



UNIVERSITÉ
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Gravity currents

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Volcanic flows

Lava flows



Cloud spreading



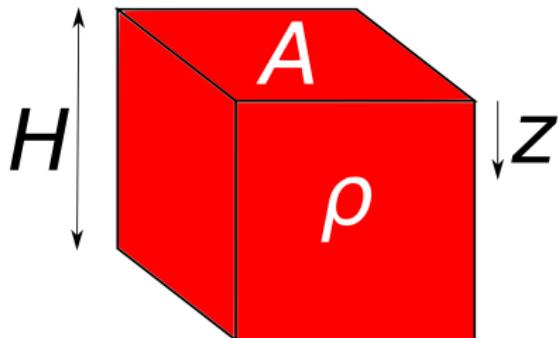
Pyroclastic density currents (PDCs)



Lahars



Hydrostatic gradients



Consider a volume of fluid of :

- Density ρ
- Height H
- Horizontal cross section A

z = Negative vertical coordinate
(depth below top surface)

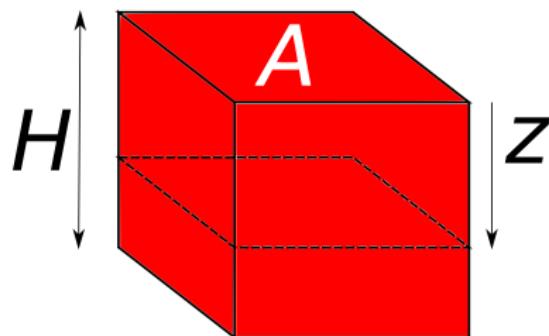
Consider horizontal plane at depth z
What are the forces acting on this plane?

- **Weight** of overlying fluid

$$W = \rho A z g$$

- Balanced by **hydrostatic pressure**

$$F_p = P A$$

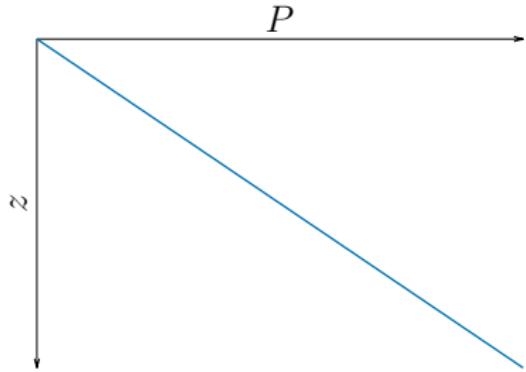
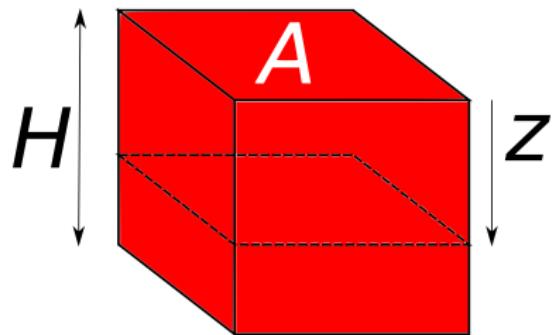


Hydrostatic gradients

Nothing is moving \Rightarrow **Mechanical equilibrium**

$$W = F_p$$

$$P = \rho g z$$



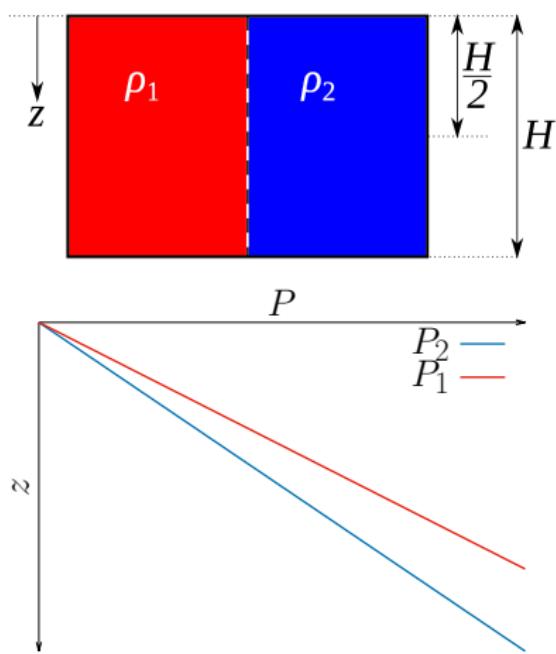
P increases linearly with z

Hydrostatic gradient:

$$\frac{dP}{dz} = \rho g$$

Gravity currents - Hydrostatic gradients

Gravity current - A horizontal flow in a gravitational field that is driven by a density difference



