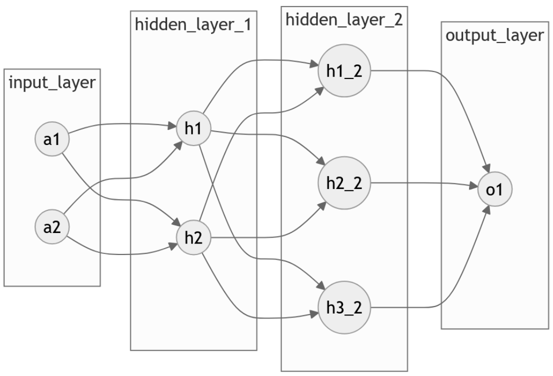
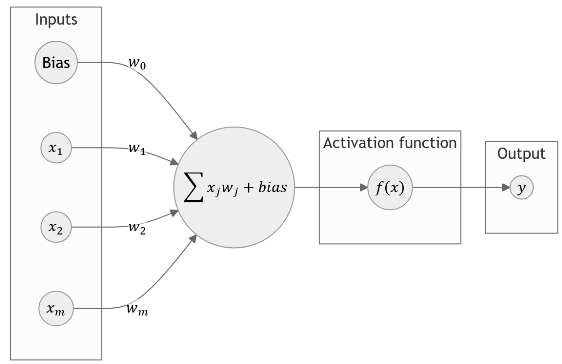
Models

February 3, 2025

FFNN MLP

FeedFoward Neural network Multilayer Perceptron.





Classical FeedFoward Neural network, multilayer perceptron. The input vector :

$$\mathbf{x} = [x_1, x_2, \ldots, x_n]$$

Then on the first hidden layer, each perceptron follow :

$$\mathbf{z} = \mathbf{W}\mathbf{x} + \mathbf{b}$$

With an acivation function typicly sigmoid, ReLU, or tanh as :

$$\mathbf{a} = \sigma(\mathbf{z})$$

Until the output layers wich follow :

$$\mathbf{z}_{\text{out}} = \mathbf{W}_{\text{out}}\mathbf{a} + \mathbf{b}_{\text{out}}$$

and same for the activation :

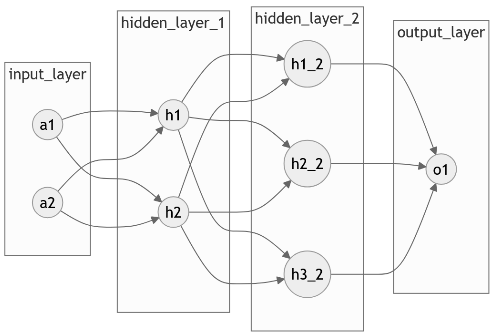
$$\mathbf{y} = \sigma_{\text{out}}(\mathbf{z}_{\text{out}})$$

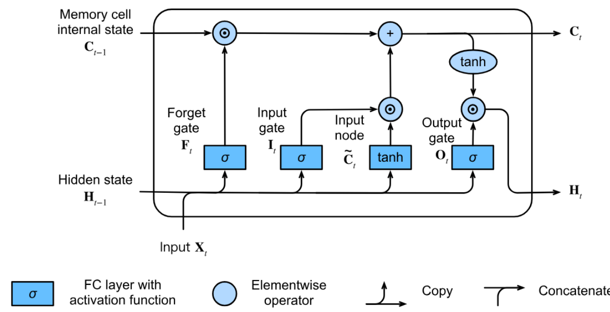
The overall relationship between the input x and the output y can be viewed as a composite function of the linear transformations and activation functions applied at each layer. This can be expressed as:

$$\mathbf{y} = \sigma_{\text{out}}(\mathbf{W}_{\text{out}}\sigma_L(\mathbf{W}_L \ldots \sigma_2(\mathbf{W}_2\sigma_1(\mathbf{W}_1\mathbf{x} + \mathbf{b}_1) + \mathbf{b}_2) \ldots + \mathbf{b}_L) + \mathbf{b}_{\text{out}}) \tag{1}$$

LSTM

Long Short Term Memory





Long Short-Term Memory (LSTM) network is a type of recurrent neural network (RNN) designed for sequential data, such as time series or natural language. LSTMs have memory cells and gates, allowing them to capture and retain dependencies over long sequences.

Each LSTM cell consists of a cell state and three types of gates: the **forget** gate, **input** gate, and **output** gate.

1. The forget gate decides which information from the previous cell state to discard
2. the input gate determines what new information to store in the cell state
3. and the output gate controls what information to pass to the next hidden state

At each time step t, the LSTM receives an input vector $\mathbf{x}_t$ and the previous hidden state $\mathbf{h}_{t-1}$.

Forget Gate Decides what information to discard from the previous cell state:

$$\mathbf{F}_t = \sigma(\mathbf{W}_f \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_f)$$

Input Gate Decides what new information to store in the cell state:

$$\mathbf{I}_t = \sigma(\mathbf{W}_i \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_i)$$

Output Gate Decides what information to output based on the cell state:

$$\mathbf{O}_t = \sigma(\mathbf{W}_o \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_o)$$

Candidate Cell State Generates new candidate values to add to the cell state:

$$\tilde{\mathbf{C}}_t = \tanh(\mathbf{W}_C \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_c)$$

Cell State Update Updates the cell state using the forget and input gates:

$$\mathbf{C}_t = \mathbf{F}_t \odot \mathbf{C}_{t-1} + \mathbf{I}_t \odot \tilde{\mathbf{C}}_t$$

Hidden State Update (Output)
Updates the hidden state using the output gate and the updated cell state:

$$\mathbf{H}_t = \mathbf{O}_t \odot \tanh(\mathbf{C}_t)$$

Or we can rewrite :

$$\begin{aligned}\mathbf{H}_t =& \sigma(\mathbf{W}_o \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_o) \\ & \odot \tanh\Big(\sigma(\mathbf{W}_f \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_f) \odot \mathbf{C}_{t-1} \\ & + \sigma(\mathbf{W}_i \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_i) \odot \tanh(\mathbf{W}_C \cdot [\mathbf{H}_{t-1}, \mathbf{X}_t] + \mathbf{b}_c)\Big) \end{aligned} \tag{2}$$

Output The output of the LSTM at time step t is the hidden state $\mathbf{H}_t$. For a sequence of inputs, the final output can be based on the hidden state at the last time step or a function of all hidden states.

