Nonlinear Optimal Tracking Control of Wind Energy Conversion System in Partial Load Region

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***Abstract*—The conversion efficiency of the wind turbine in partial load operation can be maximized if it operates at the optimal tip speed ratio and the optimal pitch angle, thus the optimal power coefficient. For this purpose, the wind turbine rotor must track an optimal reference speed. In this work, a nonlinear closed-loop finite horizon optimal tracking via state dependent Riccati equation (SDRE) is applied to track the optimal reference rotor speed of a permanent magnet synchronous generator based wind energy conversion system. The key idea in this technique is to use an approximate analytic approach to convert the state dependent differential Riccati equation (SD-DRE) into the linear differential Lyapunov equation which can be solved in a closed form at each time step of a given time period. In addition, a state dependent vector differential equation is solved simultaneously with SD-DRE at each time step to perform the optimal tracking problem.**

***Index Terms*—Finite horizon tracking, nonlinear optimal control, partial load region, state dependent Riccati equation, wind energy conversion system.**

I. Introduction

Wind, due to its clean, free and ready availability, has appeared as one of the most important sources of renewable energy during the last two decades. With the increasing demand for wind energy, investigations are focusing more and more on energy extraction and efficiency. The conversion efficiency of the wind turbine can be improved by considering some key design aspects such as blades, gearboxes, generators etc. Advanced strategies developed by the control community in the areas of nonlinear modeling, filtering and control theories can also significantly improve the efficiency of the wind turbine [1].

Generally, a wind turbine has three main operating regions as shown in Fig. 1. In Region 1, the wind speed is below the cut-in speed, and the power generation is halted in this region. Region 2 is called the partial load region because here the turbine operates at partial load below the rated power. In Region 3, the wind turbine power generation reaches its maximum value and stays there. When the wind speed exceeds the cut-out speed, the turbine shuts off due to electrical, mechanical and structural safety issues. With the technological advancement in the areas of aerodynamic design, mechanical systems and generators, more wind turbines operate in the

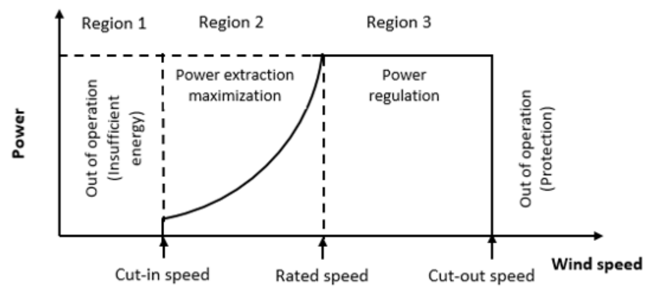

Fig. 1. Wind turbine operating regions

partial load region [2]. The operational goal of this region is to maximize the energy efficiency by keeping the power coefficient at its maximum value. For maximizing power extraction, the pitch angle and the tip speed ratio (TSR) should remain at their optimal values. Again, for keeping the TSR at its optimal value, the wind rotor speed should be controlled in such a way that it tracks an optimal reference speed.

For extracting maximum power, conventional vector control with proportional-integral loops were studied in several literature [3], [4]. But the performance of these linear control strategies is not satisfactory due to the highly nonlinear behaviour of the wind turbine and the wide range of operating points. Feedback linearizing control strategy was employed in [5], where the linear control method was used to design the current controller and the mechanical speed rotation controller. However, this strategy does not show robustness in parametric uncertainties and external disturbances. Sliding mode control (SMC) as a robust control to uncertainty and disturbance was implemented to WECS in several literature [6], [7]. However, SMC creates high mechanical stress and increases chattering phenomena due to sudden and large change of control variables [8]. Hill-climb search (HCS) control as a model free optimal point tracking technique has been studied in [9], [10]. But the effectiveness of the HCS becomes very poor when the wind turbine has to operate under a volatile wind profile [2].

Recently, the state dependent Riccati equation (SDRE) technique for designing nonlinear controller has gained great attention among the control community [11]–[14]. This method,

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by allowing nonlinearities in the system states through state dependent coefficient (SDC) matrices and offering flexibility in designing these matrices, appeared to be a highly effective tool for the controller designing of strong nonlinear systems. SDRE has been studied for both infinite and finite horizon case. In infinite horizon case, an algebraic Riccati equation with SDC matrices is solved online in order to find the control law [12]. But in the finite horizon case, the solution is time dependent and a differential equation, rather than an algebraic equation, needs to be solved for finding the control law [13].

