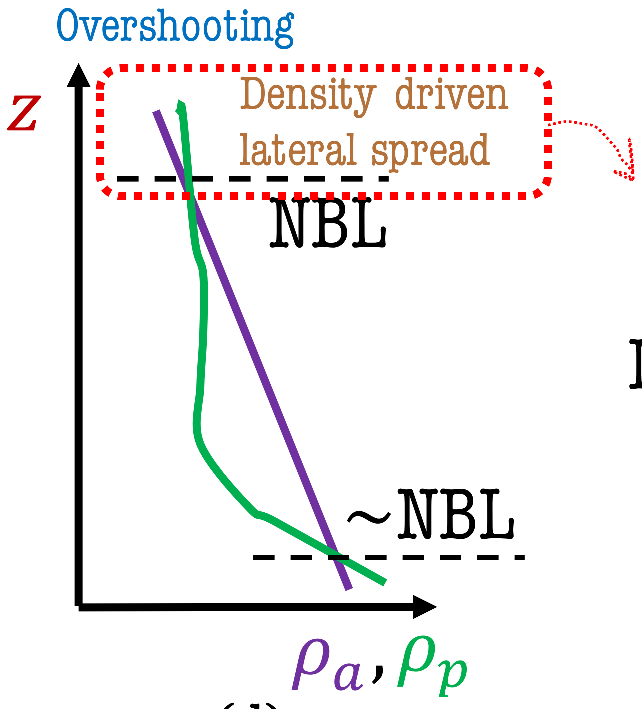
Volcanic plume density

NBL: $\rho_a = \rho_p$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

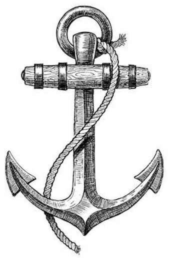
Plume density vs. atmospheric density

Overshooting and lateral spreading driven by density differences (cloud)



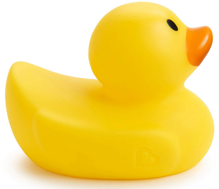
(d)

$\rho_p > \rho_a$



Plume inertia

NBL - NBL



$\rho_p < \rho_a$

Continuous injection of mass

Atmospheric density

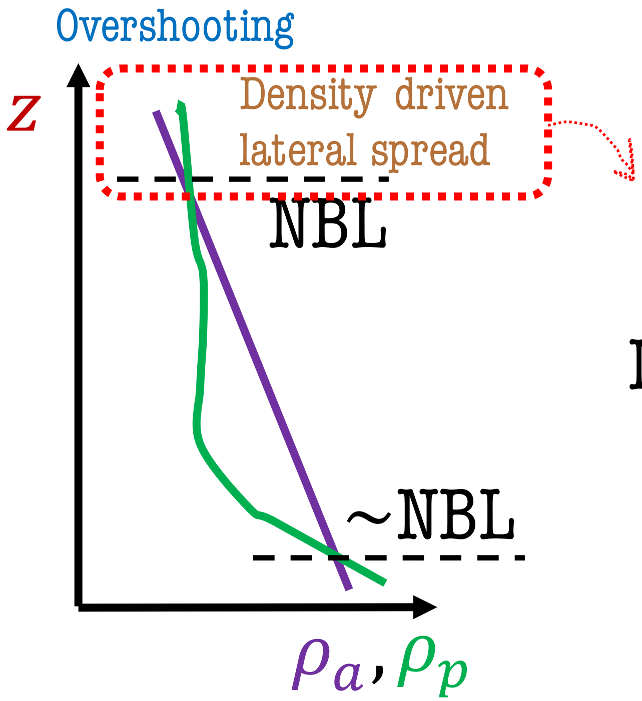
Volcanic plume density

NBL: $\rho_a = \rho_p$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

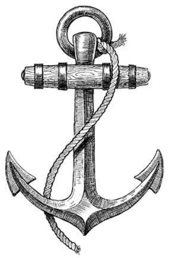
Plume density vs. atmospheric density

Overshooting and lateral spreading driven by density differences (cloud)



(d)

$\rho_p > \rho_a$



Plume inertia

Negative buoyancy

NBL - NBL

$\rho_p < \rho_a$

Continuous injection of mass

Atmospheric density

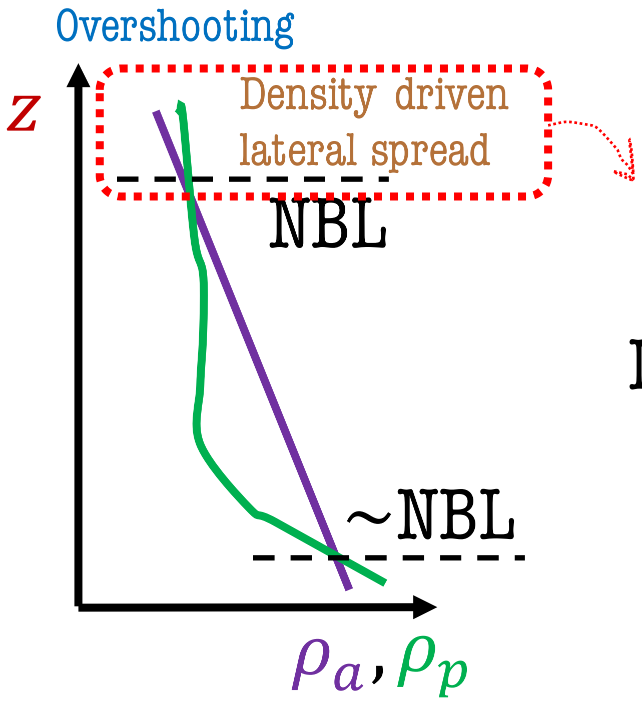
Volcanic plume density

NBL: $\rho_a = \rho_p$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

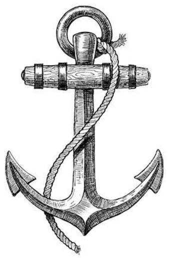
Plume density vs. atmospheric density

Overshooting and lateral spreading driven by density differences (cloud)



(d)

$\rho_p > \rho_a$



Plume inertia

Negative buoyancy

NBL - NBL

$\rho_p < \rho_a$

Continuous injection of mass

Positive buoyancy

Atmospheric density

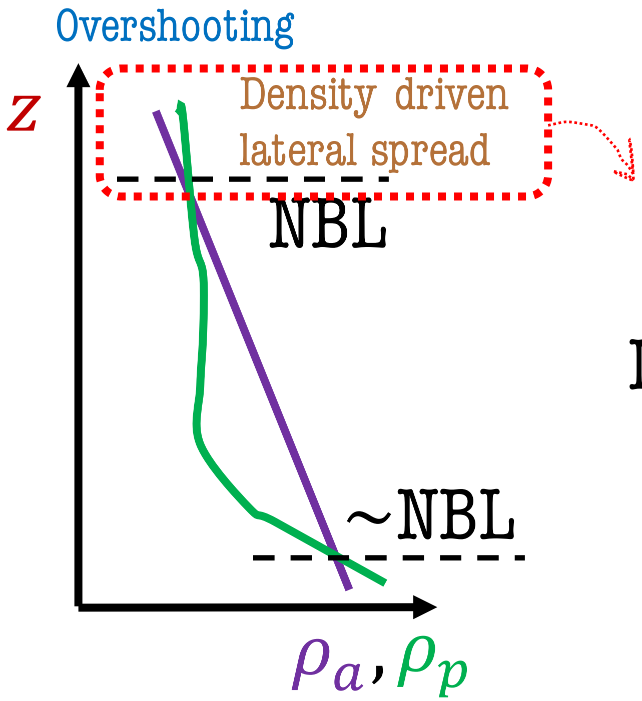
Volcanic plume density

NBL: $\rho_a = \rho_p$

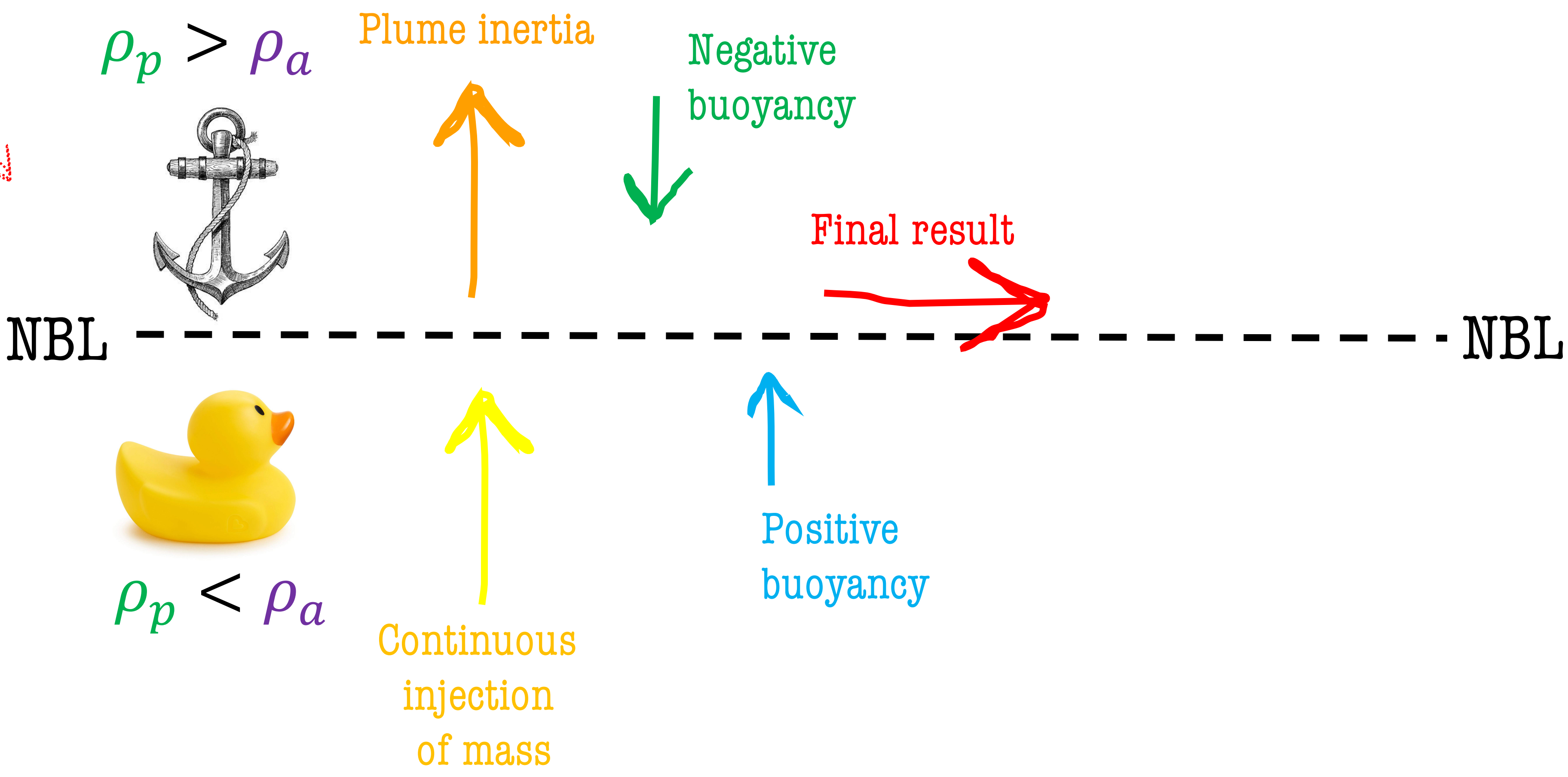
UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Plume density vs. atmospheric density

Overshooting and lateral spreading driven by density differences (cloud)



(d)



Atmospheric density

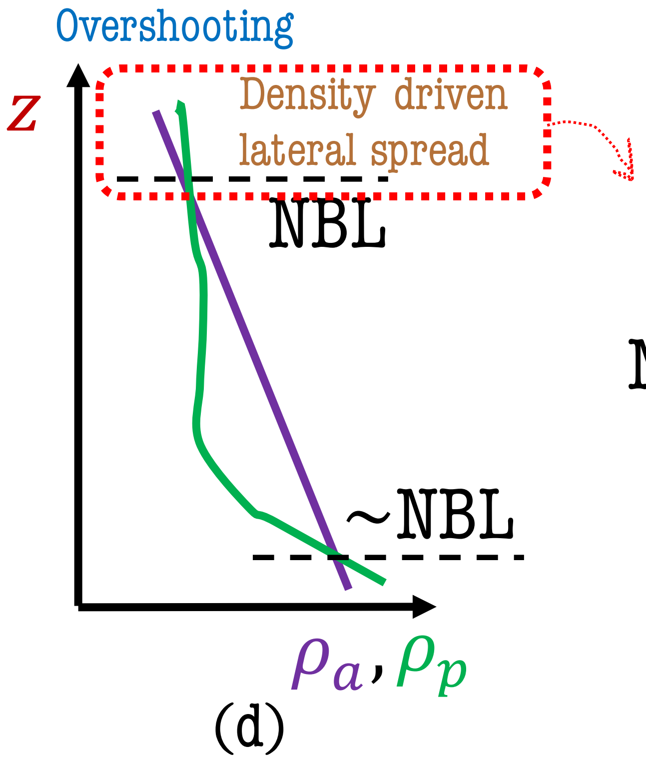
Volcanic plume density

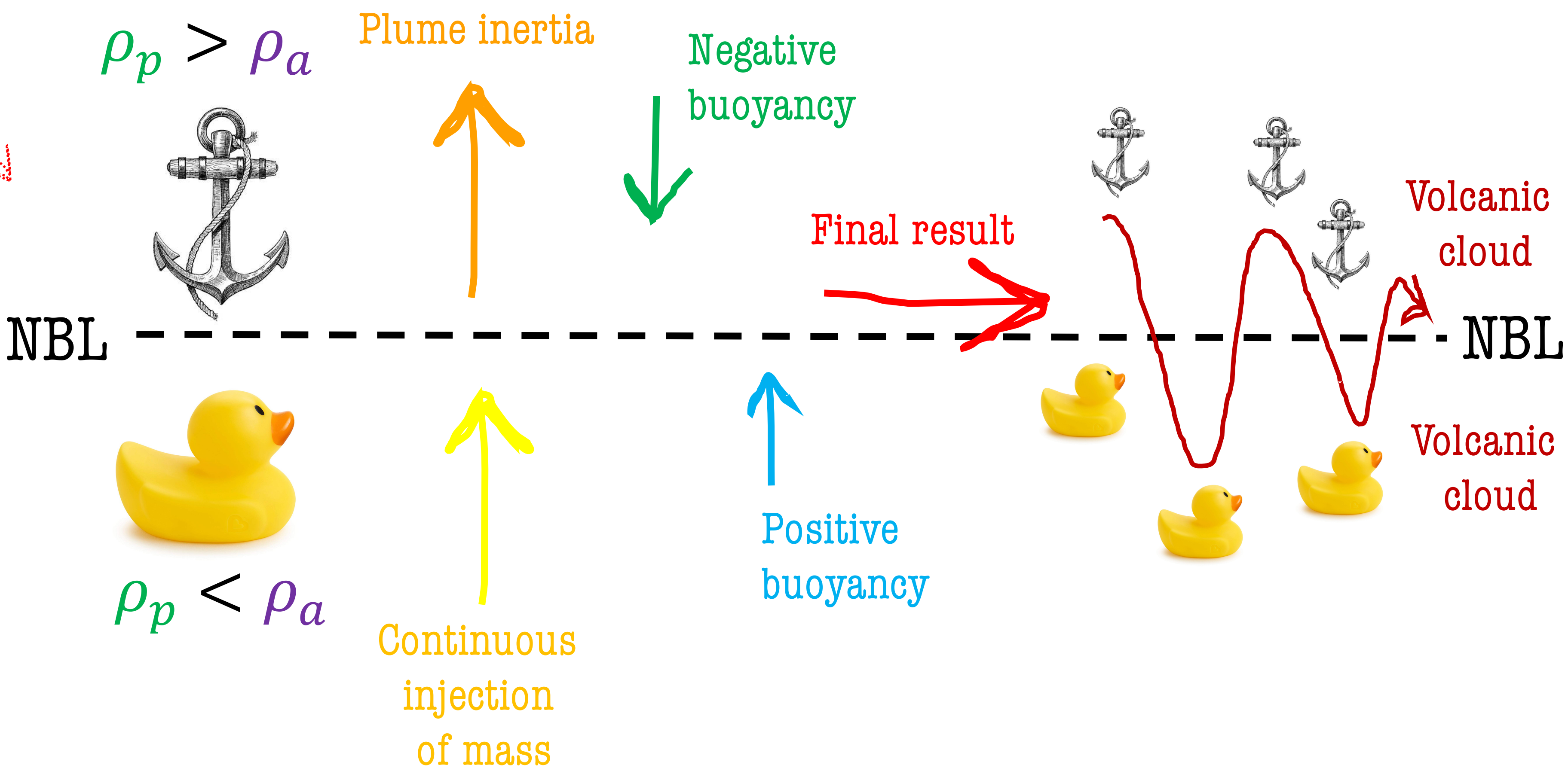
NBL: $\rho_a = \rho_p$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Plume density vs. atmospheric density

