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Gravity currents

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Volcanic flows

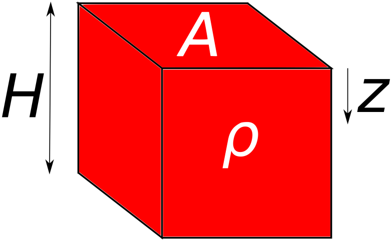
Lava flows

Cloud spreading

Pyroclastic density currents (PDCs)

Lahars

Hydrostatic gradients



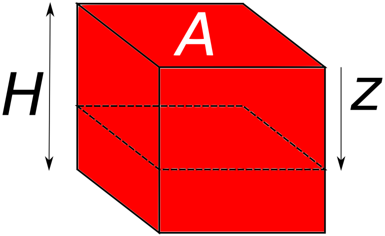
Consider a volume of fluid of :

- Density ρ
- Height H
- Horizontal cross section A

$z =$ Negative vertical coordinate (depth below top surface)

Consider horizontal plane at depth z
What are the forces acting on this plane?

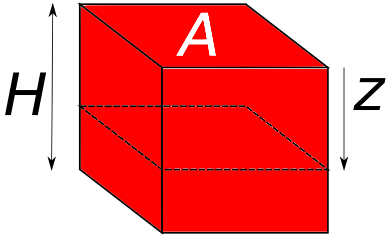
- **Weight** of overlying fluid
$$W = \rho A z g$$
- Balanced by **hydrostatic pressure**
$$F_{\mathrm{p}} = PA$$

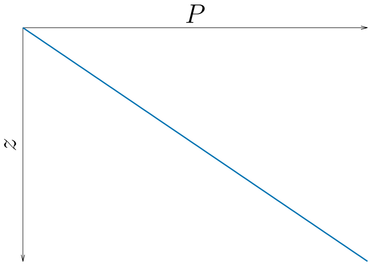


Nothing is moving $\implies$ **Mechanical equilibrium**

$$W = F_{\mathrm{p}}$$

$$P = \rho g z$$



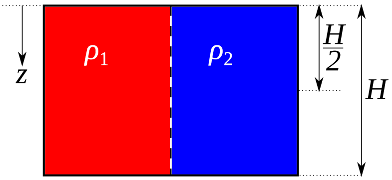


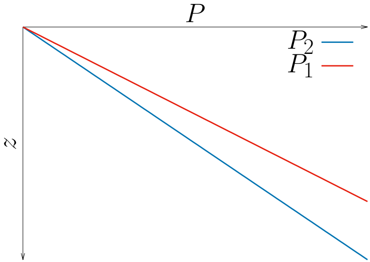
P increases linearly with z

Hydrostatic gradient:

$$\frac{\mathrm{d}P}{\mathrm{d}z} = \rho g$$

Gravity current - A horizontal flow in a gravitational field that is driven by a density difference





Consider two fluids (densities ρ_1 and ρ_2, $\rho_1 > \rho_2$) initially side-by-side and separated by a vertical barrier

Vertical pressure gradient:

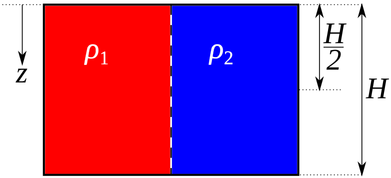
$$\frac{\mathrm{d}P}{\mathrm{d}z} = \rho g$$

