Indian Ocean Yellowfin fishblicc Analysis

Paul Medley

2023-08-01

Table of contents

1	Introduction	1
2	Data	2
	2.1 Length Frequency Data	2
	2.2 Catch Numbers	3
	2.3 Priors	5
3	Catch Curve Model	6
	3.1 Single Selectivity Model	7
	3.2 Shared-selectivity Mixture Model	8
	3.3 Estimated L_{∞}	12
	3.4 Length-Inverse Natural Mortality	14
	3.5 Dome-shaped Longline Selectivity	16
	3.6 Summary	18
4	MCMC Fit	19
5	Final Results	19
6	Conclusions	22
7	References	25

1 Introduction

This is an example analysis of Indian Ocean yellowfin tuna. This stock undergoes regular Stock Synthesis stock assessments which make use of all available data and undergoes rigorous review through the IOTC science committee. Part of the available data reported by countries

that are members of IOTC include length frequency and catch data from a variety of gears. These data are suitable for a multi-selectivity catch curve. This is a useful case study to see how the method compares with the full assessment and consider the problems that arise when dealing with limited data. There is no intention here to provide a competing stock assessment since important informative data that are available for analysis are actively excluded here.

Catch curves are a simple idea - they use a snapshot of the age structure to infer the stock status and fishing mortality. This is made more complicated when dealing with lengths as a growth model is required to link length to age. But in principle the length-based catch curve applies the same idea. Is the proportion of larger mature fish in the landings consistent with the stock being sustainably harvested or not?

The main problem with length-based catch curve models has been that the model flexibility is limited. Specifically, they cannot deal with potentially alternative selectivity functions that might be considered plausible or alternative mortality models. This analysis below illustrates the fishblicc software which applies a more flexible model that estimates a fixed mortality within each length interval, so it is able to account for different selectivity curves and the resulting different mortality schedules that will change length composition in the underlying population.

The Indian Ocean yellowfin tuna fishery is probably more complex than most fisheries in terms of the gears used, but it is not unusual for a fishery and particularly a small scale fishery, to have several gears operating. Having each gear separately modelled can produce much better results than trying to combine the data into single frequency samples or simply ignoring gears with lower catches. It will also allow gear-specific management advice when limiting fishing mortality and trying to improve overall fishing selectivity.

2 Data

2.1 Length Frequency Data

The data used are public length frequency data from [www.iotc.org] and is the same as that used in the 2019 SS3 stock assessment. The data is combined over 6 years (2013-2019).

The recognised gears reflect the available length frequency data:

- BB: Bait boats (pole and line)
- GI: gill net
- HD: handline
- LL: longline
- OT: "other" but probably mostly troll fisheries

- PS-FS: Purse seine free school
- PS-LS: Purse seine log school, but most sets on fish aggregation devices (FADs)

These gears cover the vast majority of the known catch of yellowfin in the Indian Ocean (Figure 1).

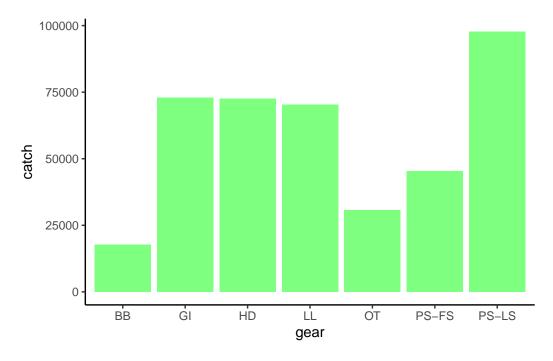


Figure 1: Total catch weight for 2013-2019 by gear type.

2.2 Catch Numbers

For multiple gears, the relative catch in numbers for each gear is required to estimate the relative fishing mortality each gear applies. For data limited cases, these relative catch data are potentially difficult to estimate. In many cases, the relative sample size of length frequency data taken for each gear may be the best indicator of the relative catches. If estimates of catches are available, these should be used.

For IO yellowfin tuna, direct estimates of catch weight are available, and these have been converted to numbers using the mean weights from the length frequency samples (Figure 3). The length-weight conversion is estimated slightly differently between longline and other gears. This alternative length-weight relationship was applied to longline and handline, which catch similar size fish.

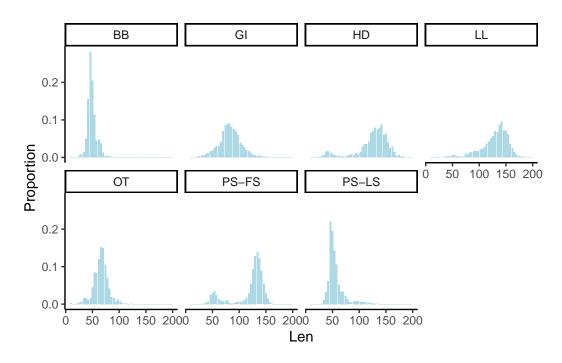


Figure 2: Length frequency data as proportions for each gear.

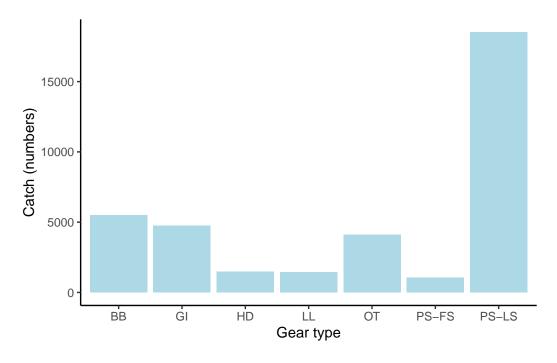


Figure 3: Estimated catch numbers 2013-2019 for each gear.

