



Figure 1: Sample of dome lava erupted from Unzen in the 1991-1995 eruption. The bulk of the lava is the white rock (Unzen¹ in Table 1), whilst the dark rock (Unzen²) exists as discrete enclaves.

Magma viscosity and density

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1. The 1991-1995 eruption of Mt. Unzen, Japan, produced a series of lava domes. A sample of this lava can be seen in Figure 1. The white rock (Unzen¹ in Table 1) forms most of the lava, whilst the dark rock (Unzen²) is present as many discrete enclaves existing within the white rock. These enclaves are interpreted to have formed prior to eruption, when a hotter magma was injected into a cooler magma storage region. Table 1 lists the melt compositions of the two magmas at their mixing temperatures and 2000 MPa. The volume V_m of 1 mol of melt of composition \mathbf{X} as a function of pressure P and temperature T is given by

$$V_m(T, P, \mathbf{X}) = \sum_i X_i(\text{mol}\%) \left[\bar{V}_i(T = T_R, P = P_R) + \left. \frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \right|_{T=T_R} (T - T_R) + \left. \frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \right|_{P=P_R} (P - P_R) \right], \quad (1)$$

where $T_R = 1673$ K and $P_R = 10^{-4}$ GPa. Values for the red, blue and green quantities can be found in Table 2. The density can then be calculated from

$$\rho_m = \frac{1}{V_m} \sum_i X_i(\text{mol}\%) M_i, \quad (2)$$

where M_i is the molar mass of each component (see Figure 2).

At 775 °C, the equilibrium crystal assemblage of the Unzen¹ magma is orthopyroxene ($\phi_{\text{opx}} = 0.07$, $\rho_{\text{opx}} = 3.52$ g cm⁻³), clinopyroxene ($\phi_{\text{cpx}} = 0.02$, $\rho_{\text{cpx}} = 3.35$ g cm⁻³) and feldspar ($\phi_{\text{fld}} = 0.30$, $\rho_{\text{fld}} = 2.62$ g cm⁻³) with minor amounts of spinel and oxides. On the other hand, at 1079°C, the Unzen² magma is at the liquidus temperature and can be considered aphyric.

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Figure 2: Periodic table.

Table 1: The melt compositions (mol%) of the Unzen magmas at their mixing temperatures and 2000 MPa.

Magma	Temperature /°C	SiO ₂	Al ₂ O ₃	TiO ₂	FeO	Fe ₂ O ₃	MnO	MgO	CaO	K ₂ O	Na ₂ O	P ₂ O ₅	H ₂ O
Unzen ¹	775	67.43	6.86	0.16	0.54	0.1	0.15	0.34	2.29	2.86	2.29	0.13	16.57
Unzen ²	1079	53.5	10.49	1.23	0.54	0.1	0.15	6.79	2.2	2.86	2.29	0.07	20

Table 2: Partial molar volume, thermal expansions and compressibilities of oxide components

	$\bar{V}_i(T = T_R, P = P_R)$ /10 ⁻⁶ m ³ mol ⁻¹	$\left. \frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \right _{T=T_R}$ /10 ⁻⁹ m ³ mol ⁻¹ K ⁻¹	$\left. \frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \right _{P=P_R}$ /10 ⁻⁶ m ³ mol ⁻¹ GPa ⁻¹
SiO ₂	26.86	0.0	-1.89
TiO ₂	23.16	7.24	-2.31
Al ₂ O ₃	37.42	0.0	-2.31
Fe ₂ O ₃	42.13	9.09	-2.53
FeO	13.65	2.92	-0.45
MgO	11.69	3.27	0.27
CaO	16.53	3.74	0.34
Na ₂ O	28.88	7.68	-2.40
K ₂ O	45.07	12.08	-6.75
Li ₂ O	16.85	5.25	-1.02
H ₂ O	26.27	9.46	-3.15
CO ₂	33.0	0.0	0.0

- Determine the density of the Unzen¹ melt.
 - Determine the density of the Unzen² melt.
 - Determine the density of the Unzen¹ magma.
 - Given the densities of the two magmas, what mixing textures would you expect to see produced if the Unzen² magma was injected into the Unzen¹ magma? Given that the Unzen² magma is preserved an enclaves, what has been neglected in this desity model that would have been important?
2. An empirical model for the viscosity of melt η_m as a function of temperature T and composition \mathbf{X} is (Giordano et al., 2008)

$$\eta_m = 10^{A+B(\mathbf{X})/[T-C(\mathbf{X})]}, \quad (3)$$

where $A = -4.55$, T is in K, and B and C are given by

$$B = \sum_{i=1}^7 b_i M_i + \sum_{j=1}^3 b_{1j} M_{1j}, \quad (4)$$

and

$$C = \sum_{i=1}^6 c_i N_i + c_{11} N_{11}. \quad (5)$$

Values for the coefficients b_i, b_{1j}, c_i and c_{11} , and the expressions for M_i, M_{1j}, N_i and N_{11} are given in Table 3.

- Calculate the viscosity of the Unzen² magma at the liquidus temperature.
- Calculate the melt viscosity of the Unzen¹ magma.

Table 3: The coefficients b_i, b_{1j}, c_i and c_{11} and expressions for M_i, M_{1j}, N_i and N_{11} as used in the melt viscosity model of Giordano et al. (2008). All X_i are in mol%. Note $X_{\text{Fe}} = X_{\text{FeO}} + X_{\text{Fe}_2\text{O}_3}$.