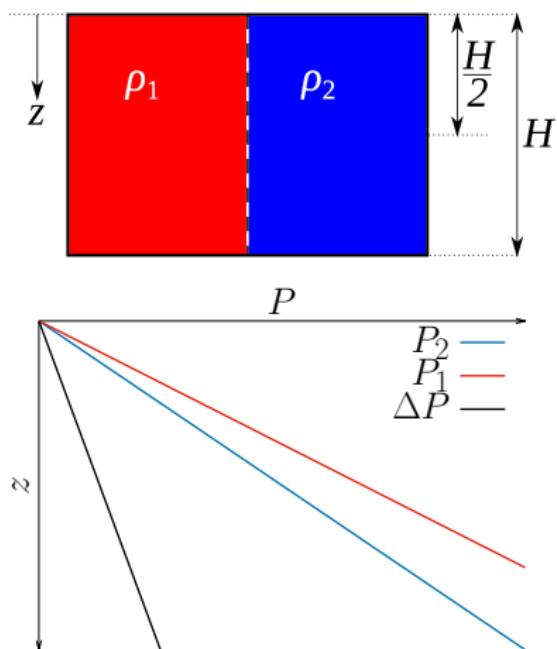
Consider two fluids (densities ρ_1 and ρ_2 , $\rho_1 > \rho_2$) initially side-by-side and separated by a vertical barrier
Pressure difference:

$$P_1 = \rho_1 g z$$

$$P_2 = \rho_2 g z$$

$$\Delta P = P_2 - P_1 = (\rho_2 - \rho_1) g z$$

Gravity currents - Horizontal force balance

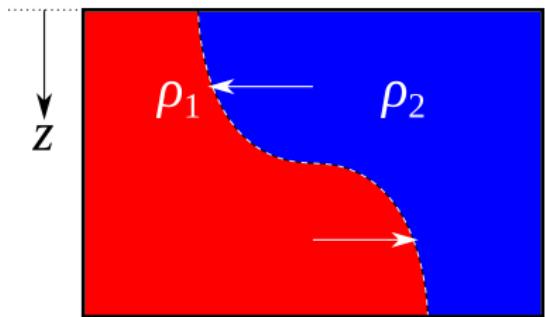


Remove barrier, and consider pressure difference ΔP across line

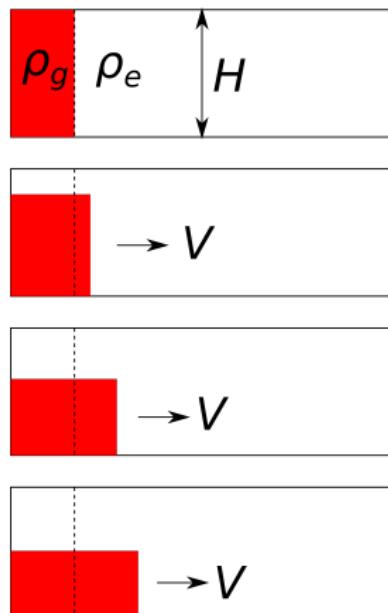
ΔP increases with depth

Flow follows a pressure gradient - but horizontal pressure gradient is greatest at the depth

This initiates from high to low pressure at the base, which is compensated by return flow at the top



Gravity currents - Box models



Assume:

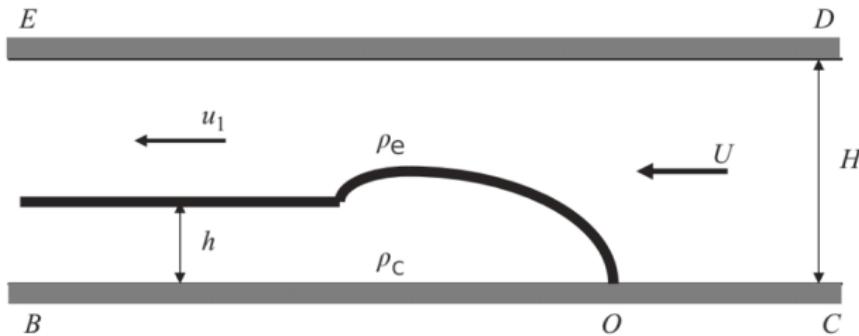
- Volume of current is conserved
- Current is always rectangular
- Energy is always conserved*

Energy balance between kinetic energy and change in GPE shows

$$V = \left(\frac{(\rho_c - \rho_e)gH}{2\rho_c} \right)^{1/2}$$

* Means assume energy only exists as kinetic or GPE. What other types of energy are there?