In this paper, finite horizon optimal tracking via state dependent Riccati equation (SDRE) has been implemented to the nonlinear sixth order model of a PMSG based WECS. Here, variable speed control schemes are investigated for the partial load region. The nonlinear tracking controller is designed to track an optimal rotor speed reference signal to capture maximum wind energy in this region. The main idea in this technique is to convert the nonlinear state dependent differential Riccati equation (SD-DRE) into the linear differential Lyapunov equation (DLE) using the change of variable technique. The coefficients of the resulted equation are evaluated at each time step and freeze from current time step to next time step [15]. The DLE is then solved at each time step in a closed form. Although, for nonlinear optimal regulation, which has recently been presented by the authors of this paper [14], control law can be found in terms of the solution of SD-DRE, for optimal tracking, one is faced with not only solving the SD-DRE but also solving the state dependent vector differential equation (SD-VDE). SD-VDE is solved simultaneously with SD-DRE at each time step, which starts with considering the vector coefficient being constant during each small time step, hence using steady-state vector coefficient by solving a vector algebraic equation (VAE). Then, applying change of variable procedures and solving differential equation, the solution of SD-VDE is obtained for each time step. Finally, the optimal control is found by in terms of the solution of SD-DRE and SD-DVE. Simulations were performed in MATLAB/Simulink environment which illustrate the validity of the proposed technique for PMSG based WECS.

The structure of the paper is as follows: Section II discusses the nonlinear modeling of PMSG-WECS. Section III presents nonlinear finite horizon tracking via SDRE technique. In Section IV, simulation results are illustrated. Finally, the conclusions of this paper are presented in Section V.

II. Nonlinear Modeling of PMSG based WECS

A WECS basically converts the kinetic energy of the wind to electric power. It can be divided into two groups in terms of physical nature (electrical and mechanical) and four groups in terms of dynamics (aerodynamics, drive train dynamics, generator dynamics and structural dynamics) as shown in Fig. 2 [16]. The aerodynamic block converts the wind's kinetic energy into rotational (mechanical) energy. The drive train block increases this slower rotational speed and then transmits the increased speed to the generator block. The generator block converts this mechanical power into electrical power.

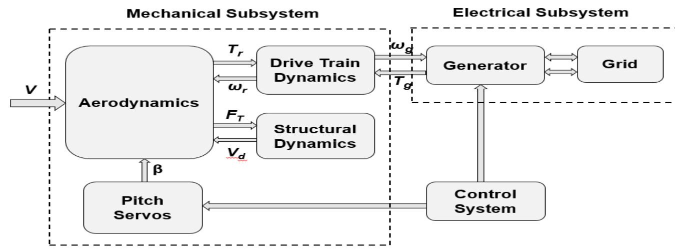

Fig. 2. WECS Block Diagram

For control purposes, only aerodynamics, drive train dynamics and generator dynamics are taken into consideration in this paper. Aerodynamics is the most complex dynamics among these three [17] which takes wind speed V and rotor speed ω_r as the input and provides the output in terms of aerodynamic torque as follows:

$$T_r = \frac{1}{2}\pi\rho R^3 C_Q(\lambda,\beta)V^2, \tag{1}$$

where, ρ is the air density, R is the radius of the wind rotor plane, C_Q is the torque coefficient as a function of tip speed ratio λ and pitch angle β. Tip speed ratio of a wind turbine can be defined as the peripheral blade speed over the wind speed and mathematically expressed as

$$\lambda = \frac{\omega_r R}{V}, \tag{2}$$

where, ω_r is the wind turbine rotor speed. The torque coefficient is related to power coefficient by the following expression

$$C_Q(\lambda,\beta) = \frac{C_P(\lambda,\beta)}{\lambda}, \tag{3}$$

where, $C_P(\lambda,\beta)$ is a nonlinear function of fixed blade pitch angle β and tip speed ratio λ. It can be expressed as [18]:

$$C_P(\lambda,\beta) = a_1(\frac{a_2}{\lambda_i} - a_3\beta - a_4)exp(-\frac{a_5}{\lambda_i}) + a_6\lambda \tag{4}$$

where,

$$\lambda_i = \Big[\frac{1}{\lambda + a_7\beta} - \frac{a_8}{\beta^3+1}\Big]^{-1}$$

with $a_1 = 0.5176, a_2 = 116, a_3 = 0.4, a_4 = 5, a_5 = 21, a_6 = 0.0068, a_7 = 0.08$ and $a_8 = 0.035$. Fig. 3 shows the power

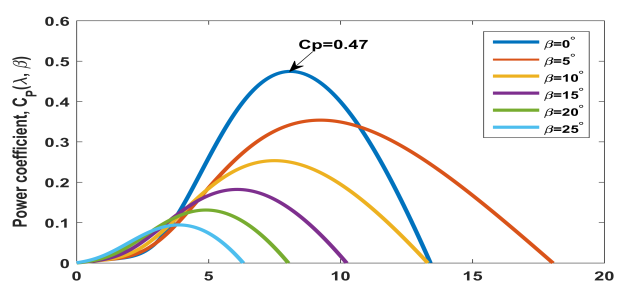

Fig. 3. Power coefficient as a function of tip speed ratio for different blade pitch angle

coefficient versus tip speed ratio for different pitch angle. It is found that the maximum power coefficient, C_P= 0.47 is obtained when the pitch angle β is zero and the optimal tip speed ratio λ_{opt} is 8.1.

