

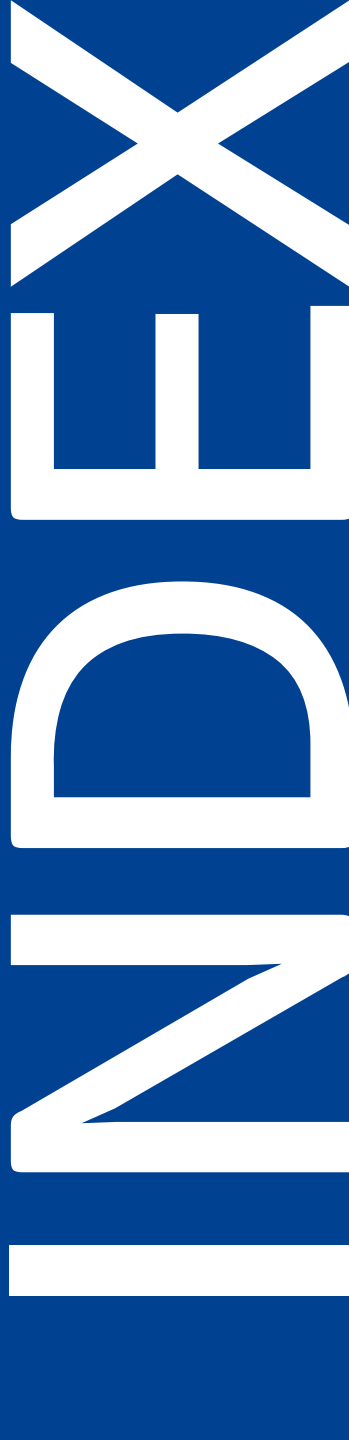
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# Collective Dynamics of ‘Small-world’ Networks

Watts, D., Strogatz, S. (1998). *Nature* 393, 440–442.

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## 01 Introduction

- Authors & Paper

## 02 Network

- Regular, Random, Small-world Network
- Characteristics ( $L$ ,  $C$ )

## 03 Examples

- Three Empirical Examples
- Infectious Disease

## 04 Summary

- Significance of Small-world Network

# 01 Introduction

1) Authors & Paper

## Authors



I think I've been contacted by someone from just about **every field**.

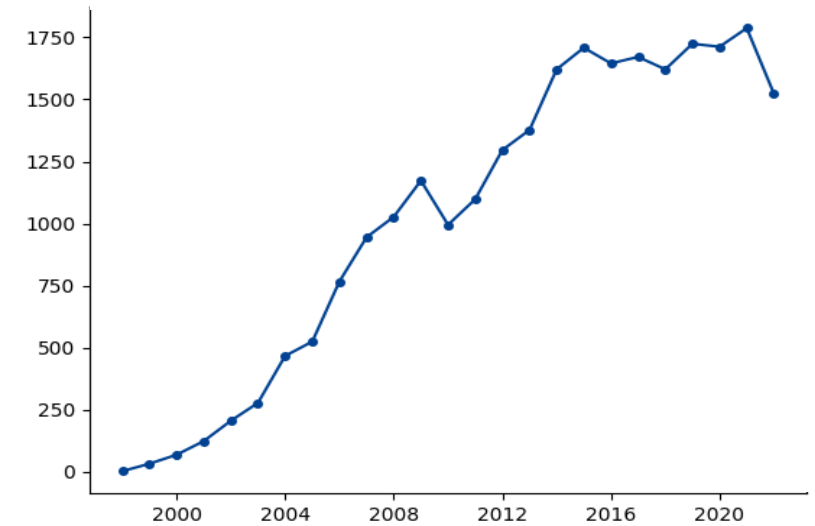
### Duncan J. Watts

- Ph.D in [Theoretical and Applied Mechanics](#) (Advisor Steven Strogatz)
- Penn Integrates Knowledge University Professor at the University of Pennsylvania (CSS Lab)