$b_1 = 159.6$	$M_1 = X_{\text{SiO}_2} + X_{\text{TiO}_2}$	$c_1 = 2.75$	$N_1 = X_{\text{SiO}_2}$
$b_2 = -173.3$	$M_2 = X_{\text{Al}_2\text{O}_3}$	$c_2 = 15.7$	$N_2 = X_{\text{TiO}_2} + X_{\text{Al}_2\text{O}_3}$
$b_3 = 72.1$	$M_3 = X_{\text{Fe}} + X_{\text{MnO}} + X_{\text{P}_2\text{O}_5}$	$c_3 = 8.3$	$N_3 = X_{\text{Fe}} + X_{\text{MnO}} + X_{\text{MgO}}$
$b_4 = 75.7$	$M_4 = X_{\text{MgO}}$	$c_4 = 10.2$	$N_4 = X_{\text{CaO}}$
$b_5 = 39.9$	$M_5 = X_{\text{CaO}}$	$c_5 = -12.3$	$N_5 = X_{\text{Na}_2\text{O}} + X_{\text{K}_2\text{O}}$
$b_6 = -84.1$	$M_6 = X_{\text{Na}_2\text{O}} + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}}$	$c_6 = -99.1$	$N_6 = \ln(1 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$
$b_7 = 141.5$	$M_7 = X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}} + \ln(1 + X_{\text{H}_2\text{O}})$		
$b_{11} = -2.43$	$M_{11} = M_1 N_3$	$c_{11} = 0.3$	$N_{11} = (M_2 + N_3 + N_4 - X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$
$b_{12} = -0.91$	$M_{12} = (N_1 + N_2 + X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}})$		
$b_{13} = 17.6$	$M_{13} = M_2 N_5$		

- (c) Neglecting the gas phase and using the Krieger & Dougherty (1959) relation, estimate the total viscosity of the Unzen¹ magma, taking 0.4 as the maximum crystal volume fraction.
- (d) Does the Unzen¹ magma have a yield stress? If so, calculate an estimate for it?
- (e) Given the temperature difference between the magmas, discuss what you expect to happen once they come into contact in terms of processes such as heat transport, crystallisation and melting. How do you expect the rheological properties of the two magmas to change?
3. During fieldwork, a geologist found a sill of thickness 135 m. The lowest 10 m of the sill was a cumulate, almost exclusively of olivine crystals with a mean diameter of 3 mm, and a volume fraction ϕ_{sed} of approximately 0.6. The geologist collected samples and performed geochemical analysis, along with thermometry to estimate that, at the time of intrusion, the magma had a melt viscosity of 8.5 Pa s. Also, the only crystals present in the magma were olivine, with a suspended volume fraction $\phi_{\text{ol}} = 0.03$. The melt density $\rho_{\text{m}} = 2670 \text{ kg m}^{-3}$ whilst the olivine density is $\rho_{\text{ol}} = 3370 \text{ kg m}^{-3}$. The gravitational settling velocity of an olivine crystal is given by

$$v_s = \frac{(\rho_{\text{ol}} - \rho_{\text{m}})gd^2}{18\eta_{\text{m}}}, \quad (6)$$

where $g = 9.81 \text{ m s}^{-2}$ is the gravitational acceleration.

- (a) Calculate the settling velocity of an olivine crystal in the sill.

A model for cumulate growth predicts that the thickness H of the cumulate grows at a rate given by

$$\frac{dH}{dt} = \frac{\phi_{\text{ol}}g(\rho_{\text{ol}} - \rho_{\text{m}})d^2}{18\phi_{\text{sed}}\eta_{\text{m}}}, \quad (7)$$

where ϕ_{sed} is the packing fraction of olivine grains in the deposit.

- (b) Find an expression for the thickness of the cumulate as a function of time.
- (c) Determine how long it takes for a cumulate to reach a thickness of 10 m.
- (d) Discuss the assumptions of this model. What processes have been neglected that would occur in reality and how might they affect the model results?
4. Consider two magmas, a basalt ($\eta = 1 \text{ Pa s}$, $\rho = 2800 \text{ kg m}^{-3}$) and rhyolite ($\eta = 10^6 \text{ Pa s}$, $\rho = 2600 \text{ kg m}^{-3}$), rising in a conduit. Assume a conduit radius of $a = 10 \text{ m}$ and 1 m for the rhyolite and basalt, respectively. Assume a pressure gradient driving magma ascent of $dP/dz = 500 \text{ Pa m}^{-1}$. For such a flow

$$\frac{dV}{dr} = -\frac{r}{2\eta} \frac{dP}{dz},$$

where r is the radial coordinate and $V(r)$ is the vertical velocity in the conduit.

- (a) Assuming $V(r = a) = 0$, find an expression for $V(r)$
- (b) For both the rhyolite and the basalt, calculate the centre-line velocity.
- (c) Evaluate the strain rate at the conduit walls in both cases. Are the strain rates large enough that the melt might be shear-thinning or undergo structural failure?
- (d) Calculate the capillary number for a 1 cm radius bubble. Assume the surface tension of melt is 10^{-3} N m^{-1} . Will the bubbles become deformed by the ascending magma?
- (e) What effect do you think the bubbles will have on magma rheology?
- (f) Calculate the rise speed of a 1 cm bubble relative to the surrounding melt. Given this speed, what flow regime is likely to characterise the magma?