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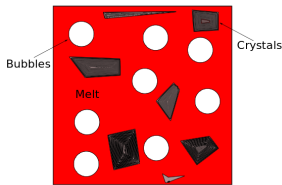
Magma density and viscosity

Paul A. Jarvis

paul.jarvis@unige.ch

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Magma density



Bulk density depends on volume fraction of crystals and bubbles

$$\rho = \rho_m \left(1 - \sum_i \phi_i \right) + \sum_i \rho_i \phi_i$$

ρ_m = Melt density

- Depends on T, P, \mathbf{X}

i = quartz, hornblende, plagioclase etc. and H_2O , CO_2 bubbles etc.

ρ_i = Density of phase i

- Depends on T, P for bubbles
- Depends on composition for crystals

ϕ_i = Volume fraction of phase i

Melt density

$$\rho_m = \sum_i X_i M_i \left(\sum_i X_i \bar{V}_i(T, P) \right)^{-1}$$

M_i = Molar mass of component i

- Mass of 1 mol of i
- $M_{\text{SiO}_2} = 28 \text{ g mol}^{-1} + 2 \times 16 \text{ g mol}^{-1} = 60 \text{ g mol}^{-1}$
- $M_{\text{H}_2\text{O}} = 2 \times 1 \text{ g mol}^{-1} + 16 \text{ g mol}^{-1} = 18 \text{ g mol}^{-1}$

\bar{V}_i = Partial molar volume of component i

- Change in mixture volume when 1 mol of i is added

Need to determine $\bar{V}_i(T, P)$ empirically

Melt density - Equation of state (EoS)

Relationship between **pressure**, **volume (density)** and **temperature**

Experiments - measure volume of a sample of **X** at a different P and T .

Find **empirical** EoS

$$V_m(T, P, \mathbf{X}) = \sum_i X_i \left[\bar{V}_i(T = T_R, P = P_R) + \frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \bigg|_{T=T_R} (T - T_R) + \frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \bigg|_{P=P_R} (P - P_R) \right]$$

T_R = Reference temperature = 1673 K

P_R = Reference pressure = 10^{-4} GPa

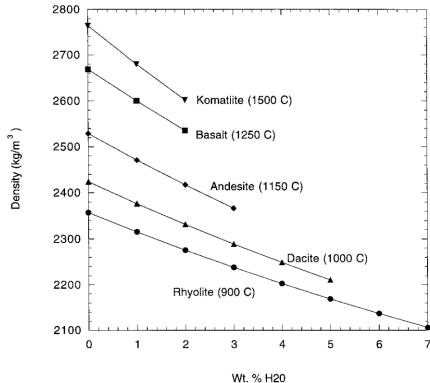
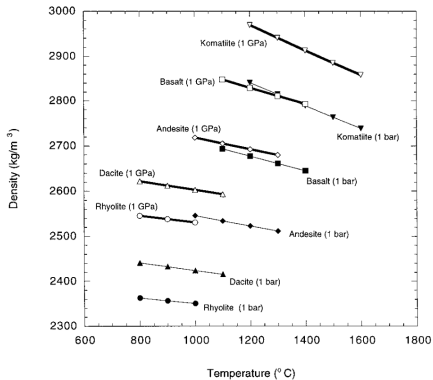
Lange & Carmichael (1990)

Lange (1997)

Ochs & Lange (1997)

	$\bar{V}_i(T = T_R, P = P_R)$ / $10^{-6} \text{ m}^3 \text{ mol}^{-1}$	$\frac{\partial \bar{V}_i(T, P = P_R)}{\partial T} \bigg _{T=T_R}$ / $10^{-9} \text{ m}^3 \text{ mol}^{-1} \text{ K}$	$\frac{\partial \bar{V}_i(T = T_R, P)}{\partial P} \bigg _{P=P_R}$ / $10^{-6} \text{ m}^3 \text{ mol}^{-1} \text{ GPa}$
SiO ₂	26.86	0.0	-1.89
TiO ₂	23.16	7.24	-2.31
Al ₂ O ₃	37.42	0.0	-2.31
Fe ₂ O ₃	42.13	9.09	-2.53
FeO	13.65	2.92	-0.45
MgO	11.69	3.27	0.27
CaO	16.53	3.74	0.34
Na ₂ O	28.88	7.68	-2.40
K ₂ O	45.07	12.08	-6.75
Li ₂ O	16.85	5.25	-1.02
H ₂ O	26.27	9.46	-3.15
CO ₂	33.0	0.0	0.0