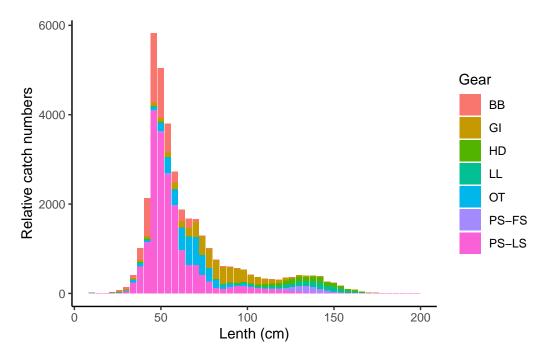


Figure 4: Length frequency in relative numbers of fish for each gear based on the estimated total catch numbers.

It would be possible to fit a catch curve to the combined length frequencies as long as they are weighted by the relative catch numbers (Figure 4). While it would be possible to fit a single catch curve to these combined frequency data, it would be necessary to have a multi-modal function to model the selectivity. Perhaps more importantly, combining the data in this way would limit the advice that can be provided on a gear-by-gear basis. It would make it difficult, for example, to predict the effect of limiting the number of FADs or gillnets.

2.3 Priors

In data poor situations, information available on the stock biology is usually also limited and so information from other stocks and fisheries, along with local expert opinion, is required to compile the likely life history parameters that are necessary to interpret length frequency data. In general, length frequency data will not contain information to support estimates of natural mortality separate from fishing mortality, or information on growth. Therefore, in a Bayesian context, informative priors on key life history parameters will be required as input to the model.

The information source for life history parameters used here is Fishbase (Froese and Pauly 2023). The life history of yellowfin tuna as a species (*Thunnus albacares*) is quite well researched, so there is reasonable confidence in values used for length-at-maturity, L_{∞} and

length-weight parameters (Table 1). The prior on M_K can also be proposed with reasonable confidence because of the values used in the main stock assessment. The default growth variability prior (10% CV) is also used, but should not make a significant difference to the results.

It should be noted that the official stock assessments do not use the von Bertalanffy growth model, but uses a variant, "Richard's model" (Maunder et al 2018), which allows two-stage growth that has been estimated to more closely match the true growth form. This may be a cause of some differences between the full stock assessment and this 'length data only' approach.

Table 1: Prior and fixed parameters values for non-gear related functions. Linf and Galpha are used in the growth model, Mk is the natural mortality and NB_phi is the scale parameter for the observation error. The fixed parameters are the length-weight exponent (b), and the logistic length-based maturity curve (L50, Ls).

Parameter	Function Type	Mu	SD
Linf	Normal	181.045	5.000
Galpha	Lognormal	4.605	0.250
Mk	Lognormal	0.539	0.100
NB_phi	Lognormal	4.605	0.500
b		2.967	
L50		103.000	
Ls		0.173	

3 Catch Curve Model

The model structure is flexible, so the first task is to explore possible structures that adequately explain the observations and cover possible underlying model assumptions. For the latter, the main aim is to determine whether the final estimate of SPR is sensitive to those assumptions. If it is not, then we do not need to worry too much about them and those alternatives can be excluded from the final fit.

Search for a model configuration that can reasonably explain the observations.

Use sensitivity model configurations to look for plausible alternative explanations for the observations and, importantly, identify which of these the final results (SPR) is most sensitive to.

The model applied here is for illustration only and the configurations are not exhaustively explored. For example, we would probably want to consider alternative data subsets such as by year or season, which would allow considering other possible effects such as recruitment or seasonal changes to selectivity.

The alternative model configurations are fitted using the Stan optimiser. This estimates the maximum posterior density. While this does not provide as much information on how well the model fits the data as the MCMC fit, it is considerably faster.

3.1 Single Selectivity Model

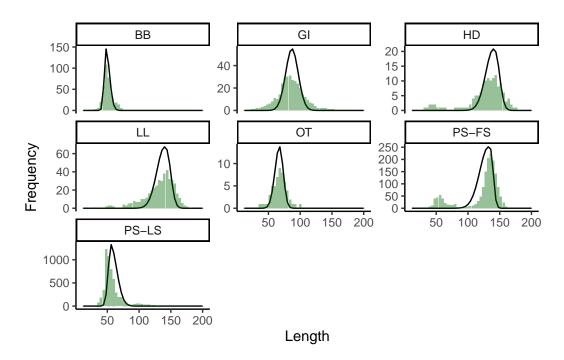


Figure 5: Estimated length frequency data using prior parameters.

The model does not fit the data well (Figure 6). With a single mode selectivity function, the selectivity model stretches between the modes to explain the higher or lower values. This increases the model's relative fishing mortality on these lengths between the modes which does not represent the data well. One of the problems with selectivity functions based on the normal distribution is that selectivity function rapidly declines, so outliers can be highly influential in the fit. While more dispersed functions such as the gamma or lognormal may be more robust to this effect, they do not usually improve the fit much and can not be well justified on any theoretical grounds.