Gravity currents - Benjamin's model



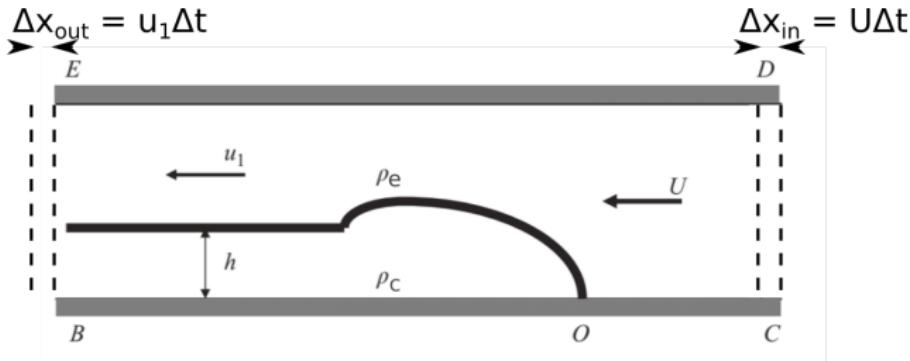
Modified from Shin et al. (2004)

Assumptions:

- Hydrostatic conditions upstream and downstream of head
- No relative flow within current

Consider a reference frame moving with the head \Rightarrow velocity inside current equals zero

Gravity currents - Benjamin's model

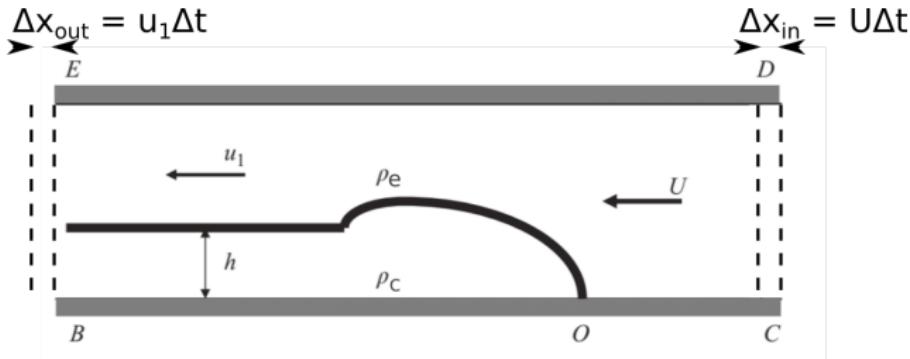


Let W = Width of channel

During time interval Δt , volume of fluid:

- entering domain = $V_{in} = \Delta x_{in} WH = U\Delta t WH$
- leaving domain = $V_{out} = \Delta x_{out} W(H - h) = u_1\Delta t W(H - h)$

Gravity currents - Benjamin's model



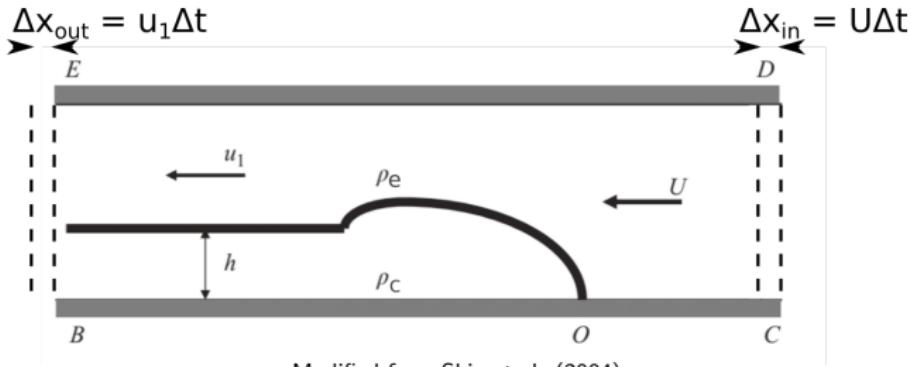
Modified from Shin et al. (2004)

During time interval Δt , mass of fluid

- entering domain = $m_{in} = \rho_e V_{in} = \rho_e U\Delta t W H$
- leaving domain = $m_{out} = \rho_e V_{out} = \rho_e u_1 \Delta t W (H - h)$

Mass conservation $\implies m_{in} = m_{out} \implies UH = u_1(H - h)$

Gravity currents - Benjamin's model



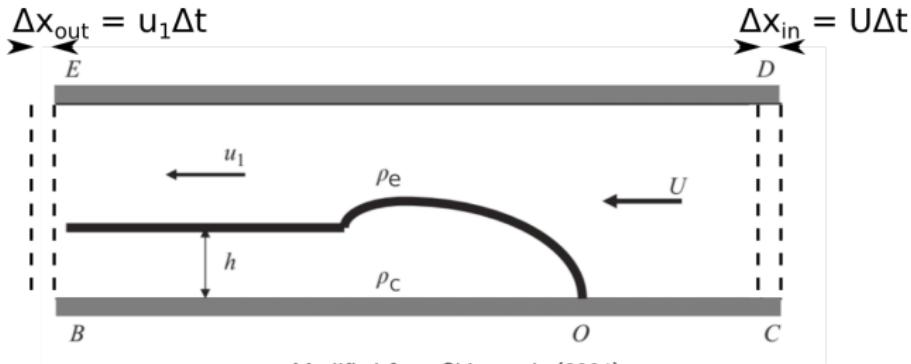
Assumptions:

During time interval Δt , momentum of fluid:

- entering domain = $p_{\text{in}} = m_{\text{in}} U = \rho_e U^2 \Delta t W H$
- leaving domain = $p_{\text{out}} = m_{\text{out}} u_1 = \rho_e u_1^2 \Delta t W (H - h)$

Change in momentum = $\Delta p = p_{\text{in}} - p_{\text{out}} = [U^2 H - u_1^2 (H - h)] \rho_e \Delta t W$

Gravity currents - Benjamin's model



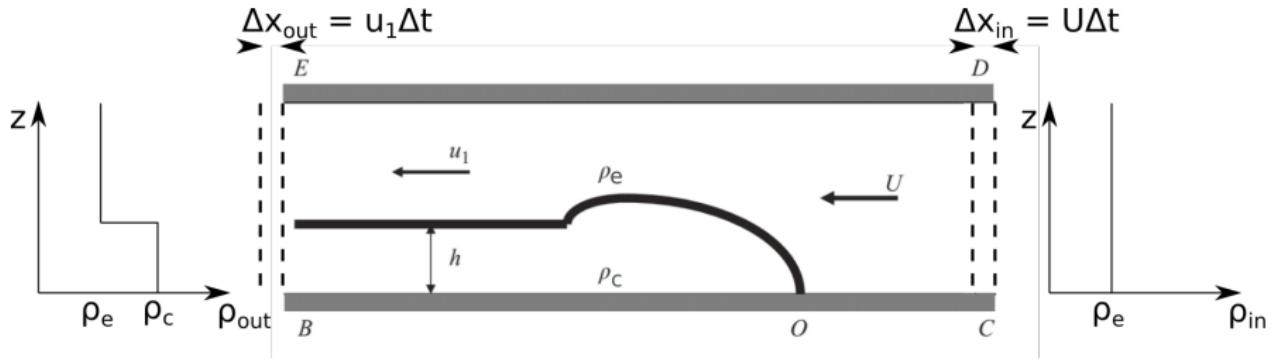
Modified from Shin et al. (2004)

Newton's second law: Force acting on substance = Rate of change of momentum

$$F = \frac{\Delta p}{\Delta t} = [U^2 H - u_1^2(H-h)]\rho_e W$$

Force F depends on pressures at the inlet and outlet

Gravity currents - Benjamin's model



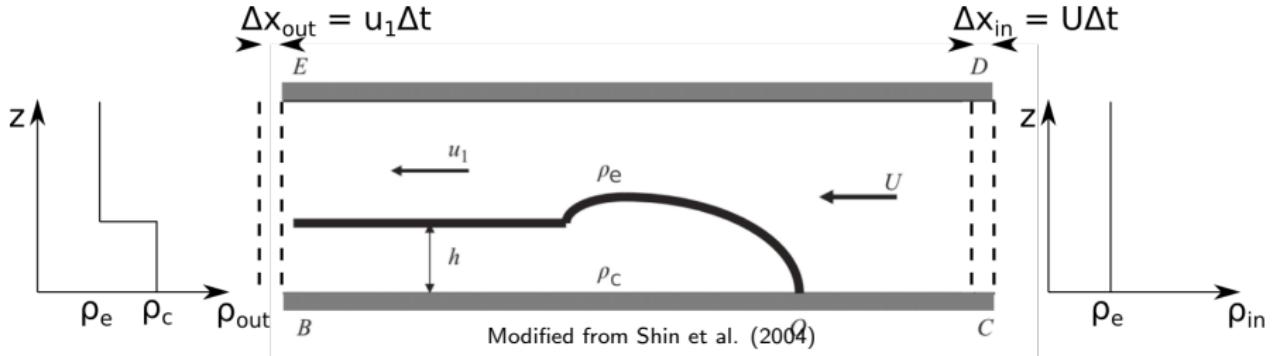
Modified from Shin et al. (2004)

$$\frac{dP_{out}}{dz} = -g\rho_{out}$$

$$\frac{dP_{in}}{dz} = -g\rho_{in}$$

$$\rho_{out} = \begin{cases} \rho_c & z < h \\ \rho_e & z > h \end{cases} \quad \rho_{in} = \rho_e$$