Gearbox is the main part of the drive train block which connects the low speed shaft with the high speed shaft. Drive train block can be represented either in flexible or rigid model. In this work, a flexible drive train is considered which has the dynamic model as follows:

$$\dot{\omega}_r = -\frac{i}{\eta J_r} T_H + \frac{1}{J_r} T_r, \tag{5}$$

$$\dot{\omega}_g = \frac{1}{J_g} T_H - \frac{1}{J_g} T_g, \tag{6}$$

$$\dot{T}_H = iK_g\omega_r - K_g\omega_g - B_g(\frac{1}{J_g} + \frac{i^2}{\eta J_r})T_H + \frac{iB_g}{J_r} T_r + \frac{B_g}{J_g} T_g, \tag{7}$$

where, T_H is the internal torque, ω_g is the generator speed, T_g is the generator electromagnetic torque, J_g is the generator inertia, i is the gearbox ratio, K_g is the stiffness coefficient of the high-speed shaft, J_r is the wind rotor inertia, B_g is the damping coefficient of the high-speed shaft and η is the gearbox efficiency.

The pitch actuator rotates the blades around the longitudinal axis and takes input from the control system which decides blade angle (desired pitch angle), β_d and gives output as the final pitch angle of the blade, β [19]. The dynamic equation of the pitch actuator is represented as

$$\dot{\beta} = -\frac{1}{\tau}\beta + \frac{1}{\tau}\beta_d, \tag{8}$$

where, τ is the time constant of the pitch actuator system.

Among the three popular types of generators used in WECS- Permanent Magnet Synchronous Generator (PMSG), Squirrel Cage Induction Generator (SCIG) and Doubly Fed Induction Generator (DFIG)- PMSG is increasingly being adopted because of its low maintenance and operating cost, high generation capacity and less weight [6]. It has the model in (d, q) axes as

$$\dot{i}_d = -\frac{R_s}{L_d} i_d + \frac{pL_q}{L_d} i_q \omega_g - \frac{1}{L_d} u_d, \tag{9}$$

$$\dot{i}_q = -\frac{R_s}{L_q} i_q - \frac{p}{L_q}(L_d i_d - \phi_m)\omega_g - \frac{1}{L_q} u_q, \tag{10}$$

$$T_g = p\phi_m i_q, \tag{11}$$

where, R_s is the stator resistance, ϕ_m is the magnetic flux of PMSG, p is the number of pole pairs, L_d and L_q are d and q axis inductance of the stator, i_d and i_q are stator's d and q axis current, u_d and u_q are the input voltage of stator's d and q axis.

III. Nonlinear Finite Horizon Tracking via SDRE

The complexity of the time-dependency of the Hamilton-Jacobi-Bellman (HJB) partial differential equation makes the finite horizon optimal control of nonlinear systems a challenging problem in the control field [13]. To handle this problem, the potential of the SDRE technique in infinite horizon control is utilized to find the solution of the finite horizon case. The infinite horizon case provides a closed form control through the online solution of state dependent algebraic Riccati equation (SD-ARE) [15]. But in finite horizon case, the optimal control is found by the solution of state dependent differential Riccati equation (SD-DRE). In this section, the solution of the finite horizon tracking via SDRE is discussed.

A. Statement of Problem

Consider the nonlinear system in the form:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}) + \mathbf{g}(\mathbf{x})\mathbf{u}(t), \tag{12}$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}). \tag{13}$$

which can be represented in the state-dependent coefficient (SDC) form as:

$$\dot{\mathbf{x}}(t) = \mathbf{A}(\mathbf{x})\mathbf{x}(t) + \mathbf{B}(\mathbf{x})\mathbf{u}(t), \tag{14}$$

$$\mathbf{y}(t) = \mathbf{C}(\mathbf{x})\mathbf{x}(t), \tag{15}$$

where, $\mathbf{f}(\mathbf{x})$ = $\mathbf{A}(\mathbf{x})\mathbf{x}(t)$, $\mathbf{B}(\mathbf{x})$= $\mathbf{g}(\mathbf{x})$, and $\mathbf{h}(\mathbf{x})$ = $\mathbf{C}(\mathbf{x})\mathbf{x}(t)$. Here, $\mathbf{A}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ are chosen in such a way that the system is controllable or at least stablizable.

Now, consider $\mathbf{z(t)}$ as the desired output or trajectory. Then, the closed loop error becomes $\mathbf{e(t)} = \mathbf{z(t)} - \mathbf{y(t)}$. The goal is to obtain a state feedback control law that eliminates the closed loop error by minimizing the cost function [20]

$$\mathbf{J}(\mathbf{x}, \mathbf{u}) = \frac{1}{2}\mathbf{e}'(t_f)\mathbf{F}\mathbf{e}(t_f) + \frac{1}{2}\int_{t_0}^{t_f} [\mathbf{e}'(t)\mathbf{Q}(\mathbf{x})\mathbf{e}(t) + \mathbf{u}'(\mathbf{x})\mathbf{R}(\mathbf{x})\mathbf{u}(\mathbf{x})]\, dt, \tag{16}$$

where, $\mathbf{Q}(\mathbf{x})$ and $\mathbf{F}$ are symmetric *positive semi-definite* matrices, and $\mathbf{R}(\mathbf{x})$ is a symmetric *positive definite* matrix.