### Steven Strogatz

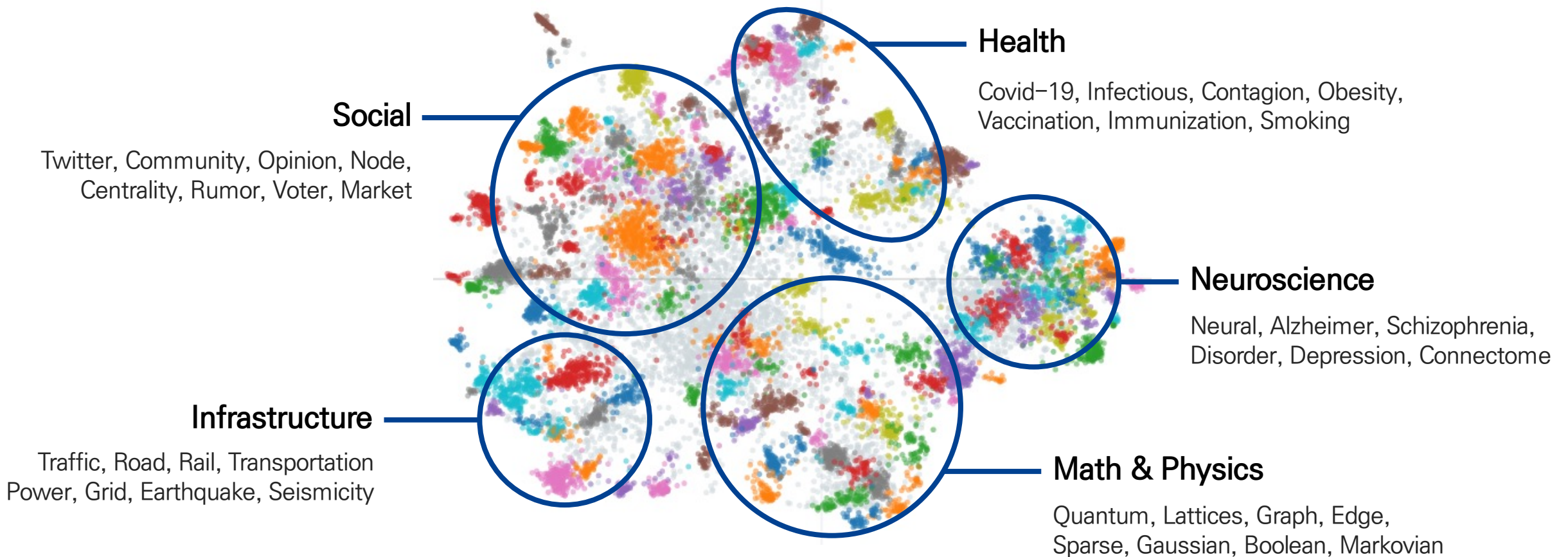
- Ph.D in [Applied Mathematics](#)
- Distinguished Professor for the Public Understanding of Science and Mathematics at the Cornell University