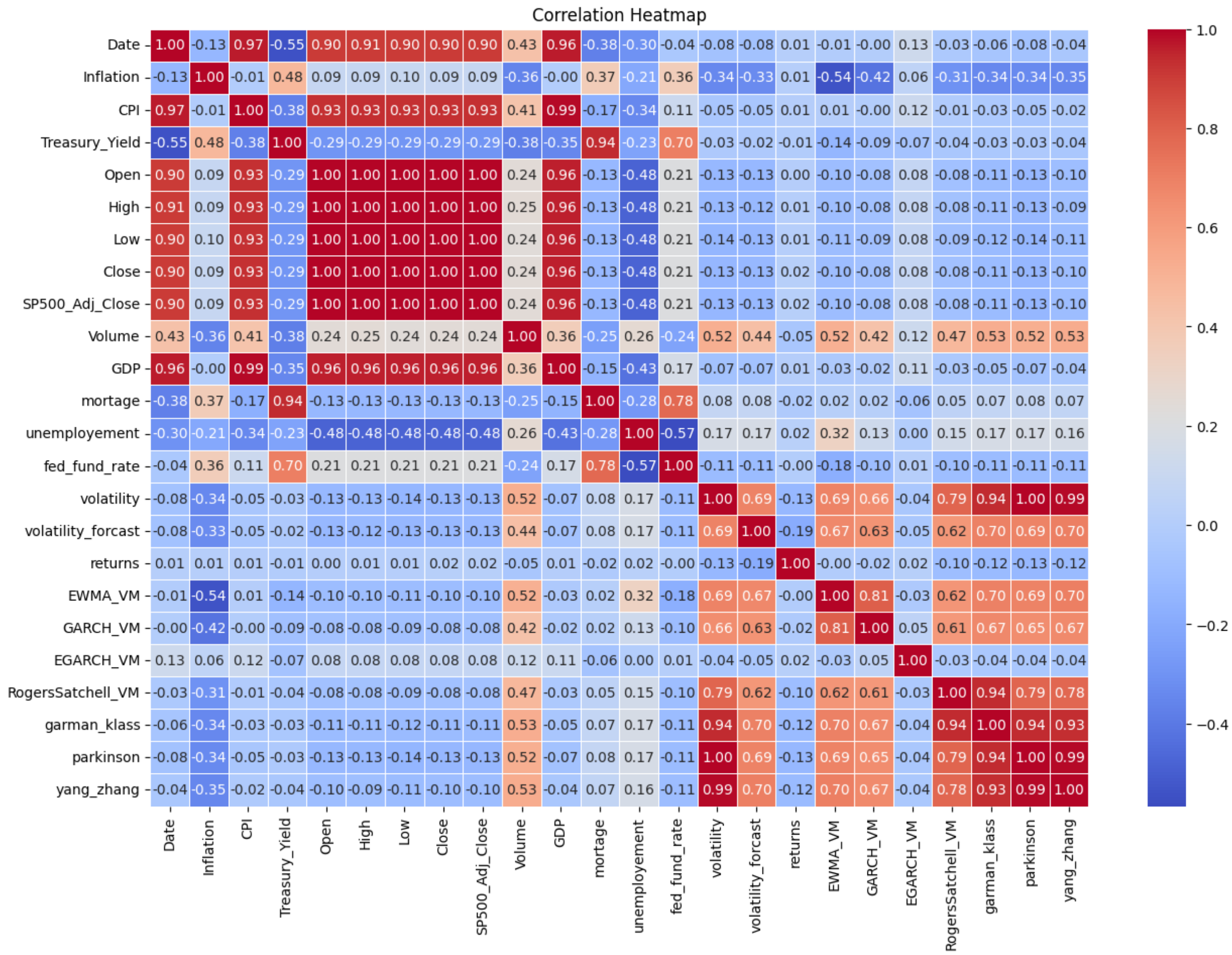
Volatility model parameters

Inputs (Features) :

1. Inflation
2. CPI
3. Treasury_Yield
4. Open
5. High
6. Low
7. Close
8. SP_Adj_Close
9. Volume
10. GDP
11. mortage
12. unemployement
13. def_fund_rate
14. Volatility(current)
15. returns
16. EWMA_VM
17. GARCH_VM
18. EGARCH_VM
19. RagerSatchell_VM
20. garman_klass
21. parkison
22. yang_zhang

Output (Target) :

1. volatility_forcast


Correlation Heatmap
Date
Inflation
CPI
Treasury_Yield
Open
High
Low
Close
SP500_Adj_Close
Volume
GDP
mortage
unemployement
fed_fund_rate
volatility
volatility_forcast
returns
EWMA_VM
GARCH_VM
EGARCH_VM
RogersSatchell_VM
garman_klass
parkinson
yang_zhang
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
−0.4