A more flexible selectivity function is required to explain multi-mode length frequencies. An obvious way to attempt this is to combine selectivity into mixtures of underlying latent func-

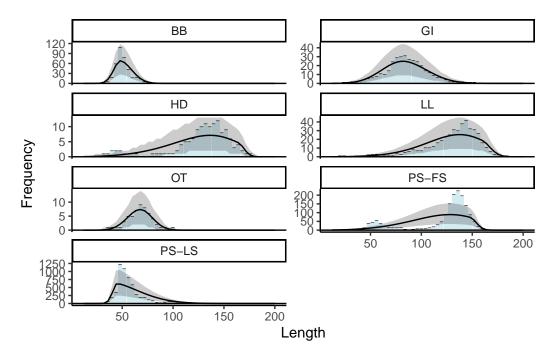


Figure 6: Expected length frequency data after the first fit.

tions. This is proposed below as an explanation for the observations from the different gears' length frequency.

3.2 Shared-selectivity Mixture Model

For the shared selectivity model, the hypothesis is that the gear selectivities are made up of a mixture of normal-like selectivity functions. For example, both pole and line and purse seine take juvenile yellowfin associated with skipjack. The availability and capture of these juveniles might follow the same length-based selectivity function. A normal or double-sided normal selectivity function might be reasonably proposed based on the central limit theorem - fish are caught due to multiple factors linked to length that, when combined, peak at a particular length and decline on either side.

- 1. Simultaneous estimation of shared selectivity functions
- 2. Correction for mortality

The functions below were developed initially through hypothetical shared selectivity and subsequently to explain outliers that otherwise might be influential in the fit. Based on the observations (Figure 6), pole and line (BB) and purse seine FAD sets (PS-LS) share a peak around 50cm. We might assume both these gears encounter juveniles that are mixing with

skipjack and therefore equally vulnerable to these gears. Handline (HD) and purse seine free-school (PS-FS) catches also seem to have minor peaks around 50cm and so likewise may have a similar selectivity shared component which can be included. This still leaves purse seine FAD sets with a minor but significant mode around 100cm which is not shared by any other gear. Another selectivity component is added around this point to explain these observations.

Using selectivity mixtures in this way does not discriminate between genuine selectivity mixture functions (fish are caught in different ways by the same gear) and mis-recording the gear. For example, in the case of differences between purse seine free school and FAD sets, it is not always clear what a set is if it is conducted near a FAD or (natural) floating log. In case the mixture may be partly an artefact of mis-recording or genuinely mixed set-types.

The actual selectivity functions should be as parsimonious as possible. Three functions (logistic, normal and single-sided normal¹) take 2 parameters only, where as the double-sided normal takes three parameters. Discovering which functions are sufficient to explain the observations is a matter of trial and error.

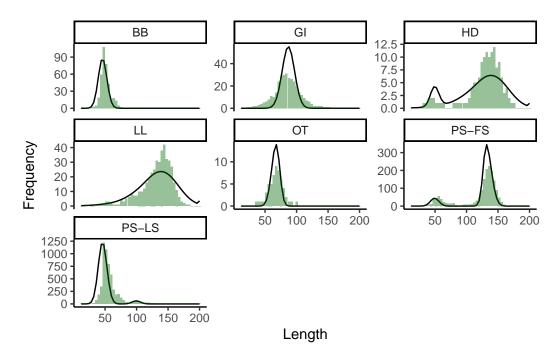


Figure 7: Prior check: Estimated length frequency data using prior parameters for the mixture model.

This model where a number of selectivity functions are included to fit the observations, and in particular the shared patterns between gears. This allows the model to explain the length

¹single-sided normal resembles the logistic but has a normal shape to the slope of the left of the selectivity. Its primary use is to test whether a selectivity might be dome-shaped as it is directly comparable with a double-sided normal: i.e. identical if the right-side slope parameter is fixed at zero.

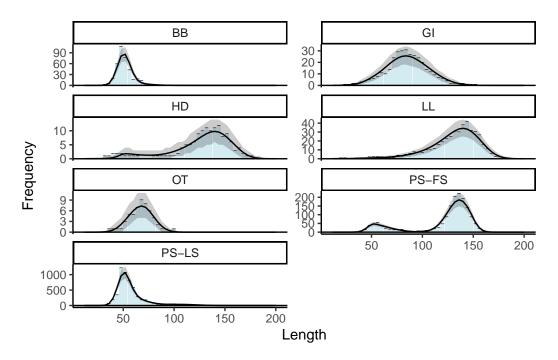


Figure 8: Estimated length frequency from the mixture model plotted against the data.

frequencies without overfitting. As this provides a basic reasonable fit to the data, it retained is retained as the base model, which can be compared to other model configurations.

Table 2: Results for the new base mixture model.

Parameter	Mean	SD	10%	50%	90%
Linf	178.993		178.993	178.993	178.993
Galpha	133.524		133.524	133.524	133.524
Mk	1.611		1.611	1.611	1.611
Fk[1]	0.651		0.651	0.651	0.651
Fk[2]	0.427		0.427	0.427	0.427
Fk[3]	3.062		3.062	3.062	3.062
Fk[4]	3.212		3.212	3.212	3.212
Fk[5]	0.474		0.474	0.474	0.474
Fk[6]	0.559		0.559	0.559	0.559