### 〈 Yearly Citation Trends for This Paper 〉



# 01 Introduction

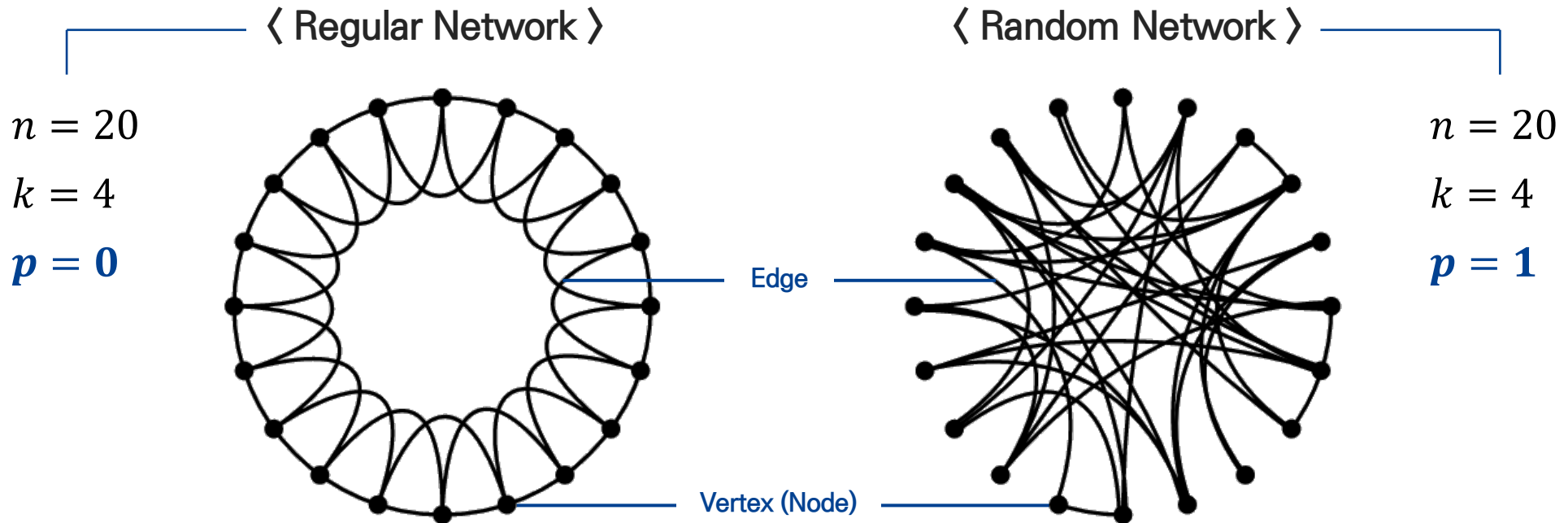
1) Authors & Paper



〈 BERTopic Results Based on Papers Citing This Paper (n=21,705) 〉

# 02 Network

## 1) Regular & Random Network



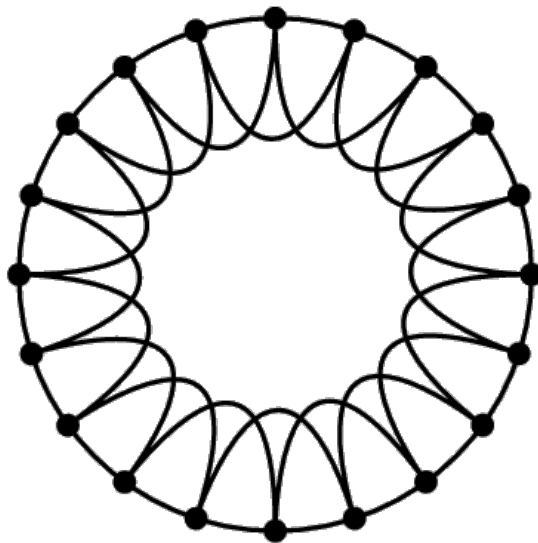
$n$  = Number of Vertices,  $k$  = Number of Edges per Vertex

$p$  = Probability of Rewire Each Edge at Random ( $0 < p < 1$ )