B. Solution of Finite Horizon Tracking via SDRE

A state feedback control law to minimize the cost function (16) can be defined as

$$\mathbf{u}(\mathbf{x},t) = -\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})[\mathbf{P}(\mathbf{x}, t)\mathbf{x}(t) - \mathbf{g}(\mathbf{x}, t)], \tag{17}$$

where, $\mathbf{P}(\mathbf{x}, t)$ is the symmetric, positive definite solution of the state dependent differential Riccati equation (SD-DRE) of the form

$$-\dot{\mathbf{P}}(\mathbf{x}, t) = \mathbf{P}(\mathbf{x},t)\mathbf{A}(\mathbf{x}) + \mathbf{A}'(\mathbf{x})\mathbf{P}(\mathbf{x},t) - \mathbf{P}(\mathbf{x},t)\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})\mathbf{P}(\mathbf{x},t) + \mathbf{C}'(\mathbf{x})\mathbf{Q}(\mathbf{x})\mathbf{C}(\mathbf{x}), \tag{18}$$

having the final condition

$$\mathbf{P}(\mathbf{x}, t_f) = \mathbf{C}'(t_f)\mathbf{F}\mathbf{C}(t_f), \tag{19}$$

and $\mathbf{g}(\mathbf{x},t)$ is the solution of the nonhomogeneous state dependent vector differential equation (SD-VDE) of the form

$$\dot{\mathbf{g}}(\mathbf{x}, t) = -[\mathbf{A}(\mathbf{x}) - \mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})\mathbf{P}(\mathbf{x},t)]'\mathbf{g}(\mathbf{x},t) - \mathbf{C}'(\mathbf{x})\mathbf{Q}(\mathbf{x})\mathbf{z}(\mathbf{x}), \tag{20}$$

with the final condition

$$\mathbf{g}(\mathbf{x},t_f)=\mathbf{C}'(t_f)\mathbf{F}\mathbf{z}(t_f). \tag{21}$$

Then, the closed-loop optimal state can be obtained as

$$\dot{\mathbf{x}}(t)=[\mathbf{A}(\mathbf{x})-\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})\mathbf{P}(\mathbf{x},t)]\mathbf{x}(t) + \mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})\mathbf{g}(\mathbf{x},t). \tag{22}$$

In finite horizon nonlinear optimal control, DRE can not be solved in real time by backward integration from final time (t_f) to initial time (t_0) since the states ahead of present time step are unknown [21]. To solve this problem, an approximate analytic approach is used which is based on the algebraic Riccati equation (ARE). Small time steps between initial time and final time are considered and during each of the these steps, $\mathbf{P}(\mathbf{x},t)$ is considered to be constant, and hence ARE is used to obtain the steady-state Riccati coefficient $\mathbf{P_{ss}}(\mathbf{x})$. Then, a new variable $\mathbf{K}(\mathbf{x},t)$ is introduced using the change of variable technique in terms of unknown $\mathbf{P}(\mathbf{x},t)$ and known $\mathbf{P_{ss}}(\mathbf{x})$ as $\mathbf{K}(\mathbf{x},t)$=$[\mathbf{P}(\mathbf{x},t)-\mathbf{P_{ss}}(\mathbf{x})]^{-1}$ which, substituting in the nonlinear DRE, leads to a linear differential Lyapunov equation (DLE) that can be solved in a closed form at each time step [22]. Finally, $\mathbf{P}(\mathbf{x},t)$ is obtained in terms of $\mathbf{K}(\mathbf{x},t)$, the analytic solution of DLE, which itself requires the solution of ARE and algebraic Lyapunov equation [23].

Similar approximate approach is performed to find the solution of vector differential equation $\mathbf{g}(\mathbf{x},t)$. The solution for $\mathbf{g}(\mathbf{x},t)$ starts with making an approximation by using the steady-state vector coefficient $\mathbf{g_{ss}}(\mathbf{x})$ and solving the vector algebraic equation (VAE). Then, using the change of variable technique, a new variable $\mathbf{K_g}(\mathbf{x},t)$ is introduced as $\mathbf{K_g}(\mathbf{x},t)$=$[\mathbf{g}(\mathbf{x},t)-\mathbf{g_{ss}}(\mathbf{x})]$, and substituting it in the VDE leads to a differential equation in terms of $\mathbf{K_g}(\mathbf{x},t)$, which is solved at each time step. The solution of VDE, $\mathbf{g}(\mathbf{x},t)$ is then obtained in terms of $\mathbf{K_g}(\mathbf{x},t)$. Once $\mathbf{P}(\mathbf{x},t)$ and $\mathbf{g}(\mathbf{x},t)$ are found, optimal control is evaluated using (17). The following steps can be followed to evaluate $\mathbf{P}(\mathbf{x},t)$ and $\mathbf{g}(\mathbf{x},t)$ at each time step for the solution of finite horizon SDRE tracking problem [21], [24]:

1) Calculate $\mathbf{P_{ss}}(\mathbf{x})$ by solving ARE.

