



Figure 1: Sample of dome lava erupted from Unzen in the 1991-1995 eruption. The bulk of the lava is the white rock (Unzen¹ in Table 4), whilst the dark rock (Unzen²) exists as discrete enclaves.

Magma viscosity and density

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1. The 1991-1995 eruption of Mt. Unzen, Japan, produced a series of lava domes. A sample of this lava can be seen in Figure 1. The white rock (Unzen¹ in Table 4) forms most of the lava, whilst the dark rock (Unzen²) is present as many discrete enclaves existing within the white rock. These enclaves are interpreted to have formed prior to eruption, when a hotter magma was injected into a cooler magma storage region. Table 4 lists the melt compositions of the two magmas at their mixing temperatures and 2000 MPa. The volume V_m of 1 mol of melt of composition \mathbf{X} as a function of pressure P and temperature T is given by

$$V_m(T, P, \mathbf{X}) = \sum_i X_i(\text{mol}\%) \left[\bar{V}_i(T = T_R, P = P_R) + \left. \frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \right|_{T=T_R} (T - T_R) + \left. \frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \right|_{P=P_R} (P - P_R) \right], \quad (1)$$

where $T_R = 1673$ K and $P_R = 10^{-4}$ GPa. Values for the red, blue and green quantities can be found in Table 2. The density can then be calculated from

$$\rho_m = \frac{1}{V_m} \sum_i X_i(\text{mol}\%) M_i. \quad (2)$$

At 775 °C, the equilibrium crystal assemblage of the Unzen¹ magma is orthopyroxene ($\phi_{\text{opx}} = 0.07$, $\rho_{\text{opx}} = 3.52$ g cm⁻³), clinopyroxene ($\phi_{\text{cpx}} = 0.02$, $\rho_{\text{cpx}} = 3.35$ g cm⁻³) and feldspar ($\phi_{\text{fd}} = 0.30$, $\rho_{\text{fd}} = 2.62$ g cm⁻³) with minor amounts of spinel and oxides. On the other hand, at 1079°C, the Unzen² magma is at the liquidus temperature and can be considered aphyric.

- (a) Determine the density of the Unzen¹ melt.

Table 1: The melt compositions (mol%) of the Unzen magmas at their mixing temperatures and 2000 MPa.

Magma	Temperature /°C	SiO ₂	Al ₂ O ₃	TiO ₂	FeO	Fe ₂ O ₃	MnO	MgO	CaO	K ₂ O	Na ₂ O	P ₂ O ₅	H ₂ O
Unzen ¹	775	67.43	6.86	0.16	0.54	0.1	0.15	0.34	2.29	2.86	2.29	0.13	16.57
Unzen ²	1079	51.23	10.49	1.23	0.0	3.70	0.15	6.79	9.88	0.62	2.47	0.07	13.59

Table 2: Partial molar volume, thermal expansions and compressibilities of oxide components

	$\bar{V}_i(T = T_R, P = P_R)$ /10 ⁻⁶ m ³ mol ⁻¹	$\left. \frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \right _{T=T_R}$ /10 ⁻⁹ m ³ mol ⁻¹ K ⁻¹	$\left. \frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \right _{P=P_R}$ /10 ⁻⁶ m ³ mol ⁻¹ GPa ⁻¹
SiO ₂	26.86	0.0	-1.89
TiO ₂	23.16	7.24	-2.31
Al ₂ O ₃	37.42	0.0	-2.31
Fe ₂ O ₃	42.13	9.09	-2.53
FeO	13.65	2.92	-0.45
MgO	11.69	3.27	0.27
CaO	16.53	3.74	0.34
Na ₂ O	28.88	7.68	-2.40
K ₂ O	45.07	12.08	-6.75
Li ₂ O	16.85	5.25	-1.02
H ₂ O	26.27	9.46	-3.15
CO ₂	33.0	0.0	0.0