Melt density - Effect of T and X_{H_2O}

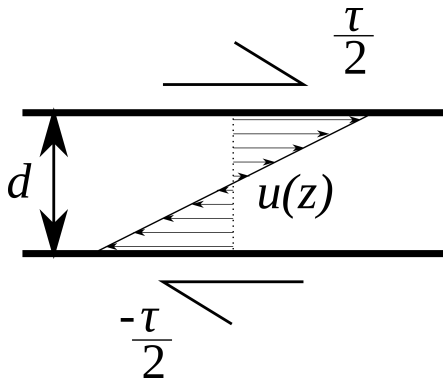


Temperature has small effect on density, particularly for high-silica melts

Water content has much stronger effect

Magma viscosity - Viscosity definition

Viscosity - A measure of a substance's resistance to flow (deformation). It relates an applied shear stress to the velocity gradient.



Consider fluid between two sheared parallel plates

τ = Applied shear stress

d = Separation between plates

$u(z)$ = Fluid velocity in gap

Viscosity η is defined as:

$$\tau = \eta \frac{dU}{dz} = \eta \dot{\epsilon}$$

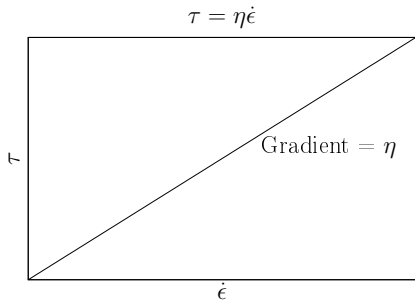
where $\dot{\epsilon} = dU/dz$ = **strain rate**

Magma viscosity - Newtonian fluids

Generally, viscosity is a function of strain rate: $\eta = \eta(\dot{\epsilon})$

Therefore, impossible to assign a single value of η to a material

Newtonian fluids - Special case where η is independent of $\dot{\epsilon}$



Flow curve - Relationship between τ and $\dot{\epsilon}$

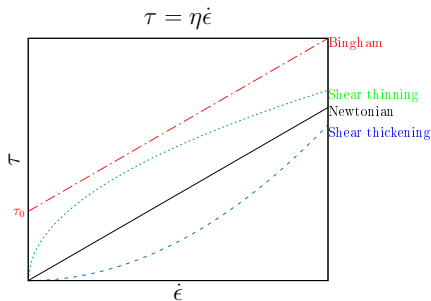
$$\tau = \eta(\dot{\epsilon})\dot{\epsilon}$$

For Newtonian fluid, $\eta = \text{constant}$

\implies flow curve is straight line

\implies gradient = η

Magma viscosity - Rheological materials



Newtonian

- Constant η
- e.g. water, magmatic melt

Shear thickening

- η increases with $\dot{\epsilon}$
- e.g. cornstarch

Shear thinning

- η decreases with $\dot{\epsilon}$
- e.g. butter

Bingham

- Fluid has **yield stress** τ_0
- For $\tau < \tau_0$ fluid does not flow ($\dot{\epsilon} = 0$)
- e.g. Mayonaise

Magma viscosity - Melt as a Newtonian fluid

Silica melt is almost perfectly Newtonian.

However, at extremely high shear rates, all materials start to undergo shear-thinning

Critical shear rate is given by

$$\dot{\epsilon}_c \approx \frac{10^{-3} G}{\eta_m}$$

G = Shear modulus $\approx 10^{10}$ Pa

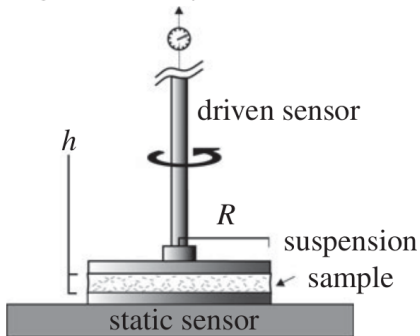
At even higher shear rates, the melt cannot deform as a fluid and snaps in a brittle manner.