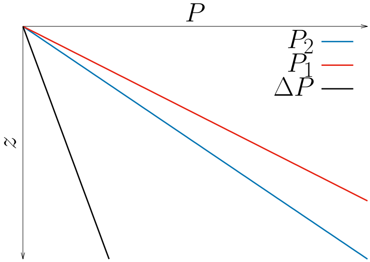
$$\mathrm{d}P = \rho g \mathrm{d}z$$

$$\int_0^P \mathrm{d}P = \rho g \int_0^z \mathrm{d}z$$

$$P = \rho g z$$

Gravity currents - Horizontal force balance



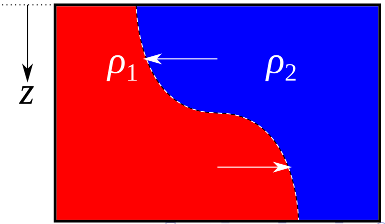


Remove barrier, and consider pressure difference ΔP across line

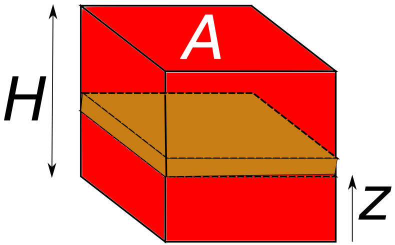
ΔP increases with depth

Flow follows a pressure gradient - but horizontal pressure gradient is greatest at the depth

This initiates from from high to low pressure at the base, which is compensated by return flow at the top



Gravitational potential energy (GPE)



GPE of thin layer:

$$\mathrm{d}U = \rho A \mathrm{d}z g z = \rho g A z \mathrm{d}z$$

GPE of total column:

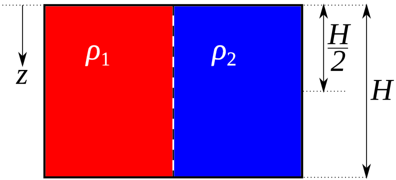
$$U = \int_{z=0}^{H} \mathrm{d}U$$

$$U = \rho g A \int_{z=0}^{H} z \mathrm{d}z$$

$$U = \rho g A \left[\frac{z}{2}\right]_{z=0}^{H}$$

$$U = \frac{\rho g A H^2}{2}$$

Initial state:



GPE of thin layer:

$$\mathrm{d}U = \frac{(\rho_1 + \rho_2)A\mathrm{d}zgz}{2} = \frac{(\rho_1 + \rho_2)gAz\mathrm{d}z}{2}$$

GPE of total column:

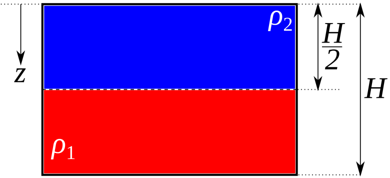
$$U_i = \int_{z=0}^{H} \mathrm{d}U$$

$$U_i = \frac{(\rho_1 + \rho_2)gA}{2} \int_{z=0}^{H} z\mathrm{d}z$$

$$U_i = \frac{(\rho_1 + \rho_2)gA}{2} \left[\frac{z}{2}\right]_{z=0}^{H}$$

$$U_i = \frac{(\rho_1 + \rho_2)gAH^2}{4}$$

Final state:



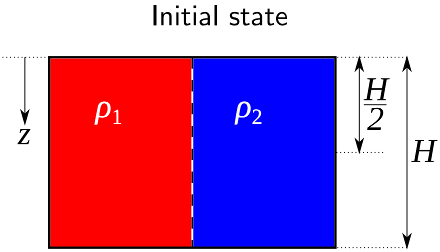
$$U_f = gA\left(\rho_1 \int_{z=0}^{H/2} z\mathrm{d}z + \rho_2 \int_{z=H/2}^{H} z\mathrm{d}z\right)$$

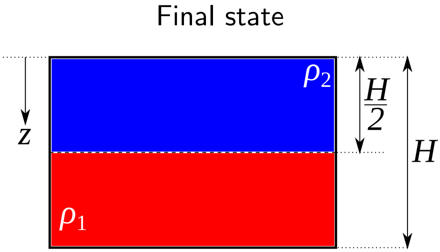
$$U_f = gA\left(\rho_1 \left[\frac{z^2}{2}\right]_{z=0}^{H/2} + \rho_2 \left[\frac{z^2}{2}\right]_{z=H/2}^{H}\right)$$

$$U_f = \frac{gA}{2}\left[\frac{\rho_1 H^2}{4} + \rho_2\left(H^2 - \frac{H^2}{4}\right)\right]$$

$$U_f = \frac{gAH^2}{2}\left(\frac{\rho_1}{4} + \frac{3\rho_2}{4}\right) = \frac{gAH^2(\rho_1 + 3\rho_2)}{8}$$