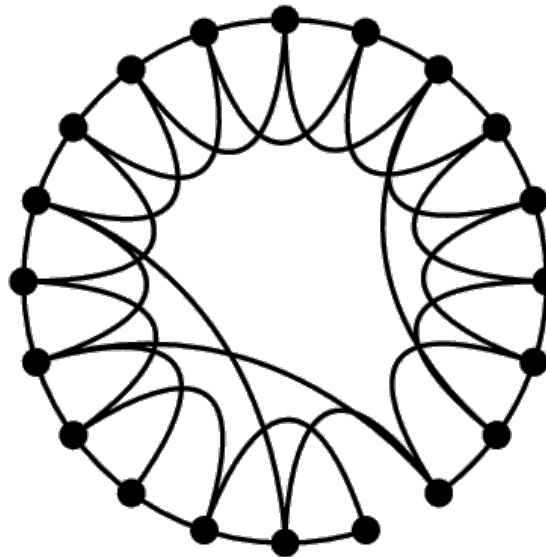
# 02 Network

## 2) Small-world Network

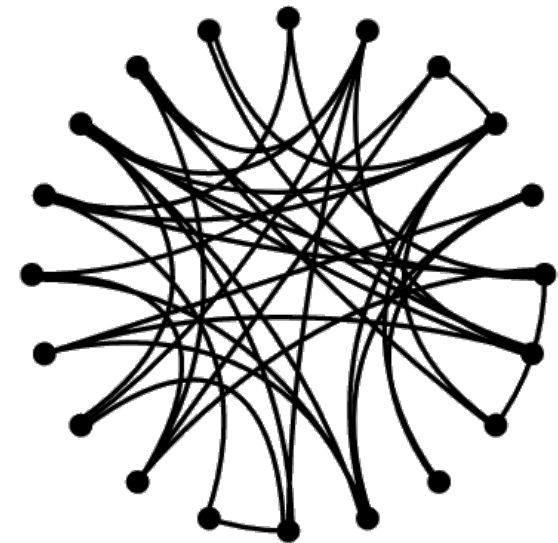
〈 Regular Network 〉



〈 Small-world Network 〉



〈 Random Network 〉



$p = 0$

————— Increasing Randomness (Disorder) —————→

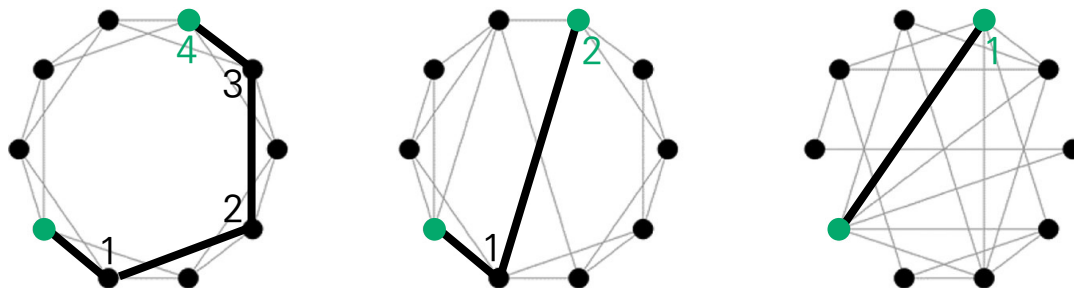
$p = 1$

# 02 Network

3) Characteristics (L, C)

## Mean Path Length (Global)

- Typical separation between two vertices
- Average of steps along the shortest paths for all possible pairs of network nodes
- How separate are the nodes in the network



< Regular Network >

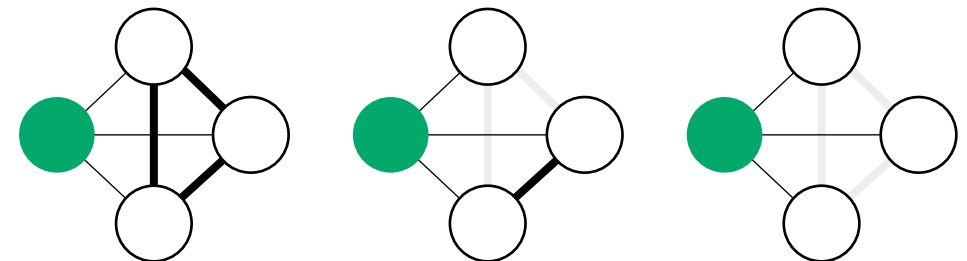
< Small-world Network >

< Random Network >

Long  $\longrightarrow$  Short

## Clustering Coefficient (Local)

- Cliquishness of a typical neighborhood
- Cliquishness: No matter which node you choose, all nodes are connected to each other
- How many strong tie in the network



$3/3 = 1$

$1/3$

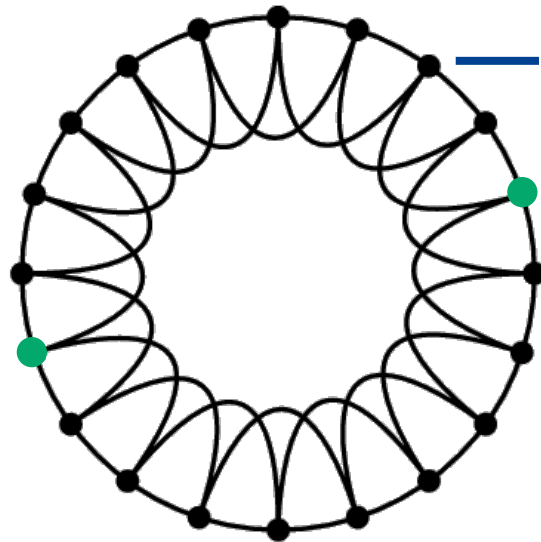
$0/3 = 0$

High  $\longrightarrow$  Low

# 02 Network

3) Characteristics (L, C)