$$\mathbf{P_{ss}}(\mathbf{x})\mathbf{A}(\mathbf{x})+\mathbf{A}'(\mathbf{x})\mathbf{P_{ss}}(\mathbf{x})+\mathbf{Q}(\mathbf{x}) - \mathbf{P_{ss}}(\mathbf{x})\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})\mathbf{P_{ss}}(\mathbf{x})=0. \tag{23}$$

2) Find $\mathbf{K}(\mathbf{x},t)$ using the change of variable technique as

$$\mathbf{K}(\mathbf{x},t)=[\mathbf{P}(\mathbf{x},t)-\mathbf{P_{ss}}(\mathbf{x})]^{-1}. \tag{24}$$

3) Calculate the value $\mathbf{A_{cl}}(\mathbf{x})$ as

$$\mathbf{A_{cl}}(\mathbf{x})=\mathbf{A}(\mathbf{x})-\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}\mathbf{B}'(\mathbf{x})\mathbf{P_{ss}}(\mathbf{x}). \tag{25}$$

4) Find $\mathbf{D}$ by solving the algebraic Lyapunov equation

$$\mathbf{A_{cl}}\mathbf{D}+\mathbf{D}\mathbf{A}'_{\mathbf{cl}}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'=\mathbf{0}. \tag{26}$$

5) Solve the differential Lyapunov equation

$$\dot{\mathbf{K}}(\mathbf{x},t)=\mathbf{K}(\mathbf{x},t)\mathbf{A}'_{\mathbf{cl}}(\mathbf{x})+\mathbf{A_{cl}}(\mathbf{x})\mathbf{K}(\mathbf{x},t) - \mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x}). \tag{27}$$

The solution of (27) is found as

$$\mathbf{K}(\mathbf{x},t)=\mathbf{e}^{\mathbf{A_{cl}}(\mathbf{t}-\mathbf{t_f})}(\mathbf{K}(\mathbf{x},t_f)-\mathbf{D})\mathbf{e}^{\mathbf{A}'_{\mathbf{cl}}(\mathbf{t}-\mathbf{t_f})}+\mathbf{D}. \tag{28}$$

6) Obtain $\mathbf{P}(\mathbf{x},t)$ from the following

$$\mathbf{P}(\mathbf{x},t)=\mathbf{K}^{-1}(\mathbf{x},t)+\mathbf{P_{ss}}(\mathbf{x}). \tag{29}$$

7) Calculate the steady-state vector coefficient $\mathbf{g_{ss}}(\mathbf{x})$ as

$$\mathbf{g_{ss}}(\mathbf{x})=[\mathbf{A}(\mathbf{x})-\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})\mathbf{P_{ss}}(\mathbf{x})]'^{-1} \mathbf{C}'(\mathbf{x})\mathbf{Q}(\mathbf{x})\mathbf{z}(\mathbf{x}). \tag{30}$$

8) Apply the change of variable technique and assume

$$\mathbf{K_g}(\mathbf{x},t)=[\mathbf{g}(\mathbf{x},t)-\mathbf{g_{ss}}(\mathbf{x})]. \tag{31}$$

9) Solve the differential equation

$$\dot{\mathbf{K}}_{\mathbf{g}}(\mathbf{x},t)=-[\mathbf{A}(\mathbf{x})-\mathbf{B}(\mathbf{x})\mathbf{R}^{-1}(\mathbf{x})\mathbf{B}'(\mathbf{x})\mathbf{P}(\mathbf{x},t)]' \mathbf{K_g}(\mathbf{x},t). \tag{32}$$

The solution of (32) is found as

$$\mathbf{K_g}(\mathbf{x},t)=\mathbf{e}^{-(\mathbf{A}-\mathbf{B}\mathbf{R}^{-1}\mathbf{B}'\mathbf{P})'(\mathbf{t}-\mathbf{t_f})}[\mathbf{g}(\mathbf{x},t_f)-\mathbf{g_{ss}}(\mathbf{x})]. \tag{33}$$

10) Obtain $\mathbf{g}(\mathbf{x},t)$ using the change of variable technique as

$$\mathbf{g}(\mathbf{x},t)=\mathbf{K_g}(\mathbf{x},t)+\mathbf{g_{ss}}(\mathbf{x}). \tag{34}$$

11) Finally, using $\mathbf{P}(\mathbf{x},t)$ and $\mathbf{g}(\mathbf{x},t)$ calculate the value of optimal control $\mathbf{u}(\mathbf{x},t)$ from (17).