Parameter	Mean	SD	10%	50%	90%
Fk[7]	1.390		1.390	1.390	1.390
Sm[1]	51.453		51.453	51.453	51.453
Sm[2]	0.012		0.012	0.012	0.012
Sm[3]	53.737		53.737	53.737	53.737
Sm[4]	0.012		0.012	0.012	0.012
Sm[5]	0.001		0.001	0.001	0.001
Sm[6]	111.963		111.963	111.963	111.963
Sm[7]	0.002		0.002	0.002	0.002
Sm[8]	90.256		90.256	90.256	90.256
Sm[9]	0.001		0.001	0.001	0.001
Sm[10]	0.001		0.001	0.001	0.001
Sm[11]	165.844		165.844	165.844	165.844
Sm[12]	0.065		0.065	0.065	0.065
Sm[13]	70.814		70.814	70.814	70.814
Sm[14]	0.003		0.003	0.003	0.003
Sm[15]	0.004		0.004	0.004	0.004
Sm[16]	144.909		144.909	144.909	144.909
Sm[17]	0.003		0.003	0.003	0.003
Sm[18]	0.007		0.007	0.007	0.007
Sm[19]	0.137		0.137	0.137	0.137
Sm[20]	0.003		0.003	0.003	0.003
Sm[21]	0.034		0.034	0.034	0.034
Sm[22]	0.522		0.522	0.522	0.522
Sm[23]	0.143		0.143	0.143	0.143
NB_phi	48.715		48.715	48.715	48.715
Gbeta	0.746		0.746	0.746	0.746
SPR	0.205		0.205	0.205	0.205

Parameter	Mean	SD	10%	50%	90%
lp	-550.414		-550.414	-550.414	-550.414

It is a good idea to check the standardised residuals. A problem with using the normal parametric selectivity function is outliers in the distributions tail can be very influential. These points can be potentially identified by large positive residuals. The residuals can be removed simply by adding additional selectivity functions at the cost of more parameters. Adding these selectivity functions will make little difference to the final mortality estimates because the weight on the function will be very low, so whether they are required or not can currently only be determined by trail and error.

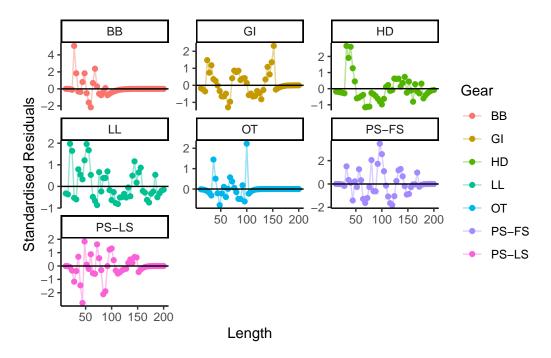


Figure 9: Standard residual plot by length bin separated for each gear.

3.3 Estimated L_{∞}

The prior on the asymptotic L_{∞} is informative. However, the parameter is estimated and it is possible that there is some support for it in the data. The model can be re-fitted while allowing greater freedom to the asymptotic length estimate by increasing the standard deviation for the normal prior.

Table 3: Results for the model allowing Linf to be fitted.

Parameter	Mean	SD	10%	50%	90%
Linf	160.315		160.315	160.315	160.315
Galpha	170.648		170.648	170.648	170.648
Mk	1.503		1.503	1.503	1.503
Fk[1]	0.231		0.231	0.231	0.231
Fk[2]	0.107		0.107	0.107	0.107
Fk[3]	0.370		0.370	0.370	0.370
Fk[4]	0.387		0.387	0.387	0.387
Fk[5]	0.138		0.138	0.138	0.138
Fk[6]	0.096		0.096	0.096	0.096
Fk[7]	0.497		0.497	0.497	0.497
Sm[1]	50.830		50.830	50.830	50.830
Sm[2]	0.013		0.013	0.013	0.013
Sm[3]	52.830		52.830	52.830	52.830
Sm[4]	0.012		0.012	0.012	0.012
Sm[5]	0.002		0.002	0.002	0.002
Sm[6]	109.877		109.877	109.877	109.877
Sm[7]	0.002		0.002	0.002	0.002
Sm[8]	87.723		87.723	87.723	87.723
Sm[9]	0.001		0.001	0.001	0.001
Sm[10]	0.001		0.001	0.001	0.001
Sm[11]	161.052		161.052	161.052	161.052
Sm[12]	0.059		0.059	0.059	0.059
Sm[13]	69.679		69.679	69.679	69.679
Sm[14]	0.003		0.003	0.003	0.003
Sm[15]	0.004		0.004	0.004	0.004
Sm[16]	143.725		143.725	143.725	143.725

Parameter	Mean	SD	10%	50%	90%
Sm[17]	0.003		0.003	0.003	0.003
Sm[18]	0.006		0.006	0.006	0.006
Sm[19]	0.138		0.138	0.138	0.138
Sm[20]	0.009		0.009	0.009	0.009
Sm[21]	0.064		0.064	0.064	0.064
Sm[22]	0.487		0.487	0.487	0.487
Sm[23]	0.092		0.092	0.092	0.092
NB_phi	47.827		47.827	47.827	47.827
Gbeta	1.064		1.064	1.064	1.064
SPR	0.614		0.614	0.614	0.614
lp	-549.203		-549.203	-549.203	-549.203

The model fits the data better, but the estimated L_{∞} is too low to be realistic. The SPR estimate is highly sensitive to this effect. The absence of larger fish in the length frequencies is either because higher mortality prevents them being in the population or they do not grow that big. The data does not support either hypothesis, so the prior on L_{∞} must be used choose between them. In this case, quite a lot is known about yellowfin, including how big they can get so the prior can be informative. If this was not known, both hypotheses could be combined as alternatives, or the more informative prior might be chosen as its results are less optimistic and would likely require more risk-averse management action.