Magma viscosity - Melt viscosity

Melt is modelled according to the VFT equation (EDIT TO INCLUDE MEANING OF VFT) (Giordano et al., 2008)

torque, $M \rightarrow$ stress, τ

angular velocity, $\Omega \rightarrow$ strain rate, $\dot{\gamma}$



$$\log \eta_m = A + \frac{B(\mathbf{X})}{T - C(\mathbf{X})}$$

$$\eta_m = 10^{A+B(\mathbf{X})/[T-C(\mathbf{X})]}$$

Measurements of the viscosity of samples of different \mathbf{X} at different T are used to determine $A, B(\mathbf{X}), C(\mathbf{X})$

N.B: This is a fitted, purely empirical equation - it is not dimensionally consistent!

Magma viscosity - Fitted parameters

$$A = \text{constant} = -4.55$$

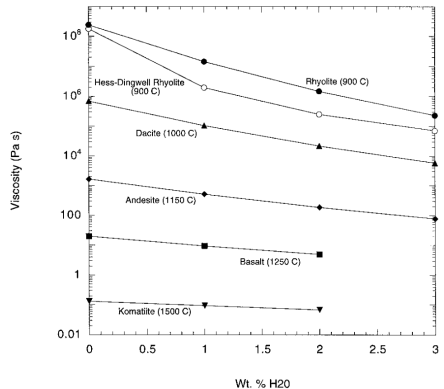
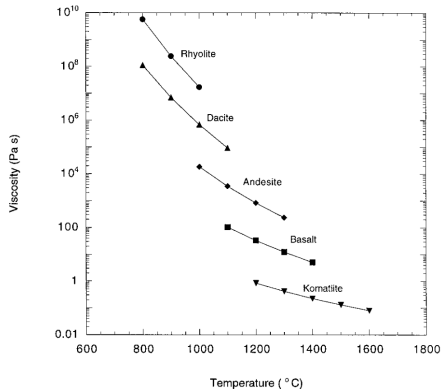
$$B = \sum_{i=1}^7 b_i M_i + \sum_{j=1}^3 b_{1j} M_{1j}$$

$$C = \sum_{i=1}^6 c_i N_i + c_{11} N_{11}$$

$b_1 = 159.6$ $b_2 = -173.3$ $b_3 = -72.1$ $b_4 = -75.7$ $b_5 = -39.9$ $b_6 = -84.1$ $b_7 = -141.5$	$M_1 = X_{\text{SiO}_2} + X_{\text{TiO}_2}$ $M_2 = X_{\text{Al}_2\text{O}_3}$ $M_3 = X_{\text{FeO}} + X_{\text{MnO}} + X_{\text{P}_2\text{O}_5}$ $M_4 = X_{\text{MgO}}$ $M_5 = X_{\text{CaO}}$ $M_6 = X_{\text{Na}_2\text{O}} + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}}$ $M_7 = X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}} + \ln(1 + X_{\text{H}_2\text{O}})$	$c_1 = 2.75$ $c_2 = 15.7$ $c_3 = 8.3$ $c_4 = 10.2$ $c_5 = -12.3$ $c_6 = -99.1$	$N_1 = X_{\text{SiO}_2}$ $N_2 = X_{\text{TiO}_2} + X_{\text{Al}_2\text{O}_3}$ $N_3 = X_{\text{FeO}} + X_{\text{MnO}} + X_{\text{MgO}}$ $N_4 = X_{\text{CaO}}$ $N_5 = X_{\text{Na}_2\text{O}} + X_{\text{K}_2\text{O}}$ $N_6 = \ln(1 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$
$b_{11} = -2.43$ $b_{12} = -0.91$ $b_{13} = 17.6$	$M_{11} = M_1 N_3$ $M_{12} = (N_1 + N_2 + X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}})$ $M_{13} = M_2 N_5$	$c_{11} = 0.3$	$N_{11} = (M_2 + N_3 + N_4 - X_{\text{P}_2\text{O}_5})(N_5 + X_{\text{H}_2\text{O}} + X_{\text{F}_2\text{O}})$