IV. Simulation Results

In this section, nonlinear sixth order PMSG based WECS is simulated with the finite horizon SDRE tracking controller. A complete nonlinear sixth order model of PMSG-WECS can be found by combining (1)-(11). By transforming this nonlinear model to SDC form using (12)-(15) and considering the state vector and control vector as $\mathbf{x}=[\omega_r\ \omega_g\ T_H\ \beta\ i_d\ i_q]^T$ and $\mathbf{u}=[u_d\ u_q\ \beta_d]^T$ respectively, $\mathbf{A}(\mathbf{x})$ and $\mathbf{B}(\mathbf{x})$ matrices become

$$\mathbf{A}(\mathbf{x})=\begin{bmatrix} A_{11} & 0 & -\frac{i}{\eta J_r} & 0 & 0 & 0\\ 0 & 0 & \frac{1}{J_g} & 0 & 0 & -\frac{1}{J_g}p\phi_m\\ A_{31} & -K_g & A_{33} & 0 & 0 & \frac{B_g}{J_g}p\phi_m\\ 0 & 0 & 0 & -\frac{1}{\tau} & 0 & 0\\ 0 & \frac{pL_q}{L_d}i_q & 0 & 0 & -\frac{R_s}{L_d} & 0\\ 0 & A_{62} & 0 & 0 & 0 & -\frac{R_s}{L_q} \end{bmatrix}$$

and,

$$\mathbf{B}(\mathbf{x})=\begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & \frac{1}{\tau}\\ -\frac{1}{L_d} & 0 & 0\\ 0 & -\frac{1}{L_q} & 0 \end{bmatrix}$$

where,

$$A_{11} = \frac{\rho\pi R^2 C_P(\lambda,\beta)V^3}{2J_r\omega_r^2},$$
$$A_{31} = iK_g + \frac{iB_g\pi\rho R^2 C_P(\lambda,\beta)V^3}{2J_r\omega_r^2},$$
$$A_{33} = -B_g(\frac{1}{J_g} + \frac{i^2}{\eta J_r}), A_{62} = -\frac{p}{L_q}(L_d i_d - \phi_m).$$

The initial conditions of the states are taken as $\mathbf{x}(0) = [20, 120, 3, 2, 5, 3]^T$. PMSG based WECS parameters are given in Table I [17]. Fig. 4 illustrates a staircase wind profile which is used as the reference wind speed in this work. For a given wind speed V, the reference rotor speed for tracking performance is found as [6]

$$\omega_{r,ref} = \frac{\lambda_{opt}V}{R}. \quad (35)$$

TABLE I
PMSG-BASED WECS PARAMETERS

Notions	Descriptions	Values
ρ	Air density	$1.25\ kg/m^3$
R	Wind rotor plane radius	$2.5\ m$
i	Gearbox ratio	6
η	Gearbox efficiency	1
J_r	Wind rotor inertia	$2.88\ kg.m^2$
J_g	Generator inertia	$0.22\ kg.m^2$
K_g	High-speed shaft stiffness coefficient	$75\ Nm/rad$
B_g	High-speed shaft damping coefficient	$0.3\ Kg.m^2/s$
p	Number of pole pairs	3
R_s	PMSG stator resistance	$3.3\ \Omega$
ϕ_m	PMSG flux linkage	$0.4382\ Wb$
L_d	PMSG stator $d-axis$ inductance	$41.56\ mH$
Lq	PMSG stator $q-axis$ inductance	$41.56\ mH$

SDRE tracking controller for PMSG based WECS system is obtained using MATLAB and SIMULINK. Simulations are performed by considering the final time (t_f) as 20 seconds and incremental time step as $0.001s$. The weighting matrices are determined based on trial and error from simulation results and chosen as, $Q = diag$(10,1000,100,1,10,1000), $R = diag$(0.1,5,0.01), and F =0.25 $diag$(1,1,1,1,1,1).

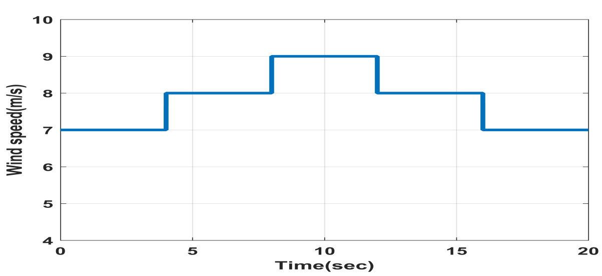

Fig. 4. Wind speed profile

The simulation results are presented in Fig. 5 - Fig. 10. In this paper, according to Fig. 3, in order to maintain the maximum power coefficient, $C_P = 0.47$, the blade pitch angle should be fixed at optimal value, $\beta_{opt} = 0°$ and the turbine should continuously operate at optimal tip speed ratio, $\lambda_{opt} = 8.1$. For this, the turbine rotor speed must be controlled in such a way that it tracks the optimal reference speed given by (35). The tracking performance via finite horizon SDRE controller is shown in Fig. 5, and control inputs for this performance are shown in Fig. 6. The solid blue line in Fig. 5 presents the actual wind turbine rotor speed and the dashed red line shows the optimal reference speed. Comparing the trajectories, it is clear that the actual wind rotor speed excellently tracks the commanded rotor speed by maintaining a negligible tracking error which is illustrated in Fig. 7. From Fig. 8, it is found that the power coefficient remains at its maximum value 0.47