3.4 Length-Inverse Natural Mortality

Lorenzen (2022) suggests that the default model for natural mortality in fish should be related to the inverse weight or length of the fish. This is the default used for most tuna stock assessments (Maunder et al. 2023) and is easily incorporated into this model by setting a reference length. This is the simple length-inverse model where natural mortality is inversely proportional to length:

$$M_L = \frac{M_1}{L}$$

Setting the reference length will set the natural mortality equal to the current prior natural mortality only at this length. The natural mortality will increase below this length and decrease

above it according to the reciprocal relationship. In this formulation, no additional parameters are required to be fitted.

Table 4: Results for the model with length-inverse M to be fitted.

Parameter	Mean	SD	10%	50%	90%
Linf	178.797		178.797	178.797	178.797
Galpha	124.759		124.759	124.759	124.759
Mk	1.676		1.676	1.676	1.676
Fk[1]	0.380		0.380	0.380	0.380
Fk[2]	0.397		0.397	0.397	0.397
Fk[3]	3.321		3.321	3.321	3.321
Fk[4]	3.484		3.484	3.484	3.484
Fk[5]	0.352		0.352	0.352	0.352
Fk[6]	0.690		0.690	0.690	0.690
Fk[7]	0.796		0.796	0.796	0.796
Sm[1]	51.920		51.920	51.920	51.920
Sm[2]	0.013		0.013	0.013	0.013
Sm[3]	54.734		54.734	54.734	54.734
Sm[4]	0.012		0.012	0.012	0.012
Sm[5]	0.001		0.001	0.001	0.001
Sm[6]	113.984		113.984	113.984	113.984
Sm[7]	0.002		0.002	0.002	0.002
Sm[8]	93.223		93.223	93.223	93.223
Sm[9]	0.001		0.001	0.001	0.001
Sm[10]	0.001		0.001	0.001	0.001
Sm[11]	160.421		160.421	160.421	160.421
Sm[12]	0.074		0.074	0.074	0.074
Sm[13]	72.189		72.189	72.189	72.189
Sm[14]	0.003		0.003	0.003	0.003

Parameter	Mean	SD	10%	50%	90%
Sm[15]	0.004		0.004	0.004	0.004
Sm[16]	145.089		145.089	145.089	145.089
Sm[17]	0.003		0.003	0.003	0.003
Sm[18]	0.006		0.006	0.006	0.006
Sm[19]	0.156		0.156	0.156	0.156
Sm[20]	0.002		0.002	0.002	0.002
Sm[21]	0.017		0.017	0.017	0.017
Sm[22]	0.602		0.602	0.602	0.602
Sm[23]	0.262		0.262	0.262	0.262
NB_phi	51.522		51.522	51.522	51.522
Gbeta	0.698		0.698	0.698	0.698
SPR	0.227		0.227	0.227	0.227
lp	-547.321		-547.321	-547.321	-547.321

The model fit slightly improves, but the SPR estimate is not sensitive to this alternative model of natural mortality. However, this model is retained because it fits a little better, it is the standard in tuna stock assessments, and is being used increasingly widely as a more accurate representation of the natural mortality process.

3.5 Dome-shaped Longline Selectivity

The current model assumes logistic selectivity function for longline and handline which catch the largest fish. This is the standard selectivity for most length-based methods. An alternative option is that the selectivity is dome-shaped (double-sided normal) which can be tested.

Table 5: Results for the model with length-inverse M to be fitted.

Parameter	Mean	SD	10%	50%	90%
Linf	178.823		178.823	178.823	178.823
Galpha	123.947		123.947	123.947	123.947
Mk	1.676		1.676	1.676	1.676