Gravity currents - Benjamin's model



$$P_{out}(z) - P_B = -g \int_0^z \rho_{out} dz$$

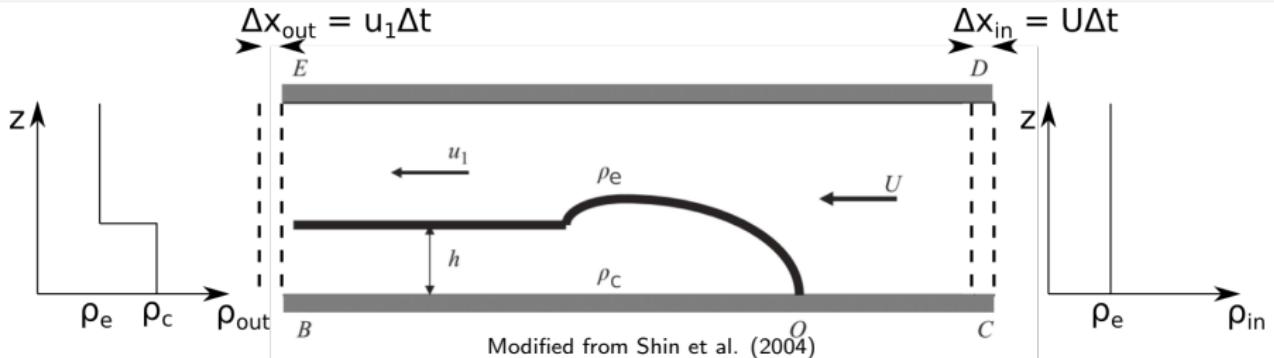
$$P_{in}(z) - P_C = -g \int_0^z \rho_{in} dz$$

$$P_{out}(z) = P_B - g \begin{cases} \rho_c \int_0^z dz & z < h \\ \rho_c \int_0^h dz + \rho_e \int_h^z dz & z > h \end{cases}$$

$$P_{in}(z) = P_C - g \rho_e z$$

$$P_{out}(z) = P_B - g \begin{cases} \rho_c z & z < h \\ \rho_c h + \rho_e(z - h) & z > h \end{cases}$$

Gravity currents - Benjamin's model

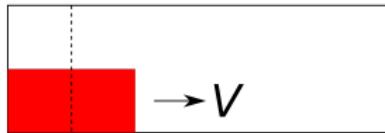
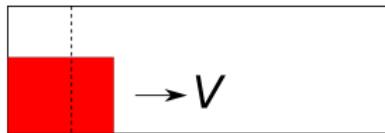
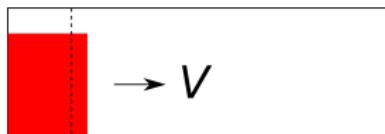
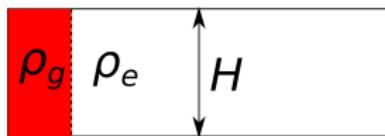


$$F_{out} = \int P_{out} dA \quad F_{in} = \int P_{in} dA$$
$$dA = W dz$$

$$F = F_{in} - F_{out} = W \int_0^H (P_{in} - P_{out}) dz$$

$$F = W \left(\int_0^h (P_{in} - P_{out}) dz + \int_h^H (P_{in} - P_{out}) dz \right)$$

Gravity currents - Reduced gravity



If environment is air, then often $\rho_e \ll \rho_c$

$$V = \left(\frac{(\rho_c - \rho_e)gH}{2\rho_c} \right)^{1/2} \approx \frac{(gH)^{1/2}}{2}$$

Therefore, we see that $(\rho_c - \rho_e)/\rho_c$ is factor by which environmental fluid reduced gravitational acceleration

Define **reduced gravity**:

$$g' = \frac{\rho_c - \rho_e}{\rho_c}$$

So generally:

$$V = \frac{(g'H)^{1/2}}{2}$$

Gravity currents - Froude number

Froude number:

$$\text{Fr} = \frac{V}{(g'H)^{1/2}}$$

Represents ratio of inertial (U) to buoyancy ($(g'H)^{1/2}$) forces

For an ideal gravity current $V = (g'H)^{1/2}/2$ so:

$$\text{Fr} = \frac{1}{2}$$

Measuring Fr gives a means of testing model assumptions

Dimensionless numbers

Froude	Fr	$\frac{V}{(g'H)^{1/2}}$	Inertia and buoyancy forces
Capillary	Ca	$\frac{\eta V r}{\sigma}$	Viscous and surface tension forces