For a given melt composition, can evaluate η

Melt viscosity - Effect of T and X_{H_2O}



Increase in T and X_{H_2O} can reduce melt viscosity by orders of magnitude

Magma viscosity - Effect of crystals

Particles suspended in a fluid increase the viscosity of the medium

Suspension - A mixture of particles and fluid

ϕ_c = **Volume fraction** of crystals

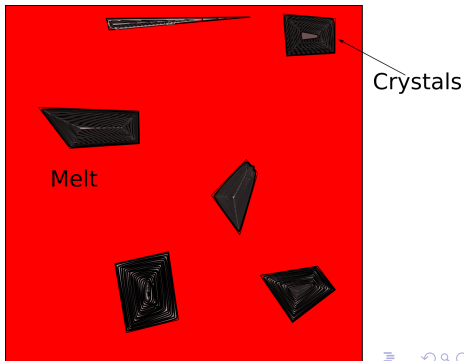
Define **relative viscosity** $\eta_r(\phi_c)$ to contain effect of ϕ_c

$$\eta = \eta_m \eta_r(\phi_c)$$

$\phi_c \lesssim 0.01$ - **Dilute**

$0.01 \lesssim \phi_c \lesssim 0.25$ - **Semi-dilute**

$\phi_c \gtrsim 0.25$ - **Concentrated**



Magma viscosity - Dilute particle suspensions

Dilute suspension: $\phi_c \lesssim 0.01$

Interactions between particles are weak and effect on viscosity is small

Viscosity depends linearly on ϕ_c (Einstein, 1906; 1911)

$$\eta_r = 1 + B\phi_c$$

Experiments suggest that $B \approx 2.5$

Suspension rheology models are only strictly valid for suspensions on **spherical** particles.

Developing models for suspensions of differently shaped particles is difficult
- particularly for magmas where crystals have many different and irregular shapes

However, for dilute and semi-dilute suspensions, models work well.

Magma viscosity - Semi-dilute suspensions

Semi-dilute suspension: $0.01 \lesssim \phi_c \lesssim 0.25$

Particles interact with each other, affecting the viscosity

Viscosity depends non-linearly on ϕ_c (Guth & Gold, 1938; Vand, 1948; Manley & Mason, 1955)

Can model the viscosity with a **polynomial** expression

$$\eta_r = 1 + B\phi_c + B_1\phi_c^2 + \dots$$

Experiments suggest that $7.35 \lesssim B_1 \lesssim 14.1$

Predictions from polynomial models get worse as ϕ_c increases.

- This is because as particles begin to touch the viscosity rises very fast

Magma viscosity - Maximum packings

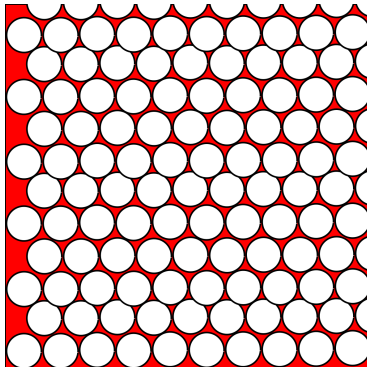
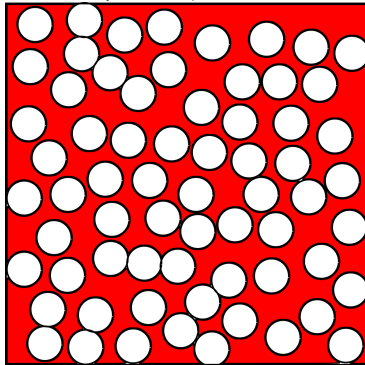
Concentrated suspension: $\phi_c \gtrsim 0.25$

Random close packing - Maximum volume fraction of crystals that can be obtained in a disordered system

For spheres: $\phi_m \approx 0.64$

Densest regular packing - Maximum possible packing if crystals are arranged orderly