Fk[1] 0.381 0.381 0.381 0.381 Fk[2] 0.399 0.399 0.399 0.399 0.399 Fk[3] 3.341 3.341 3.341 3.341 3.341 3.341 Fk[4] 3.505 3.505 3.505 3.505 3.505 Fk[5] 0.353 0.353 0.353 0.353 0.353 Fk[6] 0.692 0.692 0.692 0.692 0.692 Fk[7] 0.798 0.798 0.798 0.798 Sm[1] 51.921 51.921 51.921 51.921 Sm[2] 0.013 0.013 0.013 0.013 0.013 Sm[3] 54.734 54.73						
Fk[2] 0.399 0.399 0.399 0.395 Fk[3] 3.341 3.352 6.62 0.692 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.798 0.79	Parameter	Mean	SD	10%	50%	90%
Fk[3] 3.341 3.352 3.505 <th< td=""><td>Fk[1]</td><td>0.381</td><td></td><td>0.381</td><td>0.381</td><td>0.381</td></th<>	Fk[1]	0.381		0.381	0.381	0.381
Fk[4] 3.505 3.505 3.505 3.505 Fk[5] 0.353 0.353 0.353 0.353 0.353 Fk[6] 0.692 0.692 0.692 0.692 0.692 Fk[7] 0.798 0.798 0.798 0.798 Sm[1] 51.921 51.921 51.921 51.921 Sm[2] 0.013 0.013 0.013 0.013 Sm[3] 54.734	Fk[2]	0.399		0.399	0.399	0.399
Fk[5] 0.353 0.353 0.353 0.353 Fk[6] 0.692 0.692 0.692 0.692 Fk[7] 0.798 0.798 0.798 0.798 Sm[1] 51.921 51.921 51.921 51.921 Sm[2] 0.013 0.013 0.013 0.013 Sm[3] 54.734 54.734 54.734 54.734 Sm[4] 0.012 0.012 0.012 0.012 Sm[5] 0.001 0.001 0.001 0.001 Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074	Fk[3]	3.341		3.341	3.341	3.341
Fk[6] 0.692 0.692 0.692 0.692 Fk[7] 0.798 0.798 0.798 0.798 Sm[1] 51.921 51.921 51.921 51.921 Sm[2] 0.013 0.013 0.013 0.013 Sm[3] 54.734 54.734 54.734 54.734 Sm[4] 0.012 0.012 0.012 0.012 Sm[5] 0.001 0.001 0.001 0.001 Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 <td>Fk[4]</td> <td>3.505</td> <td></td> <td>3.505</td> <td>3.505</td> <td>3.505</td>	Fk[4]	3.505		3.505	3.505	3.505
Fk[7] 0.798 0.798 0.798 0.798 Sm[1] 51.921 51.921 51.921 51.921 Sm[2] 0.013 0.013 0.013 0.013 Sm[3] 54.734 54.734 54.734 54.734 Sm[4] 0.012 0.012 0.012 0.012 Sm[5] 0.001 0.001 0.001 0.001 Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 <td>Fk[5]</td> <td>0.353</td> <td></td> <td>0.353</td> <td>0.353</td> <td>0.353</td>	Fk[5]	0.353		0.353	0.353	0.353
Sm[1] 51.921 51.921 51.921 51.921 51.921 Sm[2] 0.013 0.013 0.013 0.013 Sm[3] 54.734 54.734 54.734 54.734 Sm[4] 0.012 0.012 0.012 0.012 Sm[5] 0.001 0.001 0.001 0.001 Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004	Fk[6]	0.692		0.692	0.692	0.692
Sm[2] 0.013 0.013 0.013 0.013 Sm[3] 54.734 54.734 54.734 54.734 Sm[4] 0.012 0.012 0.012 0.012 Sm[5] 0.001 0.001 0.001 0.001 Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097	Fk[7]	0.798		0.798	0.798	0.798
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Sm[1]	51.921		51.921	51.921	51.921
Sm[4] 0.012 0.012 0.012 0.012 Sm[5] 0.001 0.001 0.001 0.001 Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156 <td>Sm[2]</td> <td>0.013</td> <td></td> <td>0.013</td> <td>0.013</td> <td>0.013</td>	Sm[2]	0.013		0.013	0.013	0.013
Sm[5] 0.001 0.001 0.001 0.001 Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 160.457 160.457 160.457 160.457 172.192 <	Sm[3]	54.734		54.734	54.734	54.734
Sm[6] 113.988 113.988 113.988 113.988 Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[4]	0.012		0.012	0.012	0.012
Sm[7] 0.002 0.002 0.002 0.002 Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[5]	0.001		0.001	0.001	0.001
Sm[8] 93.223 93.223 93.223 93.223 Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[6]	113.988		113.988	113.988	113.988
Sm[9] 0.001 0.001 0.001 0.001 Sm[10] 0.001 0.001 0.001 0.001 Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[7]	0.002		0.002	0.002	0.002
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	Sm[8]	93.223		93.223	93.223	93.223
Sm[11] 160.457 160.457 160.457 160.457 Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[9]	0.001		0.001	0.001	0.001
Sm[12] 0.074 0.074 0.074 0.074 Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[10]	0.001		0.001	0.001	0.001
Sm[13] 72.192 72.192 72.192 72.192 Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[11]	160.457		160.457	160.457	160.457
Sm[14] 0.003 0.003 0.003 0.003 Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[12]	0.074		0.074	0.074	0.074
Sm[15] 0.004 0.004 0.004 0.004 Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[13]	72.192		72.192	72.192	72.192
Sm[16] 145.097 145.097 145.097 145.097 Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[14]	0.003		0.003	0.003	0.003
Sm[17] 0.003 0.003 0.003 0.003 Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[15]	0.004		0.004	0.004	0.004
Sm[18] 0.006 0.006 0.006 0.006 Sm[19] 0.156 0.156 0.156 0.156	Sm[16]	145.097		145.097	145.097	145.097
Sm[19] 0.156 0.156 0.156 0.156	Sm[17]	0.003		0.003	0.003	0.003
	Sm[18]	0.006		0.006	0.006	0.006
Sm[20] 0.002 0.002 0.002 0.002	Sm[19]	0.156		0.156	0.156	0.156
	Sm[20]	0.002		0.002	0.002	0.002

D	М	CD	1007	F007	0007
Parameter	Mean	SD	10%	50%	90%
Sm[21]	0.017		0.017	0.017	0.017
Sm[22]	0.602		0.602	0.602	0.602
Sm[23]	0.262		0.262	0.262	0.262
NB_phi	51.542		51.542	51.542	51.542
Gbeta	0.693		0.693	0.693	0.693
SPR	0.226		0.226	0.226	0.226
lp	-547.303		-547.303	-547.303	-547.303

The simple reason is that the longline selectivity mode is very close to the maximum size, so the downward slope of the selectivity function is only relevant to a very small proportion of fish and has little influence on the size composition of the catch. As a result, the additional parameter does not explain the observations much better and is redundant in this case. Therefore this model is rejected.

3.6 Summary

In conclusion the final retained base model is the selectivity mixture model, with an informative L_{∞} prior and length-inverse M. Applying a separate single mode selectivity function to each gear resulted in a poor fit with a very low log-probability at the mpd mode (Table 6).