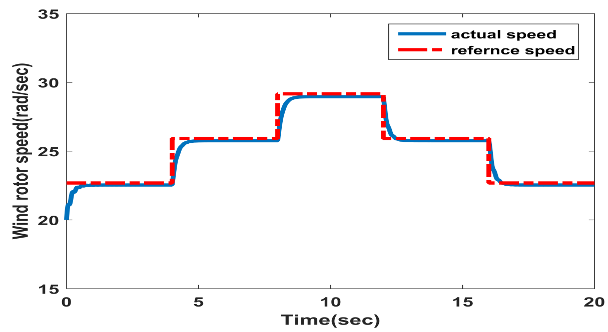

Fig. 5. Wind turbine rotor speed tracking via finite horizon SDRE

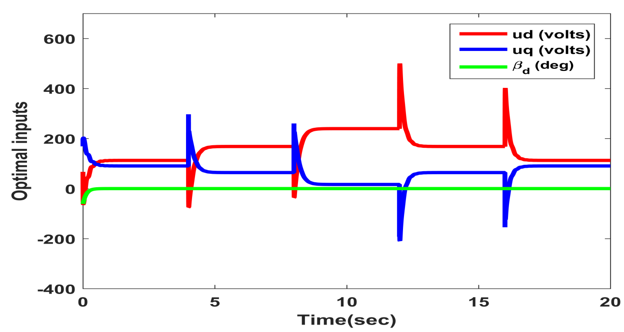

Fig. 6. Nonlinear optimal controller via finite horizon SDRE

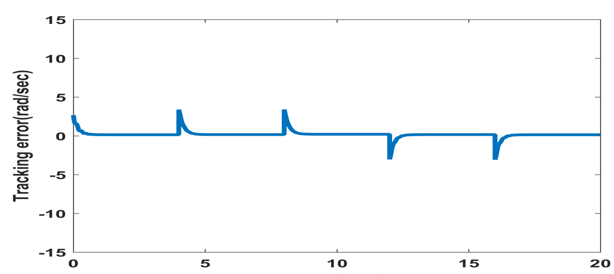

Fig. 7. Wind turbine rotor speed tracking error

throughout the partial load region. Also, the pitch angle and the tip speed ratio stay at their optimal values, which is shown in Fig. 9 and Fig. 10, respectively.

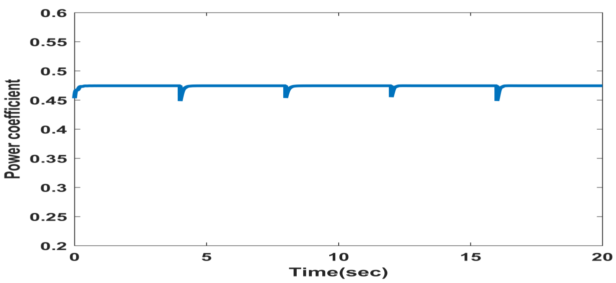

Fig. 8. Power coefficient C_P throughout the partial load region

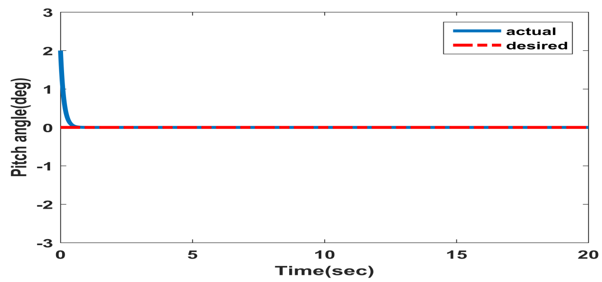

Fig. 9. Pitch angle β throughout the partial load region

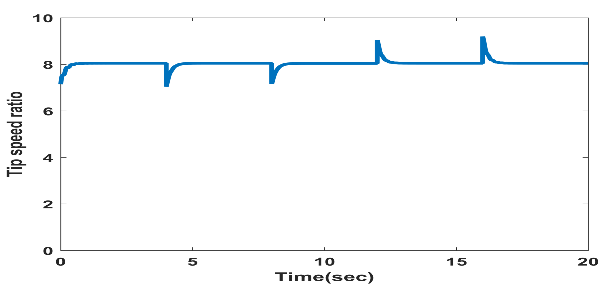

Fig. 10. Tip speed ratio λ throughout the partial load region

V. Conclusion

In this paper, a finite horizon optimal tracking controller via state dependent Riccati equation technique is presented for a variable speed wind energy conversion system with permanent magnet synchronous generator at partial load region. The purpose of the control strategy is to track the reference rotor speed and maintain the maximum power coefficient throughout the Region 2. In this strategy, using change of variable technique, the nonlinear state dependent differential Riccati equation (SD-DRE) is converted into the linear differential Lyapunov equation which is then solved forward in time. A state dependent vector differential equation (SD-VDE) is solved at the same time with the SD-DRE at each time step using an approximate analytic approach. Optimal control for tracking reference rotor speed is then found in terms of the solution of SD-DRE and SD-VDE. Simulation results validate the effectiveness of the finite horizon SDRE tracking controller for PMSG based WECS in the partial load region.

References

[1] J. F. Manwell, J. G. McGowan, and A. L. Rogers, *Wind energy explained: theory, design and application*. John Wiley & Sons, 2010.