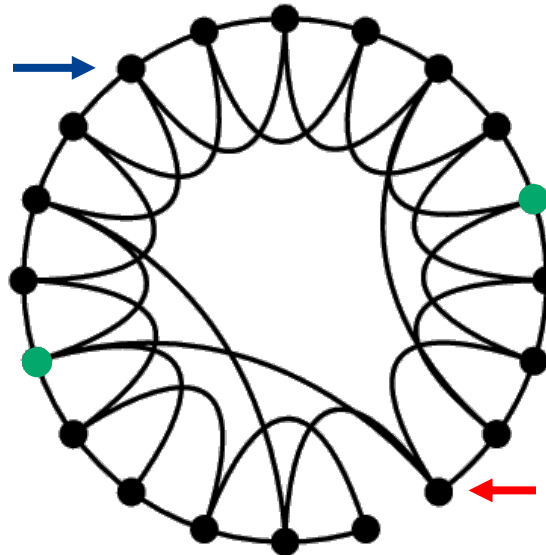
〈 Regular Network 〉



Length =  $L(p)$ : Long  
Clustering =  $C(p)$ : High

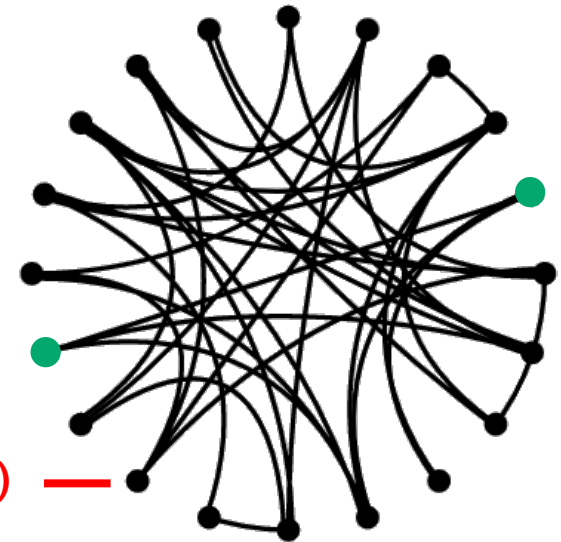
〈 Small-world Network 〉

High  $C(p)$



Length =  $L(p)$ : Short  
Clustering =  $C(p)$ : High