Gravity currents - Energy minimisation





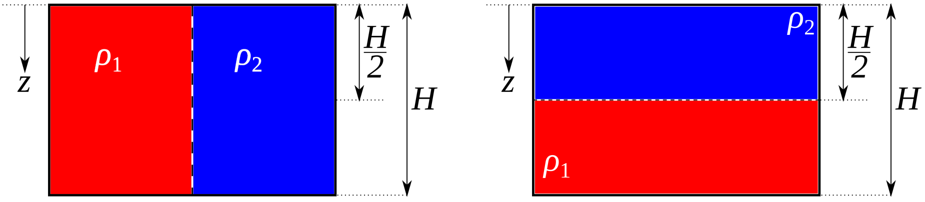
$$U_i = \frac{(\rho_1 + \rho_2) g A H^2}{4} \qquad U_f = \frac{g A H^2 (\rho_1 + 3\rho_2)}{8}$$

$$U_f - U_i = \frac{g A H^2}{4}\left(\frac{\rho_1}{2} + \frac{3\rho_2}{2} - \rho_1 - \rho_2\right)$$

$$U_f - U_i = \frac{g A H^2}{4}\left(\frac{\rho_2 - \rho_1}{2}\right) = \frac{g A H^2 (\rho_2 - \rho_1)}{8}$$

Gravity currents - Energy minimisation

Initial state | Final state

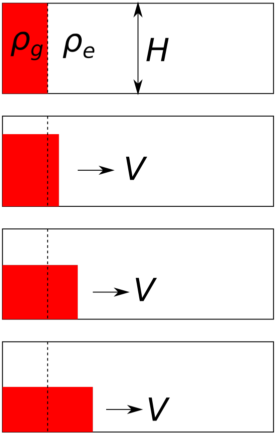


$$U_i = \frac{(\rho_1 + \rho_2) g A H^2}{4} \qquad U_f = \frac{g A H^2 (\rho_1 + 3\rho_2)}{8}$$

$$U_f - U_i = \frac{g A H^2}{4} \left(\frac{\rho_1}{2} + \frac{3\rho_2}{2} - \rho_1 - \rho_2 \right)$$

$$U_f - U_i = \frac{g A H^2}{4} \left(\frac{\rho_2 - \rho_1}{2} \right) = \frac{g A H^2 (\rho_2 - \rho_1)}{8}$$

Gravity currents - Box models



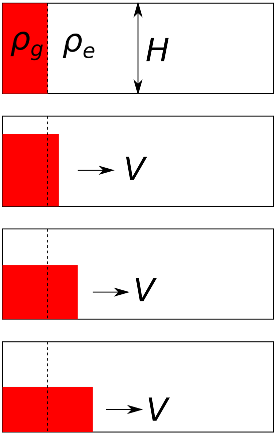
Assume:

- Volume of current is conserved
- Current is always rectangular
- Energy is always conserved*

Energy balance between kinetic energy and change in GPE shows

$$V = \left(\frac{(\rho_c - \rho_e) g H}{2 \rho_c} \right)^{1/2}$$

* Means assume energy only exists as kinetic of GPE. What other types of energy are there?



If envrionment is air, then often $\rho_e \ll \rho_c$

$$V = \left(\frac{(\rho_c - \rho_e) g H}{2\rho_c} \right)^{1/2} \approx \frac{(gH)^{1/2}}{2}$$

Therefore, we see that $(\rho_c - \rho_e)/\rho_c$ is factor by which envrionmental fluid reduced gravitational acceleration

Define **reduced gravity**:

$$g' = \frac{\rho_c - \rho_e}{\rho_c}$$

So generally:

$$V = \frac{(g'H)^{1/2}}{2}$$

Gravity currents - Froude number

Froude number:

$$\mathrm{Fr} = \frac{V}{(g'H)^{1/2}}$$

Represents ratio of inertial (U) to buoyancy ($(g'H)^{1/2}$) forces

For an ideal gravity current $V = (g'H)^{1/2}/2$ so:

$$\mathrm{Fr} = \frac{1}{2}$$

Measuring Fr gives a means of testing model assumptions

Dimensionless numbers

Froude	Fr	$\frac{V}{(g'H)^{1/2}}$	Inertia and buoyancy forces
Capillary	Ca	$\frac{\eta V r}{\sigma}$	Viscous and surface tension forces