Overshooting and lateral spreading driven by density differences (cloud)



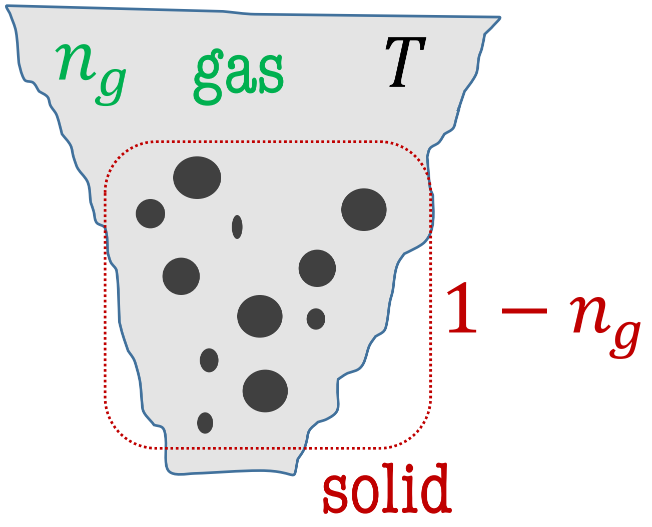


Atmospheric density
Volcanic plume density

NBL: $\rho_a = \rho_p$

Plume density vs. atmospheric density

What are the important parameters affecting the volcanic gas mixture in the plume?



$$\rho_{plume} = \left(\frac{1 - n_g}{\rho_s} + \frac{n_g R_g T}{P_g} \right)^{-1}$$

n_g = Mass fraction of gas phase

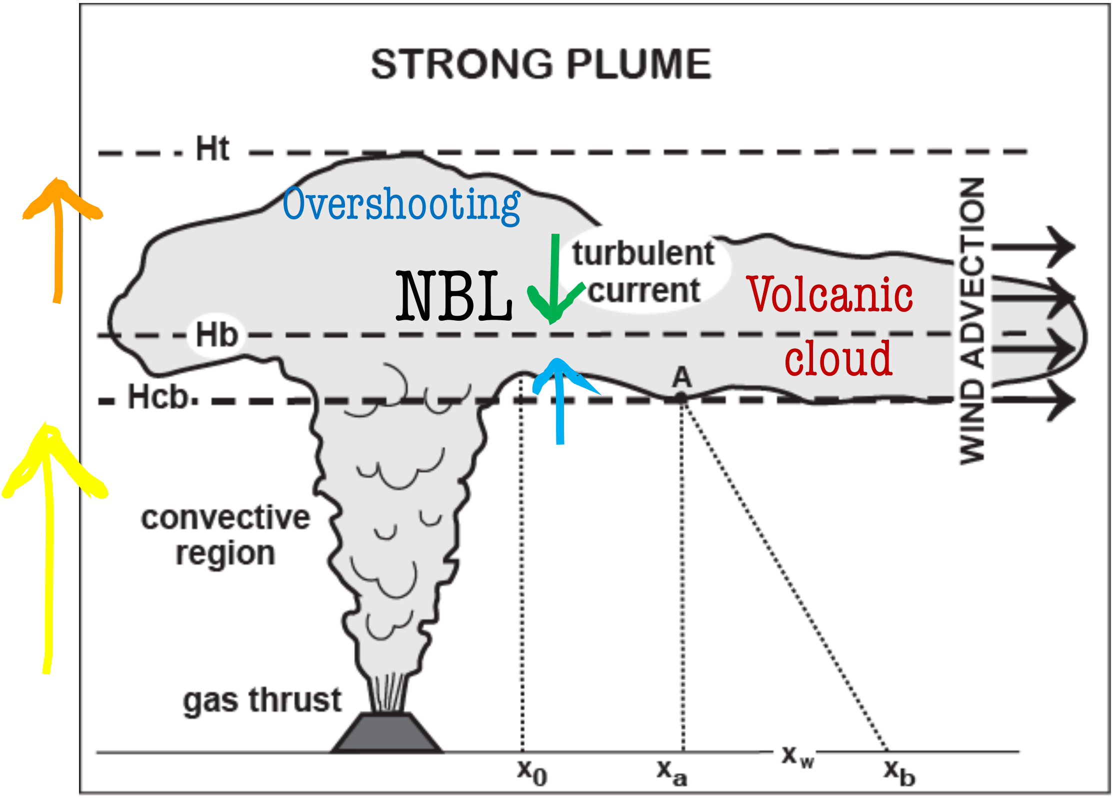
R_g = Gas constant for perfect gases

P_g = Pressure of the mixture

ρ_s = Solid particles density

Plume density vs. atmospheric density

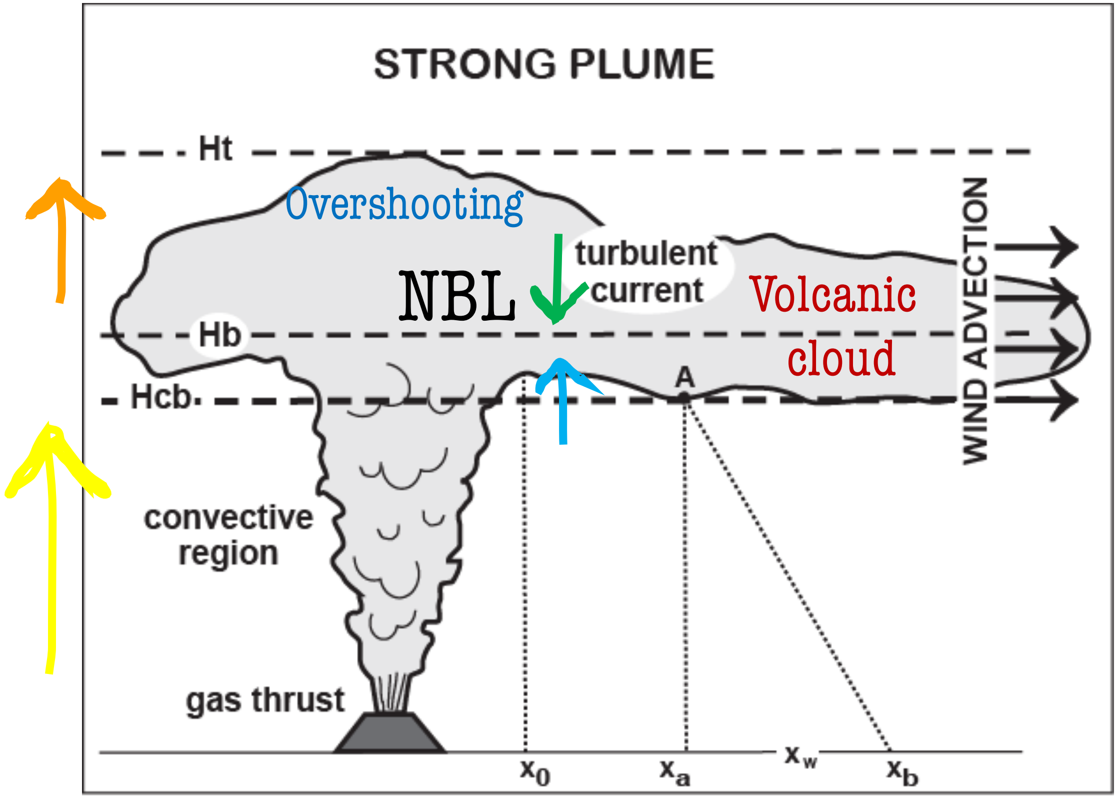
The effect of the wind: strong plumes vs. weak plumes

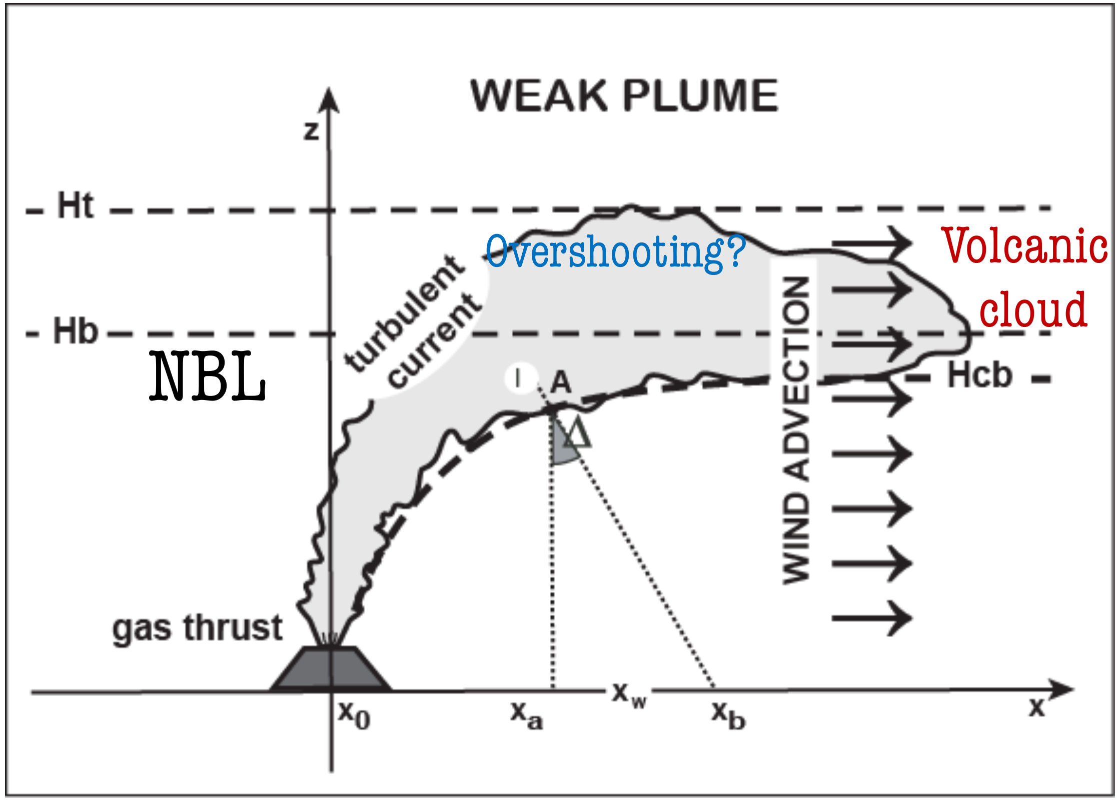


PLUME > wind

Plume density vs. atmospheric density

The effect of the wind: strong plumes vs. weak plumes





PLUME > wind

Plume <WIND

Plume tipologies

An instantaneous release of positive buoyancy (Vulcanian explosion)

Credits: www. fineartamerica.com

Anak Krakatau volcano (Indonesia)

UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Plume typologies

A continuous release of positive buoyancy (steady-state eruptions)

Eyafjatlajokull (Iceland)

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Plume typologies

A completely/partially negative buoyant plume

Credits: www.nationalgeographic.org

Mount Merapi (Indonesia)

Eruption Source Parameters (ESP)

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

What are the most important source parameters that fully characterize a volcanic eruption?

Eruption Source Parameters

Erupted mass / Erupted volume	$[M]$ / $[L^3]$
Plume height / NBL height	$[L]$
Mass Eruption Rate (MER)	$\left[\frac{M}{T}\right]$
Eruption duration	$[t]$
Total Grain Size Distribution (TGSD)	$[\%]_{per\ class}$

Eruption Source Parameters (ESP)

What are the most important source parameters that fully characterize a volcanic eruption?

Eruption Source Parameters

- Erupted mass / Erupted volume $[M]\,/\,[L^3]$
- Plume height / NBL height $[L]$
- Mass Eruption Rate (MER) $\left[\frac{M}{T}\right]$
- Eruption duration $[t]$
- Total Grain Size Distribution (TGSD) $[\%]_{per\ class}$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

MER vs. Plume height: empirical relations

Empirical relations between MER and plume height

$$MER = \mathcal{F}(H_p)$$

Important for modelling tephra dispersal

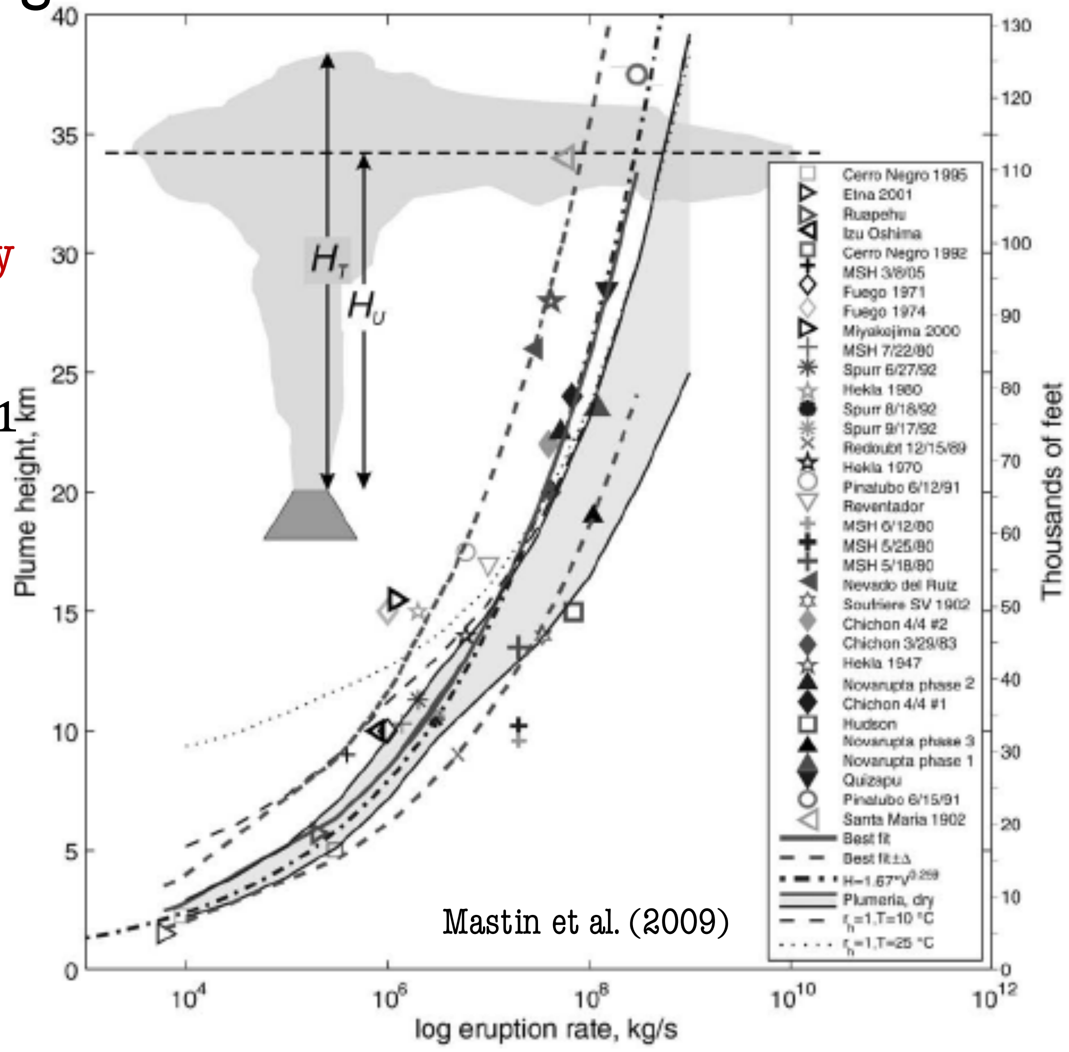
"Easy" to observe or quantify

$$H_p = 0.236 \cdot MER^{1/4}$$

Wilson and Walker (1987)

$$H_p = 0.304 \cdot MER^{0.241}$$

Mastin et al. (2009)