〈 Random Network 〉

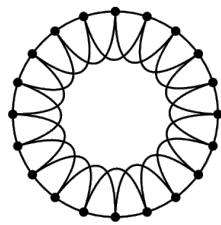


Length =  $L(p)$ : Short  
Clustering =  $C(p)$ : Low

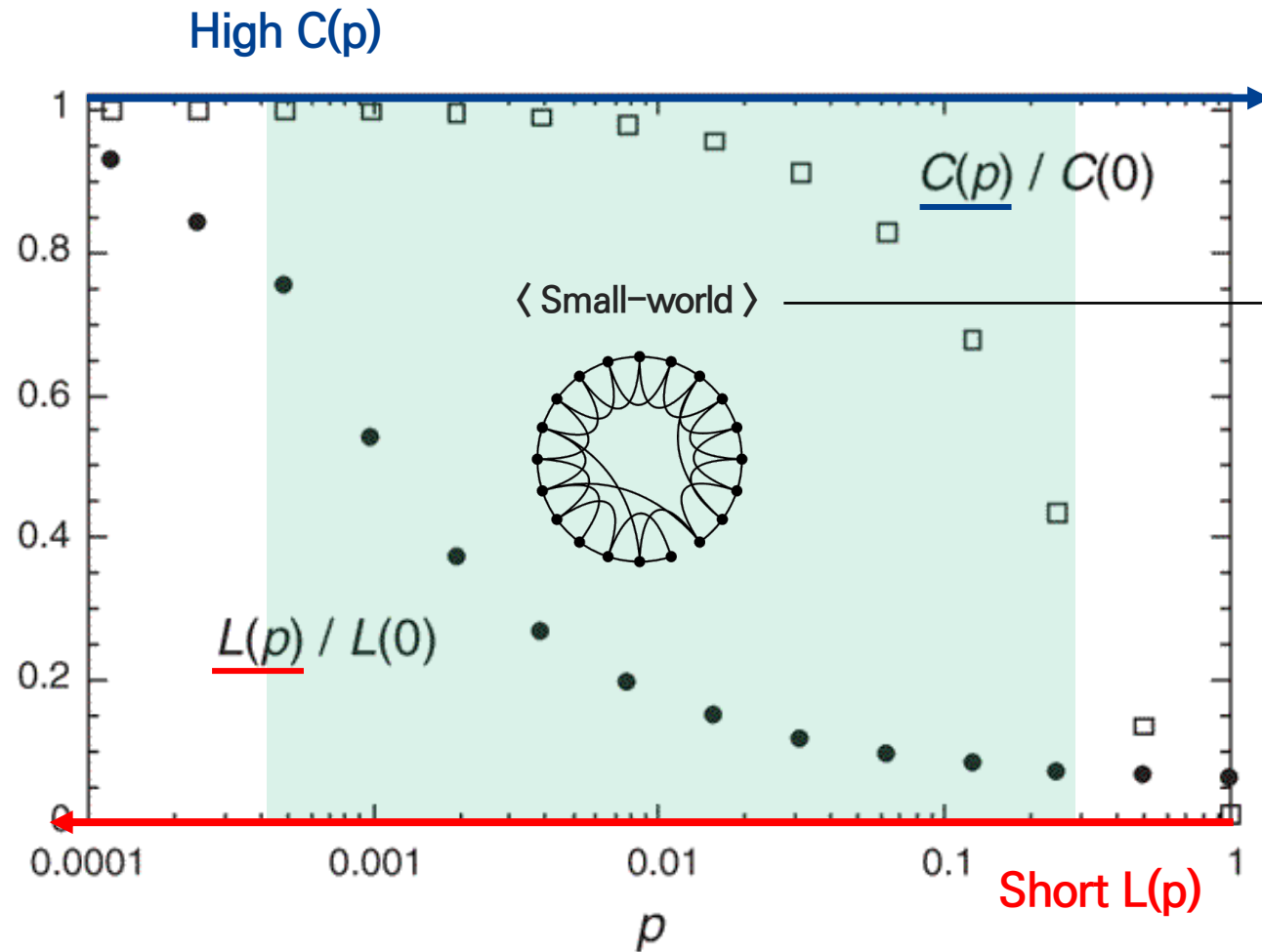


# 02 Network

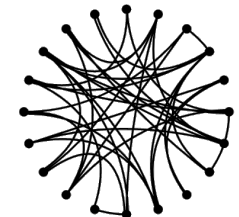
## 3) Characteristics (L, C)