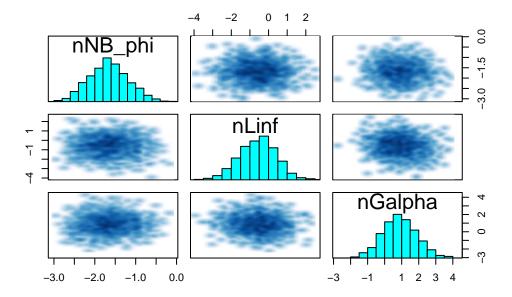
Table 6: Summary of the alternative tested sensititivites, with the final base model fitted using MCMC. 1p are the log-probabilities at the maximum posterior density points, which are almost directly comparible in terms of 'goodness-of-fit'. The final retained model is the 'Length-inverse M'.

model_name	Linf	Galpha	Mk	SPR	lpNotes
All domed single selectivity	184.971	105.739	1.389	0.469	-707.230Rejected
Selectivity Mixture Model	178.993	133.524	1.611	0.205	-550.414Base model
Allow Linf estimation	160.315	170.648	1.503	0.614	-549.203Rejected
Length-inverse M	178.797	124.759	1.676	0.227	-547.321New base
Longline dome-shaped	178.823	123.947	1.676	0.226	-547.303Rejected

4 MCMC Fit

Note about divergences...

```
Warning in par(usr): argument 1 does not name a graphical parameter
Warning in par(usr): argument 1 does not name a graphical parameter
Warning in par(usr): argument 1 does not name a graphical parameter
```



5 Final Results

The MCMC gives estimates of uncertainty in the form of probability density functions for the values of interest. This includes the observations (length frequencies). Comparing the MCMC expected frequency with the observations implies a reasonable fit, with the majority

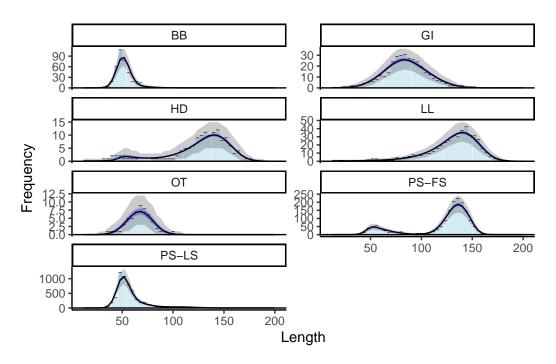


Figure 10: Estimated length frequency from the mixture model MCMC fit plotted against the data.

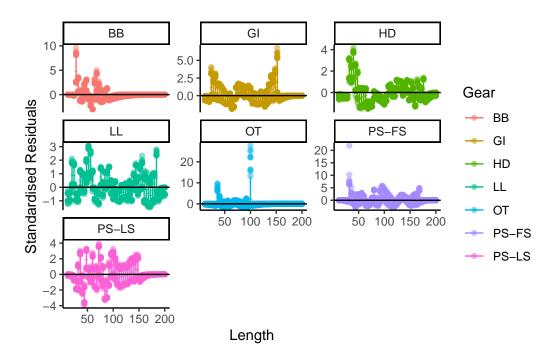


Figure 11: Residuals.

Table 7: MCMC estimates for the fitted parameters. See text for details.

Parameter	Mean	SD	10%	50%	90%
Linf	178.140	4.977	171.756	178.362	184.443
Galpha	128.763	33.206	91.842	124.424	172.202
Mk	1.685	0.161	1.492	1.674	1.894
Fk[1]	0.375	0.090	0.261	0.374	0.493
Fk[2]	0.386	0.082	0.280	0.385	0.491
Fk[3]	3.660	1.825	1.853	3.308	5.638
Fk[4]	3.840	1.928	1.955	3.478	5.922
Fk[5]	0.339	0.083	0.233	0.335	0.448
Fk[6]	0.680	0.149	0.490	0.679	0.866
Fk[7]	0.783	0.202	0.521	0.774	1.058
Sm[1]	52.107	0.807	51.055	52.139	53.099
Sm[2]	0.013	0.001	0.011	0.012	0.014
Sm[3]	54.502	1.576	52.539	54.448	56.521
Sm[4]	0.012	0.003	0.009	0.012	0.016
Sm[5]	0.001	0.000	0.001	0.001	0.002
Sm[6]	114.315	2.202	111.549	114.289	117.131
Sm[7]	0.002	0.000	0.002	0.002	0.003
Sm[8]	93.090	3.035	89.226	93.038	96.999
Sm[9]	0.001	0.000	0.001	0.001	0.002
Sm[10]	0.001	0.000	0.000	0.001	0.001
Sm[11]	160.932	6.144	153.493	160.528	168.778
Sm[12]	0.074	0.003	0.070	0.074	0.078
Sm[13]	72.177	3.400	67.713	72.353	76.386
Sm[14]	0.004	0.001	0.002	0.003	0.005
Sm[15]	0.004	0.002	0.002	0.004	0.007
Sm[16]	145.080	1.786	142.774	145.096	147.441

Parameter	Mean	SD	10%	50%	90%
Sm[17]	0.003	0.000	0.002	0.003	0.003
Sm[18]	0.006	0.002	0.004	0.006	0.009
Sm[19]	0.152	0.066	0.074	0.143	0.240
Sm[20]	0.002	0.001	0.001	0.002	0.003
Sm[21]	0.017	0.004	0.013	0.017	0.022
Sm[22]	0.607	0.178	0.414	0.576	0.833
Sm[23]	0.260	0.061	0.191	0.252	0.337
NB_phi	44.900	10.943	31.707	43.359	60.128
Gbeta	0.724	0.190	0.512	0.696	0.978
SPR	0.242	0.070	0.163	0.231	0.336
lp	-561.184	4.220	-566.787	-560.833	-556.203

The final spawning potential ratio (SPR) probability density suggests there is a high probability that the stock is overfished and it is highly likely the stock is below its target level (Figure 13).