Can use values of these quantities to define different fluid dynamical regimes

Reynolds number - Ratio of inertial and viscous forces within a fluid

$$\mathrm{Re} = \frac{\rho V L}{\eta}$$

$\rho =$ Fluid density
$V =$ Fluid velocity

$L =$ Lengthscale
$\eta =$ Viscosity

Reynolds number

$$\mathrm{Re} = \frac{\rho V L}{\eta}$$

$\mathrm{Re} \ll \mathrm{Re_c}$

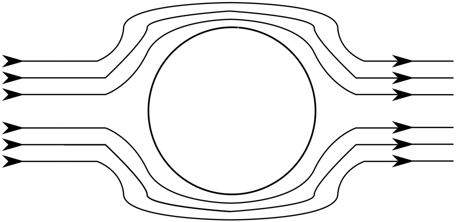
Laminar flow

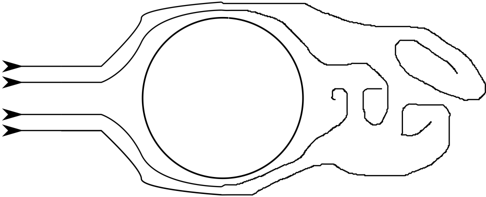
Fluid particles follow smooth paths in layers, with little or no mixing between different layers

$\mathrm{Re} \gg \mathrm{Re_c}$

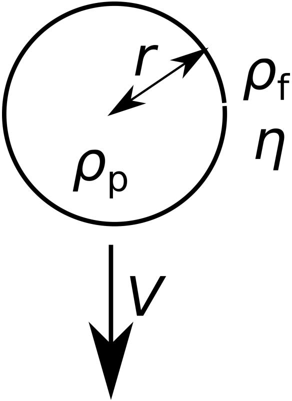
Turbulent flow

Chaotic changes in pressure and flow velocity, generating unsteady vortices

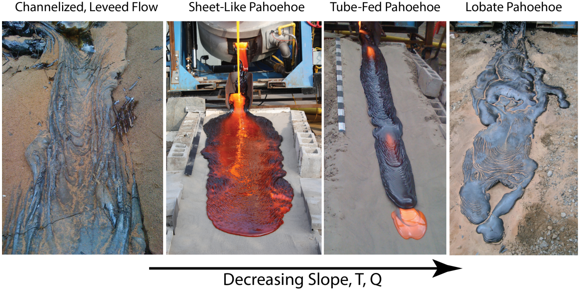




What are the Reynolds numbers for these scenarios?



Low-Reynolds number gravity currents



For fixed release volume, viscous dissipation continuosly reduces V

Varying slope angle and flow rate produces different flow morphologies

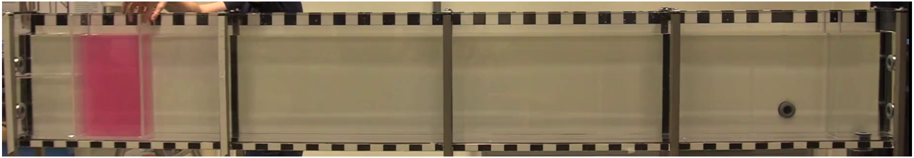
For a 2D current:

$$x \sim t^{4/5}$$

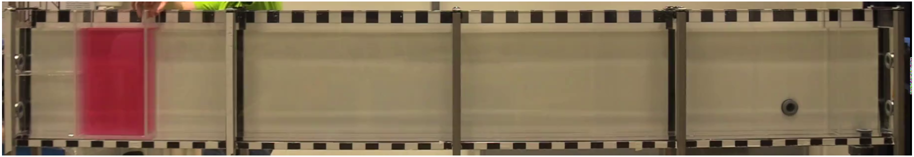
High-Reynolds number gravity currents

For Re > 1000, Re has no effect on flow
Can neglect viscous dissipation $\implies V = (g'H)^{1/2}/2$