〈 Regular 〉



Random connections make it easier to reach other nodes



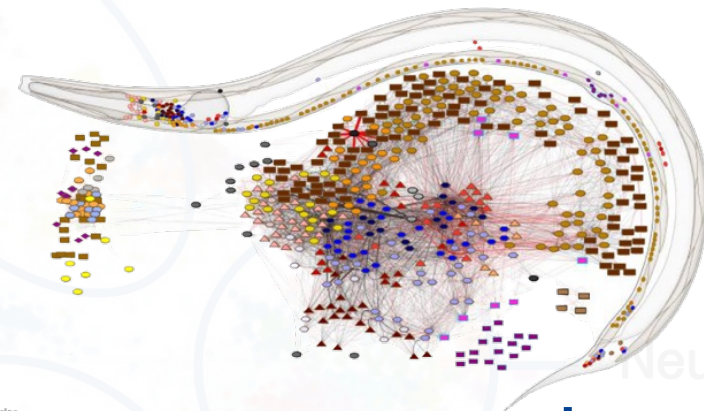
〈 Random 〉

# 03 Examples

1) Three Empirical Examples

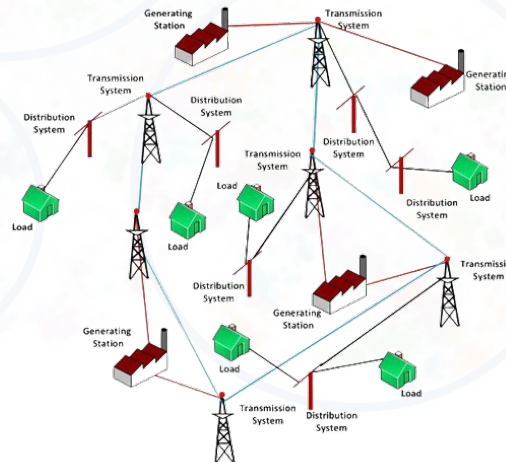
## ① Actors Network

Surrogate for a social network



## ② Power Grid Network

Relevant to the efficiency and robustness

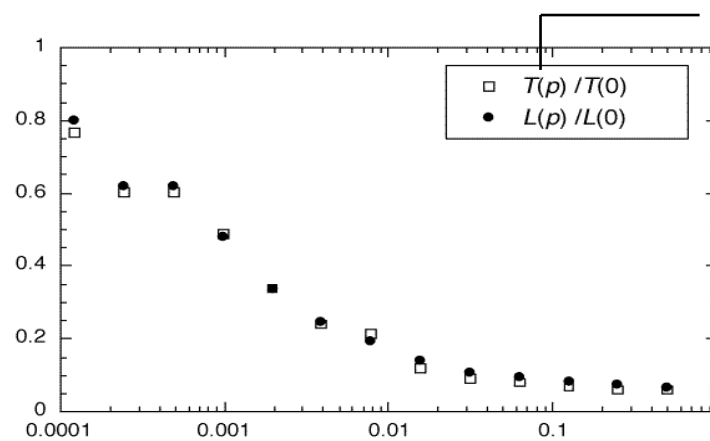


## ③ C. Elegans Network

Sole example of a completely mapped neural network

# 03 Examples

## 2) Infectious Disease (Functional Significance of Small-world Network)

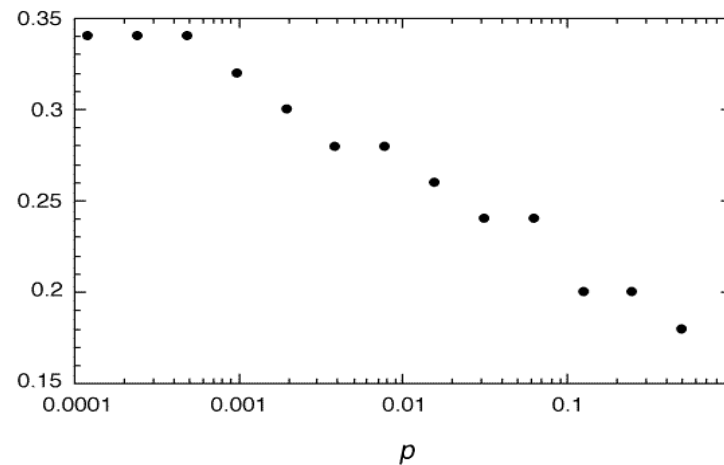


Time which takes for everyone in the network to be infected