MER vs. Plume height: empirical relations

Empirical relations between MER and plume height

$$\color{green}{MER} = \mathcal{F}(\color{red}{H_p})$$

Important for modelling tephra dispersal

"Easy" to observe or quantify

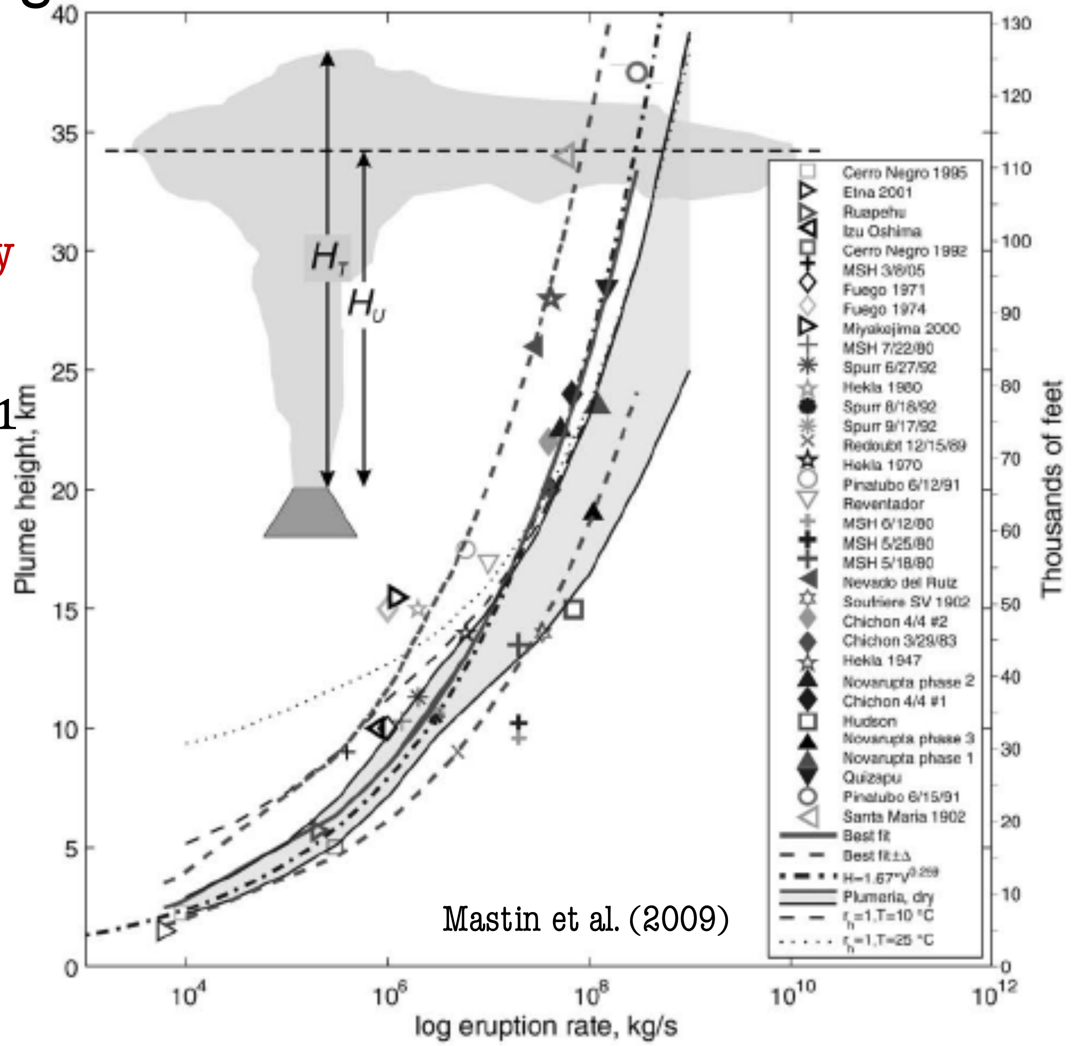
$$H_p = 0.236 \cdot MER^{1/4}$$

Wilson and Walker (1987)

$$H_p = 0.304 \cdot MER^{0.241}$$

Mastin et al. (2009)

No influence of the wind!



MER vs. Plume height: empirical relations

Empirical relations between MER and plume height

$$MER = \mathcal{F}(H_p)$$

Important for modelling tephra dispersal

"Easy" to observe or quantify

$$MER = \pi \frac{\rho_{a0}}{g'} \left(\frac{\alpha^2 \overline{N}^3}{10.9} H^4 + \frac{\beta^2 \overline{N}^2 \overline{v}}{6} H^3 \right)$$

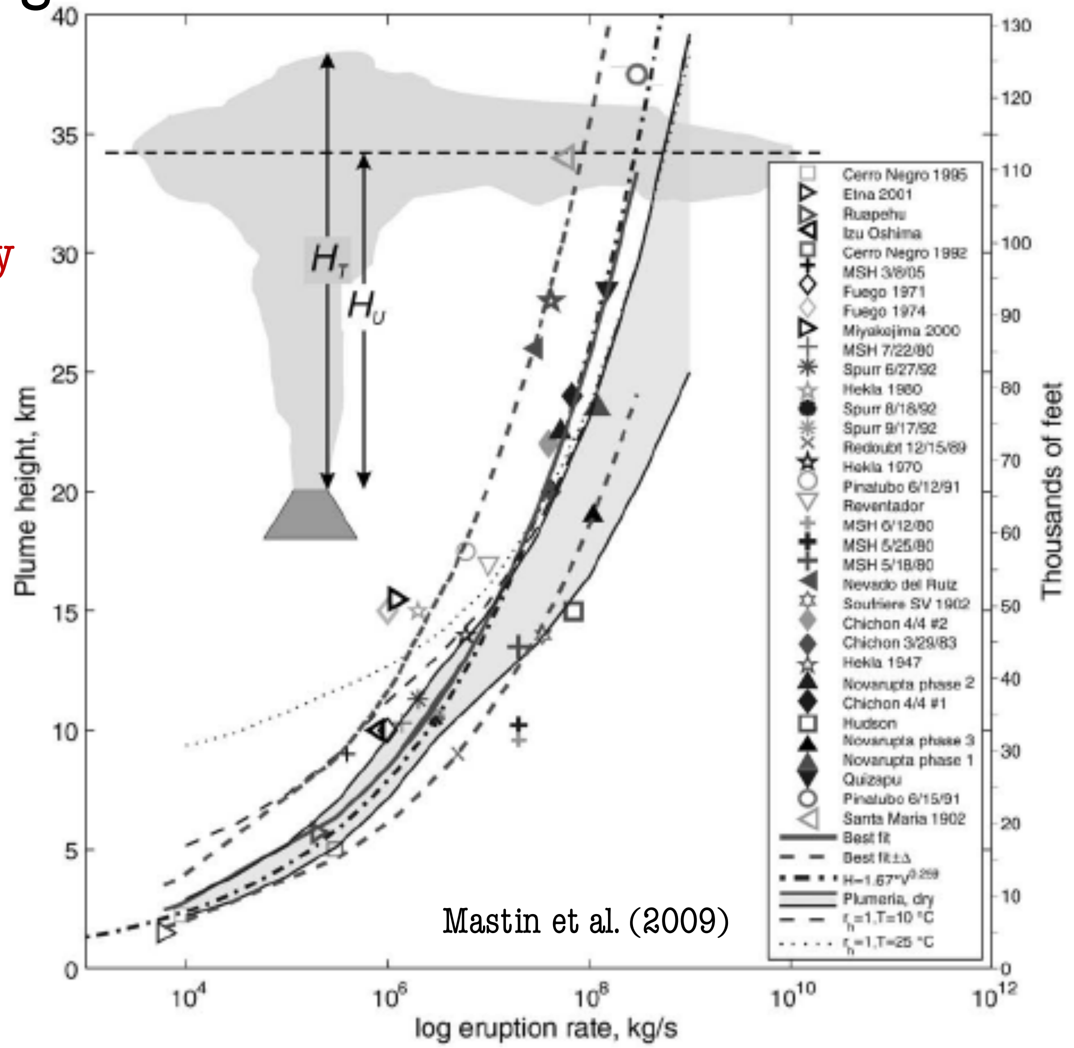
Degruyter and Bonadonna (2012)

Where the effect of the wind is summarised in the parameter Π

$$\Pi = \frac{\overline{N} H}{1.8 \overline{v}} \left(\frac{\alpha}{\beta} \right)^2$$

Wind dominant if

$$\Pi \ll 1$$



Mastin et al. (2009)

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Limits of the empirical approach

However…

$$H_p \propto MER^{1/4}$$

$$MER = \mathcal{F}(H_p)$$

Important for modelling tephra dispersal

"Easy" to observe or quantify

$$MER \propto H_p^{4}$$

A small error in height produces a large error in MER

Moreover…

$[H_p] = L$

$[MER] = MT^{-1}$

No way of making $H_p = \mathcal{F}(MER)$ dimensionally constant without other dimensional quantities

Limits of the empirical approach

1. Something missing in the physical description

UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Limits of the empirical approach

1. Something missing in the physical description

2. Real scenarios can be complex (e.g. wind)

Limits of the empirical approach

1. Something missing in the physical description

2. Real scenarios can be complex (e.g. wind)

3. Real scenarios can be complex (e.g. atmospheric profiles)

Limits of the empirical approach

1. Something missing in the physical description

2. Real scenarios can be complex (e.g. wind)

3. Real scenarios can be complex (e.g. atmospheric profiles)

4. Real scenarios can be complex (e.g. multiphase plume mixture)

Limits of the empirical approach

1. Something missing in the physical description

2. Real scenarios can be complex (e.g. wind)

3. Real scenarios can be complex (e.g. atmospheric profiles)

4. Real scenarios can be complex (e.g. multiphase plume mixture)

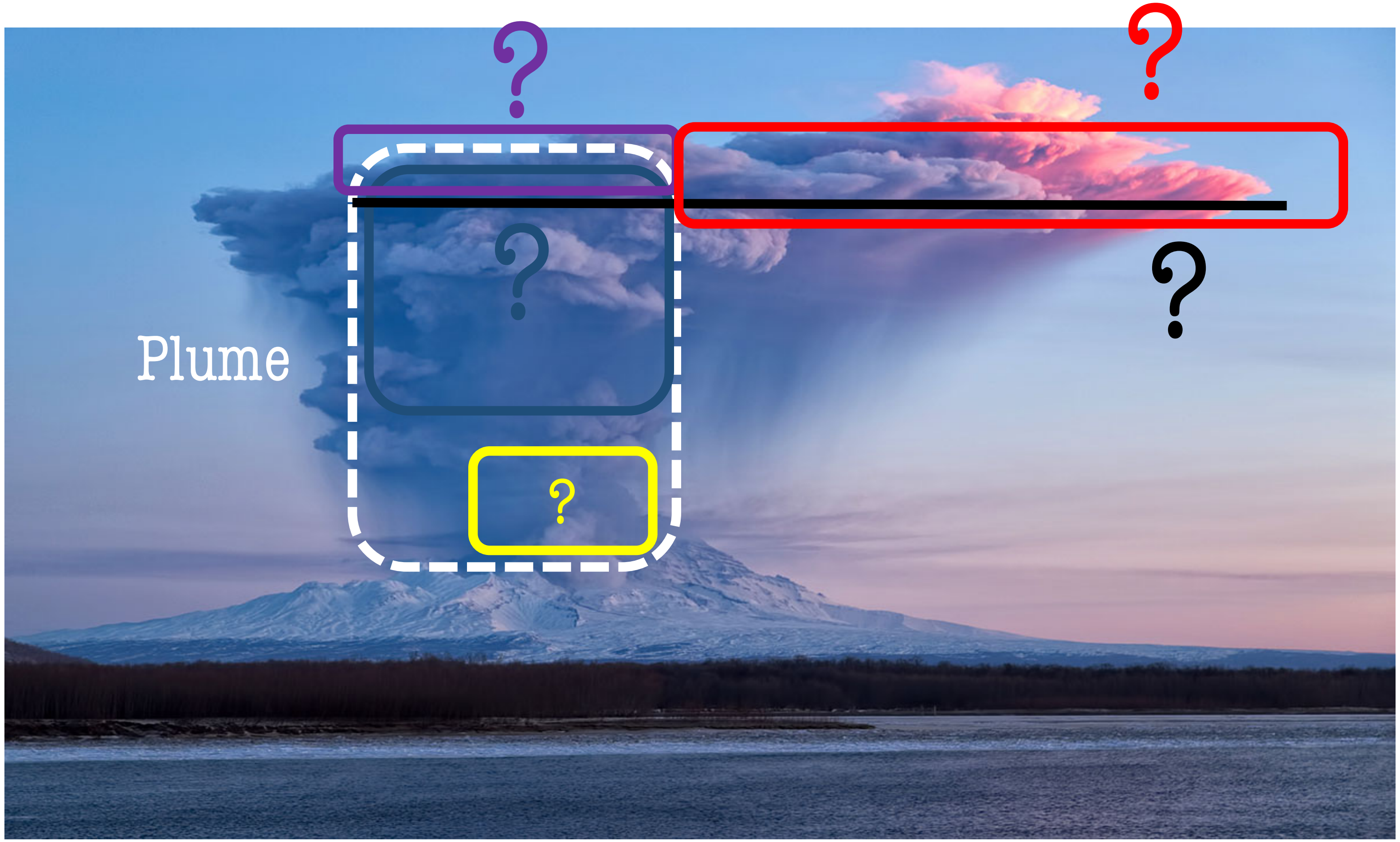
5. We can be interested in second order phenomena for plumes but first order for what concerns sedimentation (e.g. aggregation)

UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

Your first steps in modelling volcanic plumes

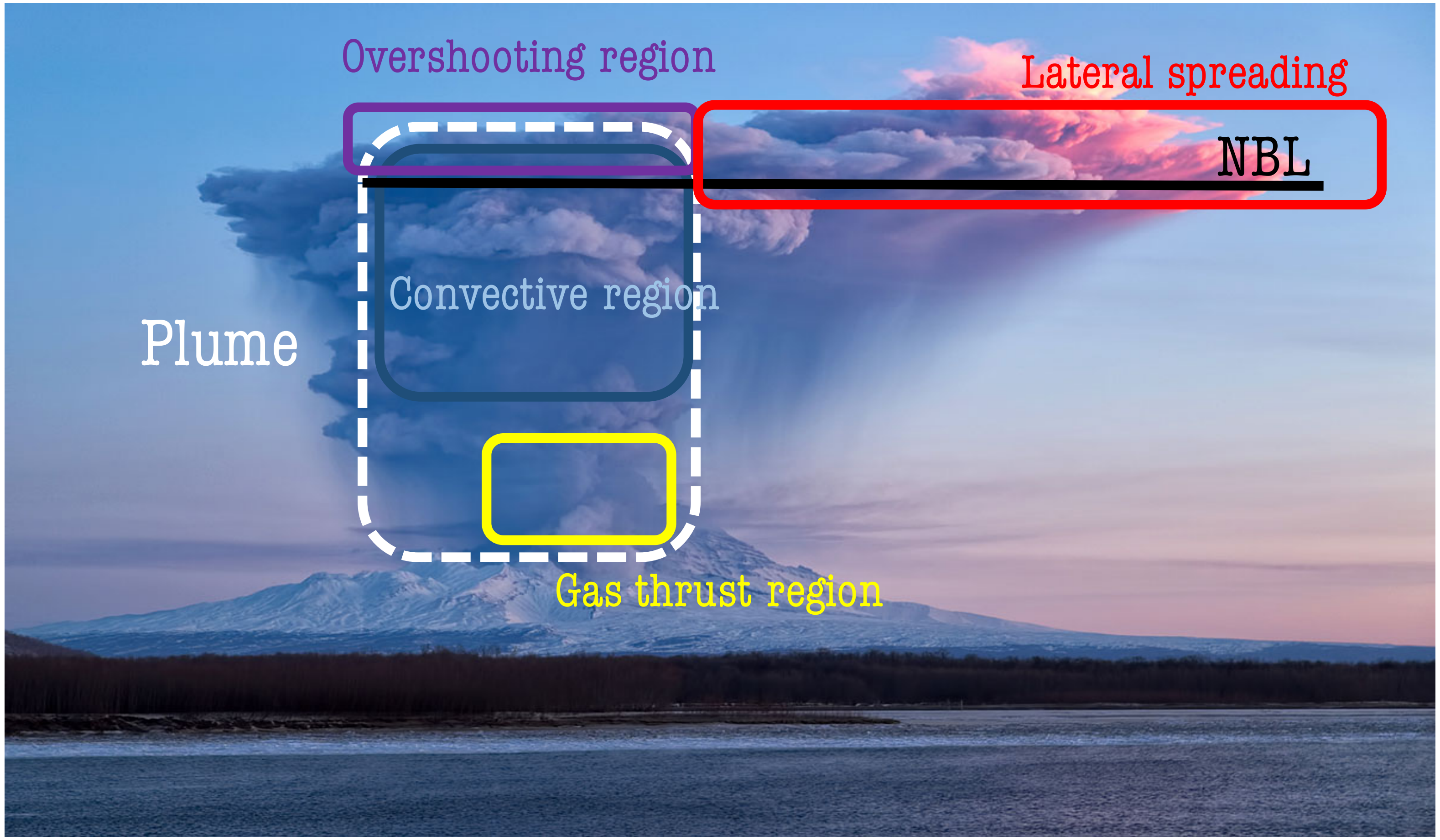
What have we seen so far?



UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Your first steps in modelling volcanic plumes

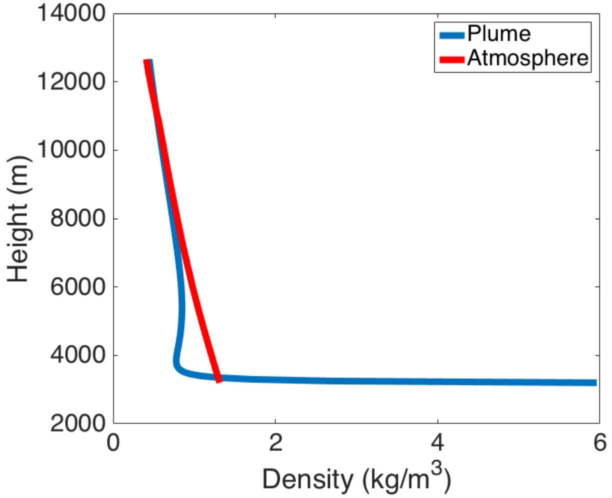
What we have seen so far

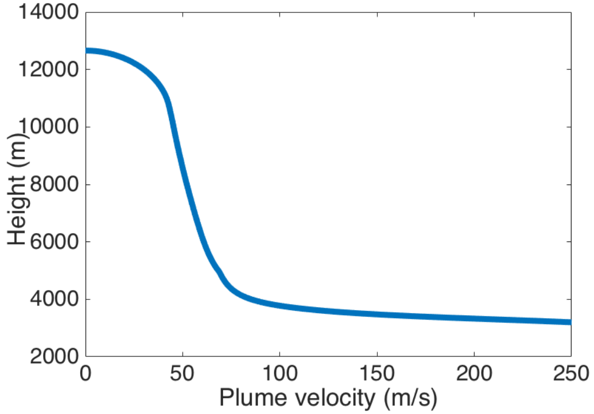


UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

1-D Models





Morton et al. (1956)

Wilson et al. (1978)

Sparks (1986)

Woods (1988)

Woods and Kienle (1994)

Woods (1995)

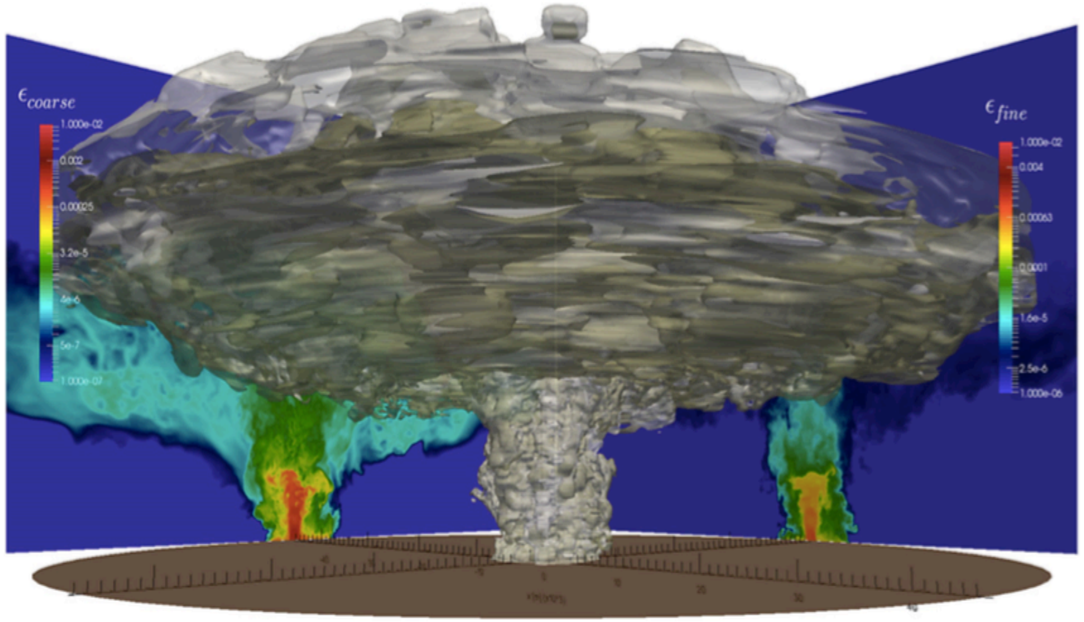
Glaze & Baloga (1996)

Bursik (2001)

Clarke et al. (2002)

Degruyter & Bonadonna (2012)

3-D Models

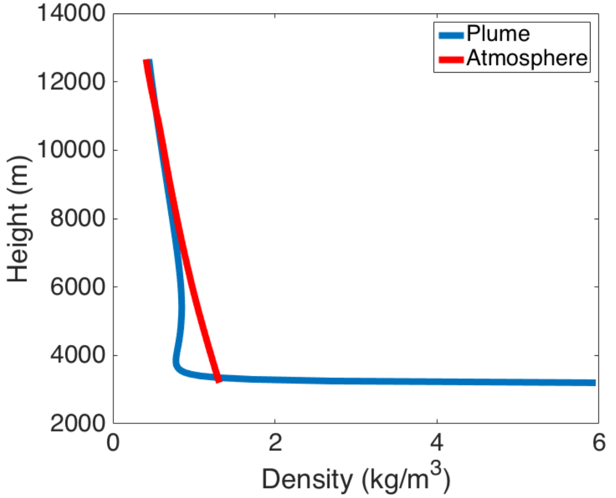


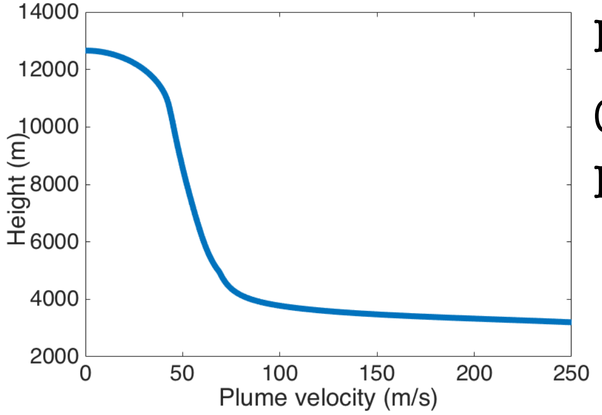
ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

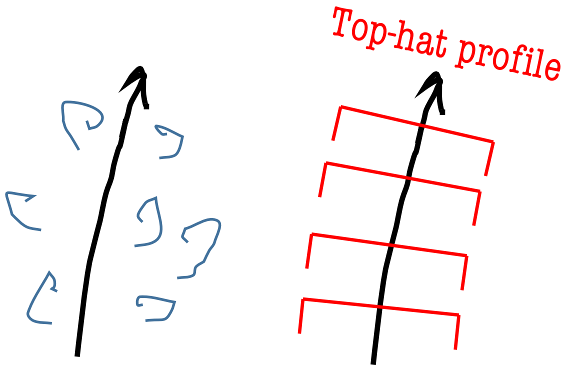
1-D Models





- Morton et al. (1956)
- Wilson et al. (1978)
- Sparks (1986)
- Woods (1988)
- Woods and Kienle (1994)
- Woods (1995)
- Glaze & Baloga (1996)
- Bursik (2001)
- Clarke et al. (2002)
- Degruyter & Bonadonna (2012)

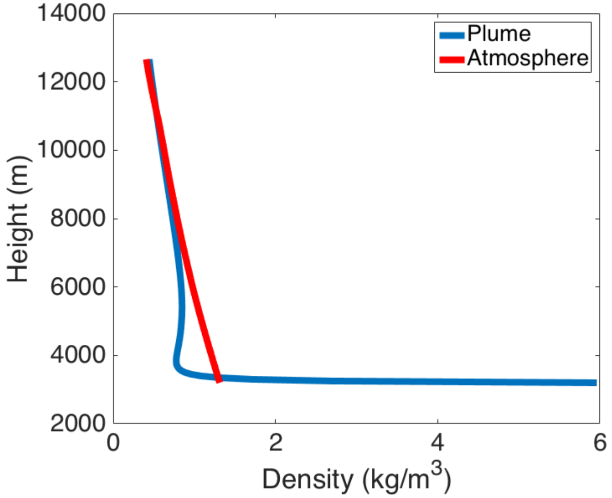
1. Integral steady-state models

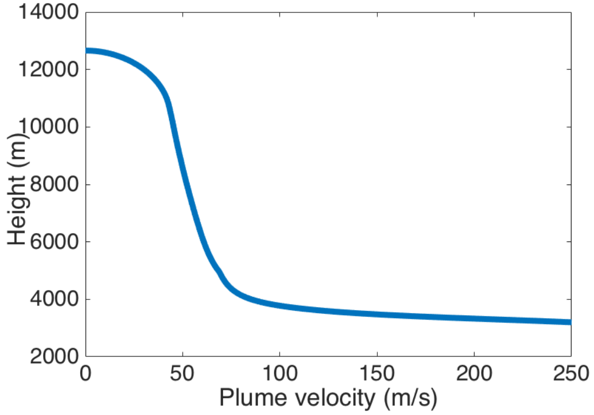


UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

1-D Models





Morton et al. (1956)

Wilson et al. (1978)

Sparks (1986)

Woods (1988)

Woods and Kienle (1994)

Woods (1995)

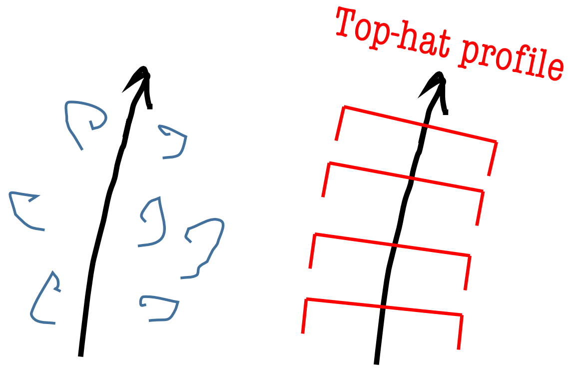
Glaze & Baloga (1996)

Bursik (2001)

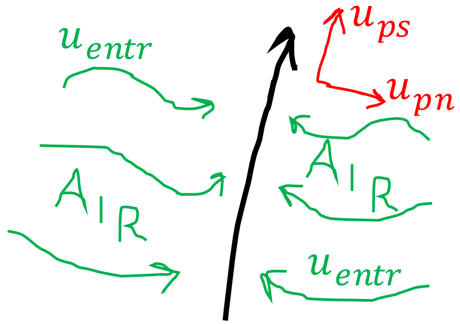
Clarke et al. (2002)

Degruyter & Bonadonna (2012)

1. Integral models
(steady-state/thermals)



2. Based on the "*entrainment assumption*"



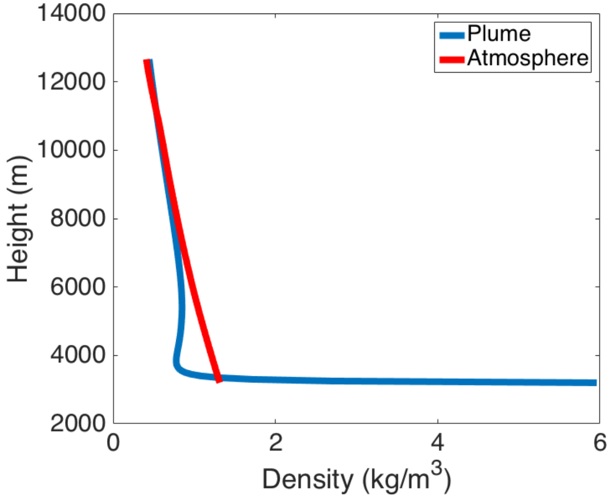
$$u_{entr} \propto \alpha u_{ps} + \beta u_{pn}$$

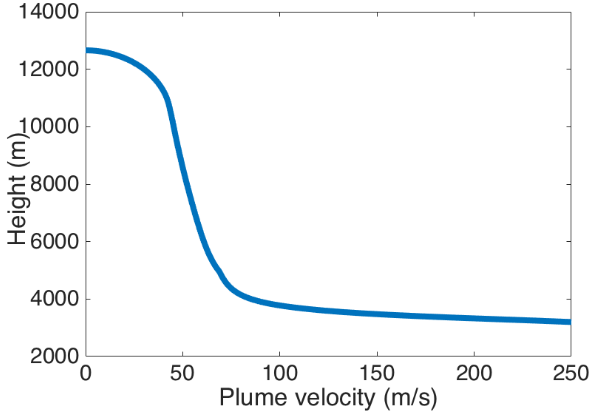
It affects the buoyancy!

Modelling volcanic plumes

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

1-D Models





Morton et al. (1956)

Wilson et al. (1978)

Sparks (1986)

Woods (1988)

Woods and Kienle (1994)

Woods (1995)

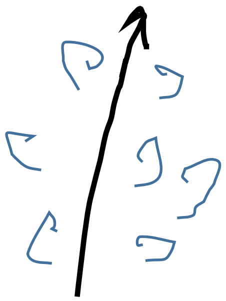
Glaze & Baloga (1996)

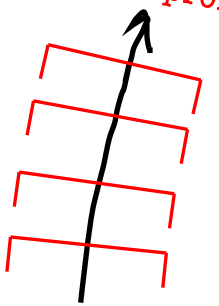
Bursik (2001)

Clarke et al. (2002)

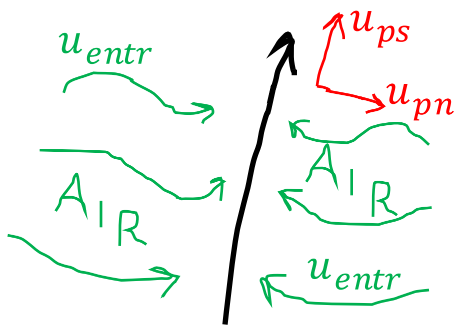
Degruyter & Bonadonna (2012)

1. Integral models (steady-state/thermals)





2. Based on the "*entrainment assumption*"



$$u_{entr} \propto \alpha u_{ps} + \beta u_{pn}$$

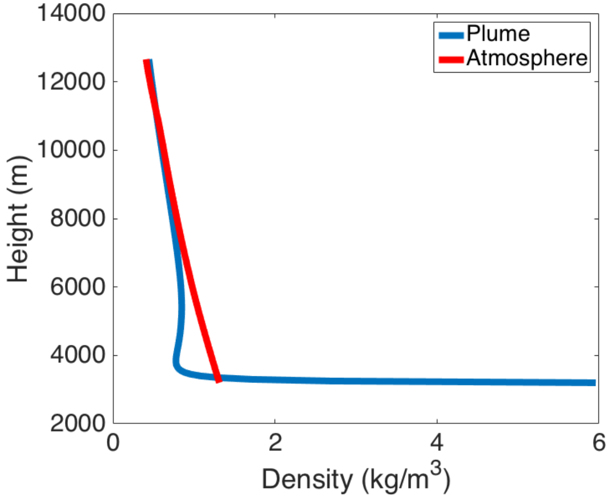
It affects the buoyancy!