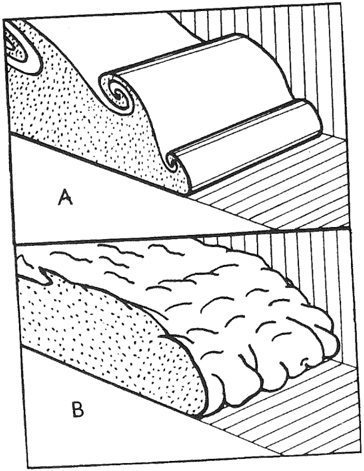
$$g' = 0.06$$



$$g' = 0.16$$



High-Reynolds number gravity currents - mixing

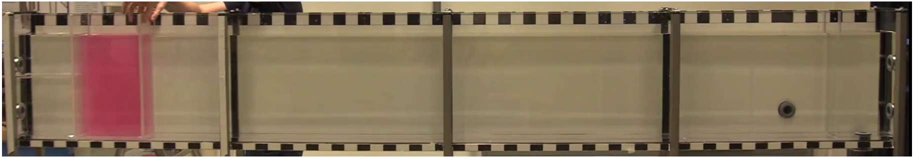


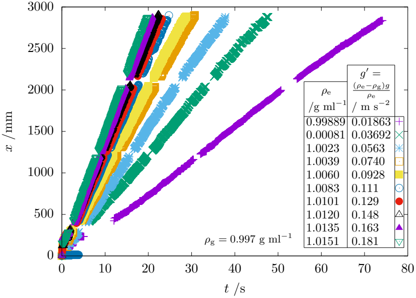
Mixing in fluid reduced density contrast

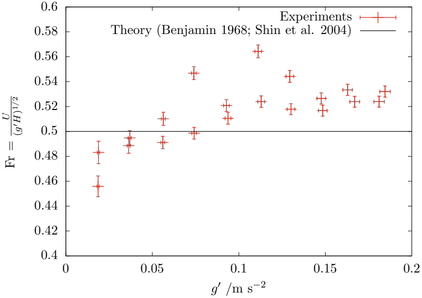
Two types of mixing:

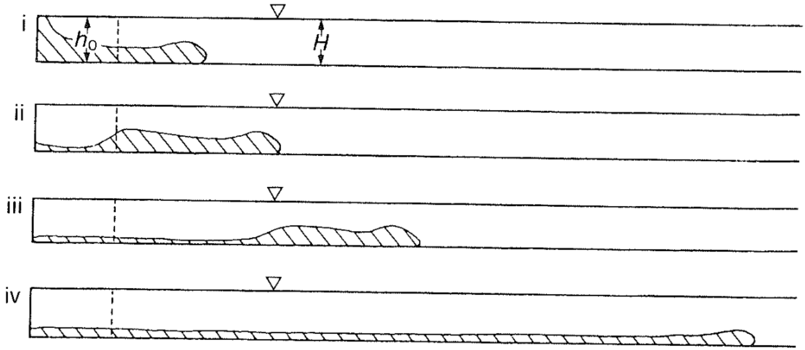
- **Billowing** - Formed by shear between current and environment
- **Lobes and clefts** - Formed by interaction with ground at contact line

Constant volume release - Constant speed phase









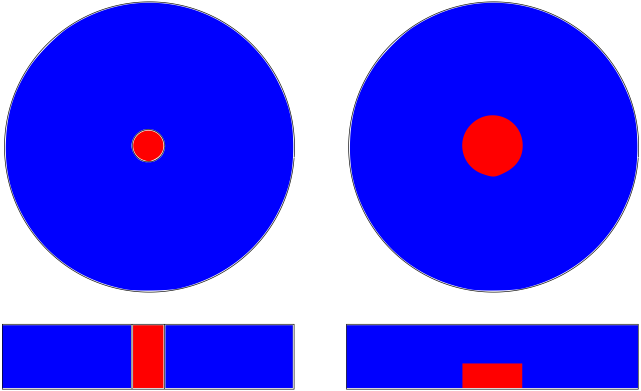
Constant speed $x \sim t$

Self-similar $x \sim t^{2/3}$

Viscous $x \sim t^{1/5}$

Constant volume release - Radial collapse

$$r \sim t^{1/2}$$



Constant radial volume flow

$$r \sim Q^{1/3} t^{1/2}$$