Time  $T(p)$  required for global infection resembles the  $L(p)$  curve

Critical infectiousness at which the disease infects half the population

$r_{half}$

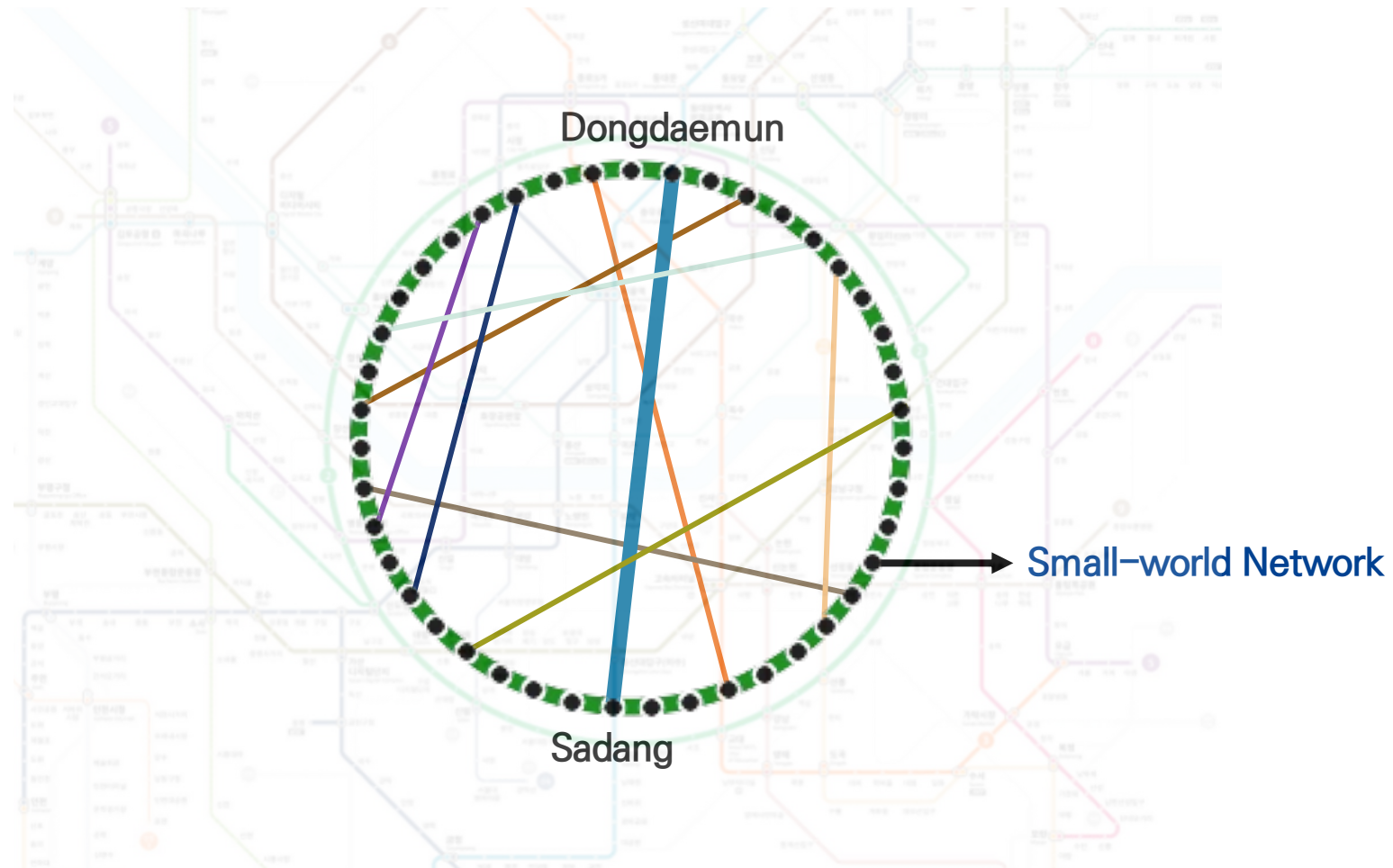


$r_{half}$  decreases rapidly for small  $p$

〈 Simulation Results for Simple Model of Disease Spreading 〉

# 04 Summary

## 1) Significance of Small-world Network

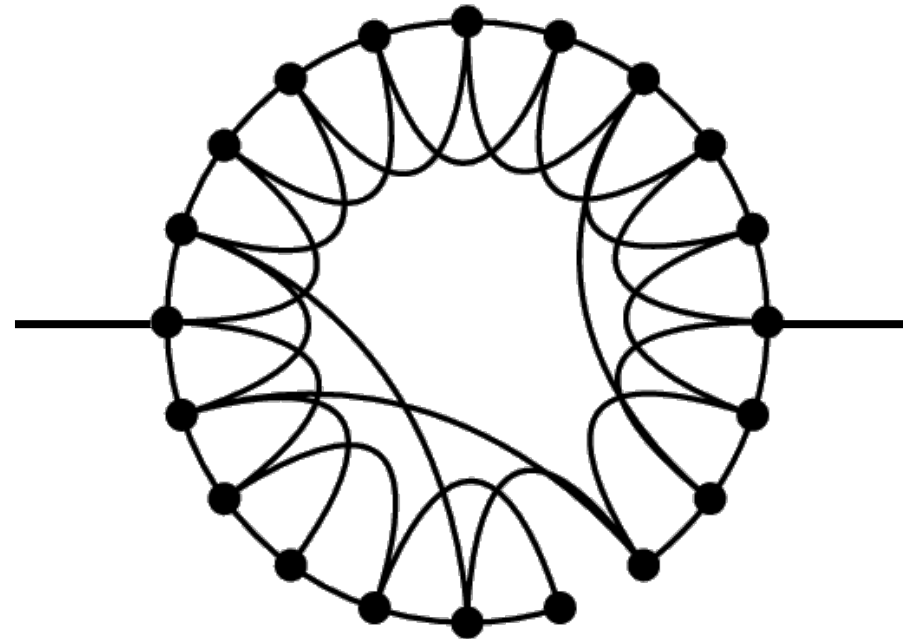


〈 Seoul Metro as ‘Small-world Network’ Analogical Example 〉

# 04 Summary

## 1) Significance of Small-world Network

High  
Clustering



Short  
Length

〈 Small-world Network 〉 → Prevalent in various fields in our real world

“ Connectivity, Efficiency, Infectivity ”