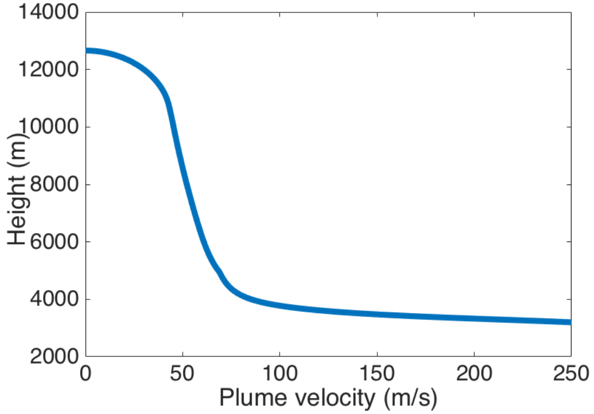
3. Low computational cost

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

1-D Models





Morton et al. (1956)

Wilson et al. (1978)

Sparks (1986)

Woods (1988)

Woods and Kienle (1994)

Woods (1995)

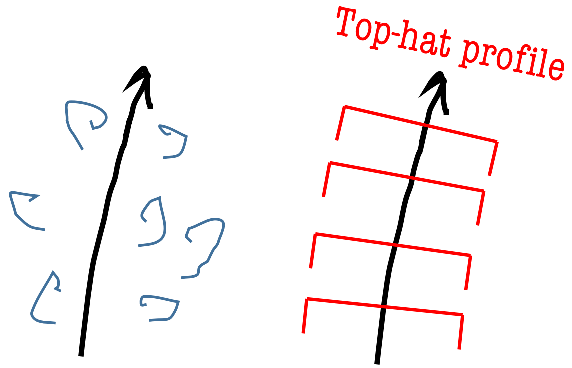
Glaze & Baloga (1996)

Bursik (2001)

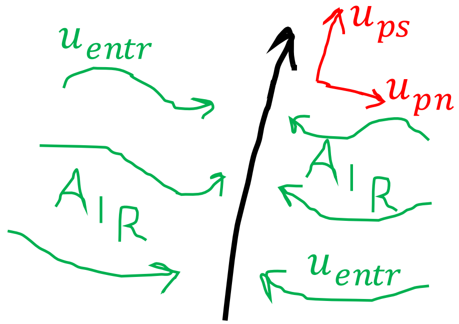
Clarke et al. (2002)

Degruyter & Bonadonna (2012)

1. Integral models (steady-state/thermals)



2. Based on the "*entrainment assumption*"



$$u_{entr} \propto \alpha u_{ps} + \beta u_{pn}$$

It affects the buoyancy!

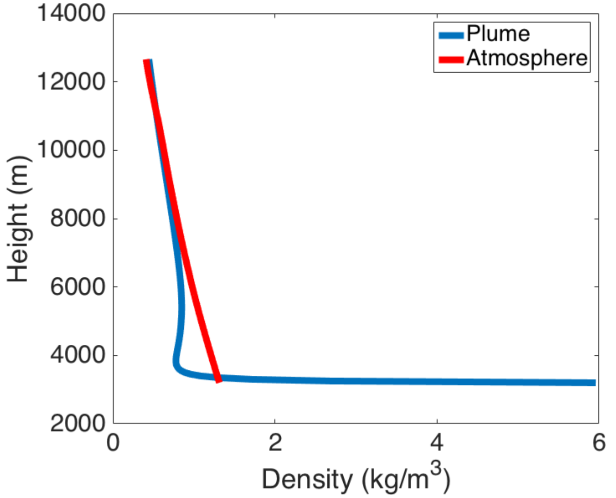
3. Low computational cost

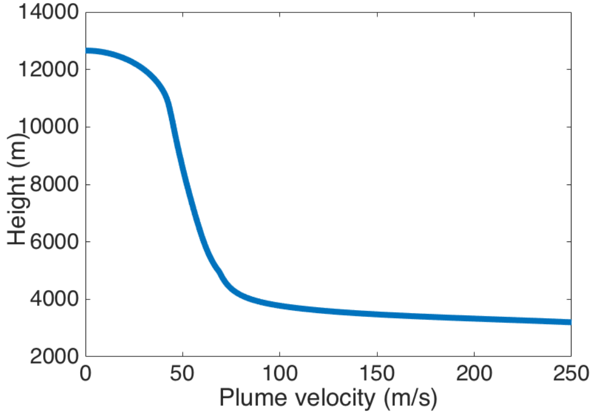
4. Reliable estimation of the NBL

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

1-D Models





Morton et al. (1956)

Wilson et al. (1978)

Sparks (1986)

Woods (1988)

Woods and Kienle (1994)

Woods (1995)

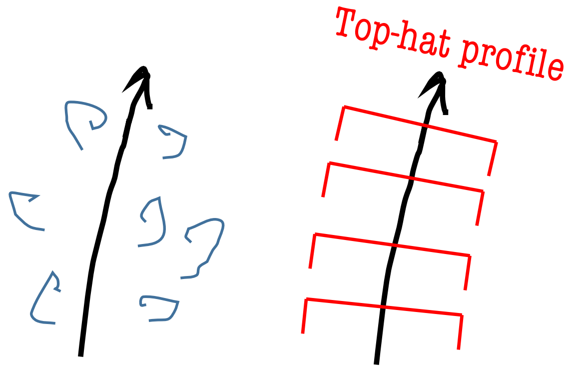
Glaze & Baloga (1996)

Bursik (2001)

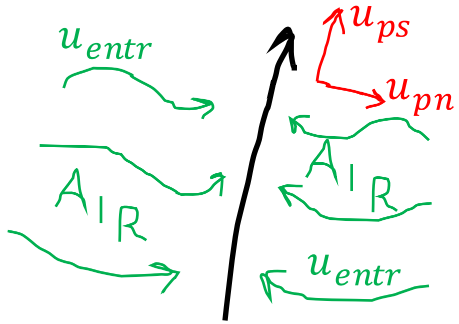
Clarke et al. (2002)

Degruyter & Bonadonna (2012)

1. Integral models (steady-state/thermals)



2. Based on the "*entrainment assumption*"



$u_{entr} \propto \alpha u_{ps} + \beta u_{pn}$

It affects the buoyancy!

3. Low computational cost

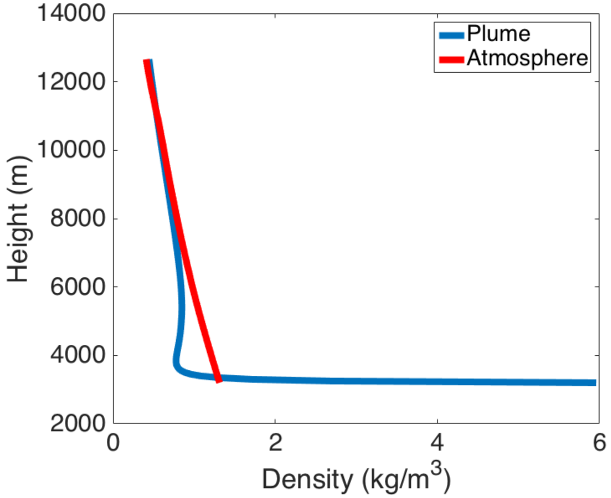
4. Reliable estimation of the NBL

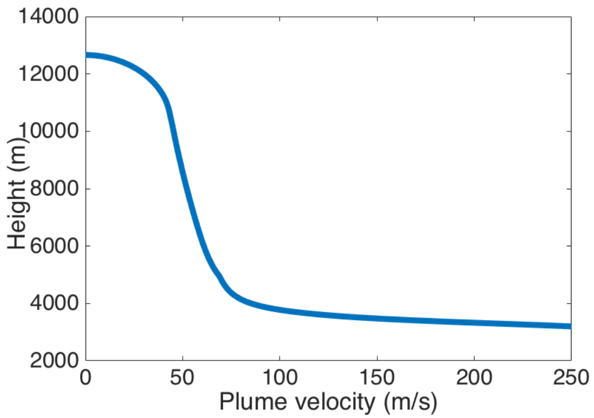
5. No turbulence

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

1-D Models





Morton et al. (1956)

Wilson et al. (1978)

Sparks (1986)

Woods (1988)

Woods and Kienle (1994)

Woods (1995)

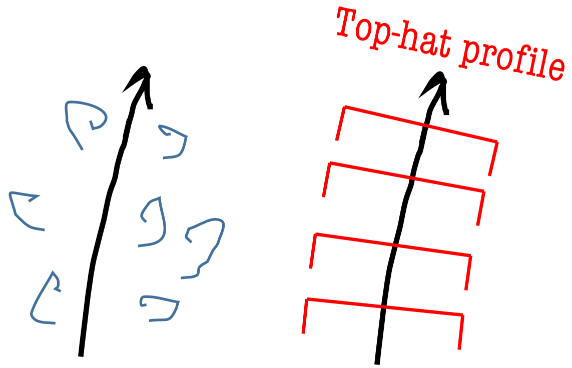
Glaze & Baloga (1996)

Bursik (2001)

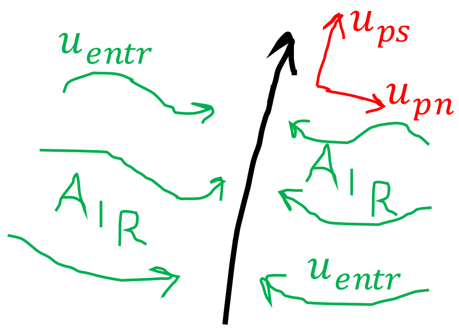
Clarke et al. (2002)

Degruyter & Bonadonna (2012)

1. Integral models (steady-state/thermals)



2. Based on the "*entrainment assumption*"



$$u_{entr} \propto \alpha u_{ps} + \beta u_{pn}$$

It affects the buoyancy!

3. Low computational cost

4. Reliable estimation of the NBL

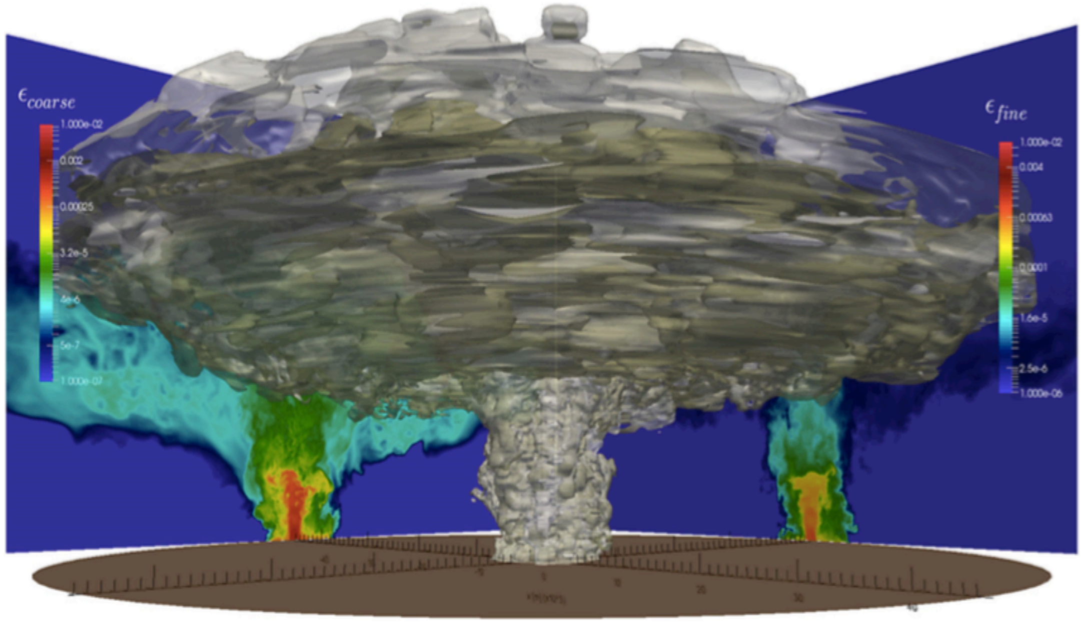
5. No turbulence

6. "Fake" multiphase flow

Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)



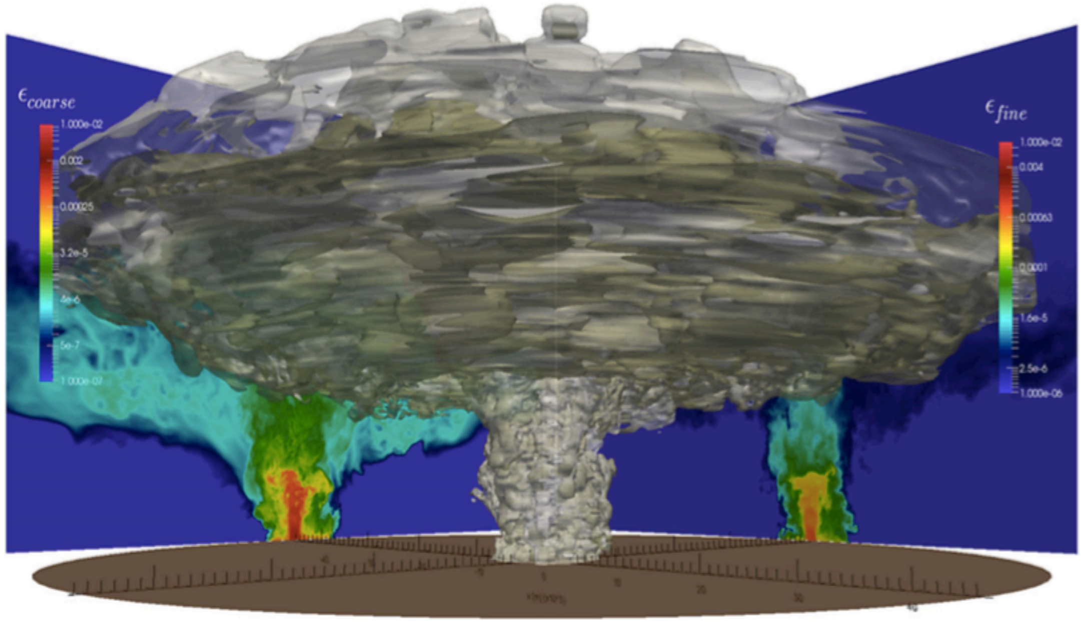
ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)

2. "Complete" description of turbulence (LES)



ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

UNIVERSITÉ
DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

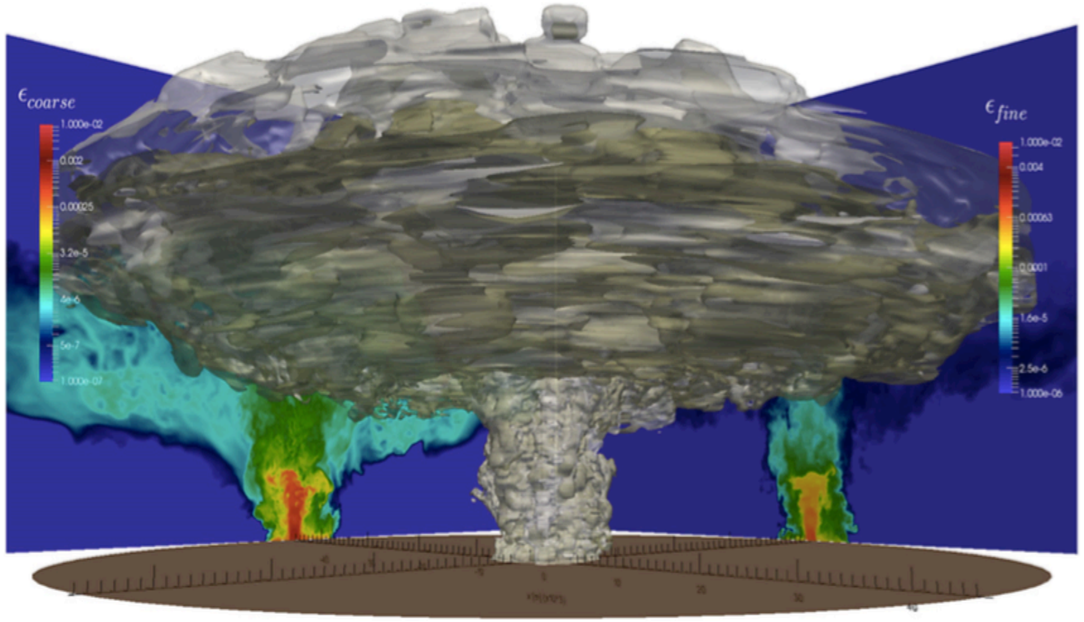
Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)