Can use values of these quantities to define different fluid dynamical regimes

Reynolds number - Ratio of inertial and viscous forces within a fluid

$$\text{Re} = \frac{\rho VL}{\eta}$$

ρ = Fluid density

V = Fluid velocity

L = Lengthscale

η = Viscosity

Reynolds number

$$Re = \frac{\rho VL}{\eta}$$

$Re \ll Re_c$

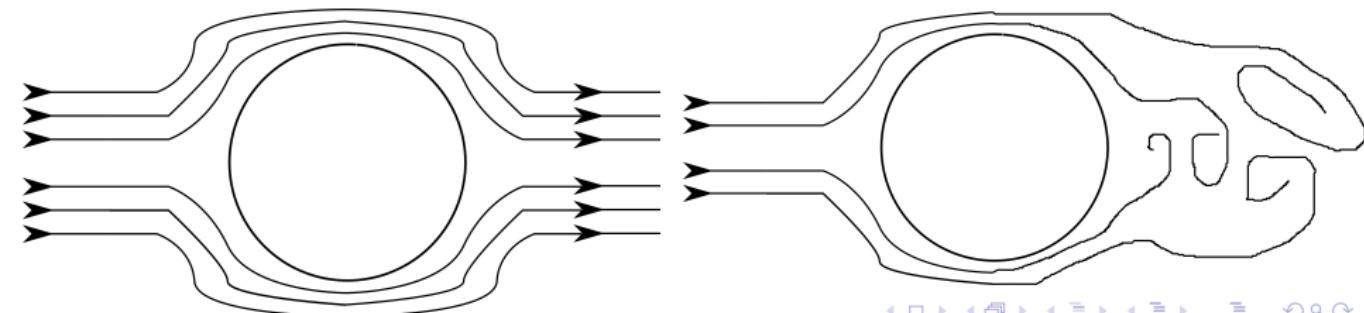
$Re \gg Re_c$

Laminar flow

Fluid particles follow smooth paths in layers, with little or no mixing between different layers

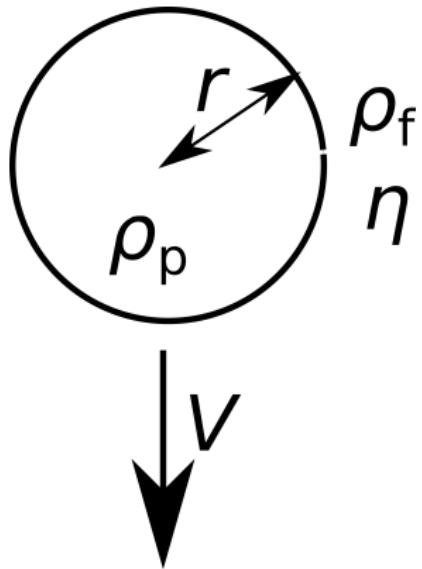
Turbulent flow

Chaotic changes in pressure and flow velocity, generating unsteady vortices

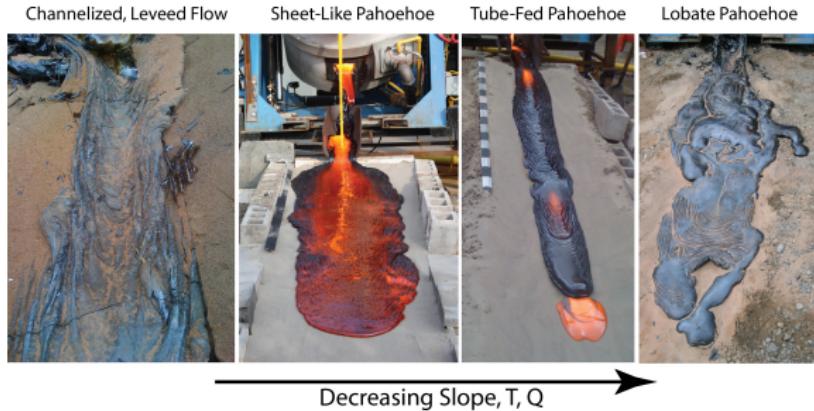


Reynolds number

What are the Reynolds numbers for these scenarios?



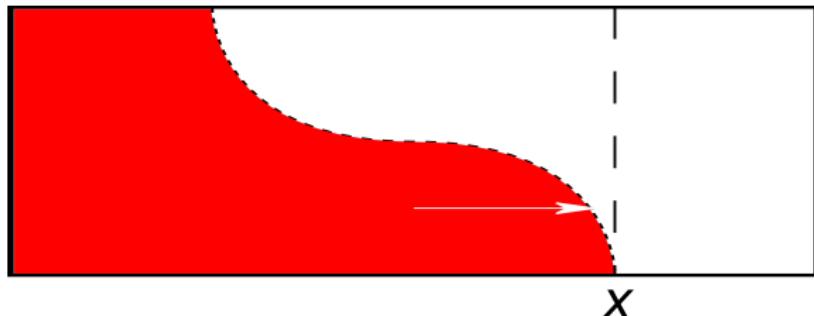
Low-Reynolds number gravity currents



For fixed release volume,
viscous dissipation
continuously reduces V

Varying slope angle and
flow rate produces
different flow
morphologies

For a 2D current:



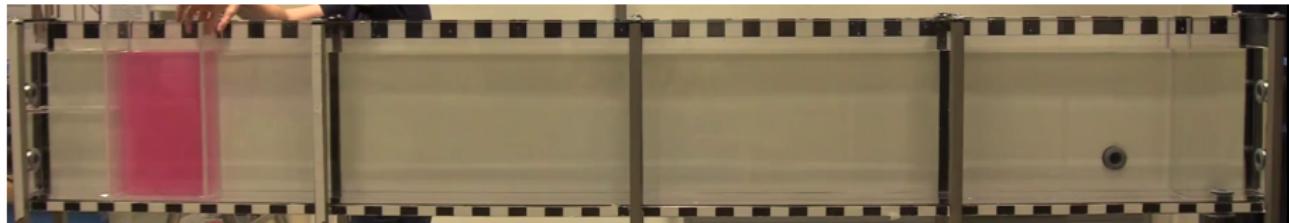
$$x \sim t^{4/5}$$

High-Reynolds number gravity currents

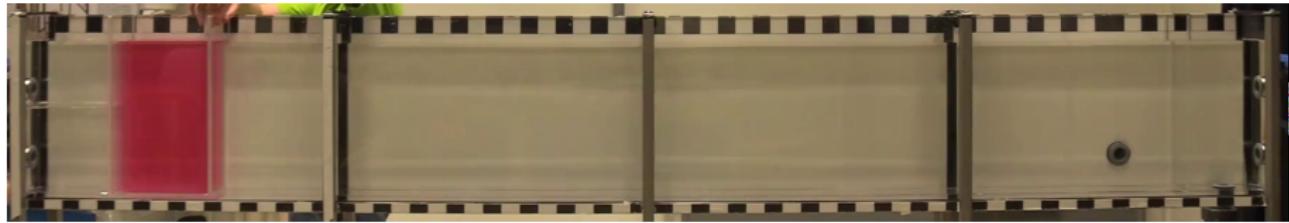
For $\text{Re} > 1000$, Re has no effect on flow

Can neglect viscous dissipation $\implies V = (g' H)^{1/2}/2$

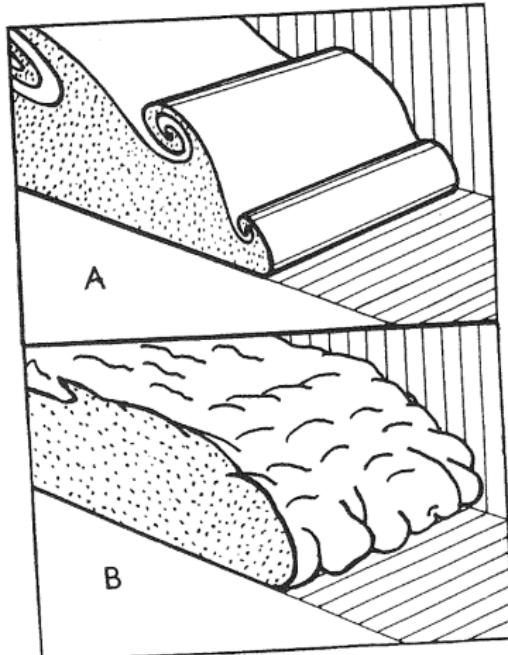
$$g' = 0.06$$



$$g' = 0.16$$



High-Reynolds number gravity currents - mixing

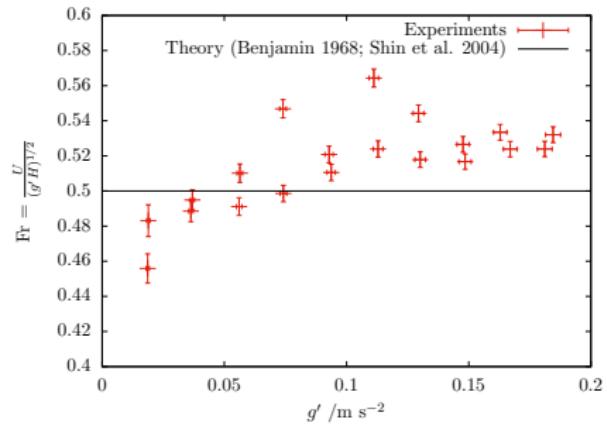
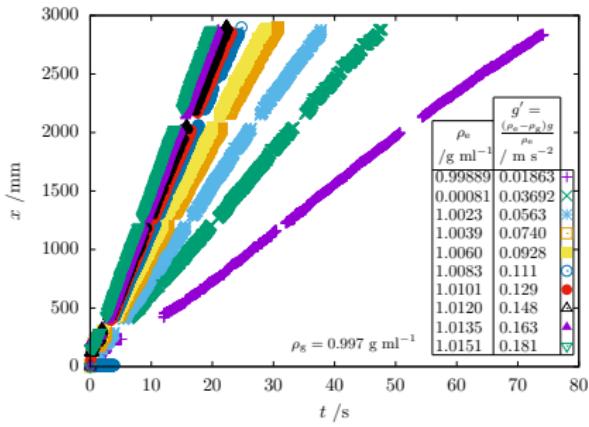
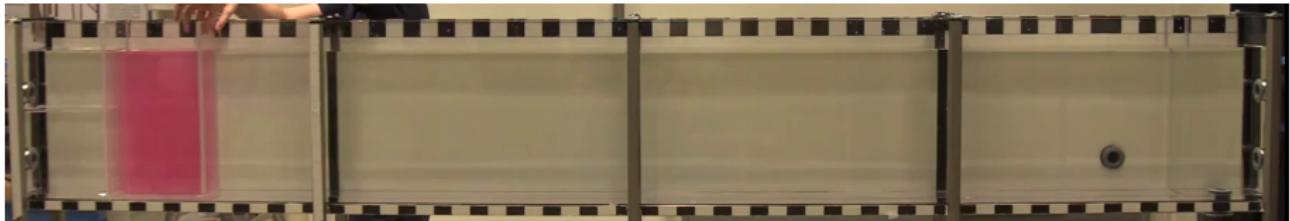