For spheres: $\phi_r \approx 0.74$



Magma viscosity - Concentrated suspensions

Concentrated suspension: $\phi_c \gtrsim 0.25$

Once $\phi_c = \phi_m$, then further deformation is impossible

- the material is **rheologically locked**
- $\eta_r \rightarrow \infty$

For magmas, ϕ_m depends on crystal shape and size distribution

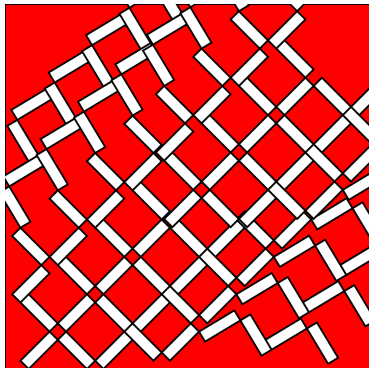
- Very difficult to accurately predict
- Many experiments, find $\phi_m \approx 0.4$

Multiple models describe rheology of concentrated suspensions, but commonly used model is **Krieger & Dougherty (1959)**

$$\eta_r = \left(1 - \frac{\phi_c}{\phi_m}\right)^{-B\phi_m}$$

Magma viscosity - Yield stress

Once a touching network of crystals exists, then the magma has a **yield stress**



Experiments show magmas with ϕ_c as small as 0.2 can have a yield stress

ϕ_y = Minimum value of ϕ_c at which yield stress exists

$$\tau_0 = 6.9 \left(\frac{\phi_c}{\phi_y} - 1 \right) \left(1 - \frac{\phi_c}{\phi_m} \right)^{-1}$$

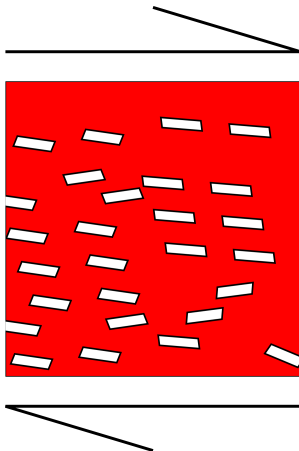
(Saar et al., 2001; Andrews & Manga, 2014)

Magma viscosity - Non-Newtonian effects

Definition of η_r assumes Newtonian or Bingham rheology

i.e. η is independent of $\dot{\epsilon}$

In reality:



Elongated particles align with the flow

Longest axis becomes parallel to flow lines

The greater the strain rate the larger the reduction in viscosity \Rightarrow **shear thinning**

More complicated models exist to describe suspensions of elongate particles e.g.

$$\eta_r = \left(1 - \frac{\phi_c}{\phi_m}\right)^{-2} \dot{\epsilon}^{0.2 r_p (\phi_c / \phi_m)^4}$$

r_p = Crystal **aspect ratio**

Magma viscosity - Effect of bubbles

Rheological behaviour is determined by the **capillary** number

$$Ca = \frac{\eta_m \dot{\epsilon} r_b}{\sigma}$$

η_m = Melt viscosity

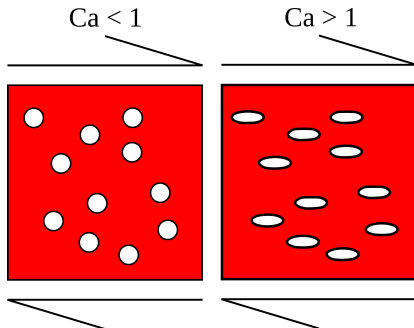
$\dot{\epsilon}$ = Shear rate

r_b = Bubble radius

σ = Surface tension

Ca is a balance between:

- **Deforming viscous stress** $\eta_m \dot{\epsilon}$
- **Restoring surface tension stress** σ/r_b



If $Ca \lesssim 1$, surface tension stress dominates and bubbles remain spherical

Bubbles behave like solid particles
 \Rightarrow increase in viscosity

If $Ca \gtrsim 1$, deforming stress dominates and bubbles can have irregular shapes

Bubbles streamline with shear
 \Rightarrow shear thinning behaviour

Magma density and viscosity - conclusions

- Empirical and semi-empirical models parameterise magma density and viscosity
- Both depend on volume fraction of solid and gas phases
- Melt density is described by an empirical equation of state and depends strongly on water content
- Melt viscosity is described by an empirical VFT equation and depends on temperature and composition
- Magma viscosity strongly depends on crystal and bubble content