2. “Complete” description of turbulence (LES)

3. Real multiphase flow



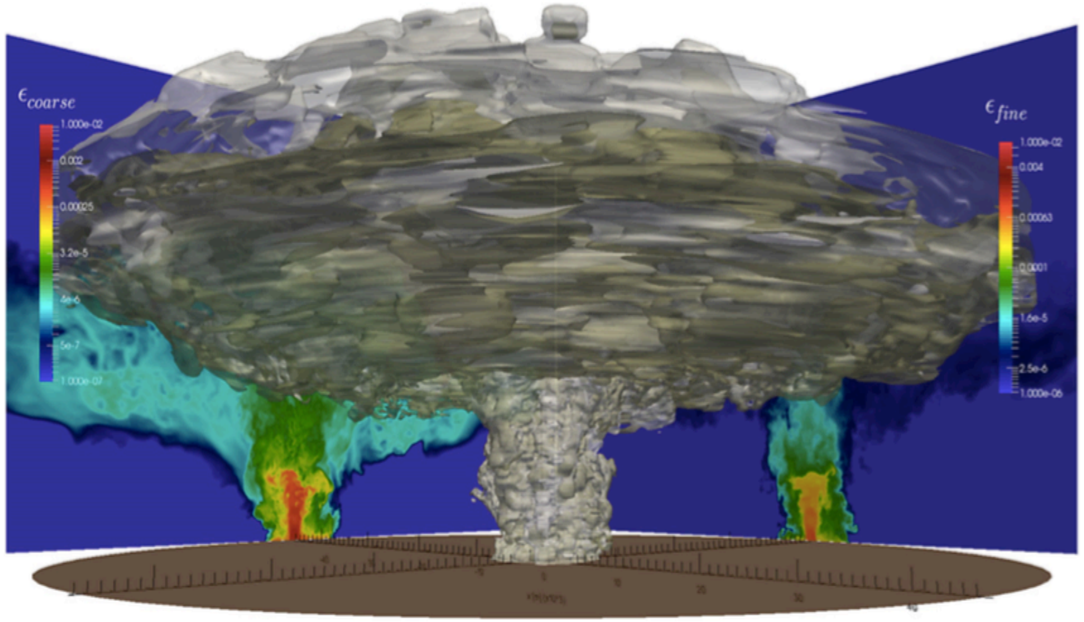
ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)
2. "Complete" description of turbulence (LES)
3. Real multiphase flow
4. Computationally heavy

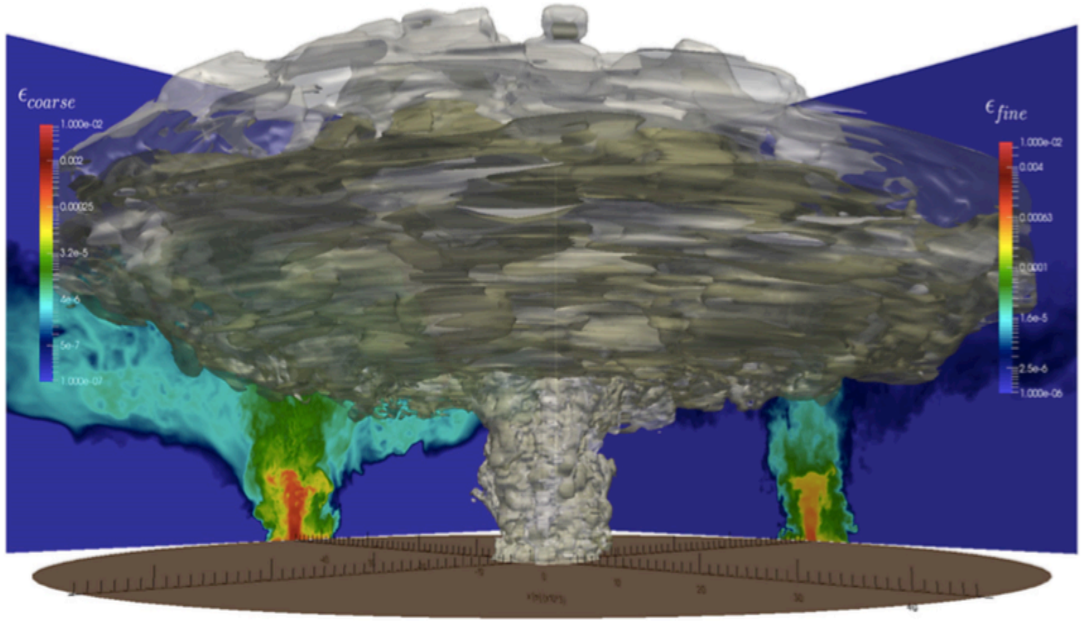


ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)
2. “Complete” description of turbulence (LES)
3. Real multiphase flow
4. Computationally heavy
5. Good description of the microphysics

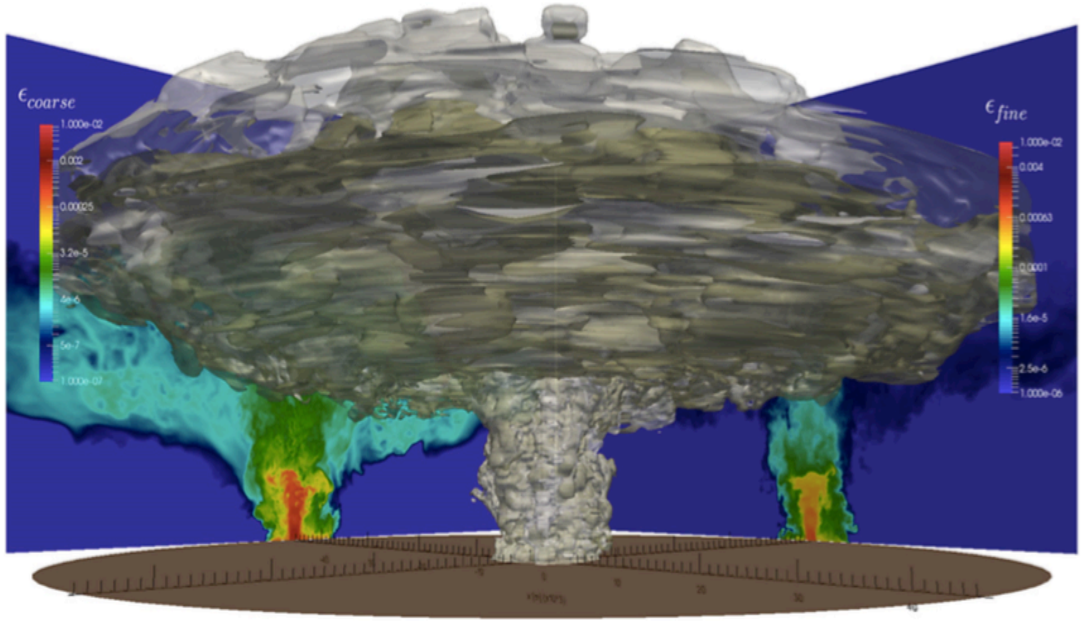


ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)
2. “Complete” description of turbulence (LES)
3. Real multiphase flow
4. Computationally heavy
5. Good description of the microphysics
6. Plume + cloud + sedimentation

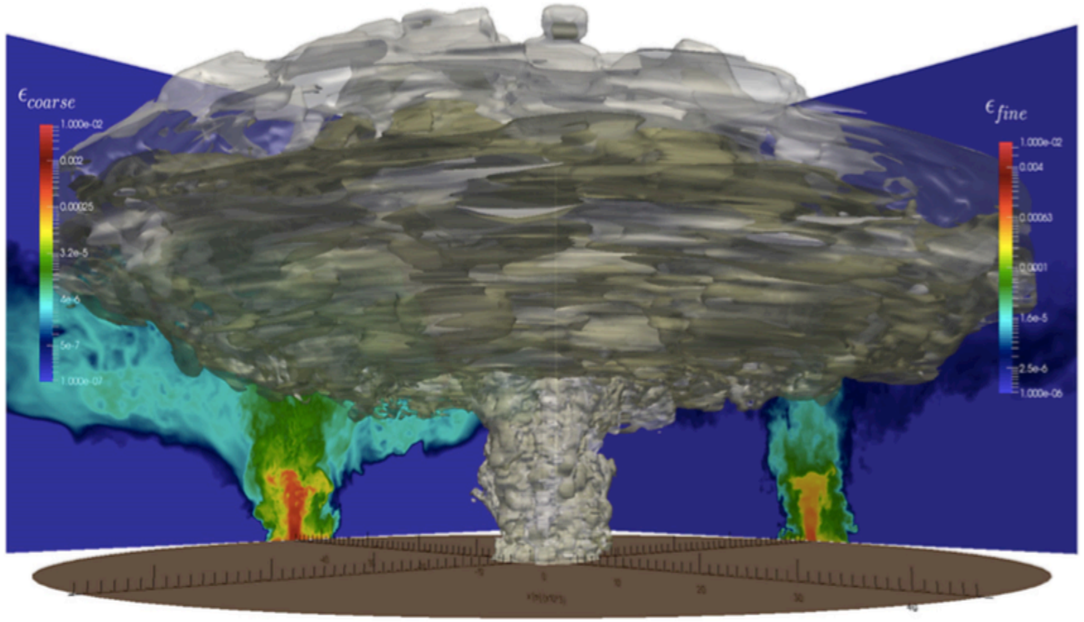


ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)
2. "Complete" description of turbulence (LES)
3. Real multiphase flow
4. Computationally heavy
5. Good description of the microphysics
6. Plume + cloud + sedimentation
7. No steady-state



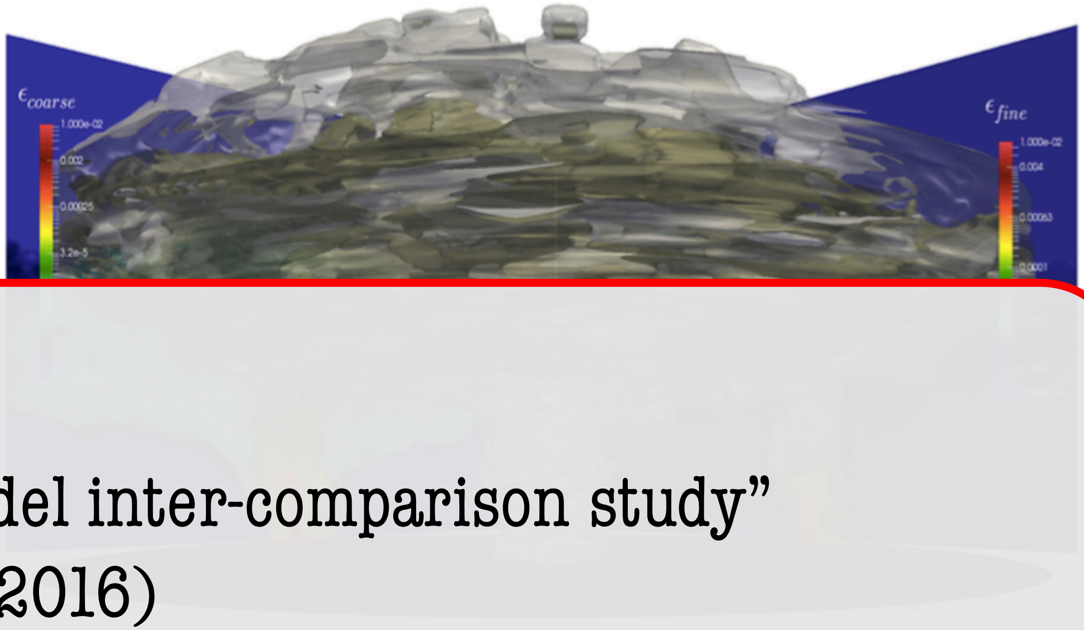
ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Modelling volcanic plumes

3-D Models

1. Eulerian (voxel) / Lagrangian (tracking)
2. “Complete” description of turbulence (LES)
3. Real multiphase flow
4. Computationally heavy
5. Good description of the microphysics
6. Plume + cloud + sedimentation



ATHAM	Oberhuber et al. (1998); Herzog et al. (2003)
SK-3D	Suzuki et al. (2005); Herzog et al. (2003)
PDAC	Neri et al. (2003); Esposti Ongaro et al., (2007)
ASHEE	Cerminara et al. (2016)

“Results of the eruptive column model inter-comparison study”
Costa et al. (2016)

The 1D model you are going to use in the exercise

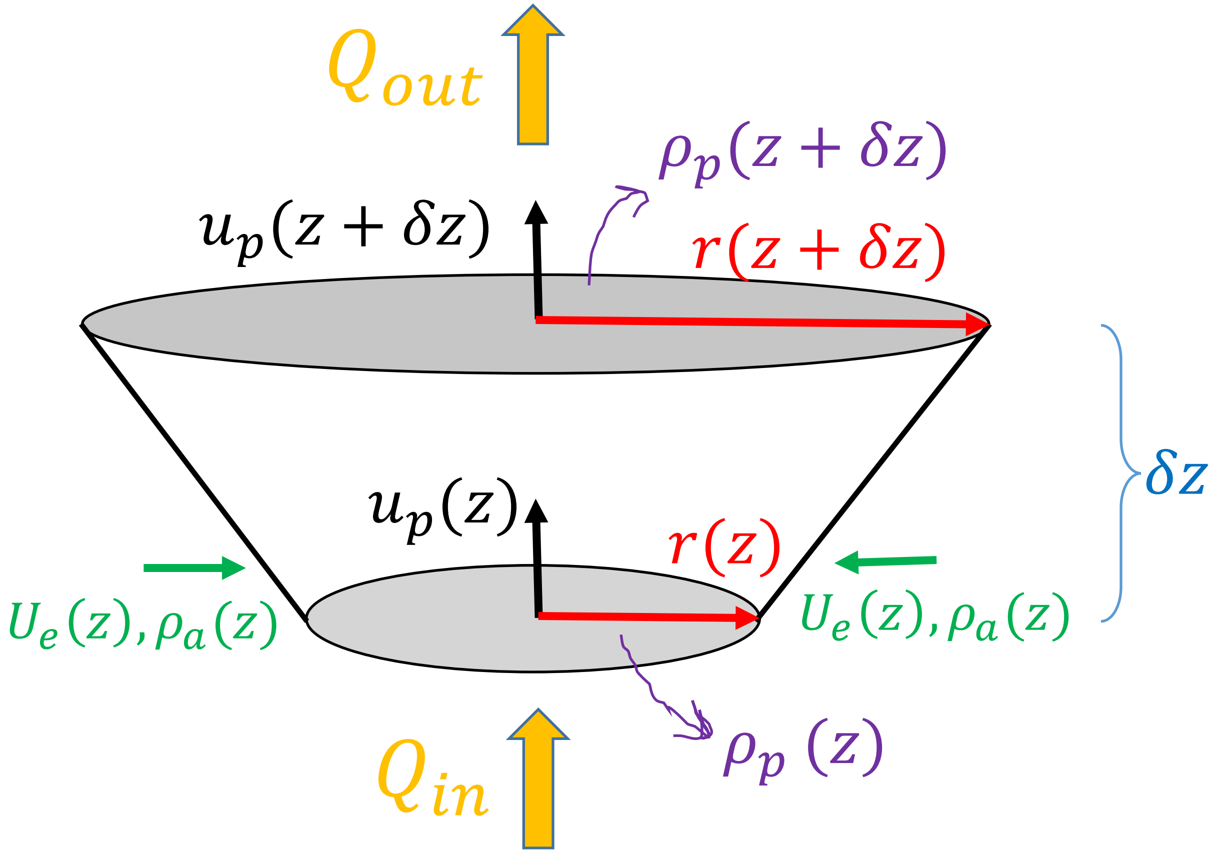
Degruyter & Bonadonna (2012)