[2] Z. Ma, Z. Yan, M. L. Shaltout, and D. Chen, "Optimal real-time control of wind turbine during partial load operation," *IEEE Transactions on Control Systems Technology*, vol. 23, no. 6, pp. 2216–2226, 2015.

[3] S. Li, T. A. Haskew, R. P. Swatloski, and W. Gathings, "Optimal and direct-current vector control of direct-driven PMSG wind turbines," *IEEE Transactions on power electronics*, vol. 27, no. 5, pp. 2325–2337, 2012.

[4] S. Li, T. A. Haskew, and L. Xu, "Conventional and novel control designs for direct driven PMSG wind turbines," *Electric Power Systems Research*, vol. 80, no. 3, pp. 328–338, 2010.

[5] F. Delfino, F. Pampararo, R. Procopio, and M. Rossi, "A feedback linearization control scheme for the integration of wind energy conversion systems into distribution grids," *IEEE systems journal*, vol. 6, no. 1, pp. 85–93, 2012.

[6] S.-W. Lee and K.-H. Chun, "Adaptive sliding mode control for PMSG wind turbine systems," *Energies*, vol. 12, no. 4, p. 595, 2019.

[7] M. Jingfeng, W. Aihua, W. Guoqing, and Z. Xudong, "Maximum power point tracking in variable speed wind turbine system via optimal torque sliding mode control strategy," in *2015 34th Chinese Control Conference (CCC)*. IEEE, 2015, pp. 7967–7971.

[8] T. L. Van, T. H. Nguyen, and D.-C. Lee, "Advanced pitch angle control based on fuzzy logic for variable-speed wind turbine systems," *IEEE Transactions on Energy Conversion*, vol. 30, no. 2, pp. 578–587, 2015.

[9] E. Koutroulis and K. Kalaitzakis, "Design of a maximum power tracking system for wind-energy-conversion applications," *IEEE transactions on industrial electronics*, vol. 53, no. 2, pp. 486–494, 2006.

[10] W. Quincy and C. Liuchen, "An intelligent maximum power extraction algorithm for inverter-based variable speed wind turbine systems," *IEEE Transactions on Power Electronics*, vol. 19, no. 5, pp. 1242–1249, 2004.

[11] J. R. Cloutier, "State-dependent riccati equation techniques: an overview," in *Proceedings of the 1997 American Control Conference (Cat. No. 97CH36041)*, vol. 2. IEEE, 1997, pp. 932–936.

[12] T. Cimen, "Survey of state-dependent riccati equation in nonlinear optimal feedback control synthesis," *Journal of Guidance, Control, and Dynamics*, vol. 35, no. 4, pp. 1025–1047, 2012.

[13] A. Heydari and S. Balakrishnan, "Path planning using a novel finite horizon suboptimal controller," *Journal of guidance, control, and dynamics*, vol. 36, no. 4, pp. 1210–1214, 2013.

[14] D. S. Naidu, S. Paul, and C. R. Rieger, "A simplified SDRE technique for regulation in optimal control systems," in *Proceedings of the 2019 IEEE International Conference on Electro Information Technology (EIT)*. IEEE, 20-22 May, 2019, South Dakota, USA.

[15] A. Heydari and S. N. Balakrishnan, "Approximate closed-form solutions to finite-horizon optimal control of nonlinear systems," in *2012 American Control Conference (ACC)*. IEEE, 2012, pp. 2657–2662.

[16] H. M. Nguyen and D. S. Naidu, "Time scale analysis and control of wind energy conversion systems," in *2012 5th International Symposium on Resilient Control Systems*. IEEE, 2012, pp. 149–154.

[17] H. M. Nguyen, *"Advanced control strategies for wind energy conversion systems"*. Ph.D. dissertation, Idaho State University, 2013.

[18] J. Liu, Y. Gao, S. Geng, and L. Wu, "Nonlinear control of variable speed wind turbines via fuzzy techniques," *IEEE Access*, vol. 5, pp. 27–34, 2017.

[19] F. D. Bianchi, H. De Battista, and R. J. Mantz, *Wind turbine control systems: principles, modelling and gain scheduling design*. Springer Science & Business Media, 2006.

[20] D. S. Naidu, *Optimal control systems*. CRC press, 2002.

[21] A. Khamis, *"Advanced tracking strategies for linear and nonlinear control systems: Theory and applications"*. Ph.D. dissertation, Idaho State University, Pocatello, Idaho, USA, 2014.

[22] T. Nguyen and Z. Gajic, "Solving the matrix differential riccati equation: a lyapunov equation approach," *IEEE Transactions on Automatic Control*, vol. 55, no. 1, pp. 191–194, 2010.

[23] Z. Gajic and M. T. J. Qureshi, *Lyapunov matrix equation in system stability and control*. Courier Corporation, 2008.

[24] I. B. Khallouf and D. S. Naidu, "Advanced control strategies for the robotic hand," in *2018 IEEE 14th International Conference on Control and Automation (ICCA)*. IEEE, 2018, pp. 698–703.