- (b) Determine the density of the Unzen² melt.
- (c) Determine the density of the Unzen¹ magma.
2. At a temperature of 767 °C and pressure of 2000 MPa, the equilibrium crystal assemblage of the wet Novarupta magma is feldspar ($\phi_{\text{fld}} = 0.5, \rho_{\text{fld}} = 2.5 \text{ g cm}^{-3}$), quartz ($\phi_{\text{qz}} = 0.3, \rho_{\text{qz}} = 2.5 \text{ g cm}^{-3}$), spinel ($\phi_{\text{sp}} = 0.007, \rho_{\text{sp}} = 4.9 \text{ g cm}^{-3}$) and biotite ($\phi_{\text{bio}} = 0.5, \rho_{\text{bio}} = 2.75 \text{ g cm}^{-3}$) whilst the melt density is 2.21 g cm^{-3} and the melt viscosity is 147910.8 Pa s . Additionally, the gravitational settling velocity of a crystal of size d and density ρ_c in melt of density ρ_m and viscosity η_m is given by

$$v_s = \frac{(\rho_c - \rho_m)gd^2}{18\eta_m}, \quad (3)$$

where $g = 9.81 \text{ m s}^{-2}$ is the gravitational acceleration.

- (a) Assuming the crystals are $500 \text{ }\mu\text{m}$ in size, calculate the settling velocities of each crystal phase in the Novarupta magma.

For the wet Kilauea composition, the equilibrium assemblage at the same pressure is olivine ($\phi_{\text{ol}} = 0.03, \rho_{\text{ol}} = 3.37 \text{ g cm}^{-3}$), orthopyroxene ($\phi_{\text{opx}} = 0.006, \rho_{\text{opx}} = 3.28 \text{ g cm}^{-3}$), clinopyroxene ($\phi_{\text{cpx}} = 8.21, \rho_{\text{cpx}} = 3.29 \text{ g cm}^{-3}$) and spinel ($\phi_{\text{sp}} = 0.02, \rho_{\text{sp}} = 4.9 \text{ g cm}^{-3}$), with a melt density of 2.31 g cm^{-3} and a melt viscosity of 28.1 Pa s .

- (b) Calculate the settling velocities of each crystal phase in the Kilauea magma.
- (c) It has been suggested that crystal cumulates can form at the base of magma chambers of sills by gravitational settling of crystals. Consider 10 m thick sills of both the Novarupta and Kilauea compositions. If the sills were maintained at constant temperatures, how long would it take for such a cumulate to form in each case? What crystals would they contain?
- (d) What are the most important parameters in determining whether such cumulates can form? Also consider processes which have been neglected here.

Table 3: The coefficients b_i, b_{1j}, c_i and c_{11} and expressions for M_i, M_{1j}, N_i and N_{11} as used in the melt viscosity model of Giordano et al. (2008). All X_i are in mol%.

$b_1 = 159.6$	$M_1 = X_{\text{SiO}_2} + X_{\text{TiO}_2}$	$c_1 = 2.75$	$N_1 = X_{\text{SiO}_2}$
$b_2 = -173.3$	$M_2 = X_{\text{Al}_2\text{O}_3}$	$c_2 = 15.7$	$N_2 = X_{\text{TiO}_2} + X_{\text{Al}_2\text{O}_3}$
$b_3 = -72.1$	$M_3 = X_{\text{Fe}} + X_{\text{MnO}} + X_{\text{P}_2\text{O}_5}$	$c_3 = 8.3$	$N_3 = X_{\text{Fe}} + X_{\text{MnO}} + X_{\text{MgO}}$
$b_4 = -75.7$	$M_4 = X_{\text{MgO}}$	$c_4 = 10.2$	$N_4 = X_{\text{CaO}}$
$b_5 = -39.9$	$M_5 = X_{\text{CaO}}$	$c_5 = -12.3$	$N_5 = X_{\text{Na}_2\text{O}} + X_{\text{K}_2\text{O}}$
$b_6 = -84.1$	$M_6 = X_{\text{Na}_2\text{O}} + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}}$	$c_6 = -99.1$	$N_6 = \ln(1 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$
$b_7 = -141.5$	$M_7 = X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}} + \ln(1 + X_{\text{H}_2\text{O}})$		
$b_{11} = -2.43$	$M_{11} = M_1 N_3$	$c_{11} = 0.3$	$N_{11} = (M_2 + N_3 + N_4 - X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$
$b_{12} = -0.91$	$M_{12} = (N_1 + N_2 + X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}})$		
$b_{13} = 17.6$	$M_{13} = M_2 N_5$		

3. The 1991-1995 eruption of Mt. Unzen, Japan, produced a series of lava domes. A sample of this lava can be seen in Figure 1. The white rock (Unzen¹ in Table ??) forms most of the lava, whilst the dark rock (Unzen²) is present as a many discrete enclaves existing within the white rock. These enclaves are interpreted to have formed prior to eruption, when a hotter magma was injected into a cooler magma storage region.