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

1-D steady state plume models

Example:

mass flow rate balance in the control volume

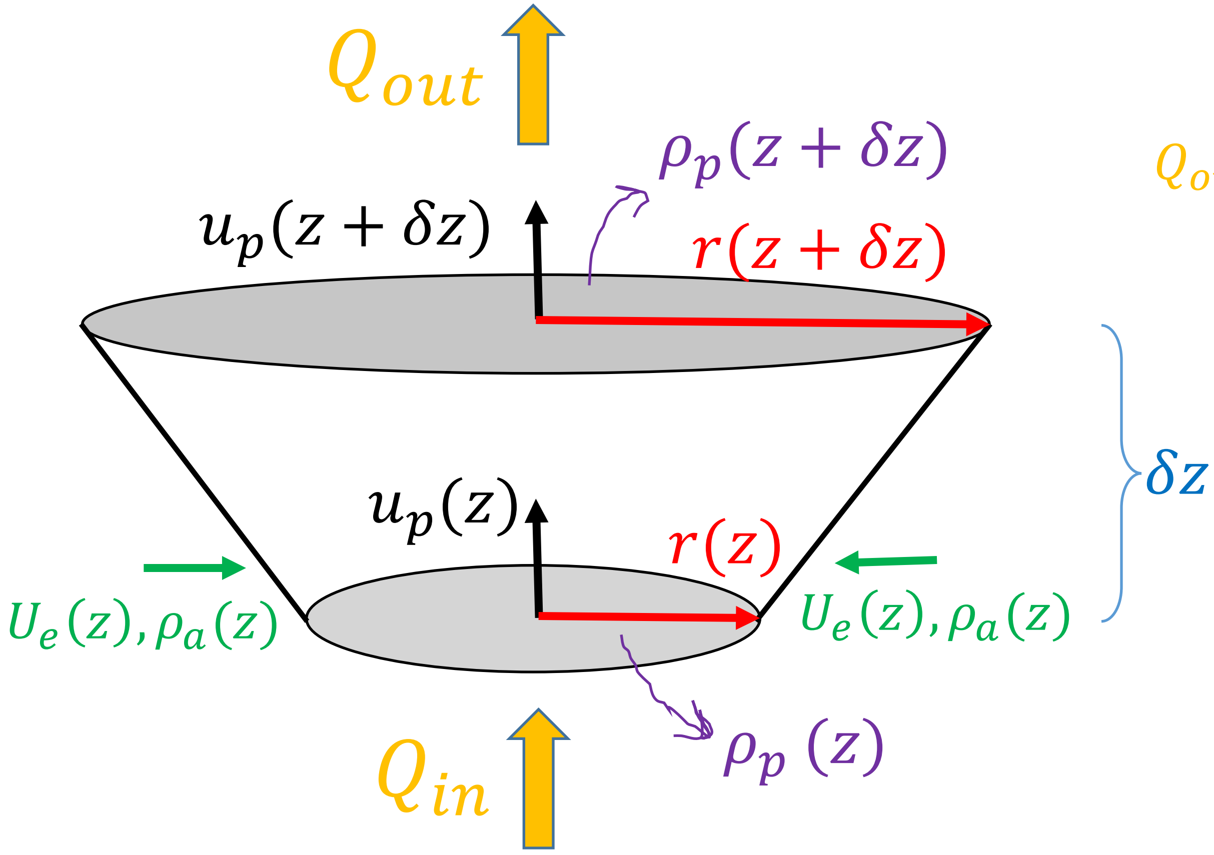


UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

1-D steady state plume models

Example:

mass flow rate balance in the control volume



Mass flow rate (inward)

$$Q_{in} = \{\rho(z) \cdot u_p(z)\} \cdot \{\pi \cdot [r(z)]^2\}$$

Mass flow rate (outward)

$$Q_{out} = \{\rho(z+\delta z) \cdot u_p(z+\delta z)\} \cdot \{\pi \cdot [r(z+\delta z)]^2\}$$

Variation in the δz (entrainment)

$$\Delta Q = 2\pi\, r(z)\, U_e\, \rho_a \delta z$$

Balance equation

$$Q_{out} - Q_{in} = \Delta Q$$

Limit for $\delta z \rightarrow 0$

$$\boxed{\frac{d}{dz}\left(\rho_p \pi r^2 u_p\right) = 2\pi\, r \rho_a U_e}$$

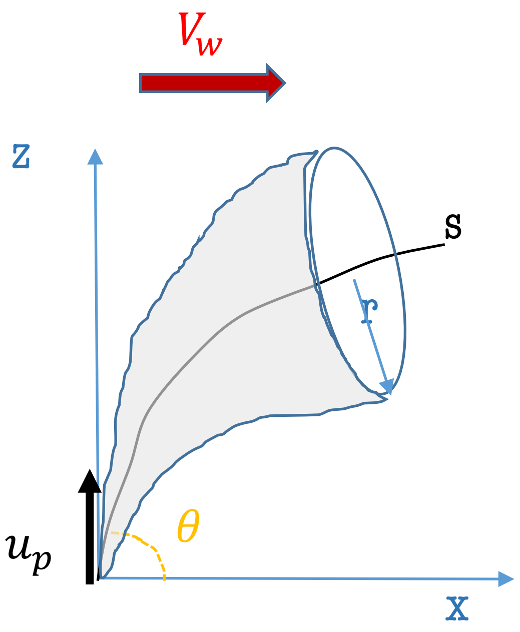
UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

1-D steady state plume models

Model of Degruyter & Bonadonna (2013)

Mass flow rates

$$\begin{cases} \frac{d}{ds}(Q_d) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_d\right) = 2\pi\, r \rho_{ad} \phi_{ad} U_e & \text{Dry air} \\ \frac{d}{ds}(Q_v) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_v\right) = 2\pi\, r \rho_{av} \phi_{av} U_e - \lambda\, \rho_p \phi_v \pi r^2 & \text{Water vapor} \\ \frac{d}{ds}(Q_l) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_l\right) = \lambda\, \rho_p \phi_l \pi r^2 & \text{Liquid water} \\ \frac{d}{ds}(Q_s) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_s\right) = 0 & \text{Solid} \end{cases}$$



UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

1-D steady state plume models

Model of Degruyter & Bonadonna (2013)

Mass flow rates

$$\frac{d}{ds}(Q_d) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_d\right) = 2\pi\, r \rho_{ad} \phi_{ad} U_e$$ Dry air

$$\frac{d}{ds}(Q_v) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_v\right) = 2\pi\, r \rho_{av} \phi_{av} U_e - \lambda\, \rho_p \phi_v \pi r^2$$ Water vapor

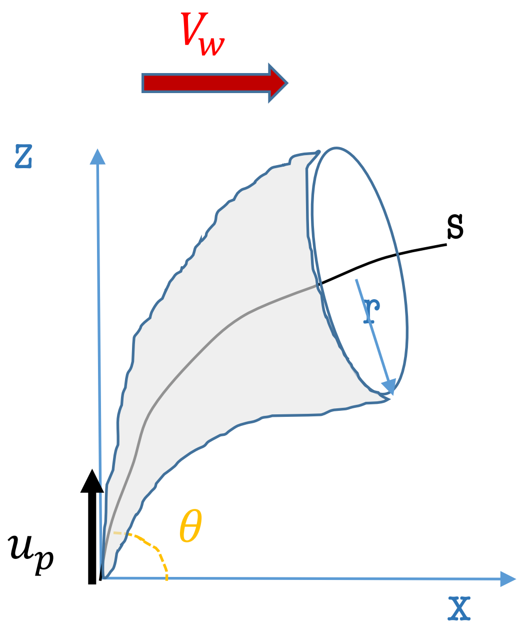
$$\frac{d}{ds}(Q_l) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_l\right) = \lambda\, \rho_p \phi_l \pi r^2$$ Liquid water

$$\frac{d}{ds}(Q_s) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_s\right) = 0$$ Solid

Momentum flow rates

$$\frac{d}{ds}(W_z) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p^2 \sin(\theta)\right) = (\rho_a - \rho_p) g \pi r^2$$ Along z-direction

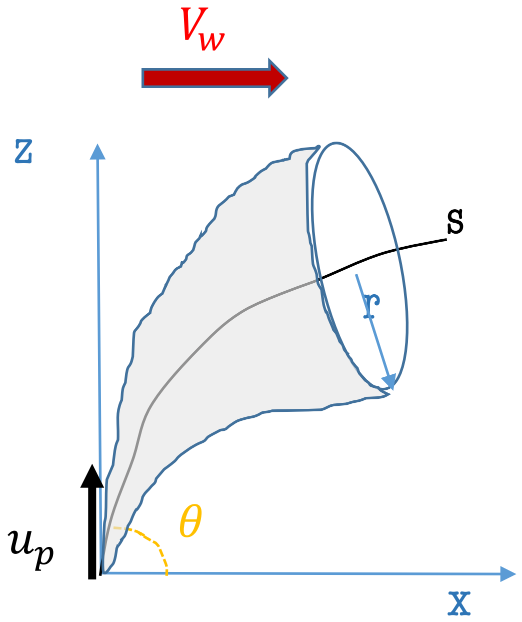
$$\frac{d}{ds}(W_x) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p^2 \cos(\theta)\right) = 2\pi \rho_a U_e\, r\, V_w$$ Along x-direction



UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

1-D steady state plume models

Model of Degruyter & Bonadonna (2013)



Mass flow rates

$$\frac{d}{ds}(Q_d) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_d\right) = 2\pi\, r \rho_{ad} \phi_{ad} U_e$$ Dry air

$$\frac{d}{ds}(Q_v) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_v\right) = 2\pi\, r \rho_{av} \phi_{av} U_e - \lambda\, \rho_p \phi_v \pi r^2$$ Water vapor

$$\frac{d}{ds}(Q_l) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_l\right) = \lambda\, \rho_p \phi_l \pi r^2$$ Liquid water

$$\frac{d}{ds}(Q_s) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p \phi_s\right) = 0$$ Solid

Momentum flow rates

$$\frac{d}{ds}(W_z) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p^2 \sin(\theta)\right) = \left(\rho_a - \rho_p\right) g \pi r^2$$ Along z-direction

$$\frac{d}{ds}(W_x) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p^2 \cos(\theta)\right) = 2\pi \rho_a U_e\, r\, V_w$$ Along x-direction

Heat flow rate

$$\frac{d}{ds}(H) = \frac{d}{ds}\left(\rho_p \pi r^2 u_p\, C_p T\right) = 2\pi \rho_a U_e\, r\, C_a T_a - \rho_p \pi r^2 u_p g \sin(\theta) + \lambda \cdot \frac{d}{ds}(Q_l)$$

Exercise 2: roll up your sleeves!
It is time for Matlab!

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

End of the lesson

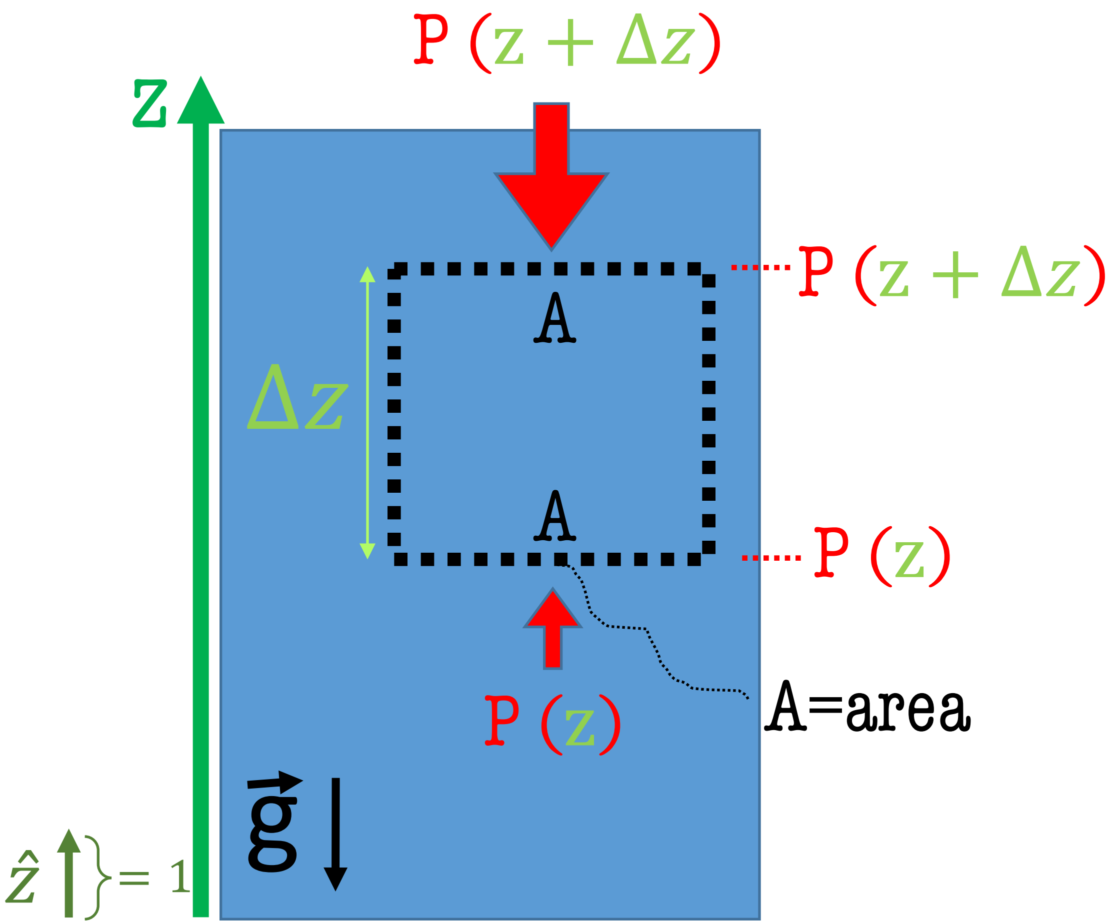
[Thank you for your attention]

{for any question or doubt Eduardo.Rossi@unige.ch}

For those of you who need more physical justifications to some of the formula used in the lesson

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Hydrostatic pressure



[...the algebra continues]

$$0 = -g \cdot \hat{\mathbf{z}} - \frac{P(z+\Delta z)\cdot V}{m \cdot \Delta z}\hat{\mathbf{z}} + \frac{P(z)\cdot V}{m \cdot \Delta z}\hat{\mathbf{z}}$$

$$\frac{P(z+\Delta z)}{\Delta z}\hat{\mathbf{z}} - \frac{P(z)}{\Delta z}\hat{\mathbf{z}} = -\frac{mg}{V}\cdot\hat{\mathbf{z}}$$

$$\frac{P(z+\Delta z) - P(z)}{\Delta z}\hat{\mathbf{z}} = -\frac{mg}{V}\cdot\hat{\mathbf{z}}$$

$$\frac{P(z+\Delta z) - P(z)}{\Delta z}\hat{\mathbf{z}} = -\rho_F \cdot g \cdot \hat{\mathbf{z}}$$