6 Conclusions

This is a non-exhaustive illustrative assessment to fitting a non-standard length-based catch curve to length frequencies from multiple gears with complex inter-related selectivity. It is possible to construct selectivity mixtures that try to capture plausible hypotheses for the actual selectivities that gears may be applying. The model can be used to provide an estimate of stock status in these circumstances and identify important sensitivities to allow careful review. In data limited situations, the review process is important as almost invariably some subjective decisions have to be made where information is lacking.

Multiple modes in length frequency data from a single gear is not necessarily unusual and can be observed in many fisheries. They seem particularly common in small scale fisheries. They can be due to a number of factors not all of which would be well modelled in this way:

- specific latent selectivity functions, as assumed here, where fish are caught in different ways due to their behaviour or shape. If these are static effects, they can be estimated as selectivity functions.
- sampling anomalies where modes represent samples. For example, if a significant number of fish are sampled from a small set of trips or frequencies have been weighted by catches

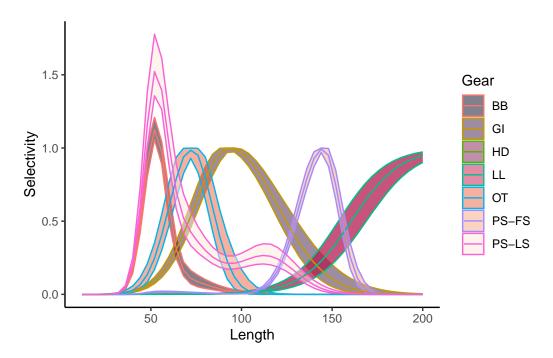


Figure 12: Estimated selectivities for each gear from the mixture model MCMC fit. The lines represent the MCMC average estimate and the ribbon the 80% credible interval. There is almost exact overlap between longline and handline producing the red ribbon.

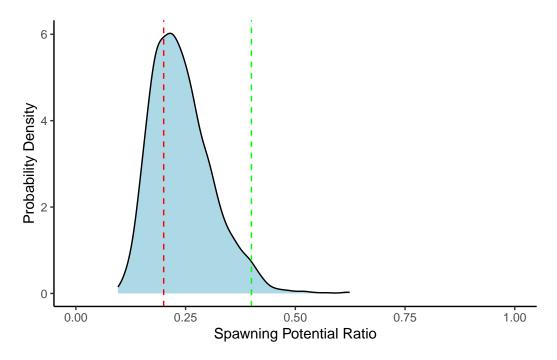


Figure 13: Spawning potential ratio estimate from the mixture model MCMC fit.

so a small sample is over-represented, these can form modes which are artefacts of the sampling and not representative of the overall gear selectivity or length composition of the population. Although selectivity functions might be used to explain such data, it is not clear how the fitted model by be used to model the overall fleet and its impact on the stock.

• temporary dynamics, such as high recruitment year classes observed in frequency data. In this case, methods such as ELEFAN might be used to track the modes and estimate growth and mortality. Attributing these to gear selectivity might still be possible if each catch in a sequence is modelled as a separate gear, but such an approach would need testing.

In this case, the main patterns are observed consistently, but sample sizes for some gears are small and data are combined over a long time period. Some of the selectivity functions' main purpose may be to explain away some observations that are not important and may not be representative of the true length frequency compositions. Whether to include these in simulations testing management actions might need to be considered.

For yellowfin tuna in the example, the results suggest that the stock could well be overfished, although this cannot be fully determined from the length frequency alone. The estimate of SB/SB_0 for the full SS3 model in 2020 was 24-38% (80% CI), compared to the SPR estimate from this assessment of around 16-34%. There is considerable overlap in this case, although

the length-based catch curve is more pessimistic which is not unreasonable in a data limited approach. As demonstrated above, other model choices would produce different estimates, so the SPR plausible range might be much wider when put through a full review process, but the conclusion would likely be to propose reductions in fishing mortality as the stock is likely to be overfished.

7 References

Maunder, M.N., Hamel, O.S., Lee, H., Piner, K. R., Cope, J. M., Punt, A. E., Ianelli, J. N., Castillo-Jordan, C., Kapur, M.S. Methot, R. D. 2023. A review of estimation methods for natural mortality and their performance in the context of fishery stock assessment. Fisheries Research 257: 106489. https://doi.org/10.1016/j.fishres.2022.106489

Kenchington, T.J. 2014. Natural mortality estimators for information-limited fisheries. Fish and Fisheries, 15, 533–562

Lorenzen, K. 2022. Size- and age-dependent natural mortality in fish populations: Biology, models, implications, and a generalized length-inverse mortality paradigm. https://doi.org/10.1016/j.fishres.2022.106454

Froese, R. and D. Pauly. Editors. 2023. FishBase. World Wide Web electronic publication. www.fishbase.org, version (06/2023).

Maunder, M.N., Deriso, R.B., Schaefer, K.M. et al. The growth cessation model: a growth model for species showing a near cessation in growth with application to bigeye tuna (*Thunnus obesus*). Mar Biol 165, 76 (2018). https://doi.org/10.1007/s00227-018-3336-9