Mixing in fluid reduced density contrast

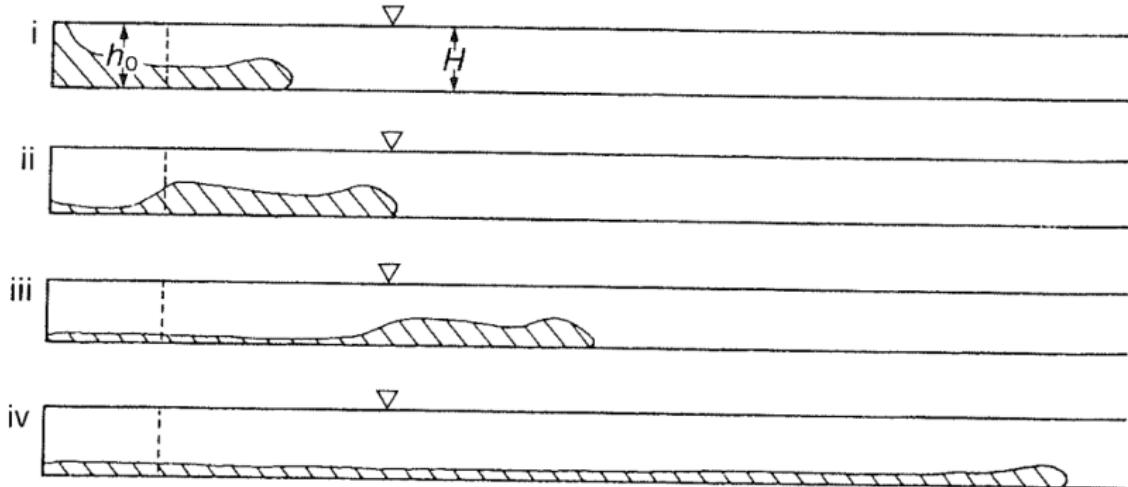
Two types of mixing:

- **Billingow** - Formed by shear between current and environment
- **Lobes and clefts** - Formed by interaction with ground at contact line

Constant volume release - Constant speed phase



Constant volume release - Self-similar phase



Constant speed

$$x \sim t$$

Self-similar

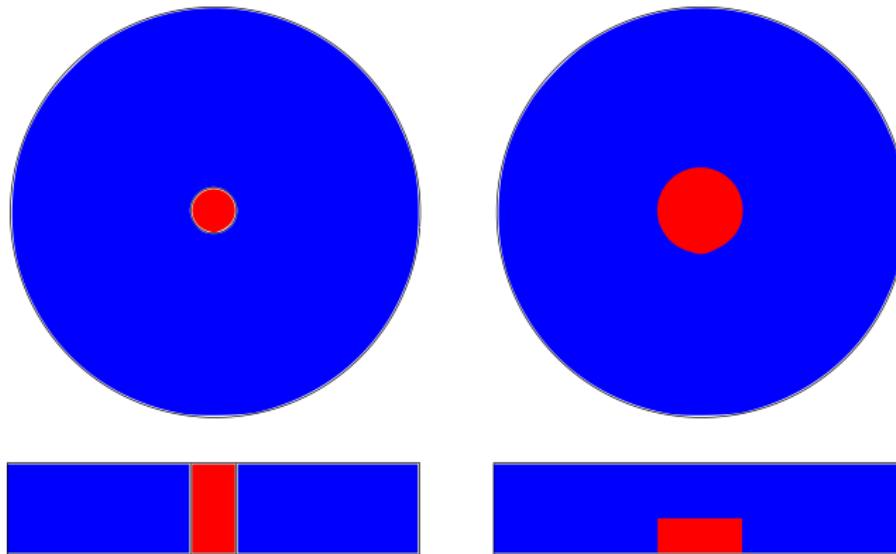
$$x \sim t^{2/3}$$

Viscous

$$x \sim t^{1/5}$$

Constant volume release - Radial collapse

$$r \sim t^{1/2}$$



Constant radial volume flow

$$r \sim Q^{1/3} t^{1/2}$$

