Small Steps Semantics for tiny arithmetic expression grammar

Paul Passeron

October 22, 2025

1 Grammar

We'll define the set of arithmetic expressions as the expressions that can be built from the following grammar:

$$E_A ::= n(\in \mathbb{Z}) \mid v(\in \mathbf{V}) \mid E_A + E_A \mid E_A \times E_A \mid E_A - E_A$$

With $\mathbf{V}$ the set of variables.

2 Small Steps Semantic

Let $\sigma \in \mathcal{V}[\mathbb{Z}]$ $(= \mathcal{F}(\mathbf{V}, \mathbb{Z}))$ a valuation.

$$(A_1)\ \frac{}{\langle n_1 \text{ op } n_2, \sigma\rangle \hookrightarrow \langle n, \sigma\rangle}\ (n_1, n_2 \in \mathbb{Z}, \text{op } \in \{+, -, \times\}, n = n_1 \text{ op } n_2)$$

$$(A_2)\ \frac{}{\langle v, \sigma\rangle \hookrightarrow \langle \sigma(v), \sigma\rangle}\ (v \in \mathbf{V})$$

$$(A_3)\ \frac{\langle e_1, \sigma\rangle \hookrightarrow \langle e_1', \sigma\rangle}{\langle e_1 \text{ op } e_2, \sigma\rangle \hookrightarrow \langle e_1' \text{ op } e_2, \sigma\rangle}\ (\text{op } \in \{+, -, \times\})$$

$$(A_4)\ \frac{\langle e_2, \sigma\rangle \hookrightarrow \langle e_2', \sigma\rangle}{\langle n \text{ op } e_2, \sigma\rangle \hookrightarrow \langle n \text{ op } e_2', \sigma\rangle}\ (\text{op } \in \{+, -, \times\}, n \in \mathbb{Z})$$