An empirical model for the viscosity of melt η_m as a function of temperature T and composition \mathbf{X} is (Giordano et al., 2008)

$$\eta_m = 10^{A+B(\mathbf{X})/[T-C(\mathbf{X})]}, \quad (4)$$

where $A = -4.55$, T is in K, and B and C are given by

$$B = \sum_{i=1}^7 b_i M_i + \sum_{j=1}^3 b_{1j} M_{1j}, \quad (5)$$

and

$$C = \sum_{i=1}^6 c_i N_i + c_{11} N_{11}. \quad (6)$$

Values for the coefficients b_i, b_{1j}, c_i and c_{11} , and the expressions for M_i, M_{1j}, N_i and N_{11} are given in Table 3.

- Assume that the enclave magma has 4 wt% water and at the time of enclave formation was at the wet liquidus temperature. Calculate the viscosity of this magma. You may find it helpful to draw a table.
- Assume that the host magma also has 4 wt% water, but is at 775°C. At this temperature, the equilibrium crystal assemblage is orthopyroxene ($\phi_{\text{opx}} = 0.07, \rho_{\text{opx}} = 3.52 \text{ g cm}^{-3}$), clinopyroxene ($\phi_{\text{cpx}} = 0.02, \rho_{\text{cpx}} = 3.35 \text{ g cm}^{-3}$) and feldspar ($\phi_{\text{fd}} = 0.30, \rho_{\text{fd}} = 2.62 \text{ g cm}^{-3}$) with minor amounts of spinel and oxides. The composition of the melt is given in Table 4. Also present is an exsolved gas phase with a volume fraction of $\phi_b = 0.07$ and density of 0.44 g cm^{-3} . Calculate the melt viscosity of this magma.
- Neglecting the gas phase and using the Krieger & Dougherty (1959) relation, estimate the total viscosity of the host magma, taking 0.4 as the maximum crystal volume fraction.
- Does the host magma have a yield stress? If so, calculate an estimate for it?

Table 4: The melt compositions (mol%) of the Unzen magmas at their mixing temperatures and 2000 MPa.

Magma	Temperature /°C	SiO ₂	Al ₂ O ₃	TiO ₂	FeO	Fe ₂ O ₃	MnO	MgO	CaO	K ₂ O	Na ₂ O	P ₂ O ₅	H ₂ O
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- (e) Given the temperature difference between the magmas, discuss what you expect to happen once they come into contact in terms of processes such as heat transport, crystallisation and melting. How do you expect the rheological properties of the two magmas to change?
4. Consider two magmas, a basalt ($\eta = 1 \text{ Pa s}$, $\rho = 2800 \text{ kg m}^{-3}$) and rhyolite ($\eta = 10^6 \text{ Pa s}$, $\rho = 2600 \text{ kg m}^{-3}$), rising in a conduit. Assume a conduit radius of 10 m and 1 m for the rhyolite and basalt, respectively. Assume a pressure gradient driving magma ascent of 500 Pa m^{-1} . For both cases:
- Calculate velocity as a function of radial position in the conduit
 - Evaluate the strain rate at the conduit walls. Are the strain rates large enough that the melt might be shear-thinning or undergo structural failure?
 - Calculate the capillary number for a 1 cm radius bubble. Assume the surface tension of melt is 10^{-3} N m^{-1} . Will the bubbles become deformed by the ascending magma?
 - What effect do you think the bubbles will have on magma rheology?
 - Calculate the rise speed of a 1 cm bubble relative to the surrounding melt. Given this speed, what flow regime is likely to characterise the magma?