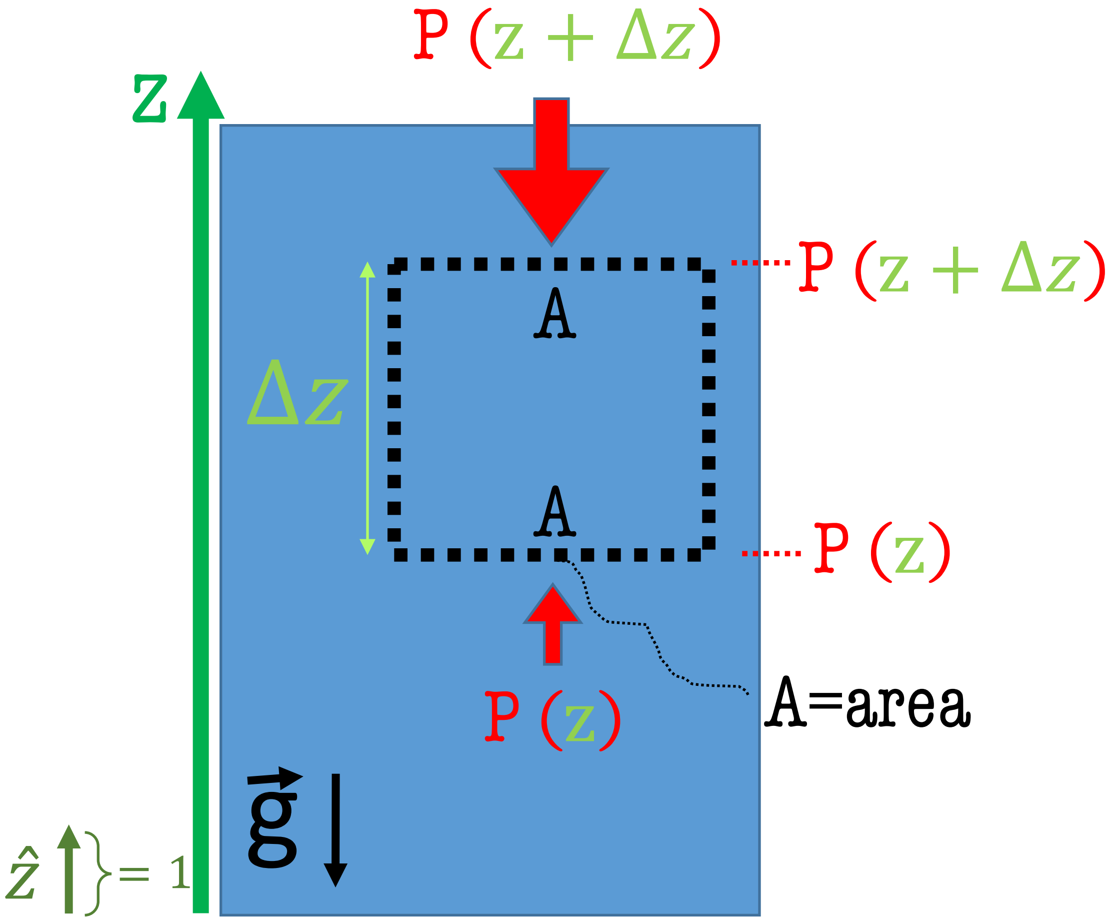
Limit for $\Delta z \to 0$

We take the limit for an infinitesimal box (i.e. $\Delta z \to 0$)

$$\lim_{\Delta z \to 0}\frac{P(z+\Delta z) - P(z)}{\Delta z}\hat{\mathbf{z}} = -\rho_F \cdot g \cdot \hat{\mathbf{z}}$$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Hydrostatic pressure



[...the limit continues]

We take the limit for an infinitesimal box (i.e. $\Delta z \to 0$)

$$\lim_{\Delta z \to 0} \frac{P(z+\Delta z) - P(z)}{\Delta z}\,\hat{\mathbf{z}} = -\rho_F \cdot g \cdot \hat{\mathbf{z}}$$

$$\boxed{\text{Hydrostatic pressure} \quad \frac{\partial P}{\partial z}\hat{\mathbf{z}} = -\rho_F \cdot g\ \hat{\mathbf{z}}}$$

Case 1: $\rho_F = \text{constant}$ — Stevin's law

Case 2: $\rho_F \neq \text{constant}$ — Barometric law

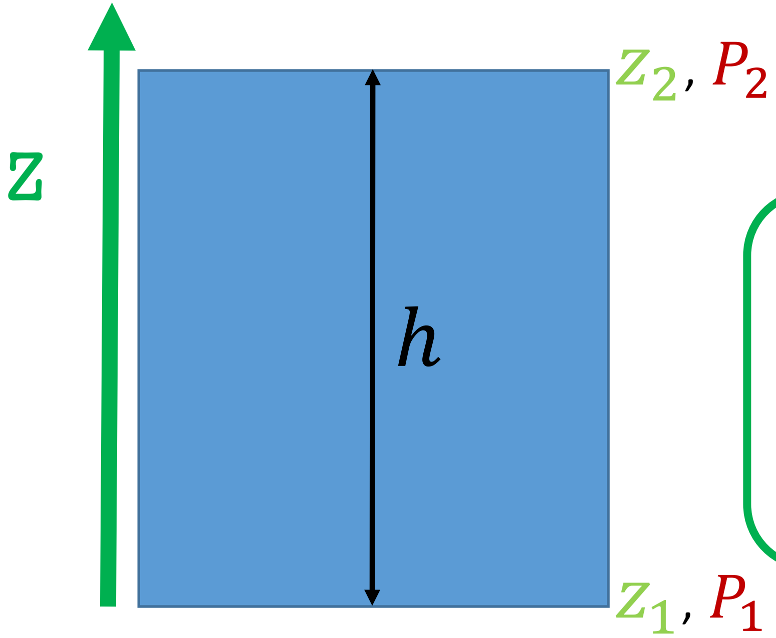
UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Hydrostatic pressure for a constant density

Hydrostatic pressure $\frac{\partial P}{\partial z}\hat{\mathbf{z}} = -\rho_F \cdot g\ \hat{\mathbf{z}}$

Case 1: $\rho_F = \text{constant}$

$$\frac{\partial P}{\partial z}\hat{\mathbf{z}} = -\rho_F \cdot g\ \hat{\mathbf{z}} \rightarrow \int_{P_1}^{P_2} dP = -\rho_F \cdot g \int_{z_1}^{z_2} dz \rightarrow P_2 - P_1 = -\rho_F \cdot g \cdot (z_2 - z_1)$$



Stevin's law $P_2 - P_1 = -\rho_F \cdot g \cdot (z_2 - z_1)$

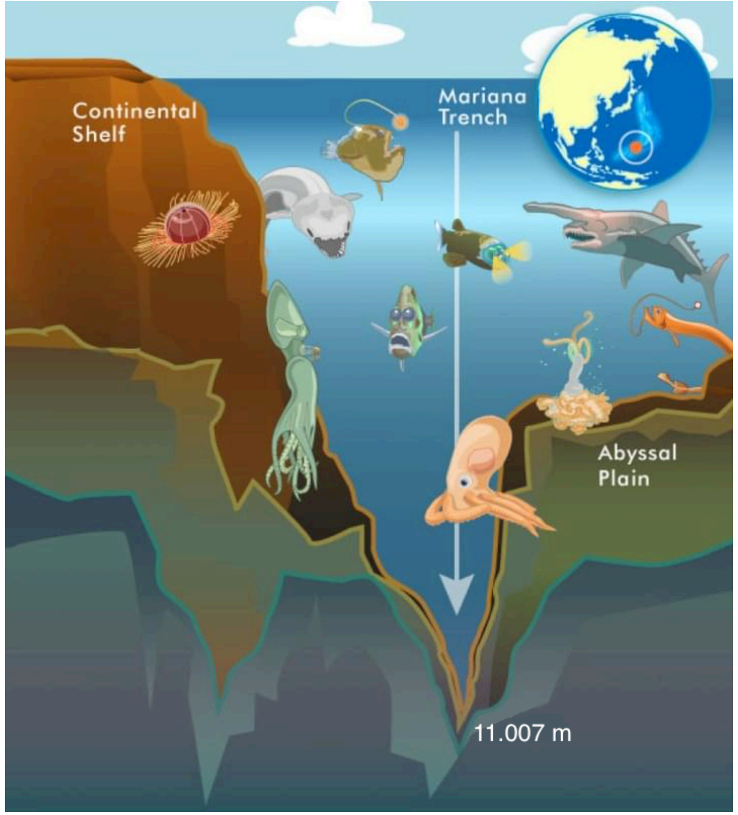
$$P_2 = P_1 - \rho_F \cdot g \cdot h$$

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

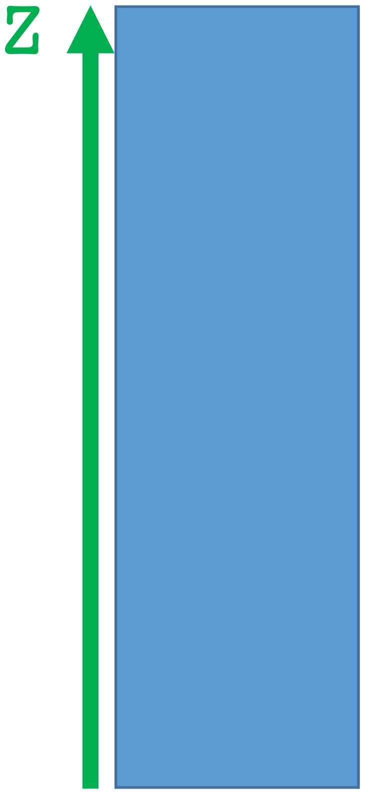
Application of Stevin's law

Stevin's law $P_2 - P_1 = -\rho_F \cdot g \cdot (z_2 - z_1)$

Example What is the pressure at the bottom of the Marianas trench?



Credits: www.travel365.it



$z_2 = 11.000$ m

$P_2 = 10^5 Pa$

$P_1 = 10^5 Pa +$

$$10^3 \frac{kg}{m^3} \cdot 10 \frac{m}{s^2} \cdot 1.1 \cdot 10^4\ m = 1.1 \cdot 10^8 Pa$$

$\underline{\boldsymbol{P_1 \approx 1000\ P_2!}}$

$z_1 = 0\ m$

$P_1 = ?$

Hydrostatic pressure for a variable density

Hydrostatic pressure $\frac{\partial P}{\partial z}\hat{\mathbf{z}} = -\rho_F(z) \cdot g\ \hat{\mathbf{z}}$

Case 2: $\rho_F \neq$ constant

Perfect gas law: $P = \rho_F \cdot \frac{RT}{\mu}$

$$\frac{\partial\left(\rho_F(z)\frac{RT}{\mu}\right)}{\partial z}\hat{\mathbf{z}} = -\rho_F(z) \cdot g\ \hat{\mathbf{z}} \longrightarrow \frac{\partial(\rho_F(z))}{\partial z}\hat{\mathbf{z}} = -\frac{\mu g}{RT}\rho_F(z)\hat{\mathbf{z}}$$

$$\frac{d\rho_F}{\rho_F} = -\frac{\mu g}{RT}\,dz \longrightarrow \int_{\rho_a}^{\rho_b}\frac{d\rho_F}{\rho_F} = -\frac{\mu g}{RT}{}^{*}\int_{z_a}^{z_b} dz \longrightarrow \log\rho_b - \log\rho_a = -\frac{\mu g}{RT}(z_b - z_a)$$

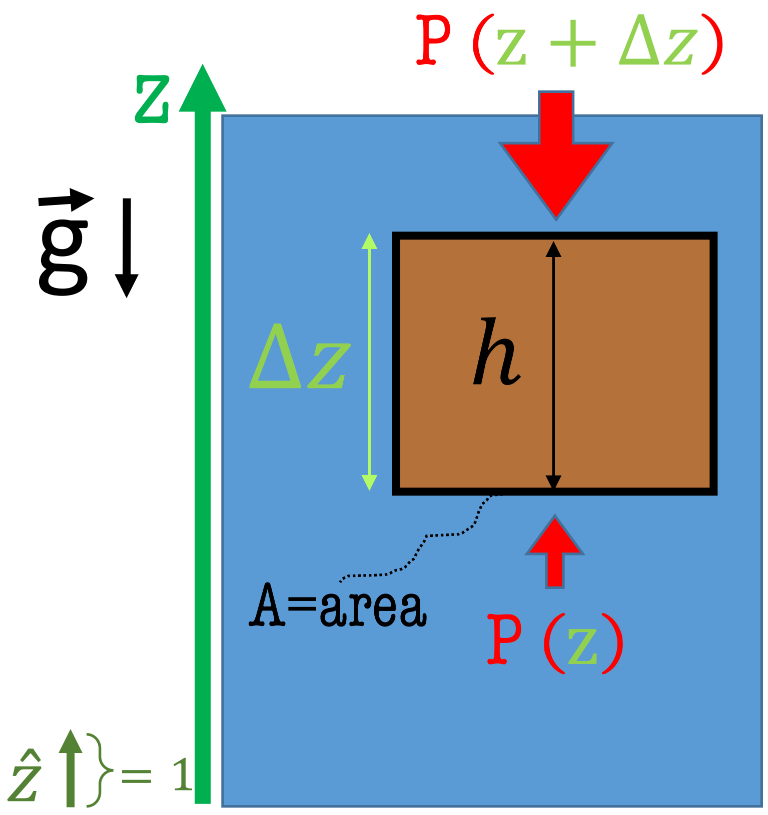
$$\log\frac{\rho_b}{\rho_a} = -\frac{\mu g}{RT}(z_b - z_a) \longrightarrow \boxed{\rho_b = \rho_a \cdot e^{-\frac{\mu g}{RT}(z_b - z_a)}}$$

Barometric law

**Be careful! Here we assumed that the temperatura is NOT a function of z…which is not true!*

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

Archimedes' principle and buoyancy



Stevin's law $P_2 = P_1 - \rho_F \, g \, h$

Newton's law $\vec{a} = \frac{1}{m}(\vec{F}_g + \vec{F}_P)$

$$ma\,\hat{\mathbf{z}} = -mg\,\hat{\mathbf{z}} - \underbrace{[P(z+\Delta z) - P(z)]}_{\text{Stevin's law}}\,A\,\hat{\mathbf{z}}$$

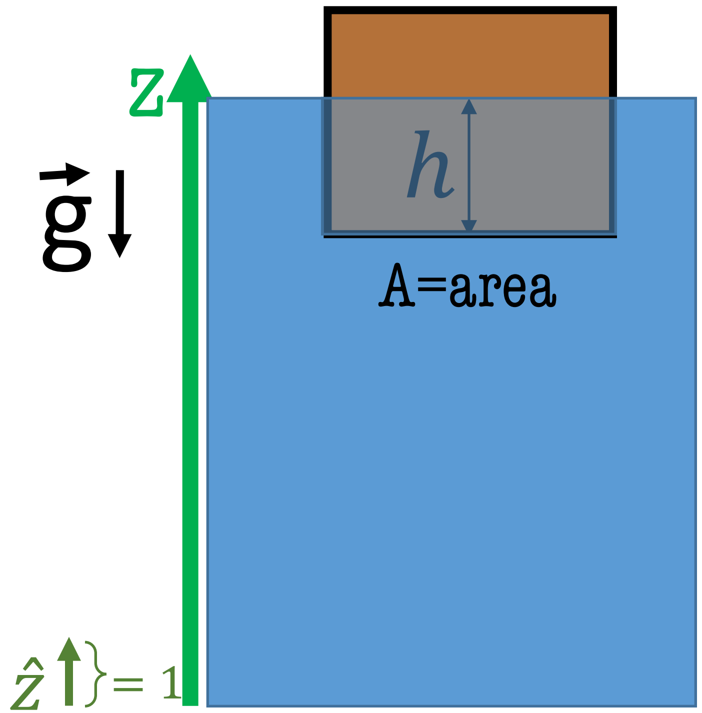
$$ma\,\hat{\mathbf{z}} = -mg\,\hat{\mathbf{z}} - [-\rho_F\, g\, h\,]\,A\,\hat{\mathbf{z}} \longrightarrow ma\,\hat{\mathbf{z}} = -mg\,\hat{\mathbf{z}} + \underbrace{\rho_F\, g\, V\,\hat{\mathbf{z}}}_{\text{Archimedes' principle}}$$

$$a\,\hat{\mathbf{z}} = -g\,\hat{\mathbf{z}} + \rho_F\, g\,\frac{V}{m}\,\hat{\mathbf{z}} \longrightarrow a\,\hat{\mathbf{z}} = -g\,\hat{\mathbf{z}} + g\,\frac{\rho_F}{\rho_p}\,\hat{\mathbf{z}}$$

$$\boxed{a\,\hat{\mathbf{z}} = -g\left[1 - \frac{\rho_F}{\rho_p}\right]\hat{\mathbf{z}}}$$ Buoyancy

UNIVERSITÉ DE GENÈVE
FACULTY OF SCIENCE
Department of Earth Sciences

The "Duck" problem



$$ma\,\hat{\mathbf{z}} = -mg\,\hat{\mathbf{z}} + \rho_F\,g\,V_{sub}\,\hat{\mathbf{z}}$$

There is no net acceleration:

$$0\,\hat{\mathbf{z}} = -mg\,\hat{\mathbf{z}} + \rho_F\,g\,V_{sub}\,\hat{\mathbf{z}}$$

$$m = \frac{\rho_F\,g\,V_{sub}}{g} \rightarrow m = \rho_F \cdot V_{sub}$$

Referring to the figure aside: $m = \rho_F \cdot (A \cdot h)$

And what about the mass of the duck?

$$m_{duck} = \rho_F \cdot V_{ds}$$

?